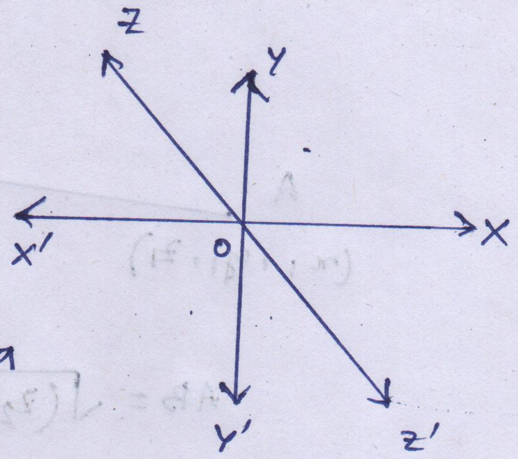
Introduction to 3D Geometry

	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−



8 octants

Q In which octant does that given point lie?

i) $(-2, 4, 3) \rightarrow$ II - octant

ii) $(3, -2, -5) \rightarrow$ VIII - octant

iii) $(-6, 3, -4) \rightarrow$ VI - octant

iv) $(-3, -1, 4) \rightarrow$ III - octant

v) $(1, -3, 6) \rightarrow$ IV - octant

vi) $(4, 7, -2) \rightarrow$ V - octant

Q A point lies on y-axis then what are x-coordinate and z-coordinate?

↳ $(0, y, 0)$.

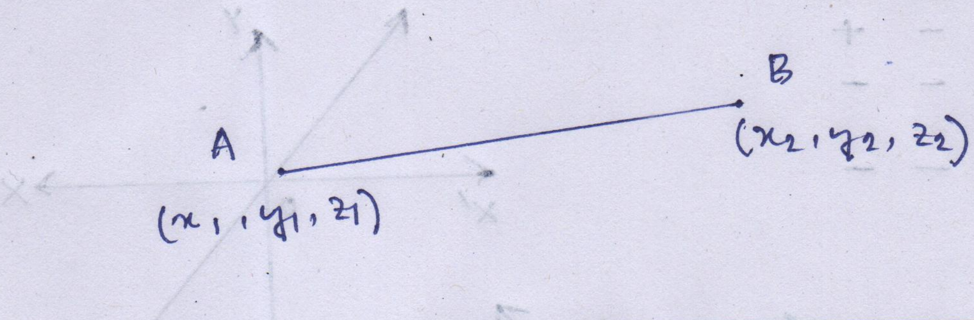
Q A point lies on x-y plane then what is its z-coordinate?

↳ 0

Q In which ~~point~~ plane does the point $(0, 5, -4)$ lie?

↳ yz plane (Ans)

Distance Formula



$$AB = \sqrt{(z_2 - z_1)^2 + (y_2 - y_1)^2 + (x_2 - x_1)^2}$$

Q Distance b/w $A(-2, 1, -3)$ and $B(4, 3, -6)$

$$AB = \sqrt{(-6 - (-3))^2 + (3 - 1)^2 + (4 - (-2))^2}$$

$$= \sqrt{9 + 4 + 36}$$

$$= \sqrt{49}$$

$$= 7 \text{ units.}$$

Q Find the coordinates of a point on y-axis which are at a distance of $5\sqrt{2}$ from the point $P(3, -2, 5)$

point on y-axis means $(0, y, 0)$

$$\therefore \text{Distance} = \sqrt{(0-5)^2 + (y - (-2))^2 + (0 - 3)^2}$$

$$5\sqrt{2} = \sqrt{25 + (y+2)^2 + 9}$$

SBS $25 \times 2 = 34 + (y+2)^2$

$(y+2)^2 = 16 \rightarrow y + 2 = \pm 4$

$y = 2$ $\quad$ $y = -6$

$\therefore$ Ans $(0, 2, 0)$ or, $(0, -6, 0)$

Q. Show that the point $A(0,7,10)$, $B(-1,6,6)$ and $C(-4,9,6)$ from an isoceles right-angled triangle.

$$AB = \sqrt{(6-10)^2 + (6-7)^2 + (-1-0)^2}$$

$$= \sqrt{18} \text{ units}$$

$$BC = \sqrt{(6-6)^2 + (9-6)^2 + (-4-(-1))^2}$$

$$= \sqrt{0+9+9}$$

$$= \sqrt{18} \text{ units}$$

$$CA = \sqrt{(10-6)^2 + (7-9)^2 + (0-(-4))^2}$$

$$= \sqrt{16+4+16}$$

$$= 6 \text{ units.}$$

$\because$ $AB = BC$

$\therefore$ ΔABC is isoceles.

$$AB^2 + BC^2 = CA^2$$

$\therefore$ ΔABC is right-angled.

Q Prove that the points A$(3, -2, 4)$ B$(1, 1, 1)$ and C$(-1, 4, -2)$ are collinear

$$AB = \sqrt{(1-4)^2 + (1-(-2))^2 + (1-3)^2}$$

$$= \sqrt{9+9+4}$$

$$= \sqrt{22} \text{ units}$$

$$BC = \sqrt{(-2-1)^2 + (4-1)^2 + (-1-1)^2}$$

$$= \sqrt{9+9+4}$$

$$= \sqrt{22} \text{ units.}$$

$$AC = \sqrt{(-2-4)^2 + (4-(-2))^2 + (-1-3)^2}$$

$$= \sqrt{36+36+16}$$

$$= \sqrt{88}$$

$$= 2\sqrt{22}$$

$\therefore \; AB + BC = AC$

The points are collinear.

Q Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, 1)

P Q.

let, $A(x, y, z)$.

$\therefore AP = AQ$. (~~we have to prove~~ As the point is equidistant from the points).

$$\therefore AP = \sqrt{(3-z)^2 + (2-y)^2 + (1-x)^2}$$

$$\therefore AQ = \sqrt{(1-z)^2 + (2-y)^2 + (3-x)^2}$$

$$\therefore (3-z)^2 + \cancel{(2-y)^2} + (1-x)^2 = (1-z)^2 + \cancel{(2-y)^2} + (3-x)^2$$

$$\cancel{9} - 6z + \cancel{z^2} + \cancel{1} - 2x + \cancel{x^2} = \cancel{1} - 2z + \cancel{z^2} + \cancel{9} - 6x + \cancel{x^2}$$

$$-6z + 2z = -6x + 2x$$

$$4z = 4x$$

$$\therefore \boxed{x = z}$$

Section Formula

Internal

$m:n$

A (x_1, y_1, z_1) —— P (x, y, z) —— B (x_2, y_2, z_2)

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$z = \frac{mz_2 + nz_1}{m+n}$$

External

$\longleftarrow m \longrightarrow$

$\longleftarrow n \longrightarrow$

P (x, y, z) —— A (x_1, y_1, z_1) —— B (x_2, y_2, z_2)

$$x = \frac{mx_2 - nx_1}{m-n}$$

$$y = \frac{my_2 - ny_1}{m-n}$$

$$z = \frac{mz_2 - nz_1}{m-n}$$

Mid point Formula.

A (x_1, y_1, z_1) —— P (x, y, z) —— B (x_2, y_2, z_2)

$$x = \frac{x_2 + x_1}{2}$$

$$y = \frac{y_2 + y_1}{2}$$

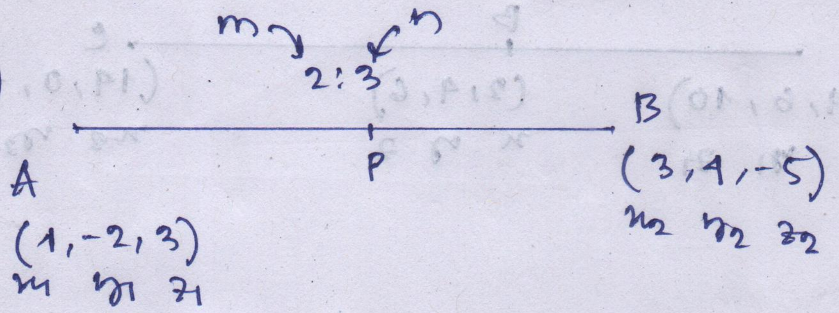
$$z = \frac{z_2 + z_1}{2}$$

Q Find the coordinates of the points which divides the line segment joining the points (1, −2, 3) and (3, 4, −5) in the ratio 2 : 3

i) internally.

ii) externally

i)

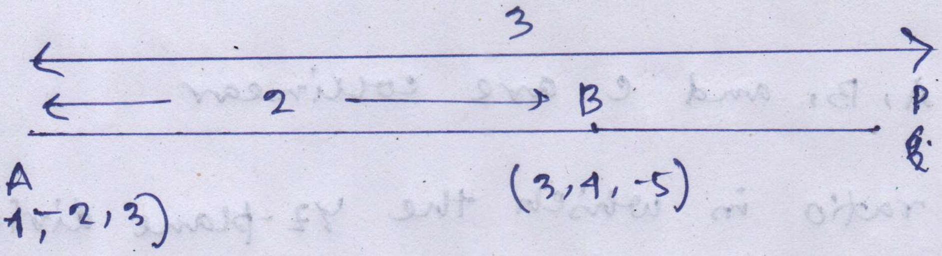


$$P(x) = \frac{2\times 3 + 3\times 1}{2+3} = 9/5$$

$$P(y) = \frac{2\times 4 + 3\times(-2)}{2+3} = \frac{8-6}{5} = 2/5$$

$$P(z) = \frac{2\times(-5) + 3\times 3}{2+3} = -1/5$$

ii)



$$P(x) = \frac{2\times 3 - 3\times 1}{2-3} = -3$$

$$P(y) = \frac{2\times 4 - 3\times(-2)}{2-3} = -14$$

$$P(z) = \frac{2\times(-5) - 3\times 3}{2-3} = +19$$

Q using section formula prove that $(-4,6,10)$, $(2,4,6)$ and $(14,0,-2)$ are collinear.

$m_1 \quad n$

$k : 1$

A $\qquad$ B $\qquad$ C

$(-4,6,10)$ $\qquad$ $(2,4,6)$ $\qquad$ $(14,0,-2)$

$x_1 \; y_1 \; z_1$ $\qquad$ $x \; y \; z$ $\qquad$ $x_2 \; y_2 \; z_2$

$$\therefore \; x = \frac{14k + (-4)}{k+1}$$

$$2k+2 = 14k-4$$

$$6 = 12k$$

$$\boxed{k = 1/2}$$

$$\therefore \; 4 = \frac{0+6}{k+1}$$

$$4k+4 = 6$$

$$4k = 2$$

$$\boxed{k = \frac{1}{2}}$$

$$\therefore \; 6 = \frac{-2k+10}{k+1}$$

$$6k+6 = -2k+10$$

$$8k = 4$$

$$\boxed{k = \frac{1}{2}}$$

$\therefore$ A, B, and C are collinear.

Q Find the ratio in which the yz-plane divides the line segment formed by joining the points $(-2,4,7)$ and $(3,-5,8)$.

$x = 0$

yz plane

$(-2,4,7)$ $\qquad$ $(0,y,z)$ $\qquad$ $(3,-5,8)$

$x_1 \; y_1 \; z_1$ $\qquad\qquad$ $x_2 \; y_2 \; z_2$

let, ~~m:n~~ ratio

$k:1$

$$0 = \cancel{\frac{3m-2n}{m+n}} \quad \frac{3k-2}{k+1}$$

$\cancel{3m = 2n}$

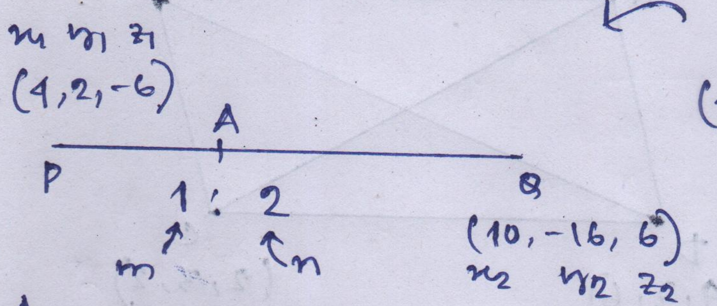
$$3k - 2 = 0$$

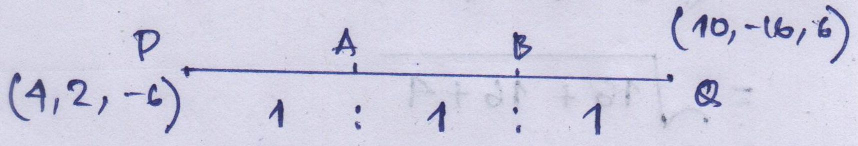
$$\Rightarrow k = 2/3$$

$$\therefore \; k:1 = 2/3 : 1 = 2:3$$

Q. Find the coordinates of the points which trisect the line segment joining the points P(4, 2, -6) and Q (10, -16, 6).

3 equal parts.





$A_x = \dfrac{mx_2 + nx_1}{m+n}$

$= \dfrac{1 \times 10 + 2 \times 4}{1+2}$

$= \dfrac{18}{3} = 6$

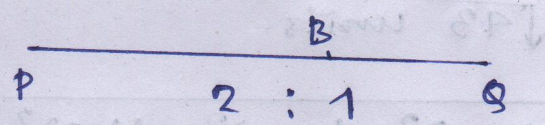
$A_y = \dfrac{my_2 + ny_1}{m+n}$

$= \dfrac{-16+4}{3} = -4$

$A_z = \dfrac{mz_2 + nz_1}{m+n}$

$= \dfrac{6 + (-12)}{3} = -2$

A (6, -4, -2)



$B_x = \dfrac{mx_2 + nx_1}{m+n} = \dfrac{2 \times 10 + 1 \times 4}{2+1}$

$= 8$

$B_y = \dfrac{my_2 + ny_1}{m+n} = \dfrac{2 \times (-16) + 1 \times (2)}{3}$

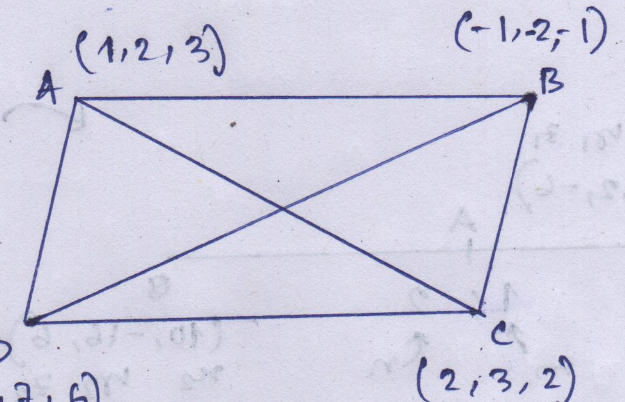
$= -10$

$B_z = \dfrac{mz_2 + nz_1}{m+n} = \dfrac{2 \times 6 + 1 \times (-6)}{3}$

$= \dfrac{6}{3} = 2$

B(8, -10, 2)

Q Show that A (1,2,3), B(−1,−2,−1), C(2,3,2) and D(4,7,6) are the vertices of a parallelogram but its not a rectangle.



$AB = \sqrt{(-1-3)^2 + (-2-2)^2 + (-1-1)^2}$

$= \sqrt{16 + 16 + 4}$

$= 6$ units.

$BC = \sqrt{(2-(-1))^2 + (3-(-2))^2 + (2-(-1))^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$ units.

$CD = \sqrt{(6-2)^2 + (7-3)^2 + (4-2)^2}$

$= \sqrt{16 + 16 + 4}$

$= 6$ units.

$AD = \sqrt{(6-3)^2 + (7-2)^2 + (4-1)^2}$

$= \sqrt{9 + 25 + 9}$

$= \sqrt{43}$ units.

$AC = \sqrt{(2-3)^2 + (3-2)^2 + (2-1)^2}$

$= \sqrt{1+1+1} = \sqrt{3}$ units.

$BD = \sqrt{(6-(-1))^2 + (7-(-2))^2 + (4-(-1))^2}$

$= \sqrt{49 + 81 + 25} = \sqrt{155}$ units.

$\therefore AB = CD$

$BC = AD$

$\therefore$ It's a parallelogram

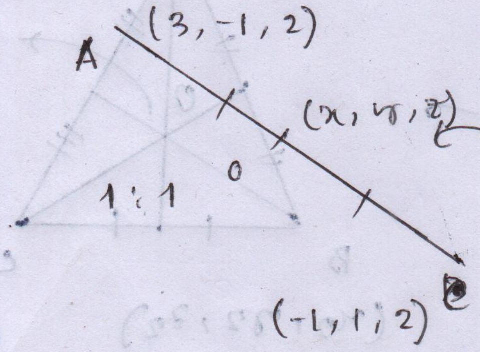
But as

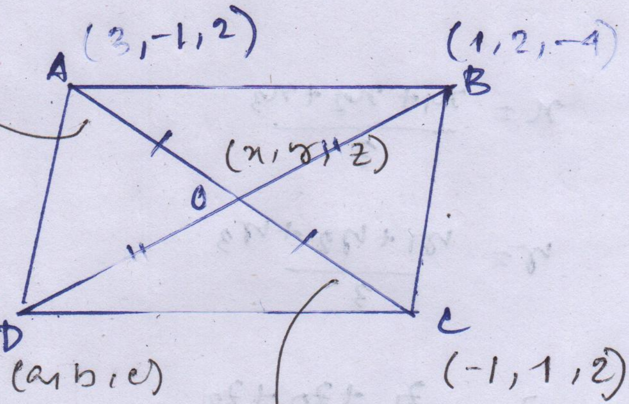
$AC \neq BD$

It's not a rectangle.

Q. Three vertices of a parallelogram ABCD are A(3, −1, 2), B(1, 2, −4) and C(−1, 1, 2). Find the coordinate of the fourth vertex D.

the ~~forth~~ fourth vertex D.



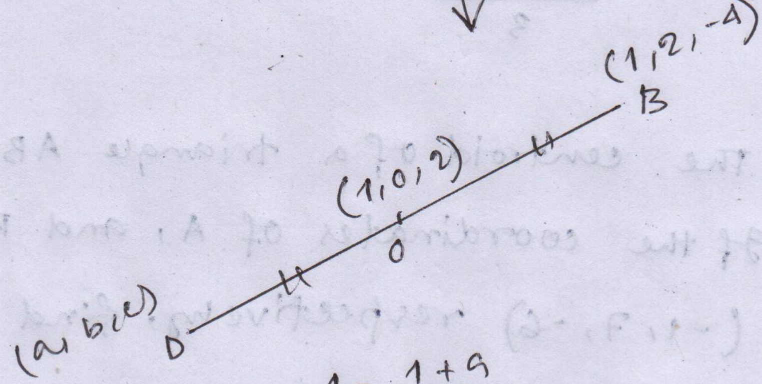


$$x = \frac{-1+3}{2} = 1$$

$$y = \frac{1-1}{2} = 0$$

$$z = \frac{2+2}{2} = 2$$

$\therefore O = (1, 0, 2)$



$$1 = \frac{1+a}{2}$$

$\therefore a = 1$

$$\therefore 0 = \frac{2+b}{2}$$

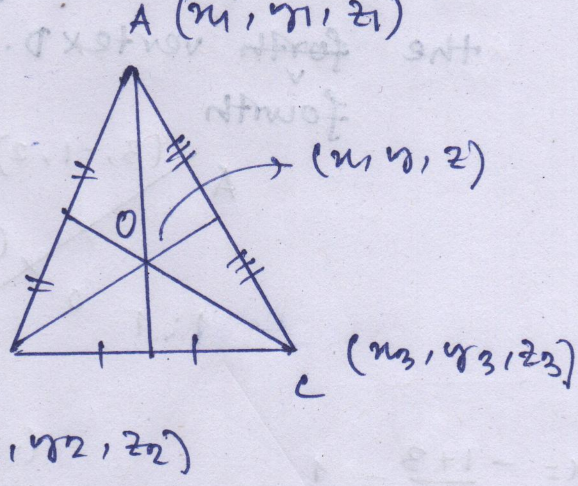
$b = -2$

$$\therefore 2 = \frac{-4+c}{2}$$

$c = 8$

$\therefore$ coordinate of the fourth vertex D $= (1, -2, 8)$

centroid of a triangle



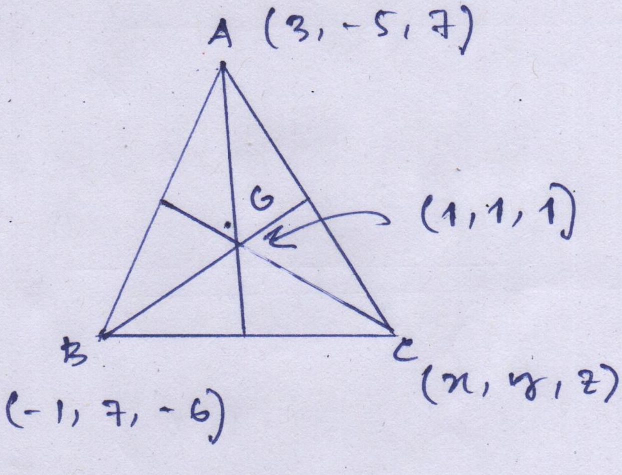
$$x = \frac{x_1 + x_2 + x_3}{3}$$

$$y = \frac{y_1 + y_2 + y_3}{3}$$

$$z = \frac{z_1 + z_2 + z_3}{3}$$

Q The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A, and B are (3, -5, 7) and (-1, 7, -6) respectively, find the coordinates of the point C.

let, $C = (x, y, z)$



$$1 = \frac{3 - 1 + x}{3}$$

$\Rightarrow 3 = 3 - 1 + x$

$\Rightarrow x = 1$

$$1 = \frac{-5 + 7 + y}{3}$$

$\Rightarrow 3 = -5 + 7 + y$

$y = 1$

$$1 = \frac{7 - 6 + z}{3}$$

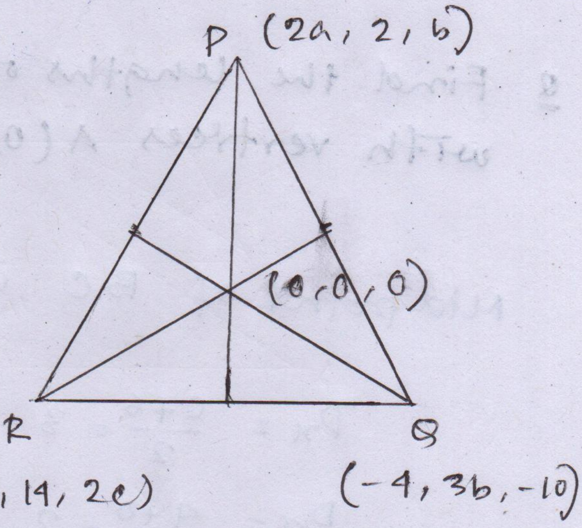
$3 - 7 + 6 = z$

$z = 2$

$\therefore$ coordinate of the point C

(1, 1, 2) (Ans)

Q If the origin is the centroid of the triangle PQR with vertices $P(2a, 2, 6)$, $Q(-4, 3b, -10)$, and $R(8, 14, 2c)$ the find the values of a, b and c.



$$\frac{2a+8-4}{3} = 0$$

$\Rightarrow 2a+4=0$

$a = -2$

$$\frac{2+14+3b}{3} = 0$$

$\Rightarrow 16+3b=0$

$\Rightarrow b = -16/3$

$$\frac{6-10+2c}{3} = 0$$

$\Rightarrow -4+2c=0$

$\Rightarrow c = 2$

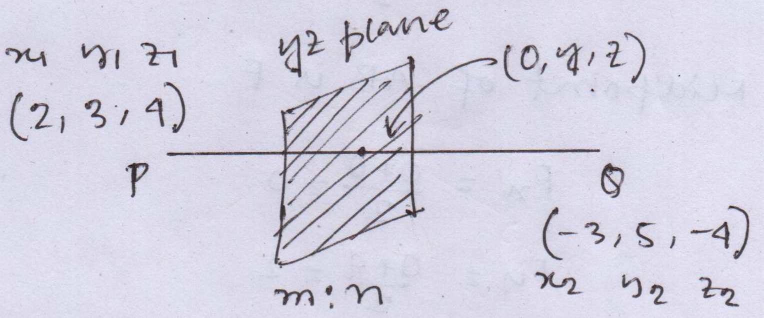
Q Find the ratio in which the line segment, joining the points $P(2,3,4)$ and $Q(-3,5,-4)$ divided by the yz plane. Also find the point of intersection.

A) ✓ $2:3; \left(0, \frac{19}{5}, \frac{4}{5}\right)$

B) $2:3; (19/5, 0, 5/4)$

C) ✗ $3:2; (0, 19/5, 4/5)$

D) ✗ $3:3; (19/5, 0, 4/5)$



$x \Rightarrow$

$$0 = \frac{m(-3)+n(2)}{m+n}$$

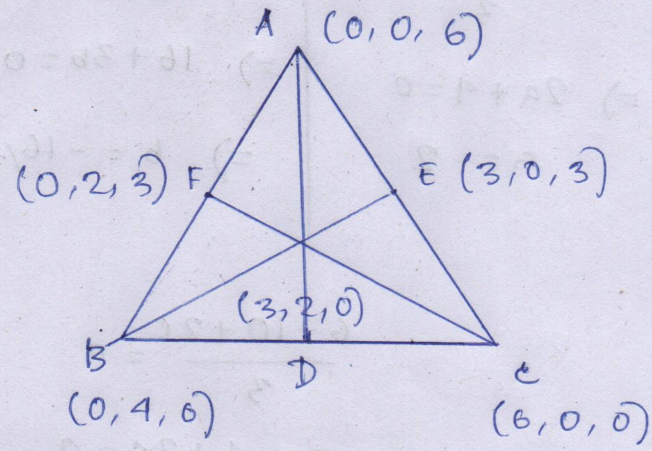
$3m = 2n$

$m/n = 2/3 \rightarrow m:n = 2:3$

$$y = \frac{2\times 5 + 3\times 3}{2+3} = \frac{19}{5}$$

$$z = \frac{2\times(-4) + 3\times(4)}{2+3} = \frac{-8+12}{5} = \frac{4}{5}$$

Q Find the lengths of the medians of the triangle with vertices A(0, 0, 6), B(0, 4, 0) and C(6, 0, 0).



Midpoint of BC is D

$D_x = \frac{0+6}{2} = 3$

$D_y = \frac{4+0}{2} = 2$

$D_z = \frac{0+0}{2} = 0$

Midpoint of AC is E

$E_x = \frac{0+6}{2} = 3$

$E_y = \frac{0+0}{2} = 0$

$E_z = \frac{6+0}{2} = 3$

Midpoint of AB is F

$F_x = \frac{0+0}{2} = 0$

$F_y = \frac{0+4}{2} = 2$

$F_z = \frac{6+0}{2} = 3$

$\therefore AD = \sqrt{(0-6)^2 + (2-0)^2 + (3-0)^2}$

$= \sqrt{36+4+9} = \sqrt{49} = 7$

$\therefore BE = \sqrt{(3-0)^2 + (0-4)^2 + (3-0)^2}$

$= \sqrt{9+16+9} = \sqrt{34}$

$\therefore CF = \sqrt{(3-0)^2 + (0-2)^2 + (6-0)^2}$

$= \sqrt{9+4+36} = \sqrt{49} = 7$

$\therefore$ the lengths of the medians of the triangle with the following vertices given in the question are,

$7, \sqrt{34}, 7$ units