

Highly Recommended

For 2024 Exam

 **OSWAAL BOOKS®**
LEARNING MADE SIMPLE

INDIAN NAVY

AGNIVEER SSR

Senior Secondary Recruit

AGNIPATH SCHEME

Chapter-wise

**QUESTION
BANK**

SCIENCE | MATHEMATICS | ENGLISH
REASONING | GENERAL AWARENESS

Get the #OswaalEdge

1

**100% Exam
Readiness**

with 2 Sample
Question Papers

2

**Concept
Clarity**

with Chapter-wise
Theory

3

**Extensive
Practice**

with more than 1500
Questions with detailed
explanations

4

**Cognitive
Learning**

with Smart
Mind Maps

5

**Examination
Analysis**

with Chapter-wise
Weightage Analysis

Highly Recommended

For **2024** Exam

 **OSWAAL BOOKS®**
LEARNING MADE SIMPLE

INDIAN NAVY

AGNIVEER SSR

Senior Secondary Recruit

AGNIPATH SCHEME

Chapter-wise

**QUESTION
BANK**

SCIENCE | MATHEMATICS | ENGLISH
REASONING | GENERAL AWARENESS

Get the #OswaalEdge

1

**100% Exam
Readiness**

with 2 Sample
Question Papers

2

**Concept
Clarity**

with Chapter-wise
Theory

3

**Extensive
Practice**

with more than 1500
Questions with detailed
explanations

4

**Cognitive
Learning**

with Smart
Mind Maps

5

**Examination
Analysis**

with Chapter-wise
Weightage Analysis



1st EDITION, YEAR 2023-24



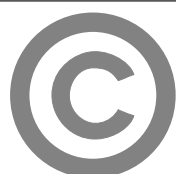
ISBN

"9789359589862"



**SYLLABUS
COVERED**

Agniveer Navy - SSR



**COPYRIGHT
RESERVED
BY THE PUBLISHERS**

All rights reserved. No part of this book may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without written permission from the publishers. The author and publisher will gladly receive information enabling them to rectify any error or omission in subsequent editions.



PUBLISHED BY



**OSWAAL BOOKS &
LEARNING PVT. LTD.**



1/11, Sahitya Kunj, M.G. Road,
Agra - 282002, (UP) India



1010, Cambourne Business Centre
Cambridge, Cambridgeshire
CB 236DP, United kingdom



0562-2857671



contact@oswaalbooks.com



www.OswaalBooks.com

DISCLAIMER

This book is published by Oswaal Books and Learning Pvt Ltd ("Publisher") and is intended solely for educational use, to enable students to practice for examinations/tests and reference. The contents of this book primarily comprise a collection of questions that have been sourced from previous examination papers. Any practice questions and/or notes included by the Publisher are formulated by placing reliance on previous question papers and are in keeping with the format/pattern/guidelines applicable to such papers.

The Publisher expressly disclaims any liability for the use of, or references to, any terms or terminology in the book, which may not be considered appropriate or may be considered offensive, in light of societal changes. Further, the contents of this book, including references to any persons, corporations, brands, political parties, incidents, historical events and/or terminology within the book, if any, are not intended to be offensive, and/or to hurt, insult or defame any person (whether living or dead), entity, gender, caste, religion, race, etc. and any interpretation to this effect is unintended and purely incidental. While we try to keep our publications as updated and accurate as possible, human error may creep in. We expressly disclaim liability for errors and/or omissions in the content, if any, and further disclaim any liability for any loss or damages in connection with the use of the book and reference to its contents".

Kindle Edition

PREFACE

In the vast expanse of the maritime domain, where the rhythmic waves echo tales of courage and commitment, the Agniveer Navy - Senior Secondary Recruits (SSR) examination sets sail as a beacon of knowledge and inspiration.

The Government had launched Agnipath scheme on June 15, 2022 to recruit both male and female aspirants into 'below the officer's rank' cadre of the three services for a period of four years as Agniveers. Candidates between the age group of 17.5 to 21 years are eligible to apply for the scheme.

In 2023, 9 Lacs students appeared for Agniveer Navy SSR exam, this presents an excellent chance for individuals with aspirations to serve the nation by joining the Indian Navy, the opportunity you get from the Indian Navy is just limitless, which helps in enhancing your personality traits.

The Agniveer Navy-SSR - Chapter-Wise book has been meticulously curated to serve as an essential tool in your quest to succeed in the Agnipath Scheme and aims to make aspirants exam-ready, boost their confidence and help them achieve better results in the examination.

Some Benefits of studying from Oswaal Agniveer Navy-SSR– Chapter-wise Question bank is:

1. **Concept Clarity** with easy to grasp theory complemented with Solved Examples
2. **Extensive Practice** with more than 1500 new Questions based on latest exam pattern and Two Sample Question Papers.
3. **Crisp Revision** with Mind Maps offering bite-sized and just-in-time revision tool
4. **Expert Tips** helps you get expert knowledge, master & crack Agniveer Navy in first attempt.
5. **Exam insights** with Chapter-wise weightage Analysis, empowering students to be 100% exam ready.
6. **Quick & Easy Techniques** the book offers Shortcuts & easy to follow Tips and Tricks to help readers solve exam questions efficiently.

May your dreams take flight, and may your journey be as rewarding as it is challenging. Here's to those who dare to dream and to the Agniveer's Navy, where the sky is not the limit but the beginning of a new adventure.



Be mindful. Be grateful. Be positive. Be true. Be kind

01

Three things that make you special

02

Three people you are grateful for and why

03

Three simple things you are grateful for

04

A challenging experience that made you stronger

05

Three ways to inject gratitude into a current challenge

06

Describe the last time you did something nice for someone

07

A fear you have overcome

08

Three activities you enjoy most and why

09

What made you smile today?

10

Three things you love about your family

11

What is your favorite place, and why?

12

Three things you love most about yourself

13

The last time you were overcome with joy

14

A risk you are grateful you took and why

15

Three everyday items you are grateful for

16

Three songs that bring you joy

17

What skill do you have that you are grateful for and why?

18

One luxury you are thankful for

19

Describe a rejection you are grateful for

20

Three things about your body you are grateful for

21

What are you most grateful for in your daily life?

22

Three things you are grateful for about where you live

23

Three items in your home you are grateful for

24

Say thank you to someone

25

Something in nature you are grateful for

26

A person in your past you are grateful for

27

Something at school you're grateful for

28

Describe the last time you laughed so hard you cried

29

What is your proudest accomplishment?

30

Three things you want to manifest



CONTENTS

■ Tips to Crack Agniveer Navy in the First Attempt	8 - 8
■ Syllabus – Agniveer Navy	9 - 10
■ Scheme - Agniveer Navy	11 - 11
■ Eligibility Criteria – Agniveer Navy	12 - 12
■ Pay and Perks – Agniveer Navy	13 - 13
➤ Mind Maps	1 - 25
➤ English	26 - 86
1. Synonyms	26 - 29
2. Antonyms	30 - 34
3. Pronoun	35 - 39
4. Verb	40 - 44
5. Phrasal Verb	45 - 46
6. Passage Comprehension	47 - 51
7. Prepositions	52 - 54
8. Correction of Sentences	55 - 59
9. Determiners: Use of Adjectives Compounds, Use of A, The, Any, Etc.	60 - 64
10. Reported Speech	65 - 72
11. Punctuation	73 - 77
12. Active & Passive Voice	78 - 86
➤ Mathematics	87 - 224
13. Relation & Functions	87 - 96
14. Complex Numbers	97 - 106
15. Quadratic Equations	107 - 112
16. Sequence & Series	113 - 115
17. Trigonometry	116 - 123
18. Cartesian System of Rectangular Coordinates, Straight Line, Family of Straight Line	124 - 126
19. Circles & Conic Sections	127 - 137
20. Permutations & Combinations	138 - 145
21. Logarithms, Exponential & Logarithmic Series	146 - 149
22. Set & Set Theory	150 - 157
23. Statistics	158 - 167
24. Introduction to Three-Dimensional Geometry	168 - 178
25. Probability	179 - 185
26. Limits, Continuity, Applications of Derivatives and Differentiation	186 - 196
27. Indefinite & Definite Integration	197 - 208
28. Determinants	209 - 213
29. Matrices	214 - 219
30. Vectors	220 - 225
➤ Science	226 - 352
31. Physical World & Measurements	226 - 230
32. Kinematics & Laws of Motion	231 - 238
33. Work, Energy & Power	239 - 245
34. Motion of System of Particles and Rigid Body	246 - 249
35. Gravitation	250 - 254

Contd....

36. Mechanics of Solids and Fluids	255 - 261
37. Heat & Thermodynamics	262 - 266
38. Oscillations & Waves	267 - 273
39. Electrostatics	274 - 278
40. Current & Electricity	279 - 284
41. Magnetic Effects of Current & Magnetism	285 - 290
42. Electromagnetic Induction & Alternating Current	291 - 298
43. Electromagnetic Waves	299 - 302
44. Kinetics	303 - 306
45. Dual Nature of Radiation & Matter	307 - 309
46. Optics	310 - 315
47. Atoms & Nuclei	316 - 320
48. Metals & Non-Metals	321 - 325
49. Electronic Devices	326 - 328
50. Organic Chemistry	329 - 333
51. Communication Systems	334 - 336
52. Food, Nutrition and Health	337 - 339
53. Physiology and Human Diseases	340 - 348
54. Computer Science	349 - 352
➤ General Awareness	353 - 433
55. Culture & Religion	353 - 358
56. Geography	359 - 367
57. Sports	368 - 372
58. Eminent Personality	373 - 376
59. Current Affairs	377 - 385
60. Important National Facts about India Heritage, Arts, Dance, History	386 - 397
61. National Language, Birds, Animal, Song, Flag, Monuments, Capital & Currency	398 - 402
62. Common Names, Full Forms & Abbreviations	403 - 407
63. Discoveries, Diseases & Nutrition	408 - 416
64. Awards & Authors	417 - 423
65. Coding & Decoding	424 - 427
66. Spatial & Numerical Reasoning	428 - 433
➤ Sample Question Paper - 1	434 - 440
➤ Sample Question Paper - 2	441 - 447

□□□



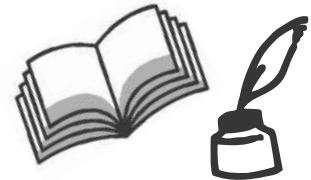
POSITIVE AFFIRMATIONS

An affirmation is a positive statement or phrase that individuals repeat to themselves with the intention of fostering a positive mindset, self-confidence, or personal development. The practice involves affirming desired beliefs or qualities, often in the present tense, to reinforce a positive self-image and encourage a more optimistic outlook.

“ EMBRACE YOUR UNIQUENESS, NOURISH SELF-CARE, AND PRESS FORWARD RELENTLESSLY. TO ACHIEVE FLIGHT, UNLOAD THE BAGGAGE THAT SLOWS YOUR ASCENT.”



I strongly trust in my vision and work diligently towards making it a reality.



I am capable of learning and growing everyday.



I embrace my mistakes, learn from them and use them to fuel my growth & resilience.

I am not defined by grades & test scores. My true worth lies in my character, compassion and positive impact I have on others.



I celebrate my quirks, for they make me beautifully unique.



My voice matters and I believe in the power of my words to shape the world around me.



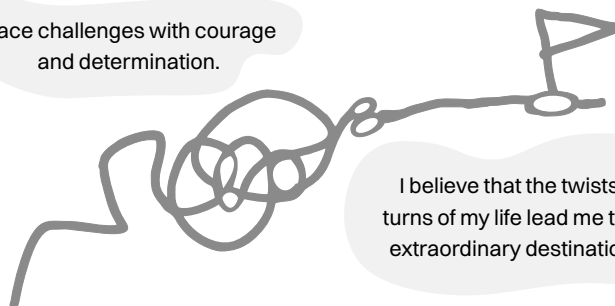
I am not the product of circumstances, I am the product of my own choices.



I face challenges with courage and determination.



I celebrate the achievements of others without diminishing my journey.



I believe that the twists & turns of my life lead me to an extraordinary destinations.

Tips to Crack Agniveer Navy - SSR in the First Attempt

The Indian Navy conducts the Agniveer Navy exam for recruitment of Senior Secondary Recruit (SSR), The Agnipath Scheme was inducted recently and is one of the National Level Examinations in India. Cracking the Agniveer Navy exam in the first attempt, given the difficulty level, is tricky but attainable if done diligently and smartly. Here are some tips that you must follow by heart to crack the exam in the very first attempt:

1 Think Right

Calming yourself and thinking positive is the first and the best course of action that one is required to take. Think and believe that the exam goal is achievable if worked upon smartly.

2 Start studying from the beginning

All the aspirants are aware of how vast, comprehensive and detailed the syllabus of the Agniveer Navy exam is. To crack the exam in the first attempt you have to start preparing for the exam from the beginning of your 10th class. It is only then that you will be able to complete the entire syllabus. Following this approach will also allow you plenty of time to revise.

3 Respect the syllabus and arrange the materials accordingly

While preparing for the Agniveer Navy exam nothing can be labelled as less important. Questions can come from the most unexpected topic too. Laying down your whole syllabus in front of you will help you to decide on the study material you require.

4 Get the right tools and study material

Gathering and preparing from the appropriate study material is something you cannot be ignorant towards. You can refer to Oswaal Books Agniveer Navy Chapter-wise Question Bank to enhance your preparation. This reference book is on the lines of the current syllabus and can be entrusted upon before the examination.

5 Schedule total time for each subject

Creating a schedule which gives due time to all the subjects is a must. Giving proper time to all the subjects daily will help you cover the syllabus on time, giving you enough time for revision.

6 Understand the concepts

No one can crack the Agniveer Navy Exam just by mugging up all the concepts and topics. The syllabus of the exam is in-depth such that you need to understand every concept.

7 Practice a lot of Sample Papers

Agniveer Navy Chapter-wise Question Bank will not only help you in understanding the examination pattern, but they will also help you in figuring out the questions that come up every year and this might give you an edge over other students. You can refer to Oswaal the Agniveer Navy book as it includes all the Examination with solution, Mind Maps, etc. Referring to various sample papers might also help you in comprehending the areas which require more work.

8 Revise whenever you get time

Make sure you revise as much as possible. The revision will help you in keeping the concepts fresh in your mind.

9 Analysing your performance

While you are solving papers, make sure you keep a track of time i.e. how much time does it take to solve one section or one question? Make a report of the sections and type of questions which take minimum and maximum time.

SYLLABUS

Agniveer Navy - SSR

Agniveer Vayu Subjects	Topics
English	Passage, Preposition, Correction of sentences, Change active to passive/passive to active voice Change direct to indirect/indirect to direct, Verbs/Tense/Non Finites, Punctuation Substituting phrasal verbs for expression, Synonyms and Antonyms, Meanings of difficult words Use of adjective, Compound preposition, Determiners (use of a, the, any etc), Use of pronouns
Mathematics	Relations and Functions, Logarithms, Complex Numbers, Quadratics Equations, Sequences and Series, Trigonometry Cartesian System of Rectangular Coordinates, Straight Lines Family of Straight Lines, Circles Conic Section, Permutations and Combinations, Vectors, Exponential and Logarithmic Series, Sets and Set Theory, Statistics Introduction to Three-Dimensional Geometry, Probability Function, Limits and Continuity, Differentiation, Applications of Derivatives Indefinite Integrals Binomial Theorem, Matrices, Determinants, Definite Integrals

Contd....

Science	Physical World and Measurement, Kinematics, Laws of Motion, Work, Energy and Power, Motion o System of Particles and Rigid Body/Gravitation Mechanics of Solids and Fluids, Heat Thermodynamics, Oscillations, Waves, Electrostatics, Current Electricity Magnetic Effect of Current and magnetism, Electromagnetic Induction and Alternating Current, Electromagnetic Waves Optics, Dual Nature of Matter and Radiations, Atomic Nucleus/ Solid and Semi-Conductor Devices, Principles of Communication Metals and Non-Metals, Organic Chemistry, Food, Nutrition and Health, Physiology and Human Diseases, Computer Science
General Awareness	Culture and Religion, Geography : Soil, Rivers, Mountains, Ports, Inland Harbours, Freedom Movement Sports : Championships/Winners/ Terms/ No. of Players, Defence, Wars and Neighbours, current Affairs Important National Facts about India-Heritage, Arts, Dance, History, National—Languages, Bird, Animal, Song, Flag, Monuments, Capitals & Currencies Common Names, Full forms and Abbreviations, Discoveries, Diseases and Nutrition, Award and Authors, Eminent Personalities Spatial, Numerical, Reasoning & Associative Ability, Sequences, Spellings Unscrambling, Coding and Decoding

Scheme of Examination - Agniveer Navy-SSR

Subjects	No. of Questions	Total Marks	Exam Duration
English	25	25	1 hour (60 minutes)
Science	25	25	
Mathematics	25	25	
General Knowledge	25	25	
Total	100	100	

MARKING SCHEME FOR AGNIVEER NAVY-SSR 2024

- (i) For every correct answer, 1 mark shall be awarded.
- (ii) For every wrong answer, 0.25 marks shall be deducted.
- (iii) For questions left blank, 0 mark shall be deducted.

□□□

Eligibility Criteria

Age Criteria	Candidates must have born between 01 May 2002 –31 Oct 2005
Educational Qualifications	- MR: Passed Matriculation Examination from a recognized Board of School Education - SSR: Passed 10+2 Examination from a recognized board with Maths and Physics along with either Computer Science/ Biology/ Chemistry
Nationality	Indian
Number of Attempts	Till the candidates have crossed their maximum age restrictions.
Experience	Previous experience is not required

Education Criteria

Post Name	Educational Qualification
Agniveer (SSR)	Qualified in 10+2 exam with Mathematics, Physics, and at least one of these subjects: Chemistry/Biology/Computer Science from an educational board recognized by Ministry of Education, Govt of India
Agniveer (MR)	Candidate must have passed Matriculation Examination from a recognized Board of School Education

Physical Fitness

Gender	1.6 Km Run	Squats	Push-Ups	Bent Knee Sit Ups
Male	6 minutes 30 sec	20	1	-
Female	8 minutes	15	-	10

Indian Navy Agniveer Medical Standards

Gender	Minimum Height (cms)
Male	157
Female	152

Visual Standards

Particulars	Without Glasses	With Glasses
Better Eye	6/6	6/6
Worse Eye	6/9	6/6

Pay and Perks

Pay, Allowances and Allied Benefits—Agniveers will be paid a customised package of ₹30,000 per month with a fixed yearly increment. In addition, risk and hardship, dress and travel allowances will be paid.

Terminal Benefits—Seva Nidhi Package. Agniveers shall be given a one-time Seva Nidhi package comprising their monthly contribution along with matching contribution by the government on completion of their engagement period, as indicated below:—

Year	Customised Package (Monthly)	In Hand (70%)	Contribution to Agniveers Corpus Fund (30%)	Contribution to Corpus fund by GoI
All figures in Rs (Monthly Contribution)				
1st Year	30,000	21,000	9,000	9,000
2nd Year	33,000	23,100	9,900	9,900
3rd Year	36,500	25,550	10,950	10,950
4th Year	40,000	28,000	12,000	12,000
Total Contribution in Agniveers Corpus Fund after four years			Rs. 5.02 lakh	Rs. 5.02 lakh
Exit after 4 year		Rs. 10.04 Lakhs as Seva Nidhi Package (absolute amount excluding interest)		

Note:

1. Agniveers will not be required to contribute to any Provident Fund of the Government.
2. There shall be no entitlement of gratuity and no pensionary benefits in the case of Agniveers.
3. The 'Seva Nidhi' package will be exempt from Income Tax.

Order Form

JEE (MAIN) 15 Mock Test Papers (For 2024 Exam)

JEE (MAIN) 15 Mock Test Papers (Mathematics, Physics, Chemistry)

JEE (MAIN) Year-wise 22 Solved Papers(All Shifts) (For 2023 Exam)

Mathematics, Physics, Chemistry

JEE (MAIN) Chapter-wise & Topic-wise Solved papers (2019-2023) Question Bank (For 2024 Exam)

Mathematics, Physics, Chemistry

JEE(Main) RMT Cards

Physics Part - I, Physics Part - II, Chemistry Part - I Chemistry Part - II, Mathematics Part -I, Mathematics Part - II

JEE Advanced (For 2024 Exam) 22 Year-wise Solved Papers (2002-2023)

Mathematics, Physics, Chemistry

JEE Advanced 10 Mock Test Papers Paper 1 & Paper 2 (For 2023 Exam)

JEE Advanced 10 Mock Test Papers (Paper-1 & 2)

GATE 14 Year-wise (For 2024 Exam) Solved Papers (2010 to 2023)

GATE 14 Yearwise Solved Paper (General Aptitude), GATE 14 Yearwise Solved Paper (Engineering Mathematics)

GATE 14 Years' (For 2024 Exam) Chapterwise & Topicwise Solved Papers (2010 to 2023)

GATE 14 Years' Chapterwise & Topicwise Solved Papers (General Aptitude), GATE 14 Years' Chapterwise & Topicwise Solved Papers (Engineering Mathematics)

NEET 36 Years' Solved Papers (For 2024 Exam) Chapterwise & Topicwise 1988-2023

Biology, Physics, Chemistry, NEET 18 Year-wise Solved Papers

NEET (UG) 15 Mock Test Papers (For 2024 Exam)

NEET (UG) 15 Mock Test Papers (Physics, Chemistry, Biology)

NEET RMT Cards

Physics Part-1, Physics Part-2, Chemistry Part-1, Chemistry Part-2, Biology Part-1, Biology Part-2

Handbook Class 11 & 12, Entrance Exams (Engineering & Medical)

Physics, Chemistry, Mathematics & Biology

Vedic Maths

Vedic Maths (Ashima Sekhri)

CUET Sample Question Papers

English, Accountancy, Biology, Business Studies, Chemistry, Mathematics/Applied Mathematics, Entrepreneurship, Computer Science/Informatics Practices, Sociology, Economics, Geography, History, Physics, Political Science, Psychology, Physical Education, (Quantitative Aptitude, Logical Reasoning General Awareness),

CDS Chapter-Wise & Topic-Wise Solved Papers (2014-2023)

Elementary Mathematics, English, General Knowledge

NDA-NA Chapter-Wise & Topic-Wise Solved Papers (2014-2023)

Mathematics, General Ability Test: English, General Ability Test: General Studies

CUET Question Bank (For 2023 Exam)

Quantitative Aptitude, Logical Reasoning, General Awareness

National Talent Search Exam (NTSE)

NTSE Mock Test 15 Sample Question Papers MAT + SAT Stage 1 & Stage 2, NTSE 10 Yearwise Solved Papers 2011-2021

NRA Question Bank 10th Pass

Quantitative Aptitude, Logical Reasoning, General Awareness, General English

NRA Question Bank 12th Pass

Quantitative Aptitude, Logical Reasoning, General Awareness, General English

NRA Question Bank Graduation Pass

Quantitative Aptitude, Logical Reasoning, General Awareness, General English

Objective General English

Objective General English

Objective Quantitative Aptitude

Objective Quantitative Aptitude

CAT

CAT 15 Mock Test Papers, CAT 25 Years' Chapterwise & Topicwise Solved Papers (1990-2008) (2017-2022), CAT 10 Years' Chapterwise & Topicwise Solved Papers Question Bank (2005-2008), (2017-2022), CAT 5 Years' Yearwise Solved Papers Shift wise (2018-2022)

UGC-NET

UGC NET Paper-1 Year-Wise 13 Solved Papers (2015-2023) For 2024 Exam General Aptitude, UGC NET Paper-1, 15 Years' Question Bank Chapterwise & Topicwise Solved Papers (2009-2023) General Aptitude

RRB NTPC 2023 1st Stage Exam

RRB NTPC 12 Previous Years' Papers 2016 (All Shifts) & 2019 (Two Shifts) +10 Mock Test Papers

UPSC Prelims 10 Years' Solved Papers (2014 -2023) English Medium (For 2024 Exam)

UPSC CSE Prelims General Studies (Paper-1), UPSC CSAT Prelims General Studies (Paper-2)

UPSC NCERT One for All (For 2024 Exam)

Polity, Economy, History, Geography, General Science

UPSC Power Bank For UPSC & State PSCs Exam (For 2024 Exam)

Ancient & Medieval History, Art & Culture, Geography, Indian Polity, Indian Economy, Science & Tech., Environment & Ecology, Modern History

UPSC Mock Test Papers CSE Prelims (For 2024 Exam)

15 Mock Test Papers for UPSC CSE Prelims General Studies Paper-1, 15 Mock Test Papers for UPSC CSE Prelims (CSAT) General Studies Paper-2

UPSC CSE (Pre) 29 Years' Solved Papers (1995-2023) (For 2024 Exam)

UPSC 29 Years' Topicwise Solved Question Paper Prelims GS Paper -1 & 2

CDS Year wise 11 Solved Papers (2018-2023)

Elementary Mathematics, English, General Knowledge

NDA-NA Year-wise 12 Solved Papers (2017-2023)

Mathematics, General Ability Test: English, General Ability Test: General Studies

Exam Preparation Books for Class 1 to 12 : CBSE, CISCE Boards & Karnataka Board | OLYMPIADS | JEE | NEET | CUET | NDA | CDS | CAT | GATE | UPSC | UGC NET | NRA CET & more

We are available in all leading e-commerce platforms and all good bookstores.



Exclusive School Books Suppliers

VIJAYAWADA	ANDHRA PRADESH Sri Vikas Book Centre, 9848571114, 9440715700,
WEST KAMENG	ASSAM Dutta Book Stall, 8729948473
BANGLORE	KARNATAKA Satish Agencies, 8861630123
RAJKOT	GUJRAT Royal Stationers, 9824207514

PUNE	MAHARASHTRA Madhusheela Books & Stationery, 7875899892
JALNA	Anil Paper Mart, 9422722522, (02482) 230733
CHENNAI	TAMIL NADU Bookmark-IT, 7305151653
HYDERABAD	TELANGANA Sri Balaji Book Depot , 9676996199, (040) 27613300
KOLKATA	WEST BENGAL United Book House, 9831344622

Our Distributors

VISAKHAPATHAM	ANDHRA PRADESH JBD Educational, 9246632691, 9246633693, Sri Rajeshwari Book Link, 9848036014
VIJAYAWADA	Akshaya Books Corner, 9666155555
PORTBLAIR	ANDAMAN & NICOBAR ISLAND Krishna Book Centre, 9474205570, Kumar Book Depot, 9932082455, Kumar Book Depot, 9932082455, Sree aditya Book Centre, 8332972720, 7013300914
GUWAHATI	ASSAM Book Emporium, 9675972993, 6000763186, Ashok Publication, 7896141127, Kayaan Enterprises, (0361) 2630443, Orchid Book house, 9864624209, Newco, 9864178188
PATNA	BIHAR Nova Publisher & Distributors, (0612) 2666404, Shri Durga Pustak Mandir, 9334477386, Sharda Pustak Bhandar, 9334259293, Vikas Book Depot, 9504780402, Alka Book Agency, 9835655005, Metro Book(E&C), Ishu Pustak Bhandar, 8294576789, Gyan Ganga Limited, 6203900312, Ishu Pustak Bhandar, (E & C), 9334186300/8294576789
MUZAFFARPUR	Pustak Bhandar, 7870834225
AMBIKAPUR	CHATTISGARH Saini Brothers, 9425582561, M.P Department Stores, 9425254264
BOKARO	Bokaro Student Friends Pvt. Ltd, Bokaro, 7277931285
BHILAI	Anil Book Depot, 9425234260
DURG	Bhagwati Bhawani Book Depot, 0788-2327620, 9827473100
KORBA	Kitab Ghar, Korba (E & C), 9425226528
RAIPUR	Shri Ramdev Traders, 9981761797, Gupta Pustak Mandir, 7974220323, Anil Publication, 9691618258/7999078802
RAIGARH	Sindhu Book Deopt, 9981935763
DELHI	DELHI Mittal Books, (011) 23288887, 9899037390, Singhania Book & Stationer, 9212028238, AoneBooks, New Delhi, 8800497047, Radhey Book Depot, 9818314141, Batheja Super Store, 9871833924, Lov Dev & Sons, Delhi (E & C), 9811182352, Zombozone, 9871274082, LDS Marketing, 9811182352/9999353491
AHMEDABAD	GUJARAT Patel Book, 9898184248, 9824386112, 9825900335, Zaveri Agency, 9979897312, 9979890330, Hardik Book Agency, (ISC) 079-24110043, 9904659821
BHAVNAGAR	Samir Book Stall, Bhavnagar (ISC) 9586305305
DAHOD	Collegian Book Corner, 9925501981
VAPI	Goutam Book Sellers, 9081790813
VALSAD	Mahavir Stationers, 9429474177
NAVSARI	College Store, (ISC) NO CALL 02637-258642, 9825099121
SURAT	Shopping Point, 9824108663
VADODARA	Umakant Book Sellers & Stationer, 9624920709
ROHTAK	HARYANA Manish Traders, 9812556687, Swami Kitab Ghar, 9355611088, Babu Ram Pradeep Kumar, 9813214692
REWARI	Sanjay book depot, 9255447231
BALLABGARH	Kashi Ram Kishan Ial, 9289504004, 8920567245
HISAR	Natraj Book Distributors, 7988917452
BHUNA	Khurana Book Store, 9896572520
JAMMU	Sahitya Sangam, 9419190177
JHARKHAND	Bokaro Student Friends, (0654) 2233094, 7360021503, Bharati Bhawan Agencies, 9431740797
BOKARO	Crown Book Distributor & Publishers, (0651) 2213735, 9431173904, Pustak Mandir, 9431115138, Vidyarthi Pustak Bhandar, 9431310228
RANCHI	KARNATAKA Renuka Book Distributor, (0836) 2244124, Vidyamandir Book Distributors, 9980773976
DUMKA	Krishna book house, 9739847334, Hema Book Stores, 9986767000,
HUBLI	Sapna Book House Pvt. Ltd., 9980513242, Hema Book World, (Chamrajpet) (ISC) 080-40905110, 9945731121
BANGLORE	Chatinya book centre, 9886064731
BELLERI	KERALA Academic Book House, (0484) 2376613, H & C Store, 9864196344, Surya Book House, 9847124217, 9847238314
ERNAKULAM	Book Centre, (0481) 2566992
KOTTAYAM	Academic Book House, (0471) 2333349, 9447063349, Ponni Book Stall, 9037591721
TRIVANDRUM	Aman Book Stall, (0495) 221282,
CALICUT	MADHYA PRADESH Pustak Bhawan, (E & C), 8982150100
CHHINDWARA	Agarwal Book Depot, 9425116210
GWALIOR	

INDORE	Bhaiya Industries, 9893326853, Sushil Prakashan,(0731) 2503333, 2535892, 9425322330, Bhaiya Store, 9425318103, Arun Prakashan, 9424890785, Bhaiya Book Centre, 9424081874, Seva Suppliers, 9826451052
JABALPUR	Vinay Pustak Sadan, 8962362667, Anand Books and Stationers, 9425323508
SAGAR	Princi Book Depot, Sagar, 9977277011
KATNI	Shri Mahavir Agency, 9425363412
UJJAIN	Shreenath Book Depot, 9827544045
BHOPAL	Gupta Brother, 9644482444
PUNE	MAHARASHTRA Natraj Book Depot, (020) 24485054, 9890054092, Vikas Book House, 9921331187, Pravin Sales, 9890683475, New Saraswati Granth Bhandar, 9422323859, Akshar Books & Stationary, 7385089789, Vardhaman Educational, 9860574354, Yash Book Centre, 9890156763, Pragati Book Centre, (ISC), 9850039311, Praveen Sales, Pragati Book Centre, Pune (E & C), 9850039311
AURANGABAD	Shree Sainath Agencies, 7350294089, Maya Book Centre, (ISC), 9372360150
MUMBAI	Vidyarthi Sales Agencies, 9819776110, New Student Agencies, 7045065799, Shivam Books & Stationery, 8619805332
JALGAON	Sharma Book Depot & Stat. (ISC), 9421393040
LATUR	Yash Book House, 9637936999, Shri Ganesh Pustakalay, 9730172188
KOLHAPUR	Granth the Book World, 9922295522
NAGPUR	Laxmi Pustakalay and Stationers, (0712) 2727354, Vijay Book Depot, 9860122094
NANDED	Renuka Book distributor, 9765406133, Novelty Book Depot, 9657690220, Karamveer Book Depot, 9923966466, Arun Book & Stationers, 9423110953
NASHIK	Abhang Pustakalaya, 9823470756/9175940756
DHULE	Rahul Book Centre, 9970849681, New India Book House, 9623123458
YAVATMAL	Navjeevan Book Stall, 7020525561
VASAI	Shri Ganesh Pustkalaya, 9423131275
ODISHA	Prime Book Centre, Vasai, 9890293662
CUTTACK	A. K. Mishra Agencies, 9437025991, 9437081319
BHUBANESHWAR	M/s Pragnya, 8847888616, 9437943777, Padmalaya, 9437026922, Bidyashree, 9937017070, Books Godown, 7894281110
BARIPADA	Trimurti Book World, 9437034735
KEONJHAR	Students corner, 7008435418
PUNJAB	Bharat Book Depot, 7988455354
AMBALA	Goel Sons, 9463619978, Adarsh Enterprises, 9814347613
PATIALA	Cheap Book Store, 9872223458, 9878258592, City Book Shop, 9417440753, Subhash Book Depot, 9876453625, Paramvir Enterprises, 9878626248
JALANDHAR	Sita Ram book Depot, 9463039199, 7696141911
FEROZPUR	Amit Book, 9815807871, Gupta Brothers, 9888200206, Bhatia Book Centre, 9815277131
LUDHIANA	Mohindra Book Depot, 9814920226
CHANDIGARH	RAJASTHAN Laxmi General Store, Ajmer, 0145- 2428942 9460652197
AJMER	Vardhman Book Depot, 9571365020, 8003221190 Raj Traders, 9309232829
KOTA	Nakoda Book Depot, (01482) 243653, 9214983594,
BHILWARA	Alankar Book Depot, 9414707462
JAIPUR	Ravi Enterprises, 9829060694, Saraswati Book House, (0141) 2610823, 9829811155, Goyal Book Distt., 9460983939, 9414782130
UDAIPUR	Sunil Book Store, 9828682260
JODHPUR	Second Hand Book Stall, 9460004745
AGARTALA	TRIPURA Book Corner, 8794894165, 9894657146, Book Emporium, 9089230412
COIMBATORE	TAMIL NADU Majestic Book House, (0422) 2384333, CBSC Book Shop, 9585979752
CHENNAI	Arraba Book Traders, (044) 25387868, 9841459105, M.R. Book Store (044) 25364596, Kalaimagal Store, (044) 5544072, 9940619404, Vijaya Stores, 9381037417, Bookmark It-Books & Stat. Store, 7305151653, M.K. Store, 9840030099, Tiger Books Pvt. Ltd., 9710447000, New Mylai Stationers, 9841313062, Prince Book House, Chennai, 0444-2053926, 9952068491, S K Publishers & Distributors, 9789865544, Dharma Book Shop, 8667227171
PUDUCHERRY	Sri Lakshmi Book Seller, 7871555145
SALEM	Pattu book centre, 9894816280
TRICHY	P.R.Sons Book Seller, 9443370597, Rasi Publication, 9894816280
THENI	Maya Book Centre, 9443929274
MADURAI	Selvi Book Shoppe, 9843057435, Jayam Book Centre, 9894658036
VELLORE	G.K book centre and collections, 9894517994
HYDERABAD	TELANGANA Sri Balaji Book Depot, (040) 27613300, 9866355473, Shah Book House, 9849564564
	Vishal Book Distributors, 9246333166, Himalaya Book World, 7032578527

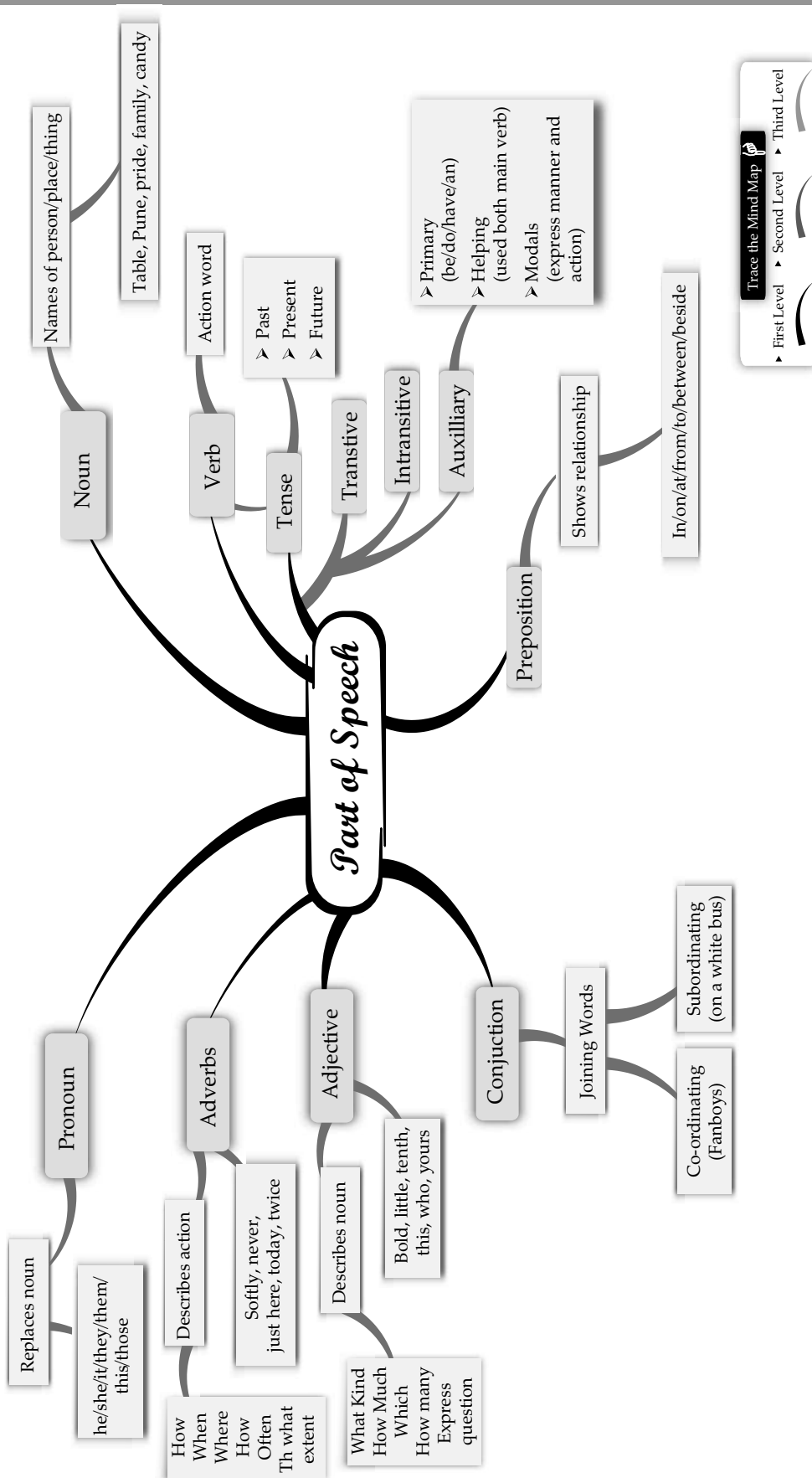
Contd....

UTTARAKHAND		GORAKHPUR	
DEHRADUN	Inder Book Agencies, 9634045280, Amar Book Depot , 8130491477, Goyal Book Store, 9897318047, New National Book House, 9897830283/9720590054	JHANSI	Central Book House, 9935454590, Friends & Co., 9450277154, Dinesh book depot, 9125818274, Friends & Co., 9450277154
MUSSORIE	Ram Saran Dass Chanda kiran, 0135-2632785, 9761344588		Bhanu Book Depot, 9415031340
UTTAR PRADESH		KANPUR	Radha News Agency, 8957247427, Raj Book Dist., 9235616506, H K Book Dis-tributors, 9935146730, H K Book Distributors, 9506033137/9935146730
AGRA	Sparsch Book Agency, 9412257817, Om Pustak Mandir, (0562) 2464014, 9319117771, Sanjay Publication, 8126699922 Arti book centre, 8630128856, Panchsheel Books, 9412257962, Bhagwati Book Store, (E & C), 9149081912	LUCKNOW	Vyapar Sadan, 7607102462, Om Book Depot, 7705871398, Azad Book Depot Pvt. Ltd., 7317000250, Book Sadan, 9839487327, Rama Book Depot(Retail), 7355078254, Ashirwad Book Depot, 9235501197, Book.com, 7458922755, Universal Books, 9450302161, Sheetla Book Agency, 9235832418, Vidyarthi Kendra Publisher & Distributor Pvt Ltd, (Gold), 9554967415, Tripathi Book House, 9415425943
ALLAHABAD	Mehrotra Book Agency, (0532) 2266865, 9415636890	MEERUT	Ideal Book Depot, (0121) 4059252, 9837066307
AZAMGARH	Sasta Sahitya Bhandar, 9450029674	NOIDA	Prozo (Global Edu4 Share Pvt. Ltd), 9318395520, Goyal Books Overseas Pvt.Ltd., 1204655555 9873387003
ALIGARH	K.B.C.L. Agarwal, 9897124960, Shaligram Agencies, 9412317800, New Vimal Books, 9997398868, T.I.C Book centre, 9808039570	PRAYAGRAJ	Kanhaiya Pustak Bhawan, 9415317109
BULANDSHAHR	Rastogi Book Depot, 9837053462/9368978202	MAWANA	Subhash Book Depot, 9760262264
BALRAMPUR	Universal Book Center, 8933826726	WEST BENGAL	
BAREILLY	Siksha Prakashan, 9837829284	KOLKATA	Oriental Publishers & Distributor (033) 40628367, Katha 'O' Kahini, (033) 22196313, 22419071, Saha Book House, (033), 22193671, 9333416484, United Book House, 9831344622, Bijay Pustak Bhandar, 8961260603, Shawan Books Distributors, 8336820363, Krishna Book House, 9123083874
HARDOI	Mittal Pustak Kendra, 9838201466	RENUKOOT	Om Stationers, 7007326732
DEORIA	Kanodia Book Depot, 9415277835	COOCH BEHAR	S.B. Book Distributor, Cooch behar, 9002670771
VARANASI	Gupta Books, 8707225564, Bookman & Company, 9935194495/7668899901	KHARAGPUR	Subhani Book Store, 9046891334
MATHURA	Sapra Traders, 9410076716, Vijay Book House , 9897254292	SILIGURI	Agarwal Book House, 9832038727, Modern Book Agency, 8145578772
FARRUKHABAD	Anurag Book Agencies, 8844007575	DINAJPUR	Krishna Book House, 7031748945
NAJIBABAD	Gupta News Agency, 8868932500, Gupta News Agency, (E & C), 8868932500	MURSHIDABAD	New Book House, 8944876176
DHAMPUR	Ramkumar Mahaveer Prasad, 9411942550		

Entrance & Competition Distributors

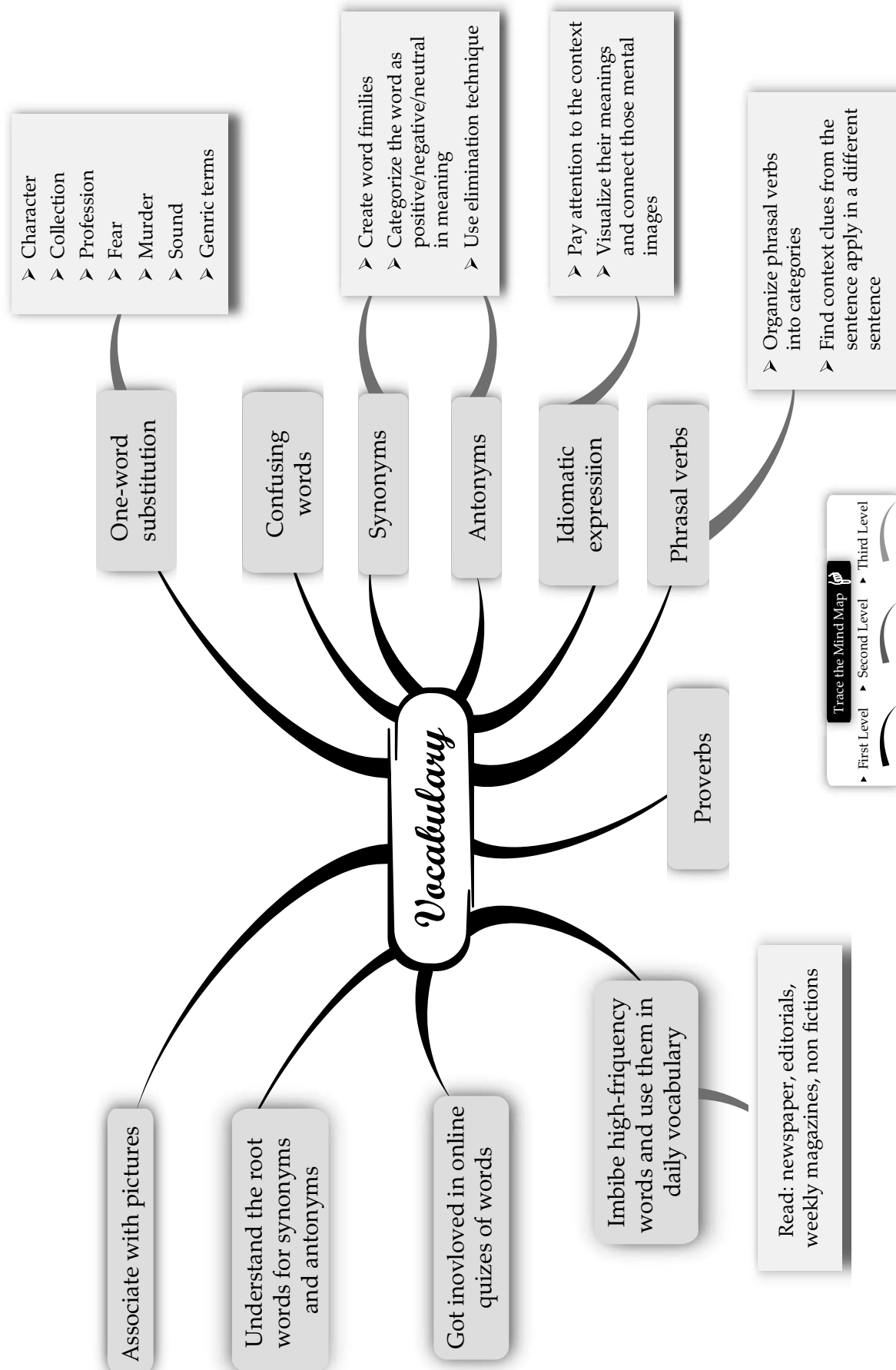
BIHAR		CUTTAK	A.K.Mishra Agencies, 9437025991
PATNA	Metro Books Corner, 9431647013, Alka Book Agency, 9835655005, Vikas Book Depot, 9504780402	BHUBANESHWAR	M/s Pragnya, 9437943777
CHATTISGARH		PUNJAB	
KORBA	Kitab Ghar, 9425226528, Shri Ramdev Traders, 9981761797	JALANDHAR	Cheap Book Store, 9872223458, 9878258592
DELHI		RAJASTHAN	
DELHI	Singhania Book & Stationer, 9212028238, Radhey Book depot, 9818314141, The Book Shop, 9310262701, Mittal Books, 9899037390, Lov Dev & Sons, 9999353491	KOTA	Vardhman Book Depot, 9571365020, Raj Traders, 9309232829
NEW DELHI	Anupam Sales, 9560504617, A ONE BOOKS, 8800497047	JAIPUR	Goyal Book Distributors, 9414782130
HARYANA		UTTAR PRADESH	
AMBALA	Bharat Book Depot, 7988455354	AGRA	BHAGWATI BOOK STORE, 9149081912, Sparsch Book Agency, 9412257817, Sanjay Publication, 8126699922
JHARKHAND		ALIGARH	New Vimal Books, 9997398868
BOKARO	Bokaro Student Friends Pvt. Ltd, 7360021503	ALLAHABAD	Mehrotra Book Agency, (532) 2266865, 9415636890
MADHYA PRADESH		GORAKHPUR	Central Book House, 9935454590
INDORE	Bhaiya Industries, 9109120101	KANPUR	Raj Book Dist, 9235616506
CHHINDWARA	Pustak Bhawan, 9827255997	LUCKNOW	Azad Book Depot PVT LTD, 7317000250, Rama Book Depot(Retail), 7355078254, Ashirwad Book Depot , 9235501197, Book Sadan, 8318643277, Book.com , 7458922755, Sheetla Book Agency, 9235832418
MAHARASHTRA		PRAYAGRAJ	Format Center, 9335115561, Garg Brothers Trading & Services Pvt. Ltd., 7388100499
NAGPUR	Laxmi Pustakalay and Stationers, (0712) 2727354	UTTAR PRADESH	
PUNE	Pragati Book Centre, 9850039311	DEHRADUN	Inder Book Agencies, 9634045280
MUMBAI	New Student Agencies LLP, 7045065799	WEST BENGAL	
ODISHA		KOLKATA	Bijay Pustak Bhandar Pvt. Ltd., 8961260603, Saha Book House, 9674827254, United Book House, 9831344622, Techno World, 9830168159
BARIPADA	Trimurti Book World, 9437034735		

MINDMAPS

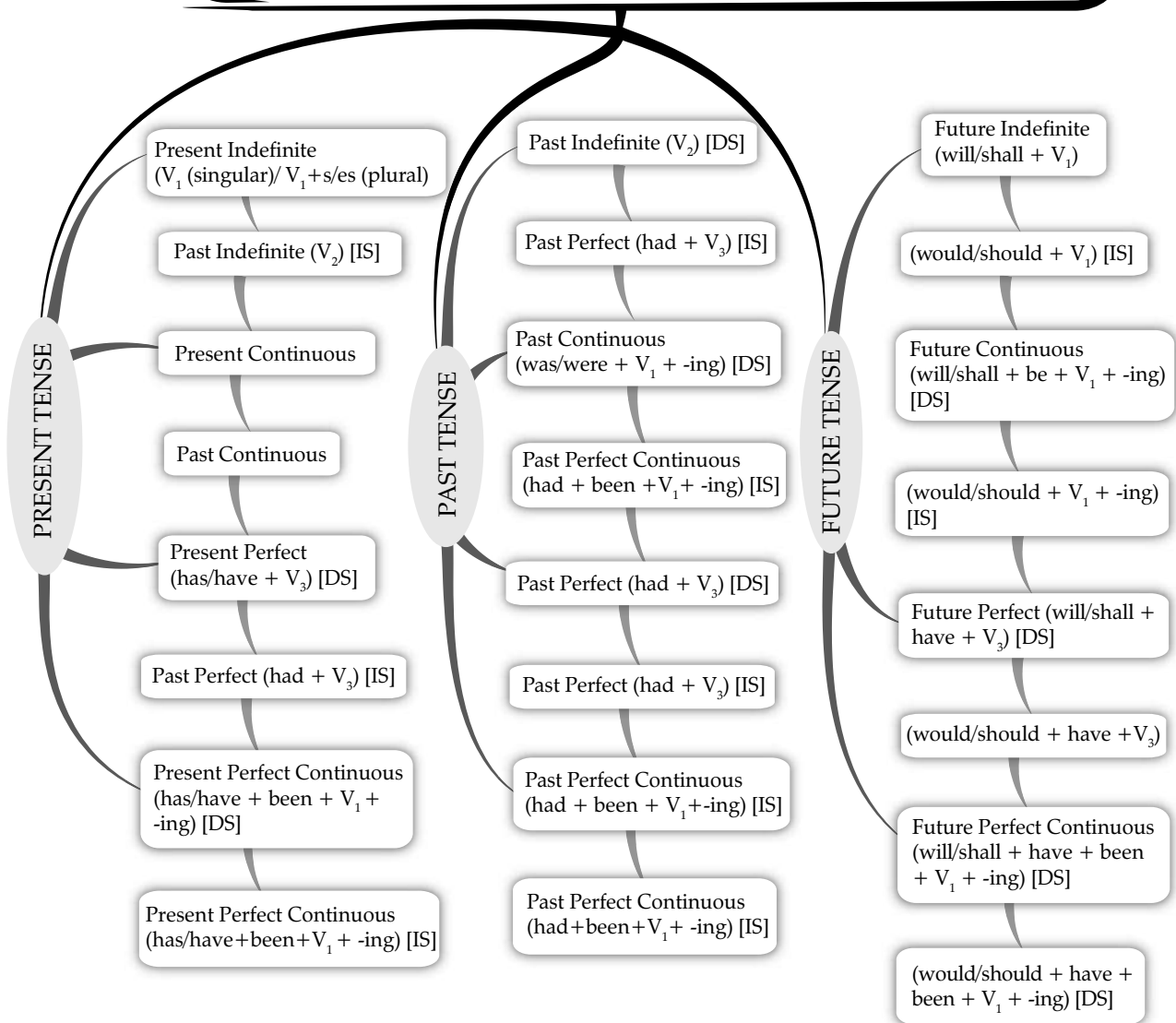


Trace the Mind Map

► First Level ► Second Level ► Third Level



Change of Tense of Verbs of Direct Narration

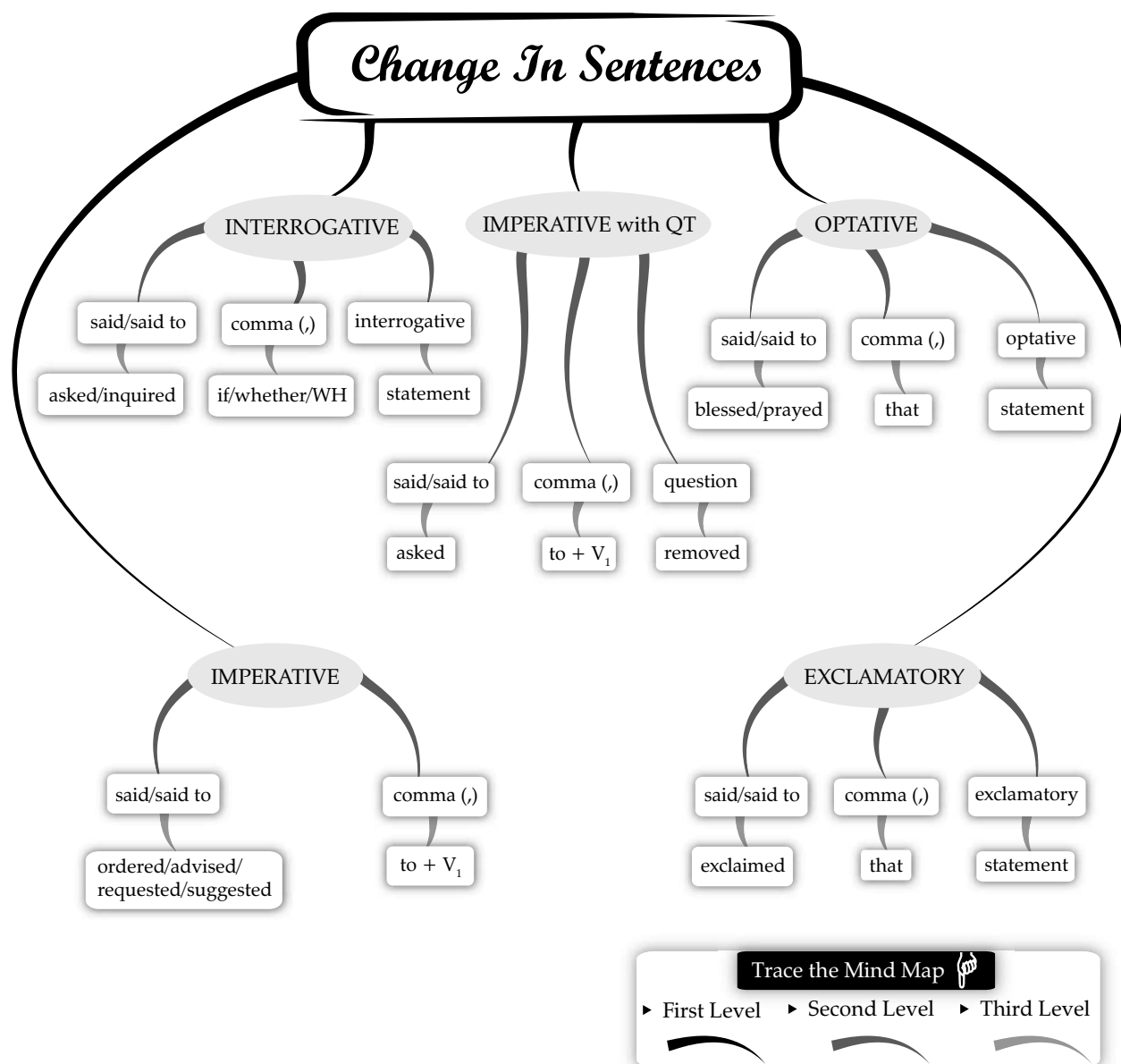


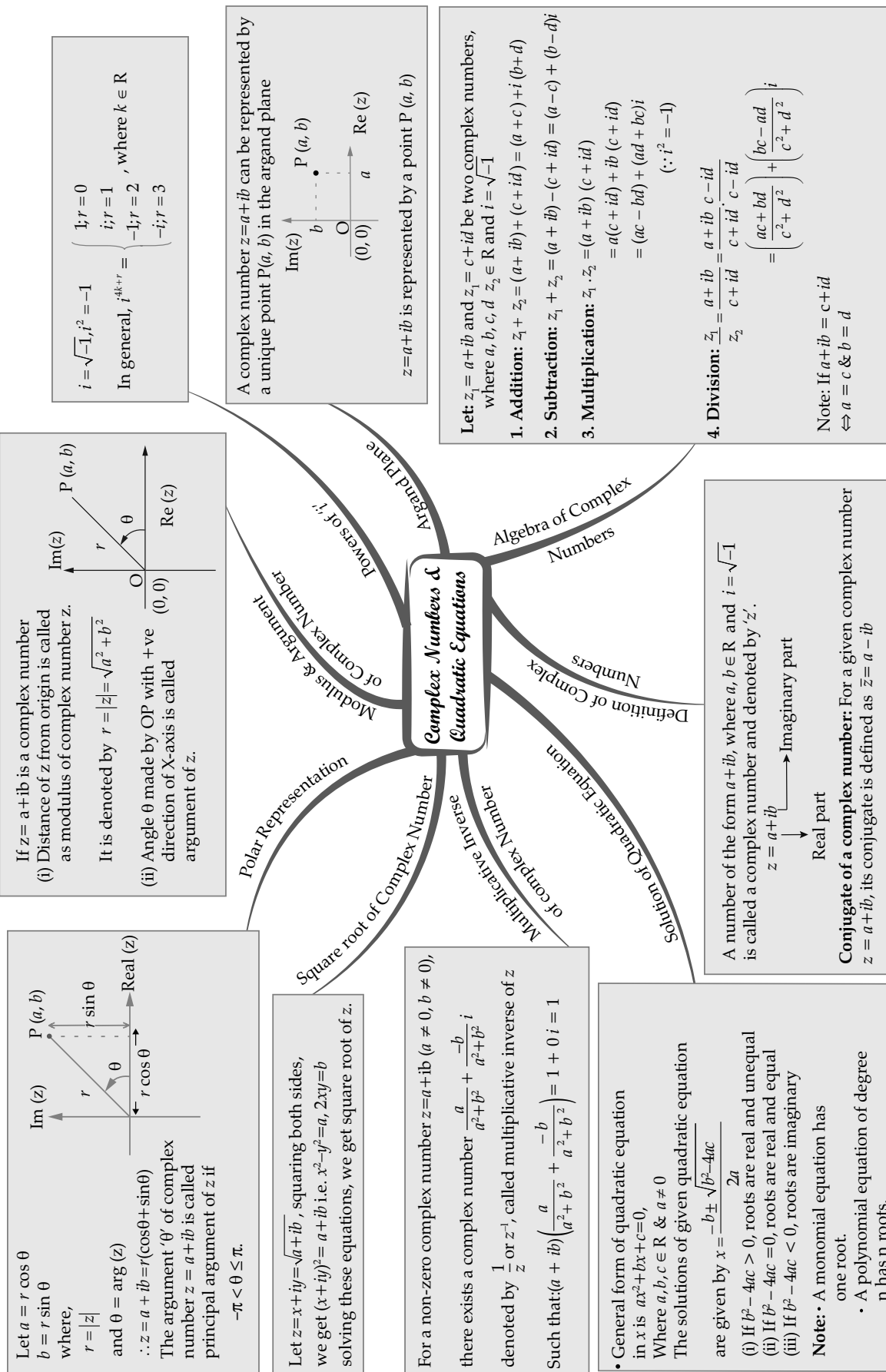
Note :

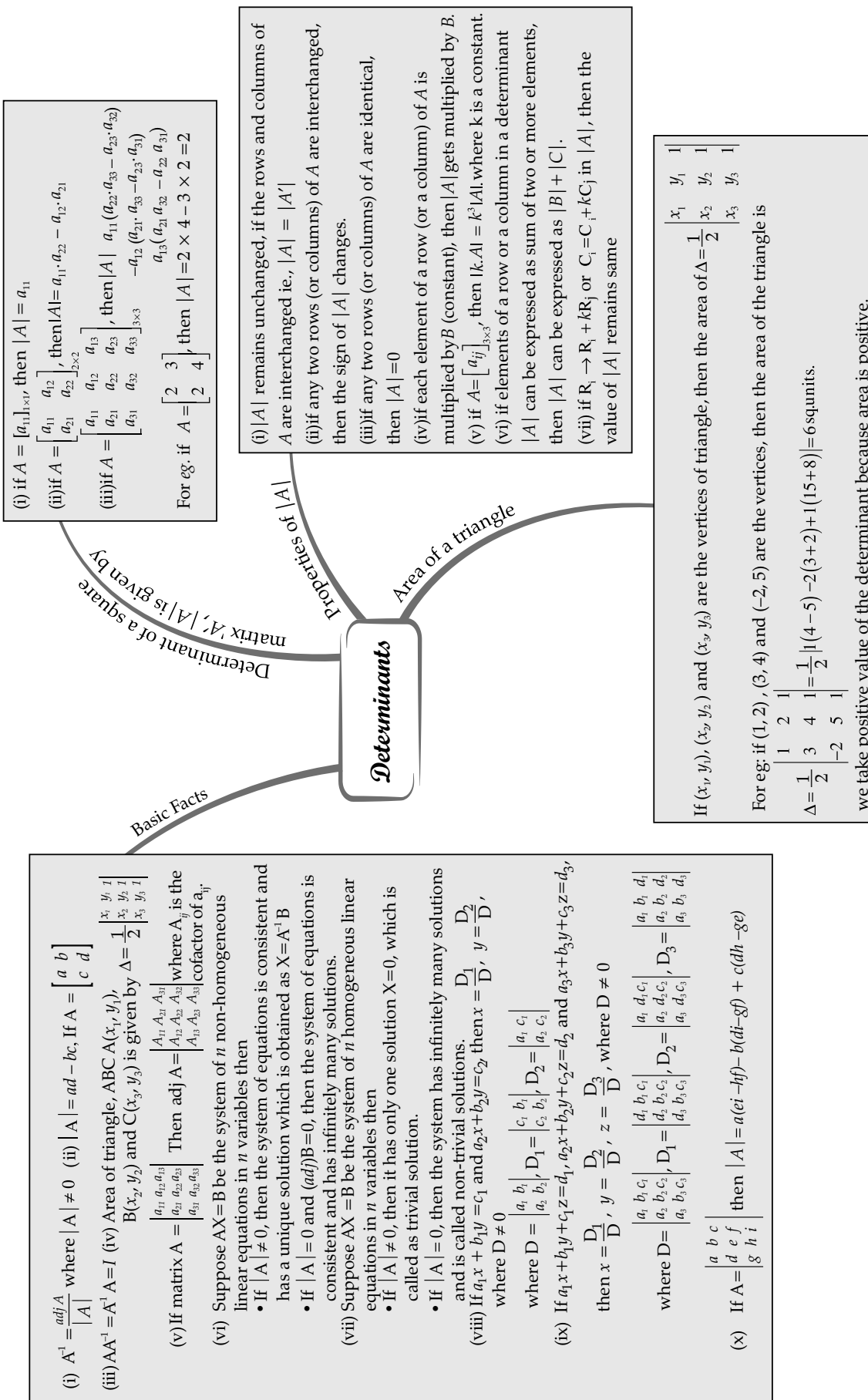
- DS – Direct Sentence
- IS – Indirect Sentence

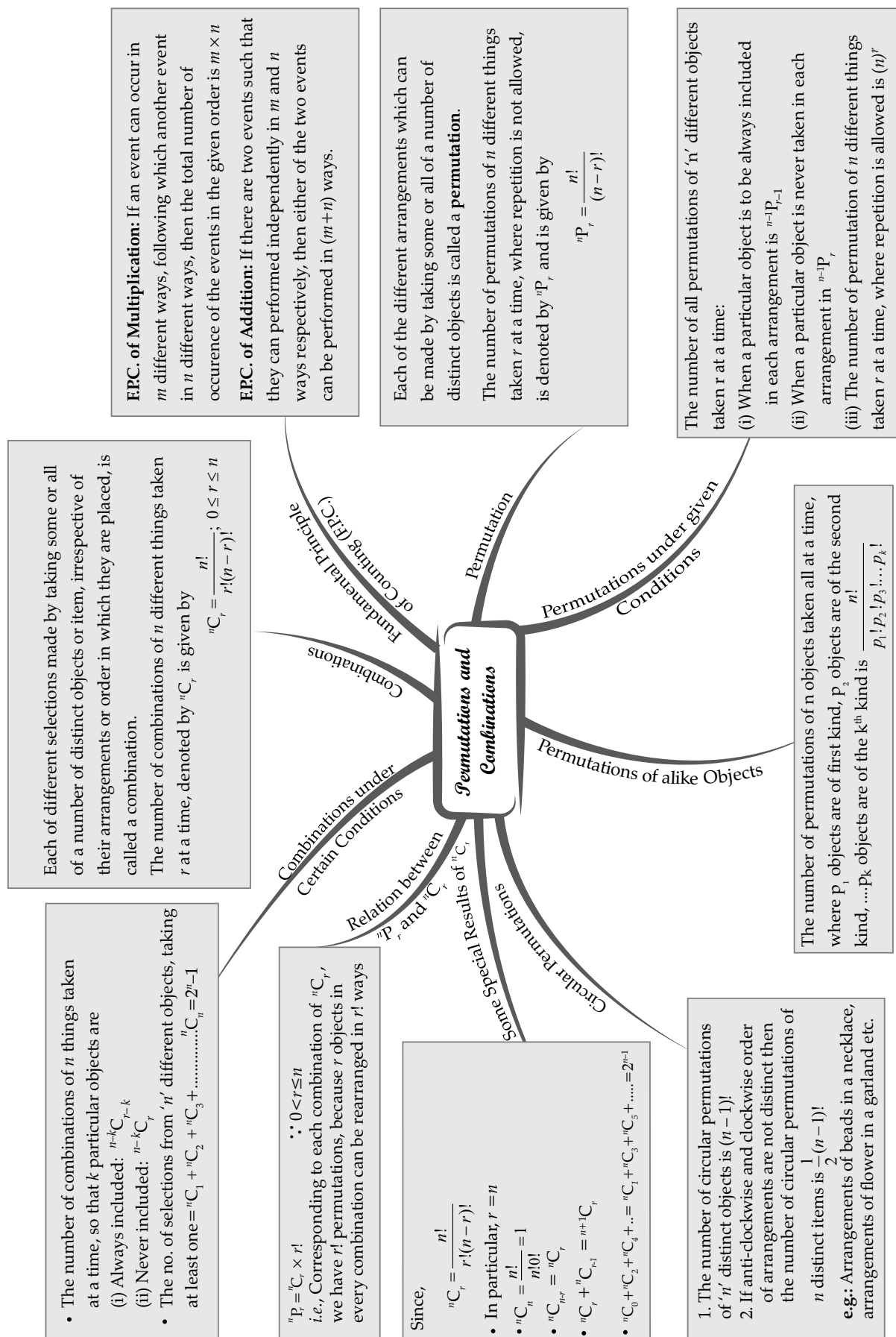
Trace the Mind Map

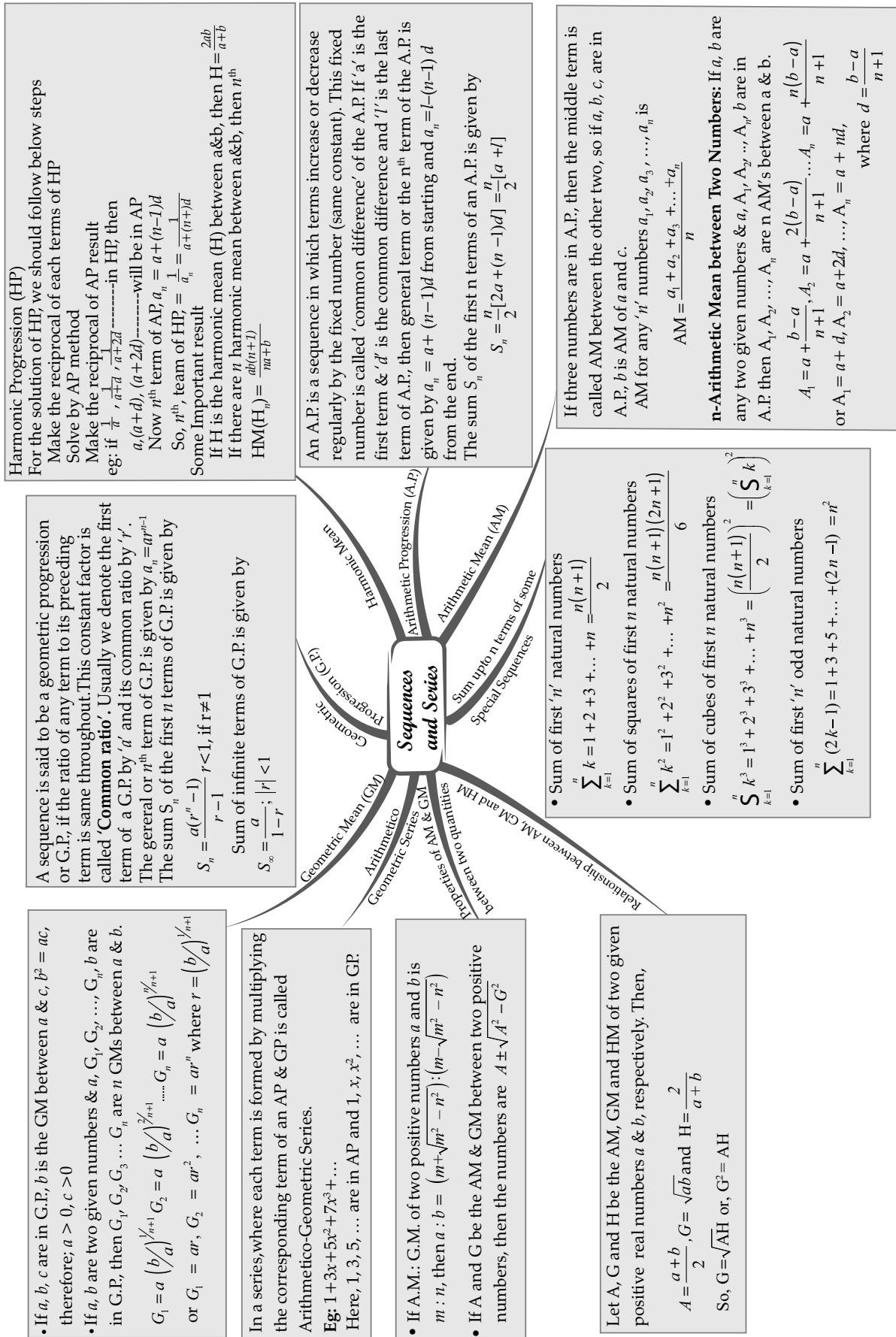
- First Level
- Second Level
- Third Level

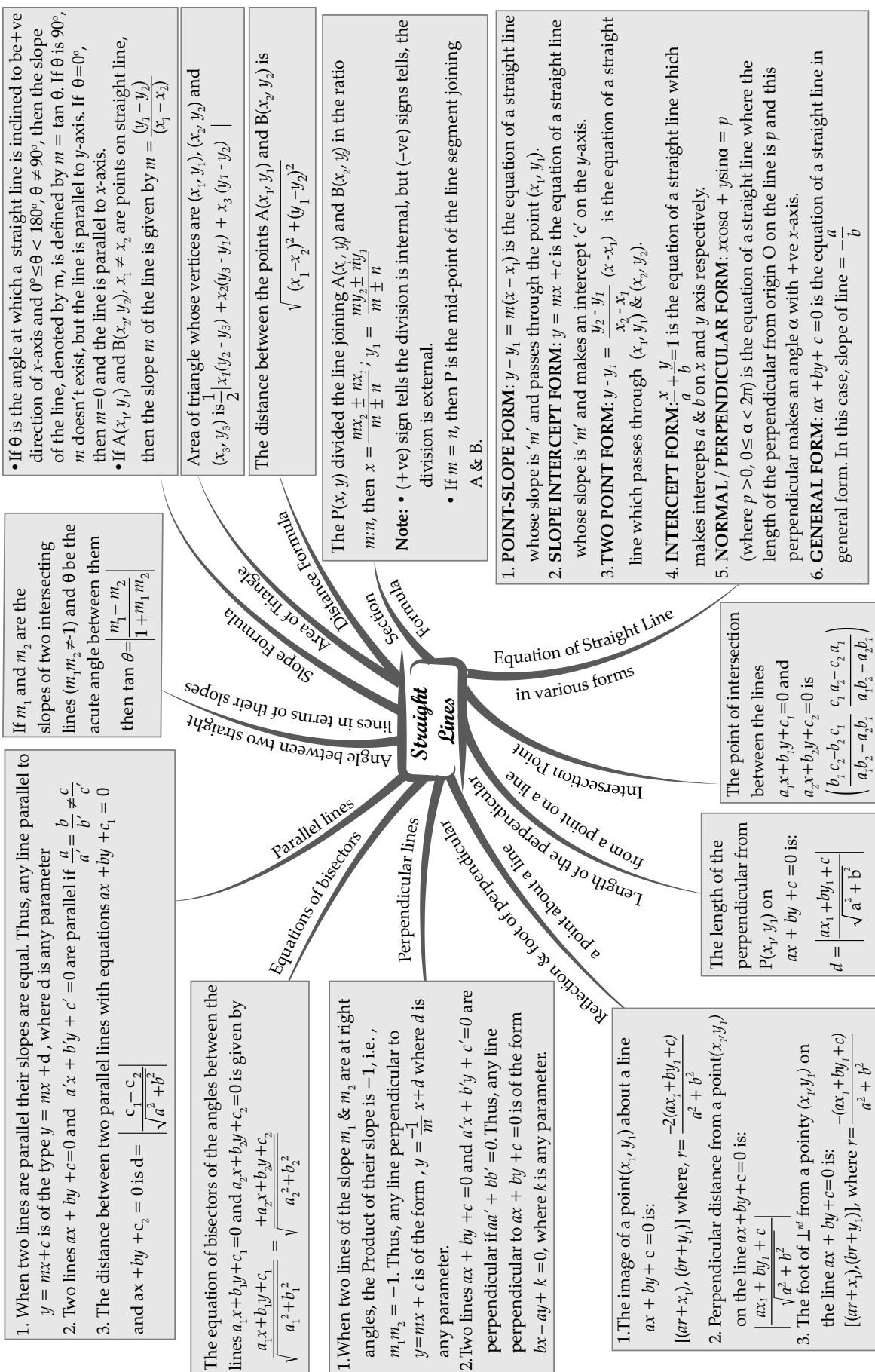






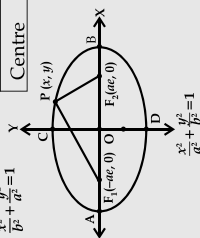
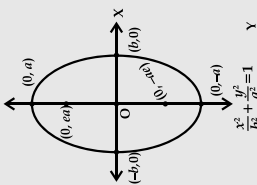






- An ellipse is the set of all points in a plane, that the sum of their distances from two fixed points in the plane is constant.
- The two fixed points are called the 'foci' of the ellipse.
- The mid-point of line segment joining foci is called the 'centre' of the ellipse.
- The line segment through the foci of the ellipse is called 'major axis'.
- The line segment through centre & perpendicular to major axis is called minor axis.

Forms of ellipse	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
Equation of major axis	$a > b$ $y = 0$ $x = 0$
Length of major axis	$2a$
Equation of Minor axis	$x = 0$ $y = 0$
Length of Minor axis	$2b$
Directrices	$x = \pm \frac{a}{e}$ $y = \pm \frac{a}{e}$
Equation of latus rectum	$x = \pm ae$ $y = \pm ae$
Length of latus rectum	$\frac{2b^2}{a}$
Centre	$(0, 0)$

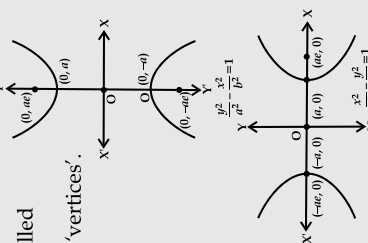


Here, $a > b$ and $b^2 = a^2(1 - e^2)$, $e < 1$

Conic Sections

- A hyperbola is the set of all points in a plane, that the difference of whose distances from two fixed points in the plane is a constant.
- The two fixed points are called the 'foci' of the hyperbola.
- The mid-point of the line segment joining the foci is called the 'centre' of the hyperbola.
- The line through the foci is called 'transverse axis'.
- Line through centre and perpendicular to transverse axis is called 'conjugate axis'.
- Points at which hyperbola intersects transverse axis are called 'vertices'.

Forms of the hyperbola	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
Equation of transverse axis	$y = 0$ $x = 0$
Equation of conjugate axis	$x = 0$ $y = 0$
Length of transverse axis	$2a$
Foci	$(\pm ae, 0)$ $(0, \pm ae)$
Equation of latus rectum	$x = \pm ae$ $y = \pm ae$
Length of latus rectum	$\frac{2b^2}{a}$
Centre	$(0, 0)$

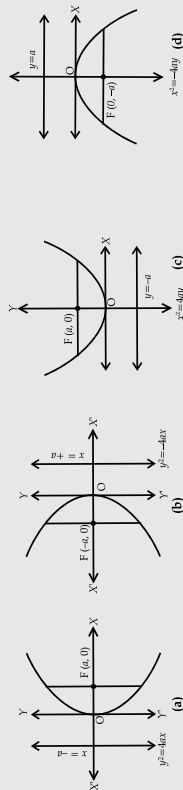


Here, $b^2 = a^2(e^2 - 1)$, $e > 1$

- A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point in the plane. Fixed line is called 'directrix' of parabola. Fixed point F is called the 'focus'. A line through focus & perpendicular to directrix is called 'axis'. Point of intersection of parabola with axis is called 'vertex'.

Main facts about the parabola

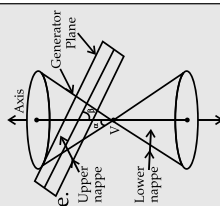
Forms of Parabolas	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = -4ay$
Axis	$y = 0$	$y = 0$	$x = 0$	$x = 0$
Directrix	$x = -a$	$x = a$	$y = -a$	$y = a$
Vertex	$(0, 0)$	$(0, 0)$	$(0, 0)$	$(0, 0)$
Focus	$(a, 0)$	$(-a, 0)$	$(0, a)$	$(0, -a)$
Length of latus rectum	$4a$	$4a$	$4a$	$4a$
Equations of latus rectum	$x = a$	$x = -a$	$y = a$	$y = -a$



Definition

Circles, ellipses, parabolas and hyperbolas are known as conic sections because they can be obtained as intersections of plane with a double napped right circular cone α . From the given figure.

- (i) Section will represent circle, if $\beta = 90^\circ$
- (ii) Section will represent an Ellipse, if $\alpha < \beta < \pi/2$
- (iii) Section will represent a parabola if $\alpha = \beta$
- (iv) Section will represent a hyperbola if $0 \leq \beta < \alpha$



A circle is a set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is called the 'centre' of the circle and the distance from the centre to a point on the circle is called the 'radius' of the circle.

The equation of a circle with centre (h, k) and the radius r is

$$(x-h)^2 + (y-k)^2 = r^2$$

The general equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

its centre is $(-g, -f)$ and radius $r = \sqrt{g^2 + f^2 - c}$

Three Dimensional Geometry-I

Introduction

Distance between Two Points

Coordinates of the Centroid of a Triangle

Coordinates of a Midpoint

The coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$.

e.g.: The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, -5, 7) and (-1, 7, -6), respectively, find the coordinates of the point C.

Sol: Let the coordinates of C be (x, y, z) and the coordinates of the centroid G be (1, 1, 1). Then $\frac{x+3-1}{3} = 1$, i.e., $x = 1$;

$$\frac{y-5+7}{3} = 1, \text{ i.e., } y = 1;$$

$$\frac{z+7-6}{3} = 1, \text{ i.e., } z = 2. \text{ So, } C(x, y, z) = (1, 1, 2)$$

Section Formula

The coordinates of the point R which divides the line segment joining two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ internally and externally in the ratio $m : n$ are given by

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right) \text{ \& } \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

respectively.

e.g.: Find the coordinates of the point which divides the line segment joining the points (1, -2, 3) and (3, 4, -5) in the ratio 2 : 3 internally.

Sol : Let $P(x, y, z)$ be the point which divides line segment joining A (1, -2, 3) and B (3, 4, -5) internally in the ratio 2 : 3. Therefore,

$$x = \frac{2(3) + 3(1)}{2+3} = \frac{9}{5}, \quad y = \frac{2(4) + 3(-2)}{2+3} = \frac{2}{5}, \quad z = \frac{2(-5) + 3(3)}{2+3} = \frac{-1}{5}$$

Thus, the required point is $\left(\frac{9}{5}, \frac{2}{5}, \frac{-1}{5}\right)$.

The coordinates of the midpoint of the line segment joining two points

$$P(x_1, y_1, z_1) \text{ and } Q(x_2, y_2, z_2) \text{ are } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

e.g.: Find the midpoint of the line joining two points $P(1, -3, 4)$ and $Q(-4, 1, 2)$.

Sol: Coordinates of the midpoint of the line joining the points P & Q are $\left(\frac{1-4}{2}, \frac{-3+1}{2}, \frac{4+2}{2}\right)$ i.e. $\left(\frac{-3}{2}, -1, 3\right)$

- In three dimensions, the coordinate axes of a rectangular cartesian coordinate system are three mutually perpendicular lines. The axes are called x , y and z -axes.
- The three planes determined by the pair of axes are the coordinate planes, called xy , yz and zx -planes.
- The three coordinate planes divide the space into eight parts known as octants.

• The coordinates of a point P in 3D Geometry is always written in the form of triplet like (x, y, z) . Here, x , y and z are the distances from yz , zx and yx planes, respectively.

- e.g.:**
- Any point on x -axis is : $(x, 0, 0)$
 - Any point on y -axis is : $(0, y, 0)$
 - Any point on z -axis is : $(0, 0, z)$

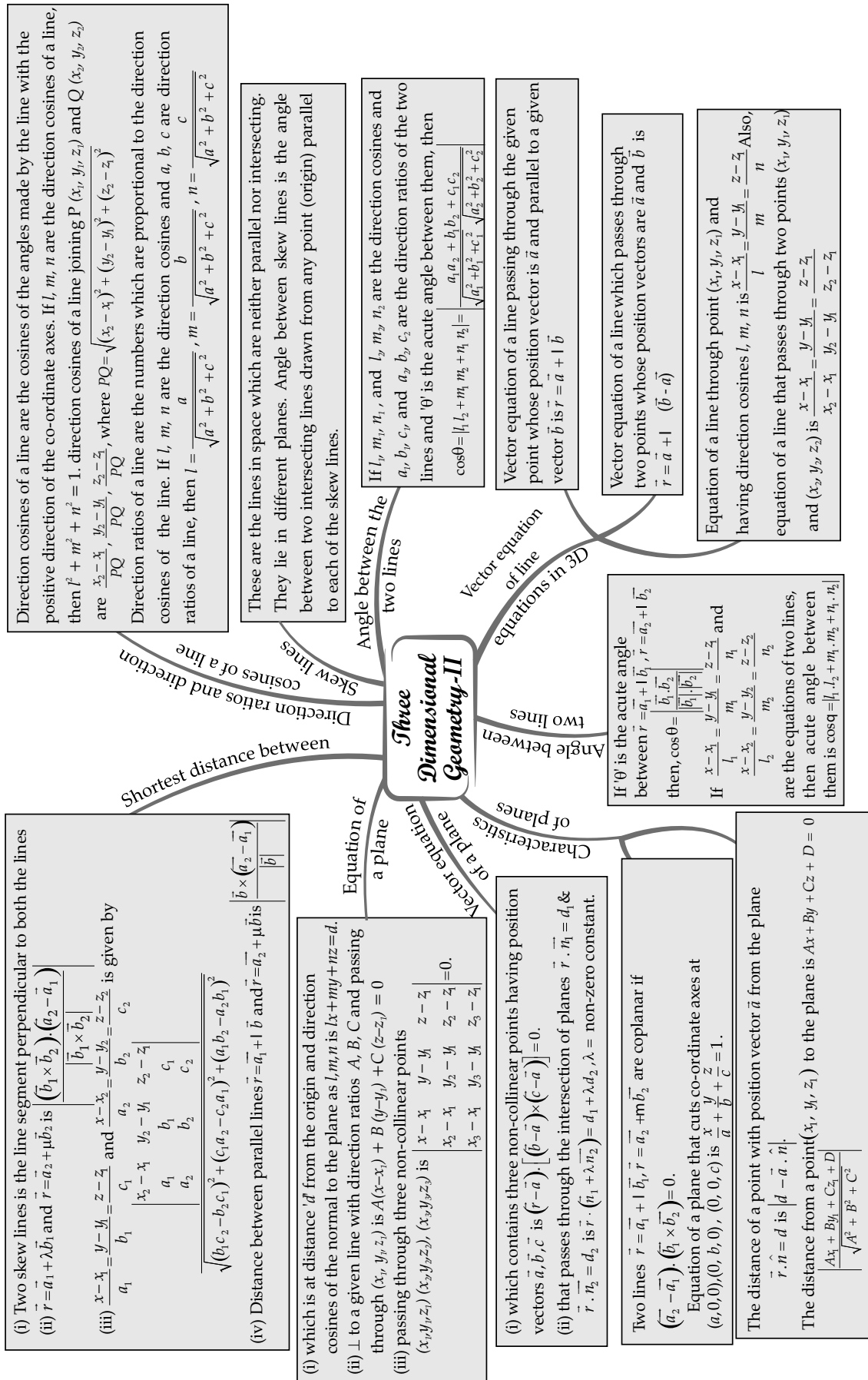
Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

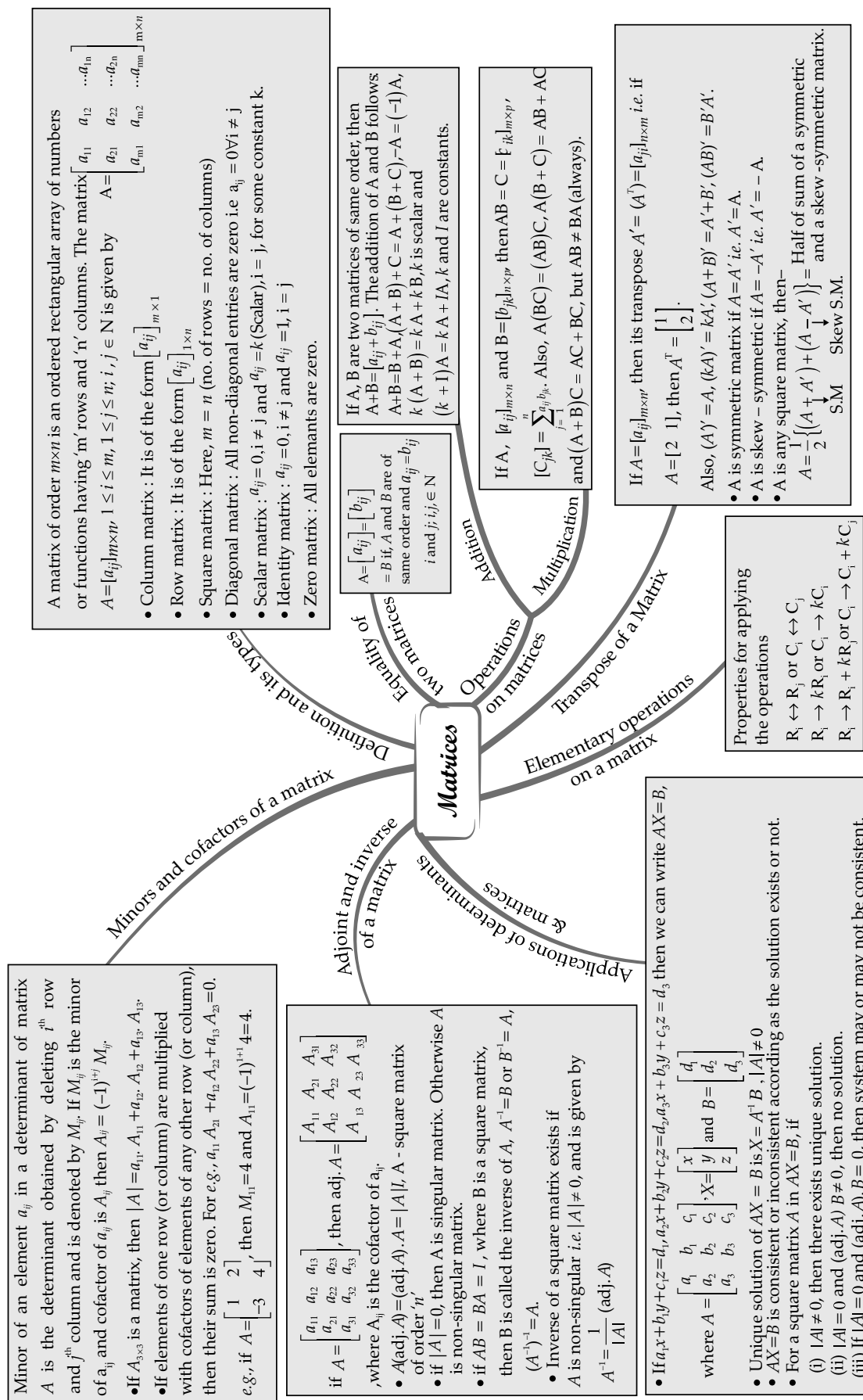
$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

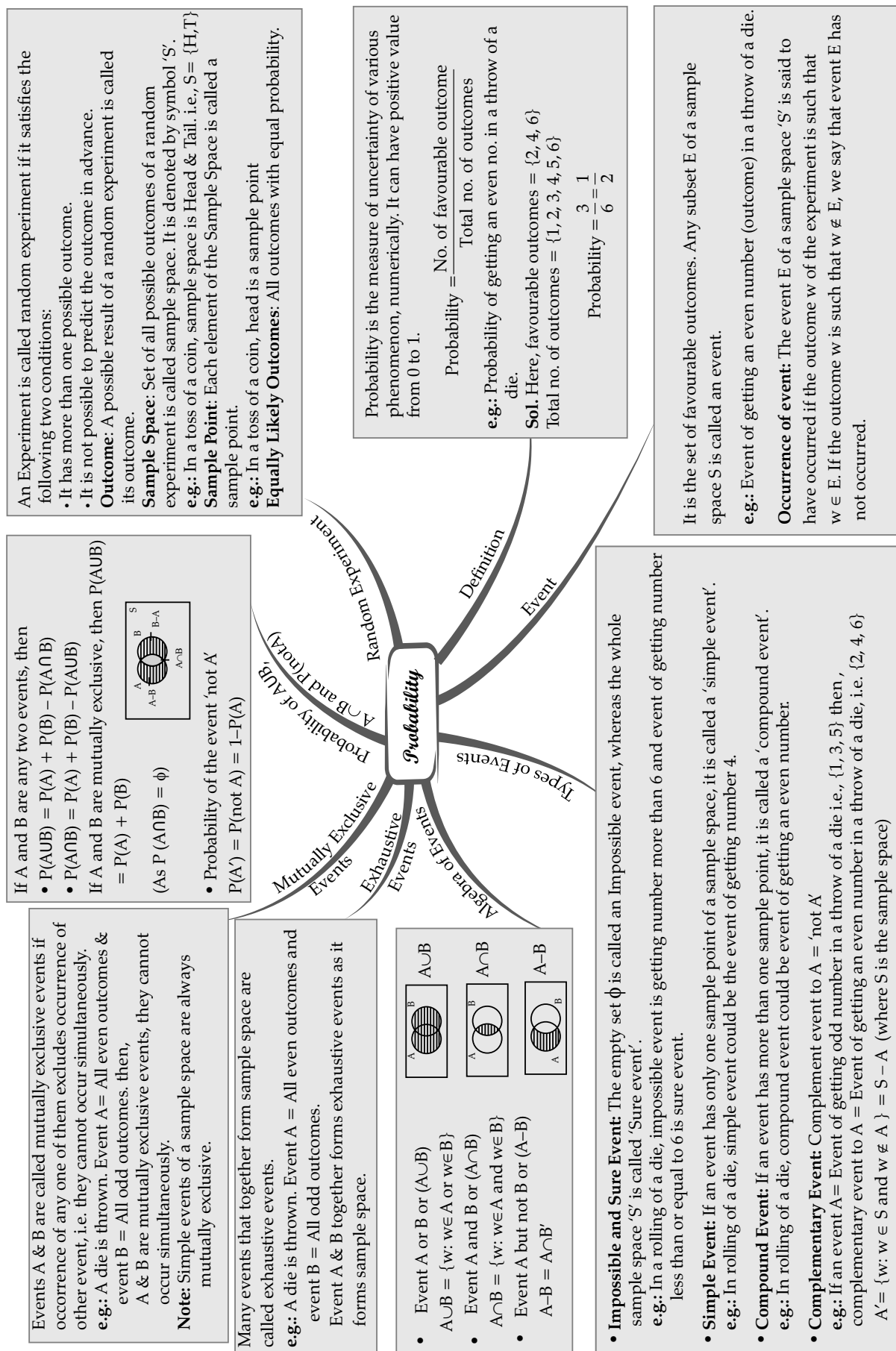
e.g.: Find the distance between the points $P(1, -3, 4)$ and $(-4, 1, 2)$.

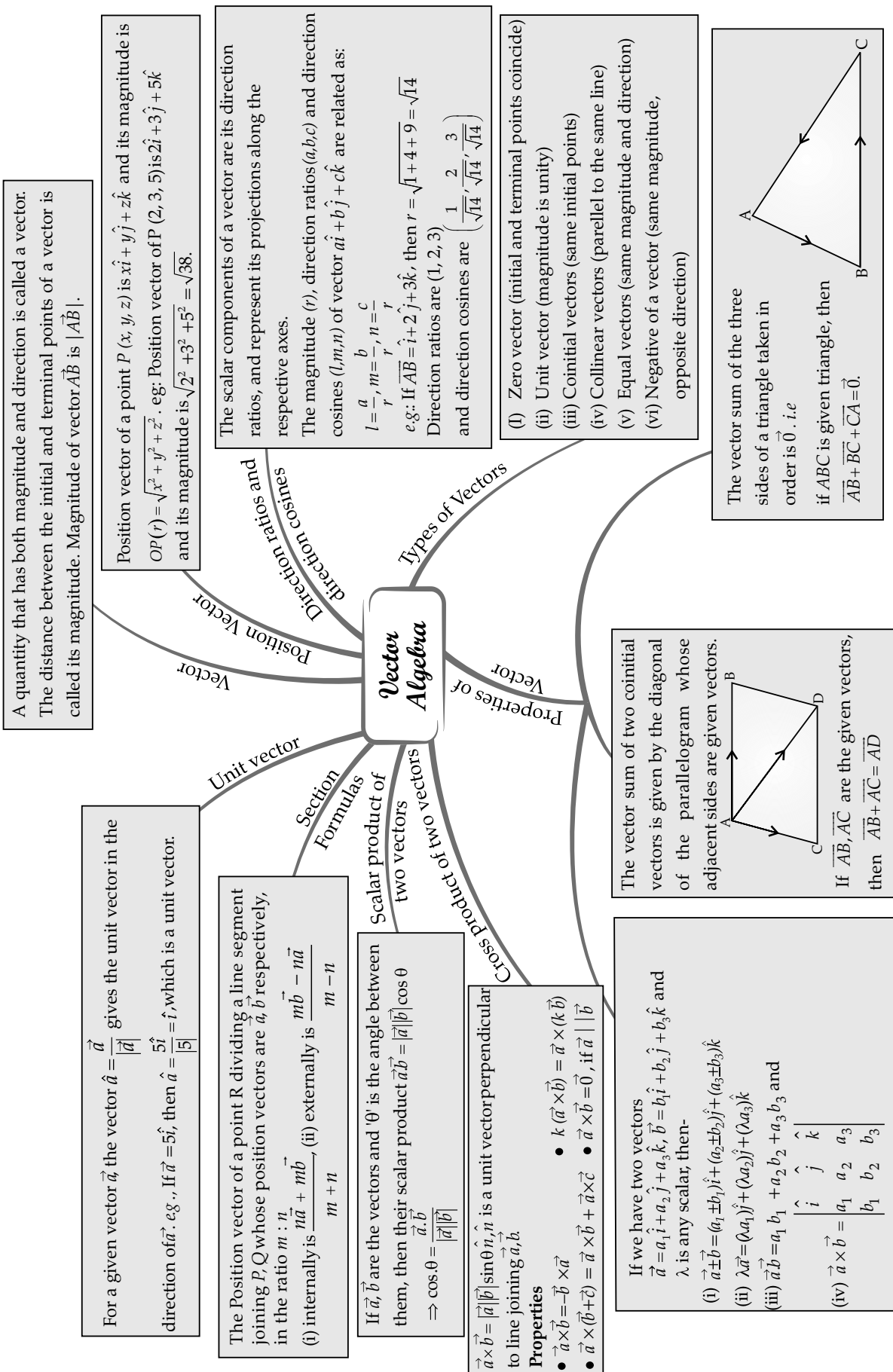
Sol: The distance PQ between the points P & Q is given by

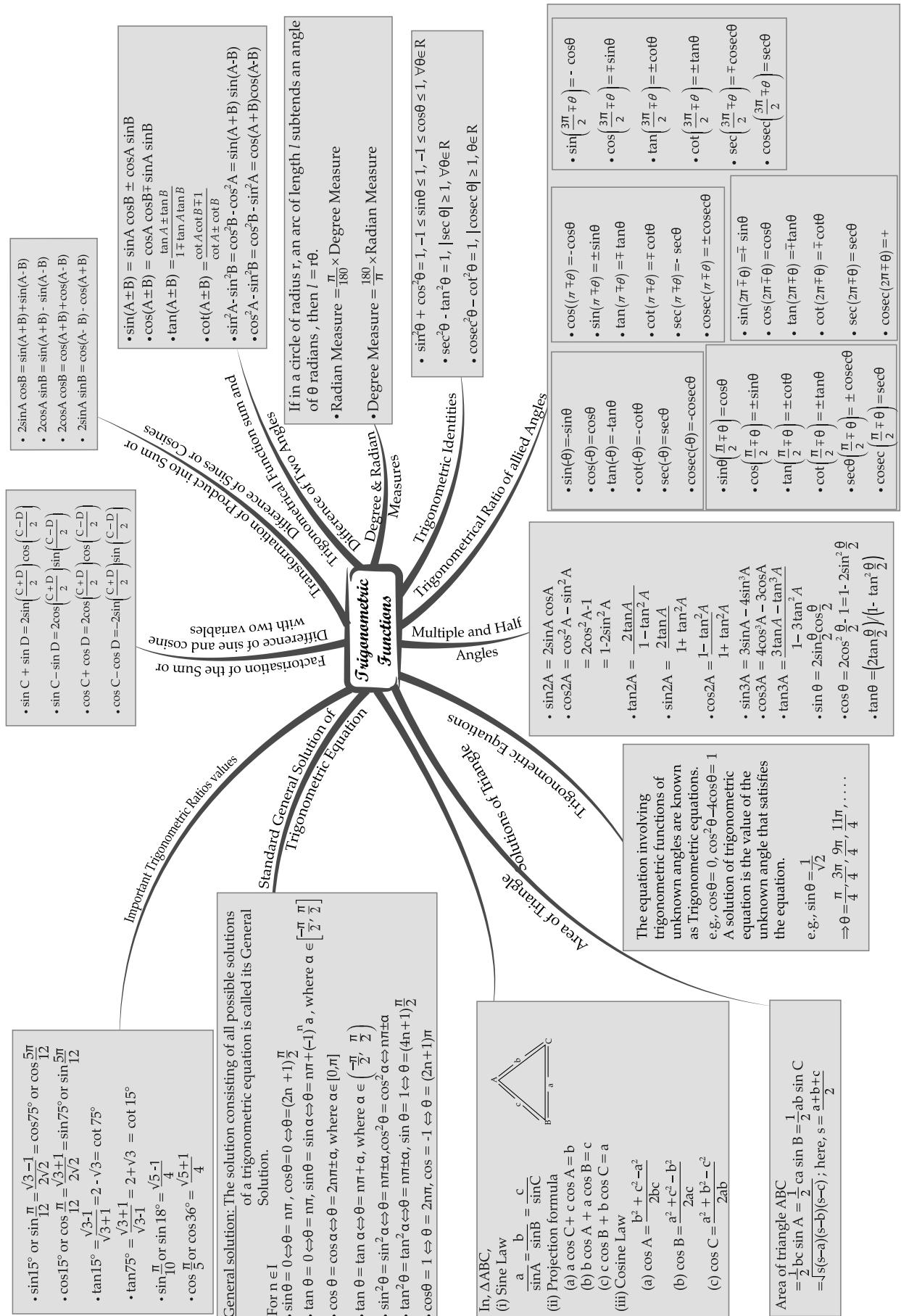
$$PQ = \sqrt{(-4-1)^2 + (1+3)^2 + (2-4)^2} \\ = \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5} \text{ units}$$

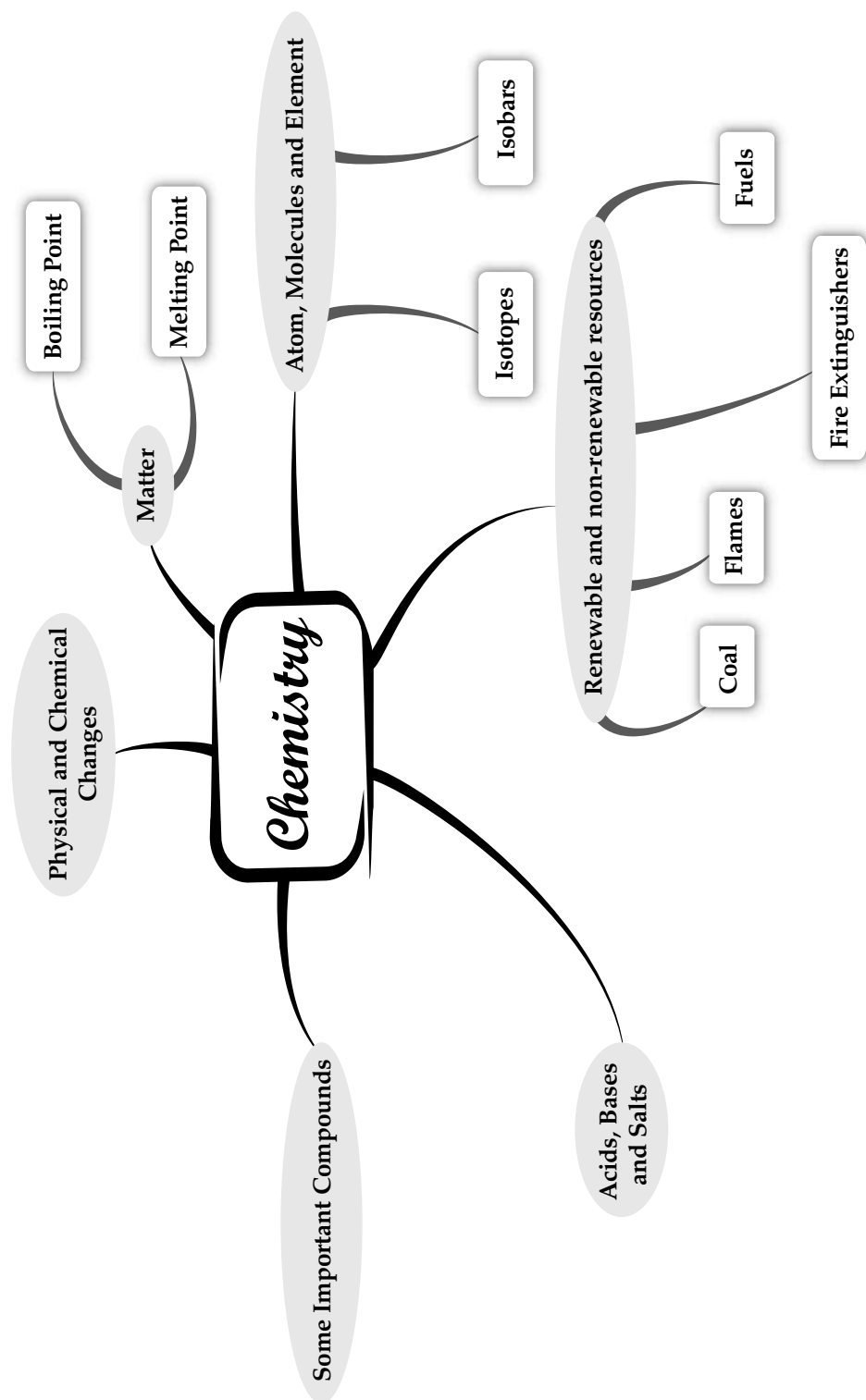








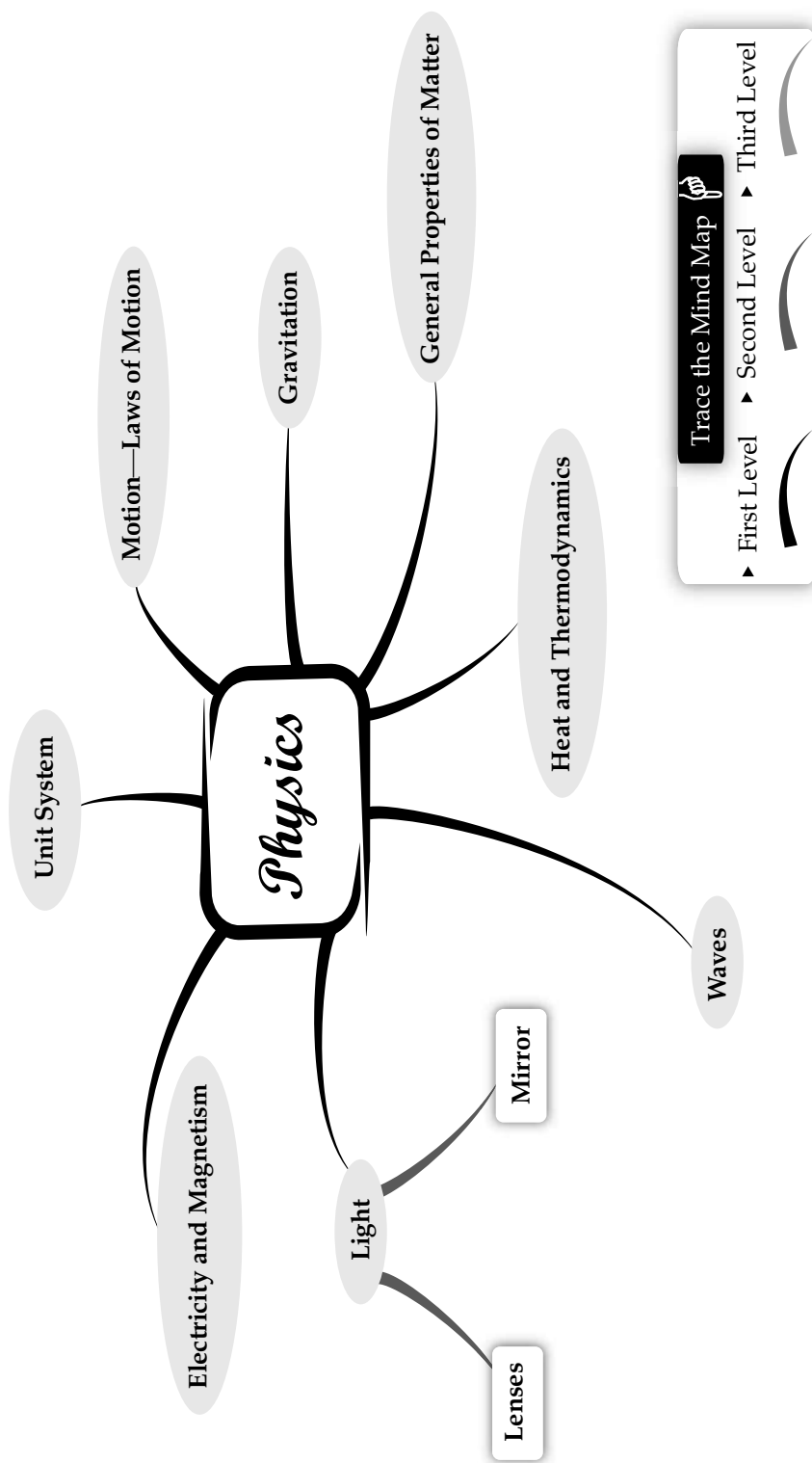


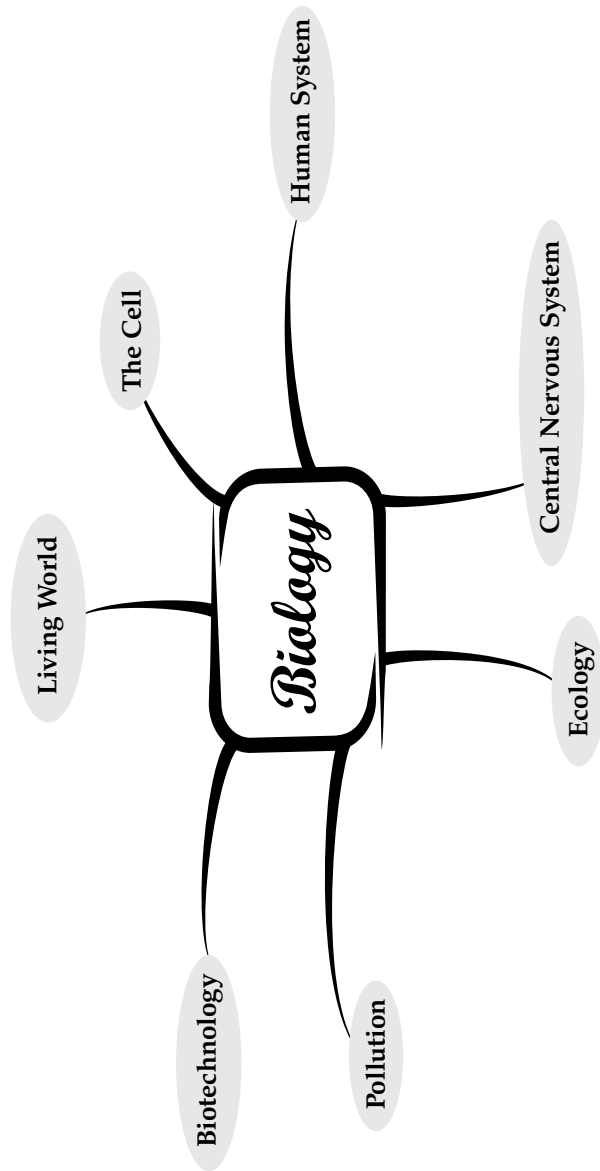


Trace the Mind Map 

► First Level ► Second Level ► Third Level



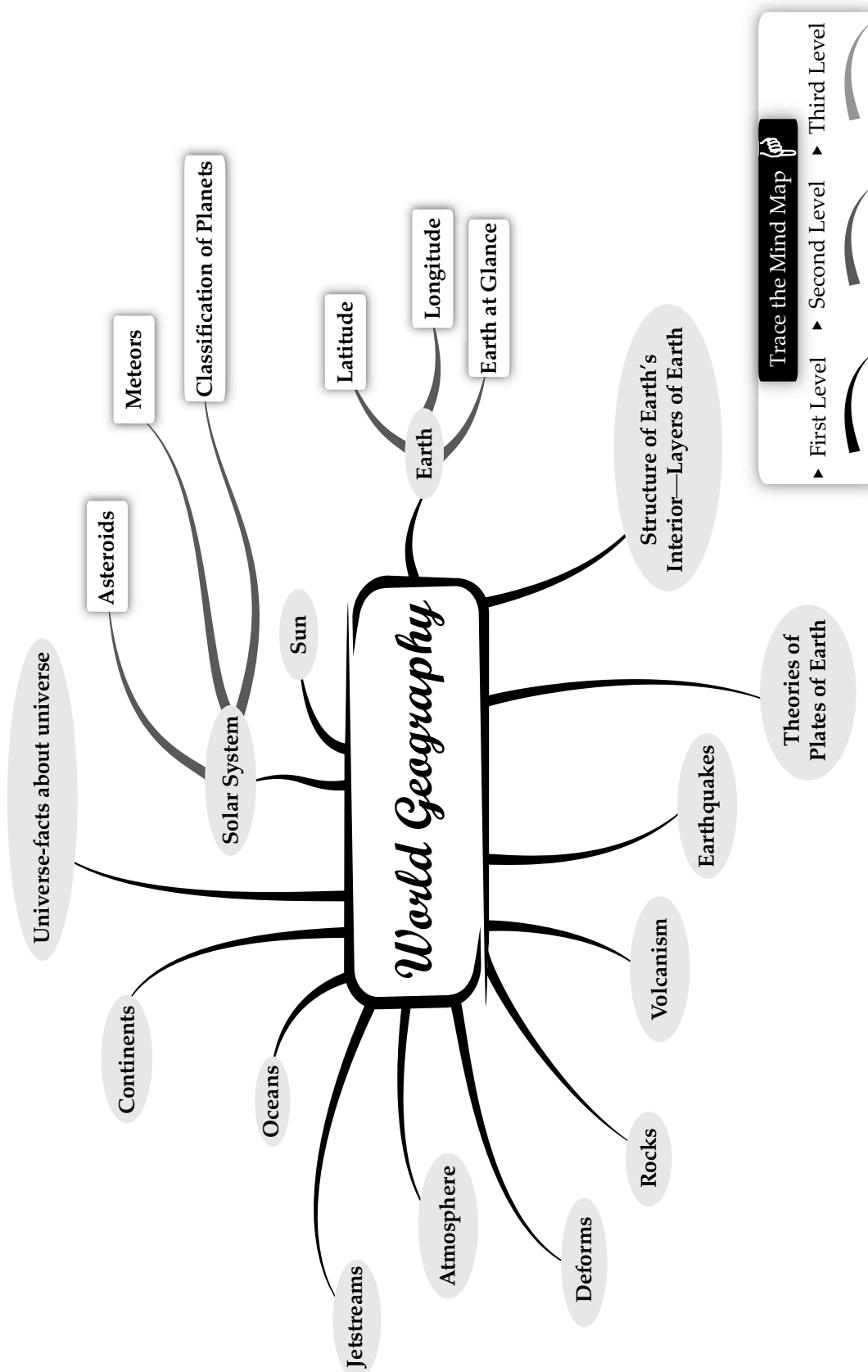


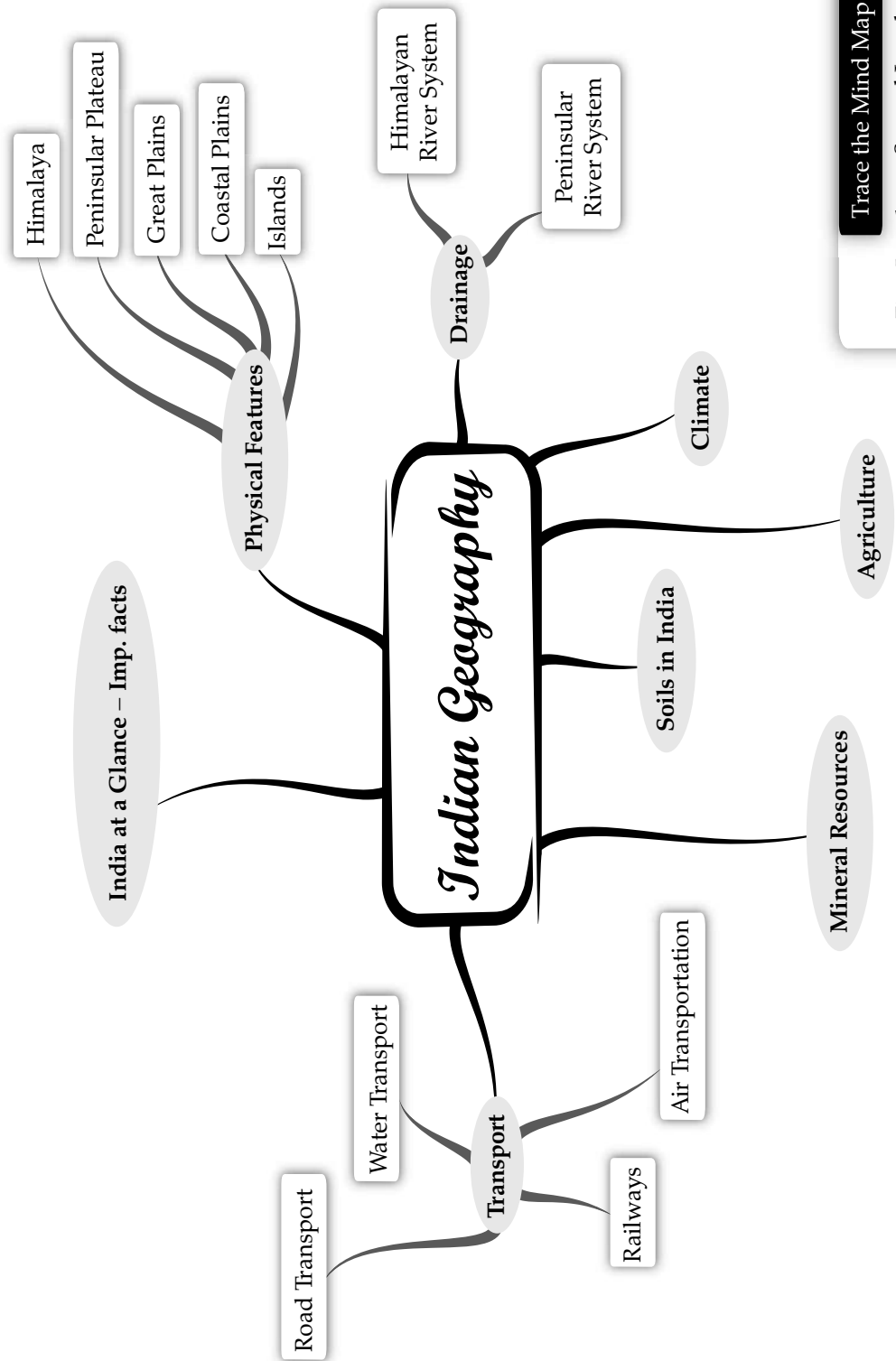


Trace the Mind Map 

► First Level ► Second Level ► Third Level

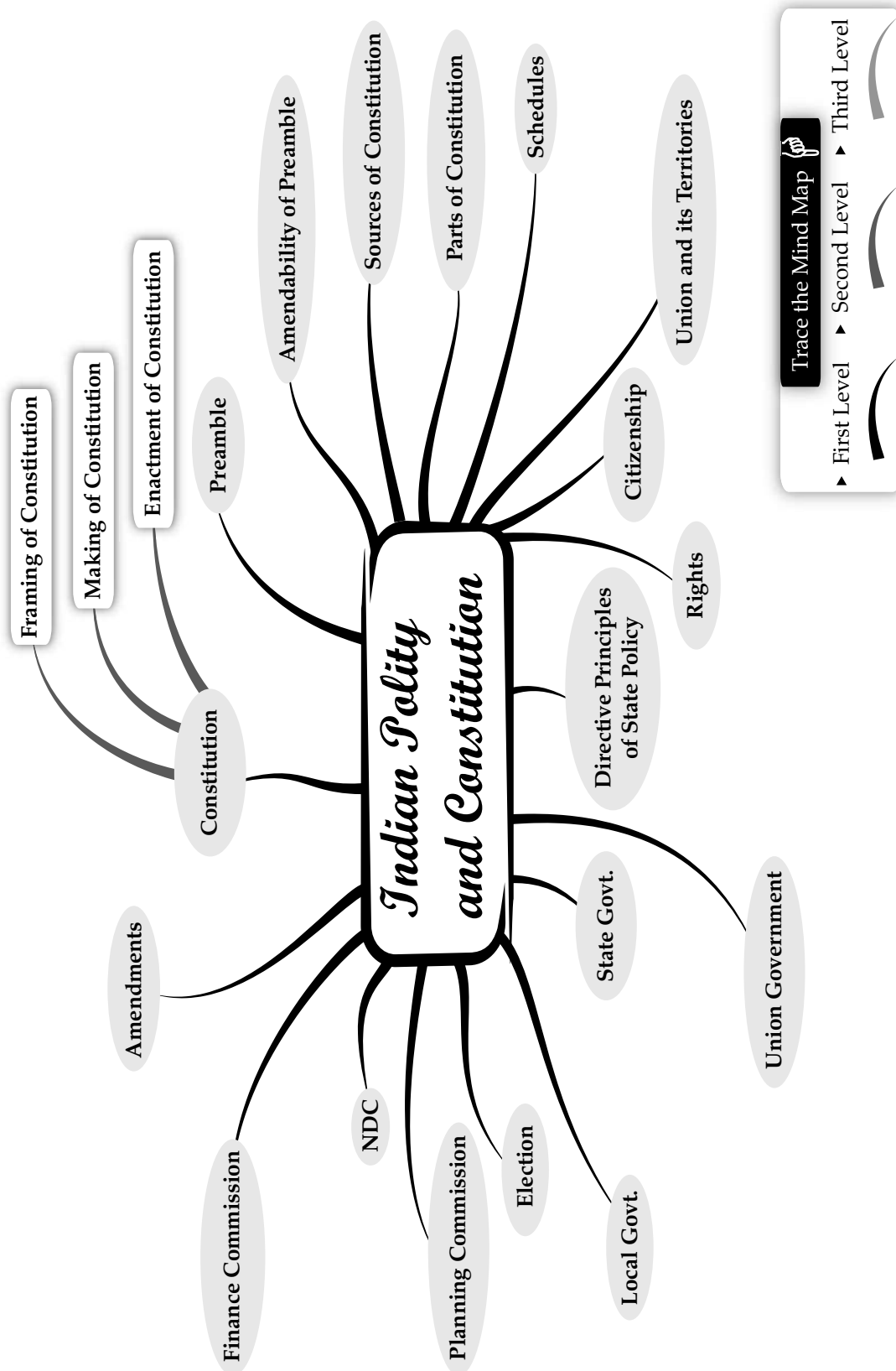


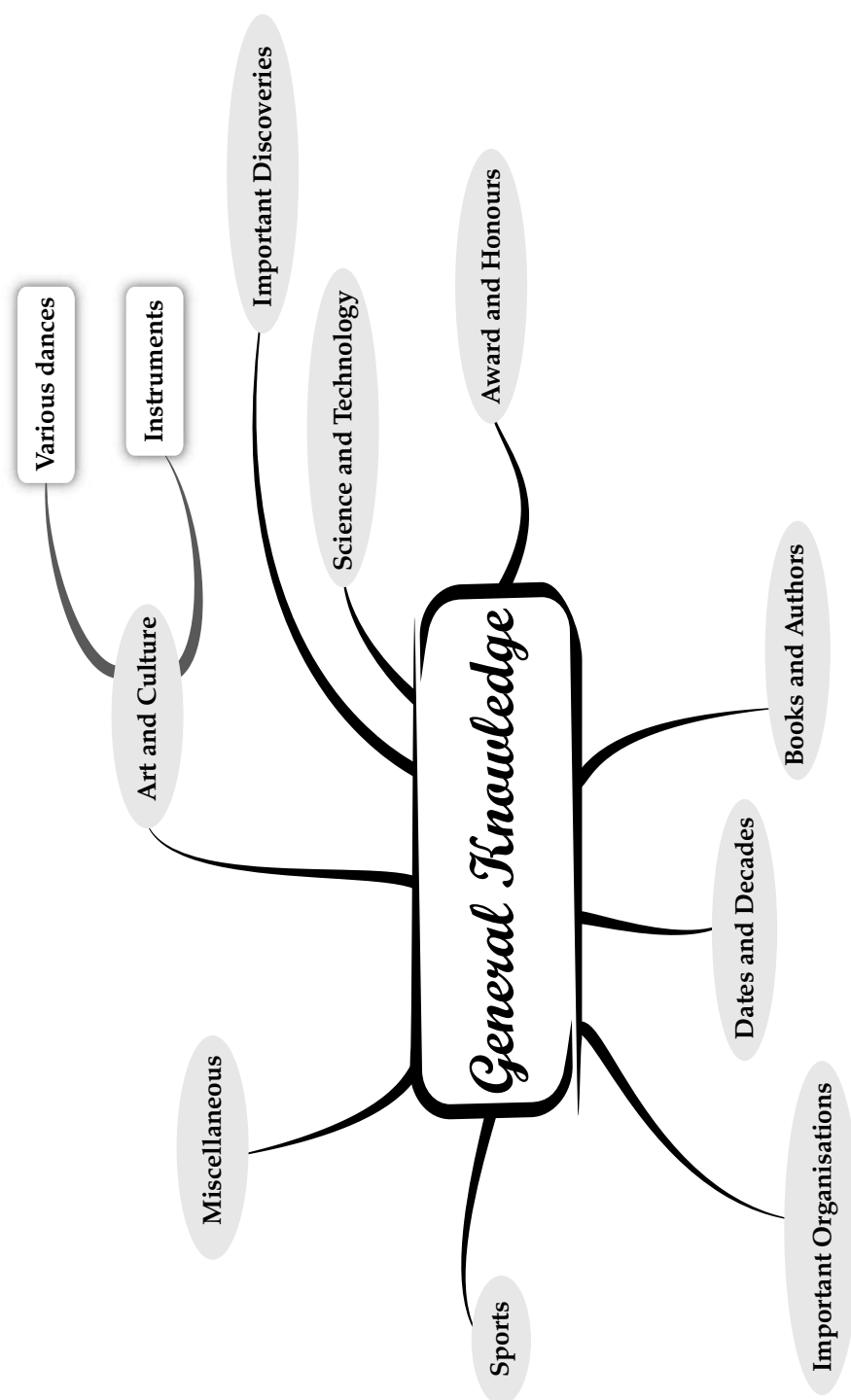




Trace the Mind Map

► First Level ► Second Level ► Third Level



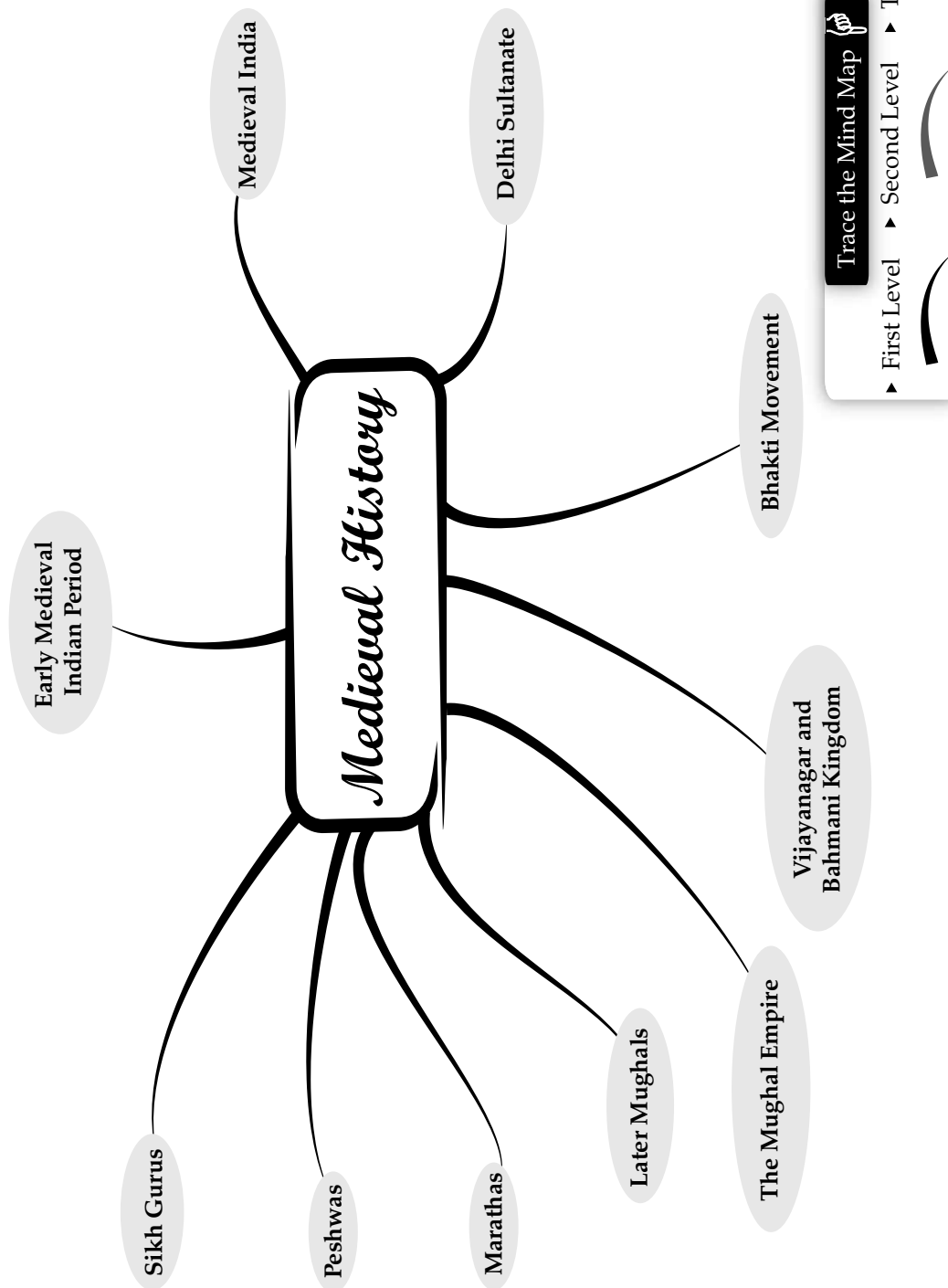


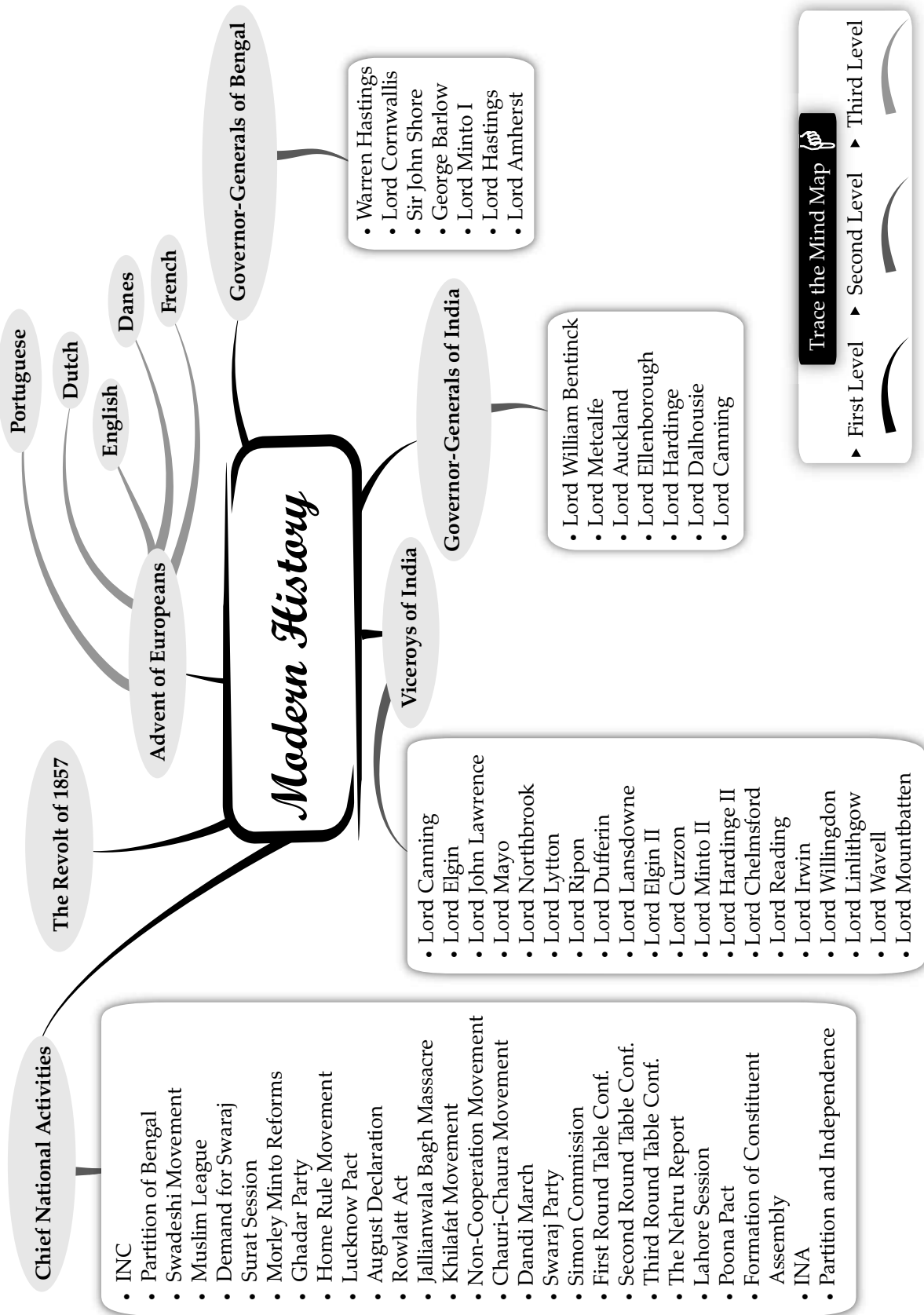
Trace the Mind Map



► First Level ► Second Level ► Third Level







SYNONYMS

- Synonyms are words that are similar, or have a related meaning, to another word.

A synonym is a word or phrase that means exactly or nearly the same as another word or phrase in the same language. For example, the words begin, start, commence, and initiate are all synonyms of one another; they are synonymous.

Some common synonyms are:

Smart – Clever, Important – Essential, Good – Excellent, Stupid – Dumb, Irrelevant – Useless, Complete – Perfect, Bad – Inferior, Interesting – Fascinating, Exact – Specific, Awful – Horrible, Commonplace – Boring, Suitable – Uniform

List of Synonyms

- **Anger** – enrage, infuriate, annoy, exasperate, madden
- **Angry** – enraged, wrathful, indignant, exasperated, aroused
- **Amazing** – incredible, improbable, fabulous, astonishing, astounding, extraordinary
- **Ask** – seek information from, put a question to, demand, request, expect, interrogate
- **Begin** – launch, initiate, commence, inaugurate, originate
- **Big** – enormous, immense, gigantic, colossal, ample
- **Bossy** – controlling, tyrannical, domineering, autocratic, high-handed
- **Brave** – dauntless, intrepid, plucky, doughty, mettlesome
- **Break** – fracture, rupture, shatter, smash, wreck, crash, demolish, atomise
- **Calm** – tranquil, mild, level-headed, unexcited, aloof
- **Come** – approach, advance, near, arrive, reach
- **Cool** – chilly, frosty, wintry, icy, frigid
- **Definite** – certain, sure, positive, determined, obvious
- **Delicious** – luscious, scrumptious, palatable, toothsome, exquisite
- **Describe** – portray, narrate, relate, represent, record
- **End** – terminate, conclude, halt, cessation, discontinuance
- **Enjoy** – delight in, be pleased, indulge in, luxuriate in, bask in, relish
- **Explain** – elaborate, clarify, interpret, justify, account for
- **Fall** – drop, descend, plunge, topple, tumble
- **False** – fraudulent, spurious, unfounded, deceptive, groundless
- **Fair** – just, objective, impartial, unbiased, open minded
- **Famous** – renowned, celebrated, eminent, illustrious, notorious
- **Good** – marvellous, pleasant, reliable, genuine, beneficial
- **Great** – noteworthy, worthy, distinguished, remarkable, mighty
- **Happy** – contented, delighted, gratified, blissful, overjoyed
- **Hardworking** – diligent, determined, industrious, enterprising, persevering
- **Hate** – despise, loathe, detest, abhor, abominate
- **Hurry** – run, speed, race, hasten, urge, accelerate, bustle
- **Interesting** – intriguing, provocative, titillating, tantalizing, absorbing
- **Introverted** – shy, bashful, quiet, withdrawn, introspective
- **Keep** – retain, withhold, preserve, sustain, sustainance
- **Kill** – slay, assassinate, murder, destroy, abolish
- **Kind** – thoughtful, considerate, amiable, gracious, tender
- **Lucky** – auspicious, fortunate, privileged, blessed, charmed
- **Make** – originate, invent, manufacture, develop, execute
- **Mark** – label, tag, trace, imprint, stamp
- **Mean** – unfriendly, unpleasant, badtempered, difficult
- **Mischievous** – prankish, playful, naughty, roguish, impish, sportive
- **Outgoing** – friendly, sociable, warm, extrovert
- **Pacify** – appease, placate, compose, abate, mitigate
- **Plan** – plot, design, device, contrivance, blueprint
- **Popular** – well-liked, approved, favourite, celebrated, current
- **Positive** – optimistic, cheerful, starryeyed, sanguine, admiring
- **Right** – accurate, factual, lawful, proper, suitable
- **Run** – race, sprint, escape, elope, flee
- **Sad** – wretched, depressed, dejected, gloomy, dismal
- **Slow** – unhurried, leisurely, late, tedious, slack
- **Stop** – cease, pause, discontinue, conclude, end
- **Strange** – odd, peculiar, unfamiliar, queer, irregular
- **Strong** – stable, secure, solid, tough, brawny
- **Take** – capture, acquire, select, prefer, occupy, consume
- **Tell** – expose, uncover, narrate, explain, declare
- **Turbulent** – disordered, violent, unstable, unsettled, stormy
- **Think** – judge, deem, assume, contemplate, mediate
- **Use** – utilise, exhaust, spend, consume, exercise
- **Valid** – authorised, legitimate, reasonable, logical, justifiable
- **Weak** – frail, infirm, puny, fragile, ailing
- **Wrong** – incorrect, inaccurate, mistaken, erroneous, improper

EXERCISE

Directions: In this section, you find a number of sentences, parts of which are underlined. For each underlined part, four words are listed below. Choose the word **nearest** in meaning to the underlined part and indicate your response in the Answer Sheet.

1. We should always try to maintain and promote communal amity.
(a) bondage (b) contention
(c) friendship (d) understanding
2. True religion does not require one to proselytise through guile or force.
(a) translate (b) hypnotise
(c) attack (d) convert
3. That the plan is both inhuman and preposterous needs no further proof.
(a) heartless (b) impractical
(c) absurd (d) abnormal
4. The attitude of the Western countries towards the third-world countries is rather callous to say the least.
(a) passive (b) unkind
(c) cursed (d) unfeeling
5. Whatever the verdict of history may be, Chaplin will occupy a unique place in its pages.
(a) judgement (b) voice
(c) outcome (d) prediction
6. He corroborated the statement of his brother.
(a) confirmed (b) disproved
(c) condemned (d) seconded
7. Whatever opinion he gives is sane.
(a) rational (b) obscure
(c) wild (d) arrogant
8. The story is too fantastic to be credible.
(a) believable (b) false
(c) readable (d) praiseworthy
9. Catching snakes can be hazardous for people untrained in the art.
(a) tricky (b) harmful
(c) difficult (d) dangerous
10. When the new teacher entered the classroom, he found the pupils restive.
(a) at rest (b) idle
(c) quiet (d) impatient
11. There is no dearth of talent in this country.
(a) scarcity (b) availability
(c) plenty (d) absence
12. The servants retired to their quarters.
(a) entered (b) went away
(c) ran away (d) mobilised
13. The navy gave tactical support to the marines.
(a) sensitiv (b) strategic
(c) immediate (d) expert
14. A genius tends to deviate from the routine way of thinking.
(a) dispute (b) disagree
(c) distinguish (d) differ
15. He was greatly debilitated by an attack of influenza.
(a) depressed (b) weakened
(c) worried (d) defeated
16. His efforts at helping the poor are laudable.
(a) welcome (b) sincere
(c) good (d) praiseworthy
17. His conduct brought him reproach from all quarters.
(a) rebuke (b) sympathy
(c) indifference (d) remorse
18. Hospitality is a virtue for which the natives of the East in general are highly admired.
(a) Duty of a doctor
(b) Generosity shown to guests
(c) Cleanliness in hospitals
(d) Kindness
19. House rent in cities like Mumbai or Delhi has risen to astronomical figures beyond the reach of even high-salaried people.
(a) exorbitant (b) commercial
(c) planetary (d) illogical
20. Appropriate technology holds the key to a nation's development.
(a) Modern (b) Suitable
(c) Effective (d) Growing
21. He was enamoured of his own golden voice.
(a) very fond of (b) concerned with
(c) obsessed with (d) imbued with
22. Some journalists are guilty of indulging in yellow journalism.
(a) misrepresentation (b) vulgarization
(c) sensational reporting (d) loud gestures
23. His truancy is detrimental.
(a) loyalty (b) integrity
(c) honesty (d) absenteeism
24. He is a maleficent person.
(a) generous (b) cunning
(c) criminal (d) friendly
25. His aversion is known to all.
(a) Hospitality (b) hostility
(c) humility (d) humbleness

Answer Key

1. (c)	2. (d)	3. (c)	4. (d)	5. (a)	6. (a)	7. (a)	8. (a)	9. (d)	10. (d)
11. (a)	12. (b)	13. (b)	14. (d)	15. (b)	16. (d)	17. (a)	18. (b)	19. (a)	20. (b)
21. (a)	22. (c)	23. (d)	24. (c)	25. (b)					

Answers with Explanations

1. **Option (c) is correct.**
Explanation: 'Bondage' means a state of being a slave, 'contention' means heated agreement and 'amity' means friendly relations. Thus, 'friendship' is the synonym of 'amity'.
2. **Option (d) is correct.**
Explanation: 'Hypnotise' means to capture the whole attention of someone 'translate' means to express the sense of (words or text) in another language, 'proselytise' means to convert or attempt to convert (someone) from one religion, belief, or opinion to another which is similar to 'convert'.
3. **Option (c) is correct.**
Explanation: Meaning of 'Preposterous' is 'contrary to reason or common sense; utterly absurd or ridiculous' which is similar to 'absurd'. 'Heartless' refers to a person who is emotionless, 'impractical' means not realistic or sensible and 'abnormal' refers to someone who is different from the normal.
4. **Option (d) is correct.**
Explanation: Passive, unkind, and cursed are not synonymous to 'callous'. 'Callous' means 'showing or having an insensitive and cruel disregard for others' which is similar to 'unfeeling'.
5. **Option (a) is correct.**
Explanation: In the context of the sentence, 'voice' and 'outcome' is not fit. 'Prediction' is also misfit in the context of history. 'Verdict' means 'a decision on an issue of fact in a civil or criminal case or an inquest' which is similar to 'judgement'.
6. **Option (a) is correct.**
Explanation: The sentence says, he supported his brother's statement. 'Corroborated' means 'confirm or give support to (a statement, theory, or finding)' which is similar to 'confirmed'.
7. **Option (a) is correct.**
Explanation: Someone's 'opinion' can be logical or illogical. 'Sane' means '(of a person) of sound mind; not mad or mentally ill' and 'rational' means 'based on or in accordance with reason or logic'. Both are similar to each other.
8. **Option (a) is correct.**
Explanation: The word 'too' has negative connotation, so the sentence means the story is not believable. 'Credible' means 'able to be believed; convincing' which is similar to 'believable'.
9. **Option (d) is correct.**
Explanation: If an untrained person catches snake, it may not necessarily be harmful, but for sure will be risky for him. 'Hazardous' means 'risky; dangerous' which is similar to 'dangerous'.
10. **Option (d) is correct.**
Explanation: 'Restive' means unable to remain silent. Options (a), (b) and (c) are the opposites of 'restive' as they all mean to be silent or at rest which leaves option (d) 'impatient' to be the correct answer.
11. **Option (a) is correct.**
Explanation: 'Dearth' means scarcity or shortage of something. 'Availability' or 'plenty' will mean 'to be in large number or quantity'. Also 'absence' is the antonym of present, thus, it is the incorrect word.
12. **Option (b) is correct.**
Explanation: 'Retired' in this sentence means to take rest or to take a break. So 'ran away' or 'mobilised' will not be its synonyms. Also 'entered' does not take the preposition 'to'. So, option (b) will be the correct answer.
13. **Option (b) is correct.**
Explanation: 'Tactical' means carefully planned, So, 'strategic' will be its synonym. 'Sensitive' means easily affected or disturbed. 'Immediate' is something done at once or quickly. Also 'expert' support is not correct usage of the word.
14. **Option (d) is correct.**
Explanation: 'Deviate' means to depart from the normal course. 'Differ' means to be of different opinion. So, option (d), 'differ' is the synonym. 'Dispute' means a fight or a disagreement. And 'distinguish' means to find differences. These words are not the synonyms of 'deviate'.
15. **Option (b) is correct.**
Explanation: 'Debilitated' means to be weak. Option (a) 'depressed' means to be in a sad state of mind. 'Worried' means to be anxious or troubled. And 'defeat' means not to win. So, only option (b) is the suitable answer.
16. **Option (d) is correct.**
Explanation: 'Laudable' means to be appreciated or praise-worthy. So, option (d) is suitable.
17. **Option (a) is correct.**
Explanation: 'Reproach' means to express disapproval, so 'rebuke' will be the synonym. 'Rebuke' also means to express disapproval. 'Sympathy' means to show concern. 'Indifference' means unconcerned and 'remorse' means sadness.
18. **Option (b) is correct.**
Explanation: 'Hospitality' means generosity shown to guests i.e. to welcome the guests and serve them wholeheartedly. So, we choose option (b). Whereas, option (a) tells about the duty or responsibility of a doctor. Also option (c) is eliminated as it tells about cleanliness in hospitals meaning the hospitals should be kept neat and tidy. 'Kindness' in option (d) is also eliminated because it means being generous or good-hearted.
19. **Option (a) is correct.**
Explanation: 'Astronomical' figures in this sentence means high price or exorbitant price. 'Commercial' means making profit or engaged in commerce trade. 'Planetary' means related to planets. And 'illogical' means without reasons. Therefore, these words can be eliminated.
20. **Option (b) is correct.**
Explanation: 'Appropriate' means suitable or correct. Option (a) 'modern' means relating to present times or contemporary. Option (c) 'effective' means having the desired result. And option (d) 'growing' means to naturally develop in size as number and changing

physically. So options (a), (c), (d) can be eliminated.

21. Option (a) is correct.

Explanation: 'Enamoured' means very fond of or to like something a lot. So, option (b) 'Concerned with' means to be worried or anxious, which does not fit in the sentence. 'Obsessed with' means to be unable to stop thinking about something, to be too interested or worried about something. Option (d), 'imbued with' means to infuse or permeate with, thus, the third option is also not suitable for the sentence.

22. Option (c) is correct.

Explanation: Yellow journalism means the kind of reporting or journalism that is meant to sensationalise matters. So option (c) 'misrepresentation' will mean to show something wrongly. 'Vulgarisation' is something, cheap and indecent. And 'loud gestures' means noisy actions. So, options (a), (b), (d) can be eliminated.

23. Option (d) is correct.

Explanation: 'Truancy' is the habit of staying out of school without permission or shirking duty. This

is a negative word. 'Absenteeism' which refers to the habit of regularly staying away from school or work, is the synonym of 'truancy'. 'Loyalty' refers to a strong feeling of alliance, 'integrity' refers to the quality of being honest and 'honesty' means to be fair and truthful all of these words are the antonyms of 'truancy'.

24. Option (c) is correct.

Explanation: 'Maleficent' means baleful, working or productive of harm or evil. It is a negative word and options (a) and (d) are positive, so, they can be ruled out. 'Cunning' means characterised by wiliness and trickery. 'Criminal' is related to crime. Out of 'cunning' and 'criminal', criminal is closer to the word 'maleficent'.

25. Option (b) is correct.

Explanation: 'Aversion' means extreme dislike or disinclination. 'Hospitality', 'humility' and 'humbleness' are positive words. 'Hostility' which refers to an unfriendly attitude, is the correct answer.

○○○

Chapter 02

ANTONYMS

- Antonyms are an important and useful section of English grammar. An antonym is a word that has the opposite meaning to a given word. In simple words - words that have the opposite meaning of another word are called antonyms.
- Most antonyms are quite simple, like 'good' and 'bad', or 'black' and 'white'. Some words can be transformed into their antonyms simply by adding the prefixes 'un', 'in', or 'non', as when 'likable' is changed into its antonym, 'unlikable'. The word antonym itself comes from the Greek word **anti** which means **opposite**, and when it is added to – **onym**, which comes from the Greek word **onoma**, which means **name**. So, antonym literally means **opposite-name**.

List of Antonyms

Word	Antonyms
Absurd	Rational, Sane
Abundance	Dearth
Accord	Disaccord, Disagreement
Acquit	Convict
Adopt	Reject
Affinity	Aversion
Amateur	Professional
Analysis	Synthesis
Appreciate	Depreciate
Arrogant	Humble
Ascent	Descent
Attract	Repel
Authentic	Spurious
Awkward	Graceful
Barbarous	Civilised
Barren	Fertile
Base	Noble
Benevolent	Malevolent
Brutal	Humane, Kind
Camouflage	Reveal
Cautious	Rash
Censure	Praise
Chaste	Impure, Unchaste
Concord	Discord
Condense	Lengthen, Expand
Confidence	Diffidence, Distrust
Delight	Displeasure, Sorrow

Dense	Sparse
Descend	Ascend
Despair	Hope
Destruction	Construction
Diffident	Confident
Diligent	Lazy
Disdain	Approve
Eclipse	Shine
Ecstasy	Despair
Elevation	Depression
Eligible	Ineligible
Emancipate	Enslave
Enmity	Friendship
Enthusiasm	Indifference
Equality	Inequality
Exceptional	Ordinary
Experience	Inexperience
Explicit	Implicit
Extensive	Intensive
External	Internal
Exterior	Interior
Extraordinary	Ordinary
Extravagant	Economical
Extrinsic	Intrinsic
Fabulous	Ordinary
Fact	Fiction
Fallacy	Truth
Fair	Foul
Familiar	Strange
Famous	Unknown, Notorious
Fanatical	Liberal
Fickle	Constant
Fictitious	True, Genuine
Fine	Coarse
Flexible	Stiff, Rigid
Folly	Wisdom
Forsake	Hold
Fragile	Enduring
Frantic	Subdued
Frugality	Lavishness

Futility	Utility
Gloomy	Gay
Glut	Scarcity
Glory	Shame
Grisly	Pleasing
Growth	Decline
Grudge	Affection, Benevolence
Guilty	Innocent
Hapless	Fortunate
Humane	Cruel
Humble	Proud
Humility	Arrogance
Hypocrisy	Sincerity
Impulsive	Cautious
Incompetent	Dexterous
Inclusive	Exclusive
Indigent	Rich, Affluent
Indolent	Active, Energetic
Industrious	Lazy, Slothful
Inferior	Superior, Royal
Infernal	Heavenly
Inspid	Pleasing
Jaded	Renewed
Jampacked	Empty
Jellying	Thawing
Jejune	Interesting
Jive	Honest, Sincere
Jocular	Depressed
Jovial	Solemn
Judicious	Irrational
Junk	Neatness
Juvenile	Dotage
Kedge	Hurt, Harm
Keen	Indifferent
Kind	Cruel, Apathetic
Kindle	Extinguish
Kinky	Straight
Knave	Innocent
Knotty	Manageable
Knowing	Ignorant
Knowledge	Ignorance
Kudos	Blame, Condemnation
Lack	Plenty
Lacuna	Sufficiency
Languid	Energetic, Vigorous

Lavish	Scarce
Lax	Reliable
Leash	Release
Lethal	Safe
Linger	Hasten
Loyal	Treacherous
Material	Spiritual
Melancholy	Gaiety
Mighty	Weak
Miserly	Generous
Mitigate	Augment
Mollify	Irritate
Momentous	Varied
Motion	Rest
Neat	Untidy
Notion	Reality
Notorious	Reputable
Noxious	Healing
Numerous	Deficient
Obstinate	Yielding
Obstruct	Encourage
Obscure	Prominent
Obstinate	Flexible
Obtain	Forfeit
Occult	Intelligible
Odious	Engaging
Offspring	Ancestor
Ominous	Auspicious
Passion	Coolness
Pathetic	Joyous
Peace	War
Persuade	Dissuade
Physical	Spiritual, Mental
Plenty	Scarcity
Premium	Discount
Pride	Humility
Profit	Loss
Prospect	Retrospect
Prosperity	Imprudent
Quell	Support, Aid
Quick	Slow, Tardy
Quiet	Noisy
Raise	Lower
Rash	Steady, Cautious
Real	False

Rear	Front
Receive	Give
Rectify	Falsify
Religious	Irreligious, Secular
Relish	Dislike
Remote	Near
Repulsive	Attractive
Sacred	Unholy, Profane
Safety	Danger
Sane	Insane
Satisfaction	Dissatisfaction
Savage	Civilised
Scanty	Plentiful
Secret	Open, Public
Sensible	Insensible, Senseless
Sensitive	Insensitive
Service	Disservice
Severe	Mild
Sharp	Blunt
Shy	Bold, Impudent
Silence	Noise
Sin	Virtue
Smart	Dull, Slow
Smile	Frown
Sober	Excited, Drunk
Sophisticated	Naive
Stale	Fresh
Stationary	Moving
Straight	Crooked
Sublime	Ridiculous
Success	Failure
Sufficient	Insufficient
Superior	Inferior
Surplus	Deficit
Swear	Forswear
Sweet	Bitter, Sour

Swift	Slow
Tame	Wild
Teach	Learn
Tedious	Lively
Temporary	Permanent
Temperate	Intemperate
Tender	Hard, Strong
Transient	Lasting, Permanent
Transparent	Opaque
Trust	Doubt
Total	Part
Unanimity	Dissent
Uniform	Variable
Uniformity	Variety
Utterly	Incomplete
Umbrage	Gratification
Unify	Diversify
Union	Disunion, Split
Vague	Definite
Valiant	Cowardly
Vain	Modest
Veteran	Novice
Vice	Virtue
Vigorous	Frail
Vigilant	Careless
Violent	Gentle
Vulgar	Refined
Wane	Wax
Want	Abundance
Warmth	Coolness
Wearisome	Refreshing
Wild	Civilised
Wilt	Revive
Woeful	Cheerful
Yearn	Satisfied

EXERCISE

Directions: In this section, each item consists of a sentence with an underlined word. It is followed by four words, one of which is closest to the **opposite** in meaning of the underlined word. Choose the word and indicate your response in the Answer Sheet.

- His short but pointed speech was applauded by all sections of the audience.
 (a) disapproved (b) misunderstood
 (c) praised (d) welcomed
- In ancient India, scholars had no interest in political power or material growth.
 (a) internal (b) spiritual
 (c) psychic (d) celestial
- A friendly dog met us at the farm gate.
 (a) helpful (b) understanding
 (c) quiet (d) hostile
- He is extremely intelligent but proud.
 (a) dull (b) weak
 (c) ignorant (d) arrogant

5. The young leader was reluctant to shoulder the responsibilities of the ministerial office.
(a) wanting (b) willing
(c) anxious (d) eager
6. He abandoned his family.
(a) supported (b) encouraged
(c) anxious (d) saved
7. History abounds in instances of courage.
(a) shines (b) lacks
(c) suffices (d) fails
8. Adversity teaches man to be humble and self-reliant.
(a) sincerity (b) animosity
(c) curiosity (d) prosperity
9. Like poverty, affluence can sometimes create its own problems.
(a) indigence (b) opulence
(c) sorrow (d) exuberance
10. The habit of squandering money should not be encouraged.
(a) discarding (b) hoarding
(c) donating (d) stealing
11. John is always shabbily dressed.
(a) decently (b) beautifully
(c) extravagantly (d) scantily
12. The new manager of the bank is urbane in his manners.
(a) civilised (b) slow
(c) rude (d) foolish
13. It is necessary to develop thrifty habits to be able to lead a comfortable life.
(a) expensive (b) extravagant
(c) economical (d) good
14. Many people suffer because of pride.
(a) lowliness (b) submission
(c) humility (d) obedience
15. He handled the machine with deft fingers.
(a) delicate (b) sturdy
(c) quick (d) clumsy
16. I was deeply affected by his urbane behaviour.
(a) rural (b) rude
(c) irrational (d) indifferent
17. His timidity proved costly.
(a) arrogance (b) boldness
(c) skillfulness (d) cunning
18. Arrangements were made to handle the mammoth gathering tactfully.
(a) significant (b) small
(c) unruly (d) noisy
19. He was engrossed in his work when I walked in.
(a) occupied (b) inattentive
(c) engaged (d) absent
20. These are the main points of the preceding paragraph.
(a) following (b) previous
(c) first (d) last
21. He made a shrewd guess.
(a) clever (b) wild
(c) incorrect (d) discriminating
22. He is suffering from a severe cough.
(a) violent (b) mild
(c) bad (d) continuous
23. Cumulatively, the effect of these drugs is quite bad.
(a) individually (b) obviously
(c) clearly (d) collectively
24. He was conspicuous because of his colourful shirt.
(a) charming (b) ugly
(c) small (d) unnoticeable
25. He hates these continual arguments with his friend.
(a) repeated (b) irrational
(c) occasional (d) regular

Answer Key

1. (a)	2. (b)	3. (d)	4. (a)	5. (b)	6. (a)	7. (b)	8. (d)	9. (b)	10. (b)
11. (a)	12. (c)	13. (b)	14. (c)	15. (d)	16. (b)	17. (b)	18. (b)	19. (b)	20. (a)
21. (b)	22. (b)	23. (a)	24. (d)	25. (c)					

Answers with Explanations

1. Option (a) is correct.

Explanation: Options (c) and (d) are the synonyms of the underlined word and thus, can be eliminated. 'Misunderstood' means incorrectly understood which is not related to 'applauded' in any sense. Thus, option (a) 'disapproved' which means to not approve of something or someone is the antonym of 'applauded'.

2. Option (b) is correct.

Explanation: 'Material' refers to 'physical objects rather than the mind or spirit' and 'spiritual' means 'relating to or affecting the human spirit or soul as opposed to material or physical things'. It is clear that they are opposite in meaning and rest of the options are not related at all.

3. Option (d) is correct.

Explanation: 'Friendly' means 'kind and pleasant' just as a friend and options (a), (b) and (c) are close

in meaning to it. 'Hostile' means 'showing or feeling opposition or dislike; unfriendly'. It is clear that 'friendly' and 'hostile' are antonyms.

4. Option (a) is correct.

Explanation: 'Intelligent' means 'having or showing sharpness of mind' and 'dull' means 'lacking interest or excitement or sharpness of mind'. It is clear that 'intelligent' and 'dull' are opposite in meaning.

5. Option (b) is correct.

Explanation: 'Wanting' is not grammatically correct, 'anxious' means 'worried' or 'nervous', 'eager' means 'keenly interested'. 'Reluctant' means 'unwilling and hesitant; disinclined' and the exact opposite of it can only be 'willing'.

6. Option (a) is correct.

Explanation: 'Abandon' means 'to desert or leave forever' and the opposite of it can only be 'supported'.

7. **Option (b) is correct.**

Explanation: 'Abounds' means 'to exist in large numbers or amounts.' 'Lacks' which refers that the state of not having enough of something is the opposite of 'abound'.

8. **Option (d) is correct.**

Explanation: 'Adversity' means 'a difficult or unpleasant situation' and 'prosperity' means 'the state of being successful and having a lot of money', which refers to a pleasant situation. It is clear that they are opposite in meaning. Rest of the options are not relevant.

9. **Option (b) is correct.**

Explanation: 'Indigence' is synonymous to 'poverty'. 'Sorrow' and 'exuberance' are not related to poverty or opulence. 'Affluence' means 'the state of having a lot of money or owning many things'. It is apparent that 'affluence' and 'indigence' are opposites.

10. **Option (b) is correct.**

Explanation: 'Squander' means 'to waste money or supplies', or 'to waste opportunities' and 'hoarding' refers to 'the act of collecting large amounts of something and keeping it for yourself, often in a secret place'. Both the words are opposite to each other.

11. **Option (a) is correct.**

Explanation: For the opposite of 'shabbily dressed', 'decently dressed' is the best option. Other options, do not fit in.

12. **Option (c) is correct.**

Explanation: 'Urbane' means refined or sophisticated. 'Rude' is the opposite of the given word.

13. **Option (b) is correct.**

Explanation: 'Thrifty' means economical, or to use one's money carefully. 'Extravagant' is the opposite of it, as it means wasteful or to spend lavishly.

14. **Option (c) is correct.**

Explanation: 'Pride' means to be arrogant or to think highly of oneself. 'Humility' which means to be humble is the correct opposite word.

15. **Option (d) is correct.**

Explanation: 'Deft' means skillful, so 'clumsy' i.e. awkward will be the opposite. 'Delicate' means soft and fragile, 'sturdy' means strong and 'quick' is fast.

16. **Option (b) is correct.**

Explanation: 'Urbane' means courteous and refined in manner. The word opposite to it is 'rude'. 'Irrational' which means not logical or reasonable and 'indifferent' which means unconcerned are not related to the given word.

17. **Option (b) is correct.**

Explanation: 'Timidity' means lacking courage. 'Arrogance' means proud and haughty, 'skillful' means talented and cunning. These words are not the opposites for 'timidity'. The correct antonym would be 'bold' which means 'to have courage'.

18. **Option (b) is correct.**

Explanation: 'Mammoth' means big, so 'small' will be the opposite word. 'Significant' means important, 'unruly' means without order and 'noisy' means full of loud sounds. Thus, options (a), (c) and (d) are not the opposites of mammoth.

19. **Option (b) is correct.**

Explanation: 'Engrossed' means to be completely absorbed or occupied in one's work. So 'occupied' and 'engaged' cannot be antonyms. 'Absent' will be the antonym of 'present'. 'Inattentive' meaning not paying attention, the most suitable option.

20. **Option (a) is correct.**

Explanation: 'Preceding' means coming or occurring immediately before in time or place, whereas 'following' means coming after or as a result of. Options (c) and (d) are not the antonyms of the given word.

21. **Option (b) is correct.**

Explanation: 'Shrewd' means clever and also cunning i.e. something that is well-thought. 'Wild' guess means taking a sudden or abrupt guess. 'Incorrect' only means something that is not correct and is not the opposite of 'shrewd'. Also 'discriminating' means differentiating. Thus, based on the meanings, 'wild' is the antonym of 'shrewd'.

22. **Option (b) is correct.**

Explanation: 'Severe' means acute or serious. 'Mild' means not so serious. 'Violent' refers to an action involving physical force and 'continuous' refers to something carried out without interruption. 'Bad' refers to something negative or not good. So, it can be concluded that based on the meaning, 'mild' is the antonym of 'severe'.

23. **Option (a) is correct.**

Explanation: 'Cumulatively' means collectively, so 'individually' which means personally or separately is the opposite of 'cumulatively'. 'Obviously' and 'clearly' mean something that is well understood and apparent. So only option (a) is the suitable answer.

24. **Option (d) is correct.**

Explanation: 'Conspicuous' mean something that is easily noticed or visible. So, option (d) will be the opposite word. 'Charming' which means pleasant, 'ugly' which refers to something which is not attractive and 'small' are not related to 'conspicuous' in the given context.

25. **Option (c) is correct.**

Explanation: 'Continual' means 'without interruptions', so option (c) i.e., 'occasional' will be the opposite word. 'Occasional' means not regular. Option (a) and option (d) are synonyms for 'continual' and thus, incorrect answers. We can eliminate option (b) because 'irrational' means without reason or logic.

Chapter 03

PRONOUN

Definition:

A *Pronoun* is a word used in place of a *Noun*. It is used to stop the repetition of a noun. It is used to refer to a noun, an individual /individuals, thing/ things (the pronoun's antecedent) whose identity is made clear earlier in the text.

1. Personal Pronoun

Read the following sentences : I am happy. We are happy. You are young. They are happy. He (she, it) is happy. I, we, you, he, (she, it), they are called Personal Pronouns because they stand for the three persons:

- the person spoken to.
- the person speaking. and
- the person spoken of.

Forms of Personal Pronoun

PRONOUN	NOMINATIVE CASE (used as subject)	ACCUSATIVE CASE (used as object of verb in a sentence)	POSSESSIVE CASE (used to show belongingness)
I (first person)	I	Me	My, mine
We (first person)	We	Us	Our, ours
You (second person)	You	You	Your, yours
He (third person)	He	Him	His
She (third person)	She	Her	Her, hers
It (third person)	It	It	Its
They (third person)	They	Them	Their, theirs

Reflexive V/S Emphatic Pronouns

Reflexive pronouns are used in the sense of 'self'. Emphatic pronouns, on the other hand, are used to just emphasize the action of the subject.

He ruined himself. (Reflexive: here the subject and object refer to the same person.)

He himself ruined the plan. (Emphatic: here the emphatic pronoun himself merely puts emphasis on the noun he.)

The core meaning doesn't change when the emphatic pronoun himself is removed from the sentence.

In the second pair of sentences, the meaning changes or becomes incomplete when the reflexive pronoun is removed.

4. Demonstrative Pronoun – These pronouns are used to point out the objects to which they refer,

- This** is the gift from my mother.
- These** are the same people who humiliated me.
- That** is my job, I have to do it.

2. Reflexive Pronoun – It is used when the action done by the subject turns back (reflects) upon the subject; as,

- I injure myself.
- We hurt **ourselves**.
- They ruined **themselves**.

3. Emphatic Pronoun – When Compound Personal Pronouns are used for the sake of emphasis, they are called Emphatic Pronouns.

- It was dictated so by the Principal **himself**.
- He lives here by **himself**.
- The town **itself** is not very large.
- They **themselves** asked me to do this.

Demonstrative Pronouns V/S Demonstrative Adjectives

Consider the examples:

This purse is mine.

This is my purse.

The first use of 'this' modifies the noun 'purse', it is followed by a noun, but not followed by the main verb.

Whereas in the second sentence, 'this' replaces the noun (the particular purse), it is not followed by a noun, but followed by the main verb (is).

Thus, we conclude that the first example is that of a demonstrative adjective and the second sentence is that of a demonstrative pronoun.

Demonstrative Adjectives	Demonstrative Pronouns
modify the noun.	replace the noun.
are followed by a noun.	are not followed by a noun.
are not followed by the main verb.	are often followed by the main verb.

5. **Indefinite Pronoun** – A pronoun that refers to a person or thing in a general way (not in a definite way) is called an Indefinite Pronoun.

- (a) **Some** have joined the party.
- (b) **All** went to the same hotel in Pune.
- (c) **Nobody** was interested in his lose talk.
- (d) Do good to **others**.
- (e) **Someone** has to take the responsibility.
- (f) One should carry one's own failures.

6. **Distributive Pronoun** – When pronouns refer to persons or things one at a time, they are called Distributive Pronouns.

- (a) **Each** of the students gets a prize.
- (b) **Each** got his turn.
- (c) **Either** of them will have his luck.
- (d) **Neither** of the girls of this institute is a topper.

7. **Relative Pronoun** – A Relative Pronoun refers to some noun going before, which is called its antecedent.

- 1. I met Reena who was returning from school.
- 2. I have discovered a new place that can be our new hangout.
- 3. The book that you gave is almost over.
The word who is used instead of the noun Reena. It, therefore, does the work of a Pronoun. The word "who" joins or connects two statements. It, therefore, does the work of a Conjunction. The word "who", therefore, does double work – the work of a Pronoun and also the work of a Conjunction.

The Relative Pronoun **who** has different forms for Accusative and Genitive. — Singular and Plural Nominative — who Genitive — whose Accusative — whom/who

- (a) They never succeed who claim to have never faced failure.
- (b) This is the boy who works hard.
- **Whose** (the Possessive form of who) is used in speaking of persons, animals and also things without life.
This is the question whose solution is still challenged by scientists of all ages.
- **Which** is used for things *without life and for animals*. It may refer to a Singular or Plural Noun.
(a) We had our college reunion dinner in the

seafood restaurant, which is now closed.

- (b) My mom asked me to clean up the box, which was filled with old clothes we no longer use.

Usage of 'That'

- ❑ After the words all, same, any, none, nothing, everything (the) only, (a, the few), a /the little; as,
(a) Not everything that glitters is gold.
(b) He is the same man that he has been.
(c) It is only donkeys that bray.
- ❑ After Adjectives in the Superlative Degree; as,
(a) He was the most eloquent speaker that I ever heard.
(b) The wisest man that ever lived made mistakes.
- ❑ After the Interrogative Pronouns who, what; as,
(a) What is it that bothers you so much?
(b) What is there that I do not know ?
- ❑ After two antecedents, one denoting a person and the other denoting an animal or a thing; as,
(a) The boy and his dog that had forcefully barged in the school premises were turned out.

8. **Interrogative Pronoun** – Interrogative pronouns are used for making queries or asking questions.

Look at the following sentences :

- (a) **What** is that?
- (b) **Who** is he?
- (c) **Which** is your pen?
- (d) I found this bag. **Whose** is this?
- (e) **Whom** do you want to talk?
- (f) **What** is your name?

Consider the following sentences: -

- Which is this place? (Nominative)
- Who sold you that book? (Nominative)
- What is that? (Nominative)
- Whose is this purse? (Possessive)
- Who did you see? To whom were you speaking? / Whom did you invite? / Who were you speaking to? (Accusative)
- What do you want? (Accusative)
- Which book do you prefer? (Accusative)

EXERCISE

Directions : In the following sentences, choose the appropriate option for the blank.

- 1. Bob and Jane decided to tie the knot because _____.
(a) it had come undone.
(b) they loved each other.
(c) they needed to save money.
(d) they wanted to make a long rope.
- 2. An Italian mayor, Mr. Kiev, has been cleaning the streets along with the councillors after their

town was left with no manual workers. In fact, _____ was sweeping the piazza in front of the local church in preparation for market day.

- (a) he (b) she
- (c) them (d) it
- 3. The plant _____ opened in a small Pennsylvania village is today the largest chocolate factory in the world.
(a) what (b) that
(c) when (d) who

4. In the beginning, the villagers were suspicious. It was hard to persuade _____ to realise that they were really getting something from the schools.
(a) him (b) us
(c) them (d) they
 5. When a waiter appears, you ask him the routine question, "_____ do you have?"
(a) when (b) why
(c) what (d) how
 6. Seoul's city government is asking people for help to correct poorly translated street signs-with prizes on offer for _____ who spot the most errors.
(a) this (b) those
(c) that (d) these
 7. Communication is a connection between people sharing information with _____ other.
(a) some (b) one
(c) every (d) each
 8. The workers, managers, consultants, and scholars _____ crack this nut will reshape industries and institutions.
(a) who (b) whom
(c) whose (d) which
(e) whomsoever
 9. The war _____ began in 1914 was a total war in which all the resources of the warring states were mobilised.
(a) which (b) whom
(c) what (d) who
 10. Dogs soon know the persons _____ they are kindly treated.
(a) by them (b) by who
(c) by whom (d) that
- A. Directions :** In the following questions, a sentence is given, a part of which may need improvement. Alternatives are given as (a), (b) and (c). In case, no improvement is needed, mark your answer as In case, no improvement is needed, mark your answer as (d).
11. They recommend this book even though **they himself** had never read it.
(a) they themselves had (b) they themself has
(c) it itself had (d) one oneself has
(e) No improvement required
 12. This is the house **that Jack built**.
(a) that Jack is building it
(b) which Jack built it
(c) whom Jack built
(d) No improvement required
 13. **Something is pretty here** that Vineeta can wear to the party.
(a) Something here is pretty
(b) Something is here pretty
(c) Here is something that
(d) No improvement required
 14. We had a grand party and we enjoyed very much.
(a) We had a grand party and enjoyed very much.
(b) We had a grand party to enjoy very much.
(c) We had a grand party and we enjoyed ourselves very much.
(d) No error.
 15. When I was working in a software company, it was mandatory to **register my legally** as an authorised software developer.
(a) register mine legally
(b) register me legal
(c) register myself legally
(d) No improvement required
 16. The second player left the field, just as the first **player had done**.
(a) One had done that
(b) One had flown away
(c) Had done
(d) No improvement required
 17. Mr. Mukherjee knows ten languages, **isn't it?**
(a) doesn't Mr. Mukherjee?
(b) hasn't he?
(c) doesn't he?
(d) No improvement required
- B. In the sentence identify the part that contains the grammatical error.**
18. Those who are late they will not be allowed inside the classroom.
(a) Those who are late
(b) they will not be allowed
(c) inside the classroom
(d) No error
 19. The Green River cuts through the Colorado plateau, gathering plant life and minerals that give them their exquisite colour.
(a) The Green River cuts through
(b) the Colorado plateau,
(c) gathering plant life and minerals that
(d) give them their exquisite colour
 20. The present what I bought for you yesterday was very expensive.
(a) Was very expensive
(b) The present
(c) For you yesterday
(d) What I bought
- C. Choose the grammatically correct sentence from the following options and correct the errors of the remaining three.**
21. (a) The members of the executive committee want the facilities of car and accommodation for themselves.
(b) None of the two girls could complete the job.
(c) Who should I talk to about labelling food in the refrigerator?
(d) The guests whom we were talking about have arrived are my colleagues.

22. (a) The farmer and his cow which lived here have died.
 (b) Many a soldier has sacrificed their life during the World War.
 (c) Shlok speaks more fluently than she.
 (d) The five sisters lived in Mumbai and they loved each other.
23. (a) Veena is fatter than either of her five sisters.
 (b) My father has given me everything that he had.
 (c) One must not be blind to his own defects.
 (d) Mr. Chaubey is the most laborious man whom I know.
24. (a) Nobody but her can come to the rescue of her friends.
- (b) Our is the only house which has all modern amenities.
 (c) This is the house in which she lives in.
 (d) All of the participants, mainly whom I have selected, have been eliminated.
25. (a) Whose is that house which looks exceptionally beautiful?
 (b) Either the teachers or the principal failed in **their** duty.
 (c) My friend forced my brother and I to stay away from the award ceremony.
 (d) He hates everybody and everything that reminds him of his mistakes of young age.

Answer Key

1. (b)	2. (a)	3. (b)	4. (c)	5. (c)	6. (b)	7. (d)	8. (a)	9. (a)	10. (c)
11. (a)	12. (d)	13. (c)	14. (c)	15. (c)	16. (b)	17. (c)	18. (b)	19. (d)	20. (d)
21. (a)	22. (c)	23. (b)	24. (a)	25. (d)					

Answers with Explanations

1. Option (b) is correct.

Explanation: The sentence concerns two people, thus 'each another' is the appropriate choice.

2. Option (a) is correct.

Explanation: The personal pronoun 'he' is the correct choice because the antecedent refers to the Italian mayor, Mr. Kiev.

3. Option (b) is correct.

Explanation: The relative pronoun 'that' is the correct choice because the antecedent refers to the 'plant'.

4. Option (c) is correct.

Explanation: The personal pronoun 'them' refers to the suspicious 'villagers'.

5. Option (c) is correct.

Explanation: The interrogative pronoun 'what' is the correct choice to ask the question.

6. Option (b) is correct.

Explanation: "Those" will be the most appropriate word for the blank. "Those" is used to refer to specific people or things that are being mentioned, known or understood in the statement. Here, "those" is used for 'People who spot the most errors'.

7. Option (d) is correct.

Explanation: The sentence implies that communication deals with people sharing information, 'between' themselves. The preposition 'between' is used to refer to the dealings within two people. Therefore, 'each other' is appropriate, which has to do with two people, unlike 'one another'.

8. Option (a) is correct.

Explanation: The subject of the sentence consists of the workers, managers, consultants, and scholars, hence 'who' is correct.

9. Option (a) is correct.

Explanation: 'Which' is used for things *without life and for animals*, the antecedent being in 'the war'.

10. Option (c) is correct.

Explanation: 'By whom' is correct because the antecedent is 'persons', thus, it will take the objective case (whom).

11. Option (a) is correct.

Explanation: Reflexive pronouns can also act as emphatic pronouns, but they are used differently. An emphatic pronoun emphasises its antecedent. Hence, the correct phrase is "they themselves had".

12. Option (d) is correct.

Explanation: In the Relative clause, the 'relative pronoun' is the subject/object of the relative clause, so we should not repeat the subject/object. And as 'that' is already used as the object of 'built', which is just object of 'built' and written ahead of 'Jack' to connect the sentences, then we should not use 'it' as the object of 'built'. Thus, we don't need the object of 'built' which is given as 'it'.

13. Option (c) is correct.

Explanation: The relative pronoun 'that' refers to 'something', therefore it should sit next to its antecedent to avoid confusion. Other sentences distort the meaning.

14. Option (c) is correct.

Explanation: In the question 'we' denotes a group, an emphatic pronoun 'ourselves' is used to emphasize the enjoyment by the group.

15. Option (c) is correct.

Explanation: Since the sentence has first person pronoun 'I', so it will take the reflexive pronoun 'myself'.

16. Option (b) is correct.

Explanation: The underlined portion of the sentence needs to be rectified. It may be replaced with one had done. So, the sentence will be as The second player left the field, just as the first one had done.

17. Option (c) is correct.

Explanation: Since, name of person is present in the given sentence, 'isn't it?' may be replaced with 'doesn't he?'

18. Option (b) is correct.

Explanation: The pronoun 'those' already refer to the people, so another pronoun 'they' is unnecessary.

19. Option (d) is correct.

Explanation: The subject of the sentence is singular (the Green River), therefore the subsequent pronoun to refer to the subject will also be in singular. The correct sentence should be : The Green River cuts through the Colorado plateau, gathering plant life and minerals that give it its exquisite colour.

20. Option (d) is correct.

Explanation: The relative pronoun 'that' must be indicating to the present, instead of 'what'.

21. Option (a) is correct.

Explanation:

- (a) Here members of the executive committee are not used as a collective noun. So it will take a plural pronoun.
- (b) 'Neither' will replace 'none' because the former is used with two people .
- (c) 'whom' should replace 'who' as the subject is I, and object is the person being talked to = him/her (object). Thus the objective case (whom).
- (d) 'who' should replace 'whom' because the players are the subject / antecedent.

22. Option (c) is correct.

Explanation:

- (a) 'Which' should be replaced by 'that' because it is used after two antecedents , when one denotes a person and the other an animal / thing.
- (b) 'Many a' is always followed by a singular noun / pronoun /verb.
- (c) 'Than' always takes pronoun in the nominative case (she).
- (d) 'one another' should replace 'each other' because the former is used for reciprocal relationship for more than two persons or things, while the latter (each other) is used for relationships between two persons.

23. Option (b) is correct.

Explanation:

- (a) 'Either' is used with two persons, while 'any' is used for more than two persons. The comparison being among five sisters, 'any' is the correct alternative.
- (b) 'That' is used after words all, same, any, none, nothing, everything.
- (c) The indefinite pronoun 'one' must be followed by 'one/one's'. Here, 'one's' is correct.
- (d) 'That' is used after superlative degree, hence, 'whom' will be replaced by 'that'.

24. Option (a) is correct.

Explanation:

- (a) 'but/except' always take the pronoun in the objective case (her/him).
- (b) 'Ours' needs to be used instead of 'our'.
- (c) If a preposition is used before 'whom /which', the preposition is not used after it. So, the correct one will be : This is the house in which she lives.
- (d) 'Those' is used before the relative pronoun or after words like 'notably / mainly / especially/ chiefly'. So the correct one is : All of the participants, mainly those whom I have selected, have been eliminated.

25. Option (d) is correct.

Explanation:

- (a) 'That' is used after interrogative pronouns 'who/ whom/ whose'.
- (b) With 'either...or/ neither...nor' the pronoun adheres to the second subject. Hence the correct one is :
Either the teachers or the principal failed in **his** duty.
- (c) A pronoun is used in objective case when used as an object in a sentence: My friend forced my brother and me to stay away from the award ceremony.
- (d) Correct.

Chapter 04

VERB

Definition:

A Verb is a word that tells or asserts something about a person or thing.

A Verb may tell us-

- (1) What a person or thing does; as,
Hari laughs. The clock strikes.
- (2) What is done to a person or thing; as,
Hari is scolded. The window is broken.
- (3) What a person or thing is; as,
The cat is dead. Glass is brittle.

Read these sentences:-1. The boy "kicks" the football. 2. The boy laughs loudly.

In sentence 1, the action denoted by the verb "kicks" passes over from the doer or Subject boy to some Object football. The verb "kicks" is, therefore, called a Transitive Verb. (Transitive means passing over.) In sentence 2, the action denoted by the verb "laughs" stops with the doer

or Subject boy and does not pass over to an Object, The verb "laughs" is, therefore, called an Intransitive Verb. (Intransitive means not passing over.)

Most verbs can be used both as Transitive and as Intransitive verbs. It is, therefore, better to say that a verb is used Transitivity or Intransitivity rather than that it is Transitive or Intransitive.

Used Transitivity 1. The ants fought the wasps. 2. The shot sank the ship. 3. Ring the bell, Rama.

Used Intransitivity 1. Some ants fight very fiercely. 2. The ship sank rapidly.

Sometimes, though the verb is used **reflexively**, the Object is not expressed. In the following examples the reflexive pronoun understood is put in brackets:-

- The bubble burst [itself].
- The guests made [themselves] merry,
- Please keep [yourselves] quiet.

CONFUSING VERBS

HANG	HUNG	HANGED
to be attached from above without support below. To cling tightly to something	past tense of hang	Kill someone by the neck.
Will you hang this picture on the wall? Don't hang your head in shame.	I hung the picture on the wall. He hung his head in shame.	The traitor was hanged for treason.

FIND	FOUND	FOUNDED
to spot or get something (VERB) a discovery of something very important and of great value.(noun)	the past tense (and past participle) of find.	to start or establish something – like a building, school, company, settlement, etc.
Uranium was a very unique find by the scientists.	I finally found my passion to be in writing.	We founded this organisation in 1996 with the aim of educating the poor.

LIE	LAY	LAIN
to put something down carefully in a flat position	Past tense of lie	Past participle of lie
I will lie in the Sun.	Shall I lay the tray on the bed?	He claimed to have lain down and pretended to be dead until the action was over.

LAY	LAID	LAID
put (something) down gently or carefully. To produce eggs.	to put something in especially a flat or horizontal position, usually carefully or for a particular purpose	to put something in especially a flat or horizontal position, usually carefully or for a particular purpose
Lay the fabric carefully on the table.	He laid the tray down on the table.	We're having a new carpet laid in the hall next week.

RISE	ROSE	RISEN
to move from a lower position to a higher one	Past tense of rise Become heartened or elated	Past participle of rose
<i>Prices are still on the rise.</i>	<i>Her spirits rose when she heard the good news</i>	<i>The cost of living has risen.</i>

MODALS**Definition:**

Modal verbs show possibility, intent, ability, or necessity. Because they're a type of auxiliary verb (helper verb), they're used together with the main **verb** of the sentence.

Can, may, might, could, should, would, will, must.

Use of Modals

- (A) **MAY** – informal request: May I borrow your pen?

Possibility: The sky is cloudy, it may rain today.

Permission: May I go now?

- (B) **MIGHT** – remote possibility: The sky is not cloudy but, a change in weather is always possible; it might rain today.

Possibility in the past: He might have been wounded. [that was a possibility in the past. (but he was not wounded)]

- (C) **CAN** – Physical capacity/capability: Can you lift this suitcase?

Express permission: You can go now.

Suggest possibility in negative / interrogative sentences – Can this be true? It can be so.

- (D) **COULD** – past ability: When I was young, I could climb any tree.

Unreal conditions: If I had wings, I could fly to the Himalayas.

Polite form of seeking permission / request: Could you pass me the plate?

- (E) **SHALL** – used with the first person – I shall go to market tomorrow.

Used with the second person for demand, threat, promise – You shall never come near my child.

- (F) **WILL** – used with all persons

Formal request - Will you bring me a glass of water?

Used with instructions/command – All the cadets will attend the parade.

SHOULD – Duty, Responsibility, Advice, Probability, Expectation, Presumption, Moral obligation, Mild command

(a) One should obey one's elders. *Duty*

(b) You should obey your parents. *Moral obligation*

(c) The rich should help the poor. *Expectation*

(d) You should work hard. *Advice*

(e) You should find her in the play-ground. *Probability*

- (G) **WOULD** – polite request: Would you accompany me to the railway station?

Used for likelihood, presumption, and probability:

(a) He would be in the field. *Likelihood*

(b) She would be in the club at this time. *Probability*

(c) She would be about 80 now. *Presumption*

Express strong desire/willingness: I wish they would agree to this proposal.

Used in unreal conditional sentences: If I were the king, I would make you any chief adviser.

Used with 'rather': I would rather study than see a movie.

In conditional sentences with past perfect tense: If he had come to me, I would have given him a job.

- (H) **MUST** – command, compulsion, order: Soldiers must obey orders without any question.

moral duty/obligation – We must respect our elders.

necessity, urgency, circumstantial compulsion – I must leave now otherwise I may miss the train.

strong probability/likelihood – She must be eighty now.

MUST NOT: prohibition, Emphatic advice –

(a) Students must not resort to copying. *Prohibition*

(b) You must not miss the classes. *Emphatic advice*

- (I) **OUGHT TO** – Advice, moral obligation, Duty

(a) You ought to consult a physician. *Advice*

(b) We ought to respect our parents. *Moral duty*

(c) We ought to help our friends. *Moral obligation*

EXERCISE

1. I _____ very much like to see my old classmates.

(a) can (b) should
(c) would (d) might

2. The peon respectfully asked if he _____ go there.

(a) may (b) might
(c) can (d) will

3. My father has been advised to _____ his smoking.

(a) cut down (b) cut off
(c) cut into (d) cut in

4. I have been _____ this problem.

(a) working into (b) working on
(c) working for (d) with after

5. If I had helped him, he _____.

(a) will not be drowned
(b) would not be drowned
(c) will not have drowned
(d) would not have drowned

6. **Unless** one is genuinely interested in a subject, one _____ never opt for it.

(a) ought (b) could
(c) would (d) should

7. You _____ finish the work by tomorrow.
(a) should (b) must
(c) will (d) ought to
 8. Children take tea at night.
(a) should (b) ought to
(c) must (d) would
 9. He _____ rather die than tell a lie.
(a) would (b) ought to
(c) must (d) might
 10. You _____ love your country.
(a) Ought to (b) must
(c) should (d) will
- A. Directions:** In the following questions, a sentence is given, a part of which may need improvement. Alternatives are given as (A), (B) and (C). In case, no improvement is needed, mark your answer as (D)/(E).
11. All people want to be happy, **do they**?
(a) don't they?
(b) are they?
(c) didn't they?
(d) No improvement required
 12. Iago **lied** to Othello about his wife.
(a) lay
(b) laid
(c) lain
(d) No improvement required
 13. Sita was cooking in the kitchen when her husband **was ringing** the bell.
(a) would have been ringing
(b) had rung
(c) rang
(d) No improvement
 14. **If the room had been brighter**, I would have been able to read for a while before bed time.
(a) If the room was brighter
(b) If the room are brighter
(c) Had the room been brighter
(d) No improvement required
 15. Never **such incidents have** taken place on our campus.
(a) have such incidents
(b) such incidents will have
(c) were such incidents
(d) No improvement
 16. **I took my mother some grapes** when she was in hospital.
(a) I took some grapes for my mother
(b) No improvement required
(c) I took for my mother some grapes
(d) I brought my mother some grapes
 17. You won't tell the secret, **won't you**?
(a) would you?
(b) will you?
(c) isn't it?
(d) No improvement required
 18. They didn't see you, is it?
(a) No improvement required
(b) did they?
(c) wasn't it?
(d) have they?
 19. It is the sort of work that **calls on** a high level of concentration.
(a) calls for (b) calls upon
(c) calls off (d) calls up
(E) No Improvement required
 20. If the evidence **bear out** the charge, he may be sentenced to death.
(a) bears out (b) bore out
(c) was born out (d) has born out
(E) No correction required
 21. **Where you left** your bag yesterday?
(a) Where did you leave
(b) Where were you leaving
(c) Where did you left
(d) No substitution
 22. I wish **if I was** the Principal of my school.
(a) No improvement required
(b) that I am
(c) if I had been
(d) I were
 23. We came back because **we had ran** out of money.
(a) we had run
(b) we have ran
(c) we run
(d) No improvement required
- B. In the sentence identify the part that contains the grammatical error.**
24. The old man did not wanted to eat any food.
(a) any food (b) The old man
(c) did not wanted (d) to eat
 25. Mohit did not went to school yesterday as he was unwell.
(a) to school
(b) was unwell
(c) yesterday as he
(d) Mohit did not went
 26. All the candidates will give a test on Friday.
(a) All the candidates
(b) will give
(c) a test on Friday
(d) No improvement required
 27. The audience watched the magicians performed their art.
(a) The audience watched
(b) the magicians
(c) performed their art
(d) No improvement required

28. The life-guard would not let the children to swim at the deep end of the pool.
 (a) The life-guard would not
 (b) let the children
 (c) to swim at the
 (d) deep end of the pool.
29. He denied to take any assistance from me.
 (a) He denied
 (b) to take
 (c) any assistance from me
 (d) No improvement required
30. His students have and are still doing commendable research in the field of medicine.
 (a) His students have
 (b) and are still doing
 (c) commendable research
 (d) in the field of medicine.
31. The sage asserted that time will be not return back once lost, so it is better to make full use of it.
 (a) The sage asserted that time
 (b) will be not return back once lost,
 (c) so it is better to make full use of it.
 (d) No improvement
32. Even after earning millions of money, his parents do not have a proper home to stay.
 (a) Even after earning
 (b) millions of money,
 (c) his parents do not have
 (d) a proper home to stay.
33. The accused was asked for his last wish before he was hung to death.
 (a) The accused was asked for
 (b) his last wish
 (c) before he was hung to death.
 (d) No improvement required

Answer Key

1. (b)	2. (a)	3. (b)	4. (c)	5. (c)	6. (b)	7. (d)	8. (a)	9. (a)	10. (c)
11. (a)	12. (d)	13. (c)	14. (c)	15. (c)	16. (b)	17. (c)	18. (b)	19. (d)	20. (d)
21. (a)	22. (c)	23. (b)	24. (a)	25. (d)	26. (b)	27. (c)	28. (b)	29. (d)	30. (d)
31. (a)	32. (c)	33. (b)							

Answers with Explanations

1. **Option (c) is correct.**
Explanation: 'Would' is used for saying what you want to happen (wish).
2. **Option (b) is correct.**
Explanation: Here, Reporting Verb (asked) is in Past Tense. Hence, might (V2) is the right option.
3. **Option (a) is correct.**
Explanation: 'cut down' means to 'reduce', 'cut off' implies to disconnect. 'Cut into' implies 'to reduce an amount of something that is available'. 'Cut in' connotes 'to interrupt'.
4. **Option (b) is correct.**
Explanation: 'working into' means 'to add or include (something) in (something)'. 'Working on' means 'spending time producing or improving something'; 'working for' connotes 'to consider yourself a dedicated employee.'
5. **Option (b) is correct.**
Explanation: The sentence is unreal/imaginary, as per the modal usage, 'would' is also used in *conditional sentences with past perfect tense* (had helped him). Therefore, 'would not have' is correct. 'Would not be drowned' is wrong usage.
6. **Option (b) is correct.**
Explanation: The word "should" expresses the meaning of obligation.
 It generally states that if the person is not interested in the subject they must or should not opt for it. Other modal auxiliary options don't fit to the context.
7. **Option (b) is correct.**
Explanation: 'Must' is used to express certainty.
8. **Option (c) is correct.**
Explanation: 'Must' is used in case of command, compulsion, order. While 'should' is mild command.
9. **Option (a) is correct.**
Explanation: 'would' is used to express choice or preference.
10. **Option (a) is correct.**
Explanation: 'Ought to' is used to express an obligation or an expectation that someone should do something.
11. **Option (a) is correct.**
Explanation: The sentence is in present simple (affirmative). Hence, question tag should be 'don't they' (negative)?.
12. **Option (d) is correct.**
Explanation: Lied means to not tell the truth.
13. **Option (c) is correct.**
Explanation: Past tense of 'ring', i.e., 'rang' is correct, as it is a short time process.
14. **Option (b) is correct.**
Explanation: The sentence speaks of an unreal condition, thus past perfect tense is preferred with 'would'.
15. **Option (a) is correct.**
Explanation: The sentence starts with never. Hence, inversion, i.e., have such incidents should be used.

16. Option (a) is correct.

Explanation: I took some grapes for my mother (subject 'I' + verb + immediate object grapes). Therefore, required answer will be option A.

17. Option (b) is correct.

Explanation: The sentence tag takes the same tense and modal as the first part of the sentence. If the verb in the first part of the sentence is positive, the question tag will be negative, and vice versa.

18. Option (b) is correct.

Explanation: The sentence tag takes the same tense and modal as the first part of the sentence. If the verb in the first part of the sentence is positive, the question tag will be negative, and vice versa. Hence, (B) is correct.

19. Option (a) is correct.

Explanation: 'Call on/upon' implies 'make use of someone or something as a source of help.' 'Call for' implies 'say publicly that something must happen'; 'call off' means to cancel'.

20. Option (a) is correct.

Explanation: The given sentence is in the present indefinite tense. Also, 'Evidence' is a singular uncountable noun. Hence, we need to use a singular verb with it. The meaning of "bear out" is: to show to be true, support, or confirm.

21. Option (a) is correct.

Explanation: The interrogative sentence format is: Question word (where) + auxiliary verb (did) + subject + verb in the base form (leave).

22. Option (d) is correct.

Explanation: "Were" is used in: If + subject + were + complement. The sentence is in past subjunctive mood. After 'I wish', we should use 'were' or 'the past tense of verbs'.

23. Option (b) is correct.

Explanation: With 'had', past participle of the verb 'run' is to be used – 'run'. To 'run out' means to have no more left of it.

24. Option (c) is correct.

Explanation: 'Did' always takes the first form of the verb in the simple past tense. However, the first form of the verb 'wanted' is want. Hence, 'did not want' will be the answer.

25. Option (d) is correct.

Explanation: According to grammatical rules, we use 'did + base form of the verb': did not go.

26. Option (b) is correct.

Explanation: It is correct to say: 'take' a test, not 'give a test'.

27. Option (c) is correct.

Explanation: 'Watch' as a verb either takes the gerund form of verb or base form of verb. Thus, '... watched the magicians perform/ performing their art.'

28. Option (c) is correct.

Explanation: 'Let' always takes the base form of verb.

29. Option (b) is correct.

Explanation: "Deny" is always followed by a gerund, thus, 'denied taking' is correct.

30. Option (a) is correct.

Explanation: Since the sentence has two different forms of verbs, the correct one will be:

His students have done and are still doing commendable research in the field of medicine.

31. Option (b) is correct.

Explanation: 'Return back' is a superfluous usage. 'Return' itself is enough.

32. Option (d) is correct.

Explanation: After the structure: noun (home) + to + verb (stay), a preposition needs to be added to the verb that follows the noun. The correct sentence is: Even after earning millions of money, his parents do not have a proper home to stay in.

33. Option (c) is correct.

Explanation: 'Hanged' (to put on gallows) should replace 'hung' (past tense of hang, meaning to place on a peg).

Chapter 05

PHRASAL VERB

Definition:

Phrasal verbs are an idiomatic phrase consisting of a verb and either an adverb, or a preposition, or a combination of both.

How are phrasal verbs different from prepositional phrases?

Be careful. While phrasal verbs can contain prepositions, these same prepositions can also be used in prepositional phrases. How do we tell the difference?

Examples: *I looked up the directions.*

Amy looked up the road.

In the first example, the preposition up is part of the phrasal verb looked up, a unit meaning “to locate information in reference materials.” The meaning of the sentence changes if the two words are not working together. In the second sentence, up the road is a prepositional phrase (the preposition up followed by the road, which is the object of the preposition). In this sentence, looked up is not functioning as a unit.

EXERCISE

A. Directions: In the following questions, three sentences are free from errors. Find out which sentence has an error. If all the sentences are error free click the “No error” option.

1. (a) The media likes to hype up and then tear down celebrities.
(b) If the business continues to lose money, I’m afraid we’ll have to close down.
(c) He was upset about his son talking back to him.
(d) He considered her lack of table manners a major put off.
2. (a) He was asked not to make fun by Vidya’s weight.
(b) He held back the tears as he waved goodbye.
(c) He signed away for the free meditation class.
(d) He vouched by the integrity of every worker in his factory.
3. (a) Sam told Bob to see the guest into her room.
(b) The two students ganged upon me and beat me up.
(c) The girls were invited for a sleep over at Tina’s house.
(d) Bob will have to see his mom for getting permission to go to the concert.
4. (a) Vishnu’s grandmother cannot bring up past events.
(b) They came over to New Orleans last summer.
(c) Please return the book when you get through it.
(d) The police let him get away with only a \$10 ticket.
5. (a) I’m leaving – I can’t put on with this situation any longer.
(b) Gandhi brought on a major change in the method of protest against the British.
(c) I heard that the president is giving away the prizes.
(d) I will look into who drew the graffiti on the wall.
6. (a) He talked me out of going to university because he thought it wasn’t the right thing for me to do.
(b) The match was called off because of heavy rain. It will be played tomorrow.
(c) The masked robbers held on a money transport and got away with millions of dollars.
(d) The police officer turned the thief over to the sheriff, who then locked him up in prison.
7. (a) The car broke down and we had to call someone to tow it away.
(b) The investigators promised that they would look through the matter and try to find new evidence.
(c) She’s been taken to hospital after being run down by a truck driver.
(d) We throw away too much paper and other packaging.
8. (a) We saw him off at the airport and wished him good luck for his new job.
(b) The scientist carried out the experiment in his own private lab.
(c) I have to back up my files regularly, so that I don’t lose them when the computer crashes.
(d) None of them.
9. (a) It took her a long time to get over her father’s sudden death.
(b) After weeks of negotiations the Prime Minister came with a new solution to the issue.
(c) Let’s roll up our sleeves and start working in order to get the project done in time.
(d) She hadn’t gone to school for over a month and fell behind in the main subjects.
10. (a) It started to rain just as we pulled through in front of the restaurant.
(b) She’s just splashed out a lot of money on a smart new car.
(c) The children were all at the table waiting for me to dish up.
(d) The burglars made off with some valuable jewellery after they had broken into the house.

Answer Key									
1. (d)	2. (b)	3. (b)	4. (a)	5. (a)	6. (c)	7. (b)	8. (d)	9. (b)	10. (a)

1. (d)	2. (b)	3. (b)	4. (a)	5. (a)	6. (c)	7. (b)	8. (d)	9. (b)	10. (a)

Answers with Explanations

1. Option (d) is correct.

Explanation: To say that he was repulsed by her table manners we can rewrite the sentence as "He considered her lack of table manners a major turn off" or "He was put off by her lack of table manners".

2. Option (b) is correct.

Explanation: The phrasal verb "hold back" has been correctly used in sentence b. Sentence A should be rewritten "He was asked not to make fun of Vidya's weight". "Signed away" means to sell or give away the right to some property. The correct term to use would have been "signed up." In (d) statement 'vouched by' is incorrect. It should be 'vouched for.'

3. Option (b) is correct.

Explanation: The correct phrasal verb in sentence B is "ganged up on me". Hence, the incorrect option is (b).

4. Option (a) is correct.

Explanation: The correct statement is: Vishnu's grandmother cannot call up past events, it means to "remember".

5. Option (a) is correct.

Explanation: The correct answer is: I'm leaving – I can't put up with this situation any longer. 'Put up' means to endure. Hence, (a) is the incorrect option.

6. Option (c) is correct.

Explanation: The correct answer is: The masked robbers held up a money transport and got away with millions of dollars. 'Help up' implies 'to delay or block the movement or progress of someone'. Hence, (c) has the error.

7. Option (b) is correct.

Explanation: The correct answer is: The investigators promised that they would look into the matter and try to find new evidence. 'Look into' means 'to examine something'; while 'look through' means 'to read or briefly examine some of the pages of (a book)'. Hence, (b) has the error.

8. Option (d) is correct.

Explanation: All the statements have the correct usage of phrasal verbs. Hence, (d) is correct.

9. Option (b) is correct.

Explanation: The correct answer is: After weeks of negotiations the Prime Minister came up with a new solution to the issue. 'Come up' means 'to be mentioned or discussed.' Hence, (b) has the error.

10. Option (a) is correct.

Explanation: The correct answer is: It started to rain just as we pulled up in front of the restaurant. 'Pull up' implies 'unexpectedly stop what you are doing'. 'Pull through' means 'to succeed in dealing with a difficult period'. Hence, (a) has the error.

○○○

What is Reading Comprehension (RC)?

In reading comprehension, a passage is given, based on which, certain questions are asked. The questions can be multiple choice or subjective questions. In reading comprehension, there is a set of directions given at the starting of the questions and those directions mention a statement that says, 'Read the given passage very carefully.'

A reading comprehension comprises a wide variety of topics and fields of study.

In competitive examinations, passages have four possible building blocks:

- (i) The motive/theme
- (ii) Background
- (iii) Support to the motive
- (iv) Implications/Inferences/Conclusions generated

Types of Questions in Reading Comprehension

Type 1: Main Idea Questions

Type 2: Tone of the Author Questions

Type 3: Specific Fact Questions

Type 4: Implied Questions

Type 5: Structural Questions

Type 6: Extrapolation Questions

Type 7: Negative or Exception Questions

Type 8: Contextual/Definition of a Term or Word Questions

A few of them are discussed below:

Central theme/Main idea

In such questions option based on the central theme or the main idea of the passage is chosen as the correct answer.

Examples:

- Which of the following most accurately states the central theme/main idea of the passage?
- Select the most appropriate/suitable title for the passage from the following:
- Which of the following best states/expresses the central idea/theme of the passage?
- The most appropriate/apt title for the passage is:
- Summarize the passage in one line.

Factual Questions/Specific Fact Questions

These questions are based on the facts given in the passage. Sometimes, you may need to re-read some parts of passage or do some minor calculations to identify the right option.

Examples:

- According to the author, what is _____?
- Which, out of the following, is true/false?
- By a _____, the author means _____
- The passage states that:

Inference/Implied Questions

Inferences may refer to a restatement or conclusion or consequences that author tries to state indirectly; they are implicit in the passage.

Examples:

The sentence, '_____', implies that:

It can/cannot be inferred from the passage:

The given passage uses '_____' to imply that '_____'

What can be inferred when the author states _____?

Contextual/Definition of a Term or Word Questions

Through these types of questions the examiner tests a candidate's vocabulary. They ask the meaning/synonym/antonym of a word or phrase. To tackle this type of questions, it is necessary to build up a strong vocabulary base and pay special attention to the contextual meaning of the given words.

Examples:

In the sentence, _____, what is the meaning of '_____ '?

Identify the meaning of the word, '.....', in the given context.

In the given passage, the phrase '_____' refers to

Important Strategies to Solve Comprehension Passages

- The reading of the passage must be quick.
- While reading, important lines or parts of the passage that are relevant to answer the questions must be underlined. This will capture the main idea of the passage or the tone or mood of the author.
- A complicated line/paragraph can be paraphrased in your own words and language. This will help in understanding the main idea of the paragraph.
- It is important to underline the keywords. This will help to discover the logical connections in the passage and make understanding better.
- Try to understand the unfamiliar words by reading the lines thoroughly and applying them to the context.
- Determine the main idea, tone or mood, inferential reasoning, and other details from the paragraph.
- Refrain from applying your own beliefs as understanding of the passage needs to be rational.
- Refer back at the passage when in doubt.
- Read the questions and all the given alternative options provided and choose the most appropriate option.

Tricks to Solve Comprehension Passages

- Jump to the question
- Improve vocabulary
- Note the 'keywords'.
- Read the passage carefully.
- Practice making summary of passages.
- Read English newspapers daily, especially the editorial section for almost half an hour. This will not only enhance your general knowledge and vocabulary, but will surely prepare you for reading comprehension as this part demands a quick reading quality.

- Practice five passages daily. Read the questions first and then, read the passage.

- Do not leave any sentence in order to answer the questions.

EXERCISE

- I. The rule of the road means that in order that the liberties of all may be preserved, the liberties of everybody must be curtailed. When the policeman, say, at a road-crossing steps into the middle of the road and puts out his hand, he is the symbol not of tyranny but of liberty. You have submitted to a curtailment of private liberty in order that you may enjoy a social order which makes your liberty a reality. We have both liberties to preserve our---individual liberty and our social liberty. That is, we must have a judicious mixture of both. I shall not permit any authority to say that my child must go to this school or that, shall specialise in science or arts. These things are personal. But if I say child shall have no education at all, then society will firmly tell me that my child must have education whether I like it or not.
1. According to the author, the "rule of the road" implies:
 - (a) the rule regulating the traffic on the road.
 - (b) the principle on which a road is constructed to ensure safe traffic.
 - (c) unrestricted freedom for all to lead a happy life.
 - (d) restricted individual freedom to ensure freedom for all.
2. The author thinks that when a policeman signals you to stop on a road-crossing, he is:
 - (a) behaving in a whimsical manner.
 - (b) interfering with your freedom to use the road.
 - (c) protecting the liberty of all to use the road.
 - (d) mischievously creating hurdles in your way from some personal motive.
3. The author is of the view that we should:
 - (a) have absolute individual liberty without any restrictions imposed by the society.
 - (b) have everything controlled by the society without any kind of individual liberty.
 - (c) try to strike a sensible balance between our individual liberty and our social liberty.
 - (d) have more of social liberty than individual liberty.
4. The author holds that:
 - (a) educating or not educating his child is a matter of personal liberty.
 - (b) educating or not educating his child is also a matter of social liberty.
 - (c) choosing the school for his child is a matter of social liberty.
 - (d) choosing the subject of study for his child is a matter of social liberty.
5. The most suitable title of the passage would be
 - (a) The Policeman at a Road Crossing
 - (b) The Laws of the Road
 - (c) Importance of Liberty
 - (d) Education of Children
- II. My most interesting visitor comes at night, when the lights are still burning---- a tiny bat who prefers to fly in through the open door, and will use the win-

dow only if there is no alternative. His object in entering the house is to snap up the moths that cluster around the lamps. All the bats I have seen fly fairly high, keeping near the ceiling; but this particular bat flies in low, like a dive-bomber, zooming in and out of chair legs and under tables. Once, he passed straight between my legs. Has his radar gone wrong, I wondered, or is he just plain crazy?

6. Consider the following statements:

- (1) The tiny bat flew in low like a dive-bomber.
- (2) The tiny bat like all bats keeps near the ceiling.
- (3) It has lost direction because its radar has gone wrong.
- (4) It wants to entertain the author with its skill in flying.

Which of the above statements may be assumed to be true from the information given in the passage?

- (a) 1 only
- (b) 1 and 3
- (c) 2 and 4
- (d) 3 and 4

7. The bat entered the room:

- (a) because there was no alternative.
- (b) to eat the moths round the lamps.
- (c) as it had gone mad.
- (d) as it preferred to fly in through the open door.

8. After comparing the habits of the tiny bat with those of other bats, the author was:

- (a) sure that this bat had lost its direction.
- (b) not sure of its preferences.
- (c) surprised to find that it was an expert flier.
- (d) unable to give the correct explanation for its behaviour.

9. The author calls the tiny bat an "interesting visitor". This means:

- (a) the bat visits him at night.
- (b) the bat is interested in the moths.
- (c) the bat has peculiar qualities.
- (d) the bat surprises him by zooming in and out like a dive-bomber.

10. What, according to you, can be the most suitable title for the passage?

- (a) Someone visits me
- (b) Night of Mysteries
- (c) My Nocturnal Visitor
- (d) A funny Visitor

- III. I was beginning to tire a little now. I had been cutting steps continuously for two hours, and Tenzing, too, was moving very slowly. As I chipped steps around still another corner, I wondered rather dully just how long we could keep it up. Our original zest had now quite gone and it was turning more into a grim struggle. I then realised that the ridge ahead, instead of still monotonously rising, now dropped sharply away, and far below I could see the North Col and Rongbuk glacier. I looked upwards to see a narrow ridge running up to a snowy summit. A few more whacks of the ice-axe in the firm snow and we stood on top.

11. Which of the following is the most appropriate description of the area in which the narrator had been

- moving?
- A steep rocky slope
 - A peak with a flight of steps
 - A high peak covered with snow
 - A long ridge
12. 'I had been cutting steps continuously for two hours' suggests that
- the climber was unskilled in the art of making steps.
 - he had to cut the steps out of snow before climbing up which was necessarily a slow process.
 - the slowness was caused by the fact that as soon as he cut the steps they were again covered with snow.
 - he had to work suspended from a ledge.
13. 'It was turning more into a grim struggle' would mean
- that after the long exertion he was feeling so weak that any further climbing meant a hard struggle with the forces of nature.
 - that Tenzing was making the task more difficult.
 - that strong cold wind was pushing him down from above and that he had to struggle with the forces of nature.
 - that he was in a mood to give up ultimately.
- IV. The dog is a very intelligent animal and it can be trained for many duties. Some dogs work with the police and the army as sentries and trackers, and there are medals that are awarded to dogs for bravery in action. Other dogs act as seeing-eyes for the blind, and some breeds are specially trained for rescue work among mountains and snow. Jobs like herding sheep and cattle and guarding homes are also done by them. Whenever man lives and works in the Arctic Circle, in the desert, in the jungle, in the moors or mountains, the dog is his faithful companion and help. A common mongrel may possess these qualities just as much as the aristocratic bloodhound.
14. The passage tells us that dogs _____.
- can be trained to do different kinds of work
 - are capable of difficult work
 - are of different types
 - alone are a great help to man
15. Medals are awarded to some dogs for
- their brave deeds
 - working with the police
 - serving as sentries
 - being able to perform difficult work
16. The passage implies that
- only dogs are faithful but not man
 - man would have felt helpless without dogs
 - the dog has certain qualities that makes it man's trustworthy companion
 - dogs can do almost anything
17. The phrase 'aristocratic bloodhound' in the passage means
- the dog kept by wealthy people
 - a pedigree dog
 - a dog fond of blood
 - any big dog
- V. The first time that Agha Shahid Ali spoke to me about his approaching death was on 25 April, 2001. The conversation began routinely. I had telephoned to remind him that we had been invited to a friend's house for lunch. Although he had been under treatment for cancer for some fourteen months, Shahid was still on his feet and perfectly lucid, except for occasional lapses of memory. I heard him thumbing through his engagement book and then suddenly he said: 'Oh dear. I can't see a thing.' There was a brief pause and then he added: 'I hope this doesn't mean that I'm dying...' I did not know how to respond: his voice was completely at odds with the content of what he had just said, light to the point of jocularity. I mumbled something innocuous: 'No Shahid — of course not. You'll be fine.' He cut me short. In a tone of voice that was at once quizzical and direct, he said: 'When it happens I hope you'll write something about me.' I was shocked into silence and a long moment passed before I could bring myself to say the things that people say on such occasions. 'Shahid, you'll be fine; you have to be strong...' From the window of my study, I could see a corner of the building in which he lived, some eight blocks away. It was just a few months since he had moved there: he had been living a few miles away, in Manhattan, when he had a sudden blackout in February 2000. After tests revealed that he had a malignant brain tumour, he decided to move to Brooklyn, to be close to his youngest sister, Sameetah, who teaches at the Pratt Institute—a few blocks away from the street where I live. Shahid ignored my reassurances. He began to laugh and it was then that I realised that he was dead serious. I understood that he was entrusting me with a quite specific charge: he wanted me to remember him not through the spoken recitatives of memory and friendship, but through the written word. He knew that my instincts would have led me to search for reasons to avoid writing about his death: I would have told myself that I was not a poet; that our friendship was of recent date; that there were many others who knew him much better and would be writing from greater understanding and knowledge. All this Shahid had guessed and he had decided to shut off those routes while there was still time. 'You must write about me.' Finally, I said: 'Shahid, I will: I'll do the best I can.'
18. Shahid's voice was full of 'jocularity'. This means it was:
- bitter
 - sorrowful
 - humorous
 - melodious
19. Why had the narrator called Shahid Ali?
- To give him an important task
 - To enquire about his health
 - To cheer him up in his depression
 - To remind him about a lunch invitation
20. What made Shahid think his end was near?
- He was unable to stand on his feet.
 - He suddenly couldn't see anything.
 - He couldn't utter a word.
 - He was unable to think clearly.

21. Which of these was NOT an excuse that the narrator thought of to decline Shahid's request?
 - (a) That he was not a poet
 - (b) That he was too busy
 - (c) That their friendship was quite recent
 - (d) That others knew Shahid better
22. What did Shahid request the narrator to do?
 - (a) Take him along for lunch at their friend's place
 - (b) Move to his house in Manhattan
 - (c) Write about him after his death
 - (d) Be strong and bear the loss bravely
23. The narrator was reluctant to write about Shahid because:
 - (a) he wouldn't be paid for it
 - (b) the subject did not interest him
 - (c) it would be emotionally tough for him
 - (d) he didn't have enough time
24. What disease was Shahid Ali suffering from?
 - (a) Brain tumour
 - (b) Loss of memory
 - (c) Blood cancer
 - (d) Blindness
25. Shahid had moved to Brooklyn to:
 - (a) get treatment for cancer
 - (b) fulfil certain engagements
 - (c) teach at Pratt Institute
 - (d) be close to his youngest sister
26. What kind of person was Shahid Ali?
 - (a) Depressed
 - (b) Full of life
 - (c) Dangerous
 - (d) Forgetful
27. Shahid was perfectly 'lucid'. This suggests he was:
 - (a) in a delirium
 - (b) confused
 - (c) not intelligible
 - (d) speaking coherently

Answer Key

1. (d)	2. (c)	3. (c)	4. (b)	5. (b)	6. (a)	7. (b)	8. (d)	9. (d)	10. (c)
11. (c)	12. (b)	13. (a)	14. (a)	15. (a)	16. (c)	17. (a)	18. (c)	19. (d)	20. (b)
21. (b)	22. (c)	23. (c)	24. (c)	25. (d)	26. (b)	27. (d)			

Answers with Explanations

1. **Option (d) is correct.**
Explanation: According to the author, the rule of the road means that the liberties of everyone may be maintained and the liberties of everybody must be reduced. Individual liberty should be curtailed because it may affect public liberty. When a reasonable person's actions affect other persons' liberty, he would have submitted to a curtailment of private liberty in order that he might enjoy the social order. The author says liberty is not a personal affair. It is a social commitment. It should not touch others' liberty.
2. **Option (c) is correct.**
Explanation: According to the author, the role of a police officer, who controls the traffic, imposes constraints on personal freedom. But it ensures public freedom.
3. **Option (c) is correct.**
Explanation: According to the author, the role of a police officer, who controls the traffic, imposes constraints on personal freedom. But it ensures public freedom.
4. **Option (b) is correct.**
Explanation: The author is of the opinion that the choice of a school in which your child will study is a matter of personal freedom, but whether your child will get educated or not is a matter of social liberty. This can be derived from the last line of the passage. Hence, option b is the correct answer.
5. **Option (b) is correct.**
Explanation: The passage starts with the rule of the road and the author has used rules of the roads to explain the main idea.
6. **Option (a) is correct.**
Explanation: Statement 1 is mentioned in the passage in the last third line hence, is true as per the
- passage. Statement 2 is contradicted in the passage. Statement 4 is not mentioned in the passage at all. Statement 3 seems to be true but is false as it is mere speculation of the author and not an assertion.
7. **Option (b) is correct.**
Explanation: The author mentions that the bat's objective in entering the house was to snap up the moths clustered around the lamp.
8. **Option (d) is correct.**
Explanation: From the last line of the paragraph it is clear that the author was not sure whether the bat's radar had gone wrong or he was simply crazy. 'sumptuous'
9. **Option (d) is correct.**
Explanation: The author notices that the tiny bat that visits him at night is different from other regular bats because the regular bats fly close to the ceiling. This bat is interesting because it flies close to the floor; it comes in like a dive bomber and swoops under tables and chairs.
10. **Option (c) is correct.**
Explanation: 'Nocturnal' means something that turns active in the night. The passage talks about the tiny bat that visits the author in the night and how that bat is different from other bats.
11. **Option (c) is correct.**
Explanation: The question is to describe the area in which the narrator had been moving, so (c) is the most suitable option.
12. **Option (b) is correct.**
Explanation: The answer is clear in the first and the second lines, 'I was cutting steps for two hours and Tenzing, too, was moving very slowly'.
13. **Option (a) is correct.**
Explanation: 'Grim' means without hope. The answer is clear in the second, third, and fourth lines of the

paragraph 'I wondered rather dully just how long we could keep it up. Our original zest now had quite gone and it was turning more into a grim struggle'.

14. Option (a) is correct.

Explanation: The writer has given various examples of how dogs act as companions to humans. This explanation makes option (a) the correct answer.

15. Option (a) is correct.

Explanation: Medals are usually awarded for achieving something. In this case, these are rewarded when dogs perform a brave deed, which makes option (a) to be the correct answer.

16. Option (c) is correct.

Explanation: The entire paragraph revolves around the various roles played by dogs; thus, we can say that option (c) is the correct answer.

17. Option (a) is correct.

Explanation: 'Aristocratic' means related to the upper class or the elite class. So, here it can refer to the dogs kept by the wealthy people.

18. Option (c) is correct.

Explanation: The word 'jocularity' means 'to be humorous'.

19. Option (d) is correct.

Explanation: As per the passage, the narrator had called Shahid Ali to tell him about a lunch invitation.

20. Option (b) is correct.

Explanation: When Shahid suddenly couldn't see anything, he thought that his end was near.

21. Option (b) is correct.

Explanation: As per the passage, the 2nd option is the only correct answer here, as the narrator did not think of being too busy to be given as an excuse to decline Shahid's request.

22. Option (c) is correct.

Explanation: As per the passage Shahid requested the narrator to write about him after his death.

23. Option (c) is correct.

Explanation: As per the passage, the narrator was reluctant to write about Shahid because it would be emotionally tough for him.

24. Option (c) is correct.

Explanation: As per the passage, Shahid was suffering from blood cancer and had even gone for its treatment for about fourteen months.

25. Option (d) is correct.

Explanation: As per the passage, Shahid had moved to Brooklyn so that he can be close to his youngest sister.

26. Option (b) is correct.

Explanation: As per the passage, Shahid was full of life and highly spirited.

27. Option (d) is correct.

Explanation: The word 'lucid' means 'clear and logical'. So, the 4th option is the correct answer here which states that Shahid was perfectly 'lucid'.

○○○

Definition: Prepositions are words that tell where or when a thing is in relation to another thing in terms of time, place, direction and location.

There are various types of prepositions. Some of them have been discussed below.

- (1) **Prepositions of Place:** These prepositions show the relationship between the noun and a place in a sentence.

Examples: I live in a **good house**.

- I went **to Jaipur**.

- (2) **Prepositions of Time:** These prepositions relate the relationship between time and noun in a sentence.

Examples: I can do it **in a day**.

- He returned after **half an hour**.

- I have been suffering **for a week**.

- (3) **Prepositions of Agency:** It shows the relation between a noun (doer) and an action.

Examples: These poems were written **by** William Blake.

- The man opened the lock **with** a hair pin.

- (4) **Prepositions of Instruments, Devices or Machines:** These prepositions are used to join nouns, i.e., the name of instruments, devices and machines with the rest of the sentence.

Examples: He killed the lion **with his gun**.

- The snake was killed **with a stick** by the farmer.
- This story was written by him **with a peacock feather** in ink.

- (5) **Prepositions of Purpose, Cause, or Reason:** These prepositions are used to express the cause, reason or purpose of something.

Examples: Many people were against the newly selected Deputy **as** he was corrupt.

- Every member worked hard **so that** the project could be completed on time.

- (6) **Prepositions of Possession:** These prepositions are used to show possession or the fact that something belongs to someone.

Examples: This is the handkerchief **of a woman**.

- He is a **man of** means.

- (7) **Prepositions of Manner:** These prepositions are used to describe the way in which something is

done.

Examples: He always works **with care**.

- The old man is dying **by inches**.

- (8) **Prepositions of Source or Origin:** These prepositions are used to describe the source or origin of something.

Examples: He comes **from** a rich family.

- I think this poem is **from** William Shakespeare's 'Sonnets'.
- This is a quotation **from** Munshi Premchand's 'Godan'.

- (9) **Prepositions of Contrast or Concession:** These prepositions are used to make comparison between two or more objects/persons (contrast) or between different aspects of a single object/person (concession).

Examples: **Although** he exercises regularly, he does not seem to be losing weight.

- **In spite of** being so clever, he was easily duped by the thieves.

- (10) **Prepositions of Measure, Standard, Rate, Value:** These prepositions are used in relation to the quantity of something.

Examples: Cloth in this shop is sold **by metre**.

- It is twelve **by my clock**.
- He was appointed **on one lakh rupees a month**.
- He charges interest **at fifteen percent**.

- (11) **Prepositions of Movement:** These prepositions are used to show movement and are generally, used with verbs of movement or motion.

Examples: The rabbit jumped **into** the lake.

- The bird flew **away** from the tree.

Prepositions With Adjectives

Prepositions are often used with adjectives and come after them in a sentence and form prepositional phrase. Although there is no general rule for this, there are certain patterns.

- (1) They are afraid of going out during lockdown.
- (2) I am fond of eating mangoes.
- (3) This policy is introduced with a view to improving the status of woman in the society.
- (4) I am not interested in learning French.
- (5) She objected to our shouting in the class.
- (6) I am looking forward to meeting her son.

EXERCISE

Directions: In the following sentences, choose the appropriate preposition for the blank.

1. Mr. Ghosh is very happy _____ his son's excellent result.
(a) for seeing (b) to see
(c) by seeing (d) see
2. Sunlight filtering _____ the stained glass window created a mosaic of colours on the floor.

- (a) in (b) through
(c) at (d) into
3. Are you feeling doubtful _____ your decision ?
(a) about (b) upon
(c) at (d) for
4. In the 1980s, many women _____ the country spoke out against dowry deaths.
(a) through (b) above
(c) over (d) across

5. Sometimes a person's intelligence is not fully appreciated ____ he/she gets an opportunity.
(a) upto (b) later
(c) until (d) unless
6. I continued to smile _____ his threats.
(a) over (b) on
(c) at (d) upon
7. The work was completed _____ sunset.
(a) for (b) in
(c) at (d) about
8. The average density of population is less than three persons per square kilometre _____ Canada.
(a) at (b) in
(c) on (d) into
9. Suganya writes _____ her left hand.
(a) on (b) with
(c) in (d) by
10. It is dangerous to enter ____ the enemy's camp.
(a) through (b) on
(c) in (d) into
11. A good judge never gropes ____ the conclusion.
(a) for (b) to
(c) at (d) on
12. India is committed ____ a policy of peaceful existence.
(a) of (b) to
(c) with (d) for
13. He was eager _____ the pursuit of knowledge as he was _____ distinction.
(a) for, with (b) in, with
(c) into, for (d) at, on
14. I shall entrust my money _____ him.
(a) on (b) with
(c) in (d) to
15. He ran _____ the room and sat _____ his father.
(a) to, besides (b) in, beside
(c) into, beside (d) through, besides
16. The patient is now free _____ danger. He is free _____ his money. The goods, were passed free _____ duty (from, of, with)
17. He parted _____ his friends in high spirits. He parted _____ his property and went on pilgrimage to Dwarka. (from, with)
18. He has no good cause _____ complaint. Darkness was the cause _____ his losing his way. (for, of)
19. He acted _____ fear. He acted _____ my suggestion. He acted _____ compulsion, (from, under, upon)
20. We are all slaves _____ convention. No man should be a slave _____ his passions, (of, to)

Directions: Select the most appropriate option to substitute the underlined segment in the given sentence. If there is no need to substitute it, select No improvement.

21. Dispose off the garbage
(a) (b)
from the shelves. No error.
(c) (d)
22. From thirty years he devoted himself to
public affairs
(a) (b)
without taking a No error.
holiday. (c) (d)
23. Opening his reading them carefully and
letters sending for his clerk,
(a) (b)
he dictated
answers with No error.
them. (c) (d)
24. I prefer seafood
(a) (b)
than Mediterranean food. No error.
(c) (d)
25. This building comprises of six
houses,
(a) (b)
three parking lots and No error.
one basement. (c) (d)

Answer Key

1. (b)	2. (b)	3. (a)	4. (d)	5. (c)	6. (c)	7. (c)	8. (b)	9. (b)	10. (d)
11. (a)	12. (b)	13. (b)	14. (b)	15. (c)	16.	17.	18.	19.	20.
21. (a)	22. (a)	23. (c)	24. (c)	25. (b)					

Answers with Explanations

1. Option (b) is correct.

Explanation: The sentence is in simple present tense. So, the infinitive form i.e. 'to see' best fits the structure.

2. Option (b) is correct.

Explanation: The word 'filter' means to pass through. So, option (b) will be the best fit.

3. Option (a) is correct.

Explanation: 'Doubtful' takes the preposition 'of' or 'about'. Since, 'of' is not one of the given options, so, option (a) 'about' is the correct answer.

4. Option (d) is correct.

Explanation: "Across the country" means "many or most parts of the country". Here, in the sentence, (across) should be used to talk about the expanse of the women's actions.

5. Option (c) is correct.

Explanation: 'Until' is both a preposition and a conjunction.

'Until' means 'up to the event mentioned'.

In the following sentence it is given that Intelligence is not appreciated 'upto the event mentioned' which is when opportunity is not given to a person.

6. Option (c) is correct.

Explanation: 'Smile at' something/someone is the correct usage. Hence, (c) is correct.

7. Option (c) is correct.

Explanation: 'At' sunset is correct. "At" is used to express a point. Hence, (c) is correct.

8. Option (b) is correct.

Explanation: 'In' is used for a state/country. Hence, (b) is correct.

9. Option (b) is correct.

Explanation: "With" is used for denoting an agent (left hand). Hence, (b) is correct.

10. Option (d) is correct.

Explanation: "Into" denotes movement, as expressed in the word 'enter'. Hence, (d) is correct.

11. Option (a) is correct.

Explanation: The word 'gropes' is followed by 'for'. Hence, (a) is correct.

12. Option (b) is correct.

Explanation: 'Committed' is followed by 'to'. Hence, (b) is correct.

13. Option (b) is correct.

Explanation: 'Eager in' is the correct expression, meaning 'strongly wanting to do or have something.' Hence, (b) is correct.

14. Option (b) is correct.

Explanation: 'Entrust with' implies to give someone a responsibility you assume the person will fulfill. Hence, (b) is correct.

15. Option (c) is correct.

Explanation: The act of running demands the use of 'into', and sitting next to someone implies 'beside'. Hence, (c) is correct.

16. Free of – clear of

Free from – lacking; without

Free with – exploit or take liberties with someone or something

The correct format is: from, with, of.

17. Parted from – move away from each other.

Parted with – a person/thing

The correct format is: with, from.

18. Cause of – so as to support, promote, or defend something.

Cause for – a person or thing that gives rise to an action, phenomenon, or condition.

The correct format is: for, of.

19. Acted from – be motivated by.

Act under – as a result of being forced.

Act upon – to use (something, such as a feeling or suggestion) as a reason or basis for doing something.

The correct format is: from, on, under.

20. Slave to – to be influenced too much by something:

Slave of – a person held in forced servitude.

The correct format is: of, to.

21. Option (a) is correct

Explanation: The usage of 'off' is incorrect in this sentence. Because 'of' is preposition and 'off' is an adverb and if we use dispose off there will not be any sense, off is just opposite of 'on' as we use for switch on and off the TV. 'of' completes the phrasal verb 'dispose of'.

22. Option (a) is correct.

Explanation: The preposition 'for' usually indicates a purpose or duration of time. 'From' is used to show the place, direction or time from that somebody/ something starts or started. Here, the sentence talks about time duration. So, replace 'from' with 'for'. The sentence should be 'For thirty years he devoted himself to public affairs without taking a holiday'.

23. Option (c) is correct.

Explanation: The word dictate is usually followed by 'to'. Its correct usage is 'to dictate someone' or 'dictate for someone'. Replace 'with' with 'to'. The correct sentence will be, 'Opening his letters, reading them carefully and sending for his clerk, he dictates answers to them'.

24. Option (c) is correct

Explanation : When used to state a preference for one thing over another, the verb prefer is usually followed by the preposition to: prefer one to the other. The given sentence should be, 'I prefer seafood to Mediterranean food.'

25. Option (b) is correct

Explanation : Usage of 'of' after comprises is incorrect, because 'comprise', in itself, includes the meaning of the preposition "of", so there is no point in adding "of".

1. Errors in a sentence make an expression meaningless or ambiguous in its context. Several problems related to sentence improvement have been discussed in this chapter.

Some Important Rules Pertaining to

Subject-Verb Agreement:

- (i) Nouns used after lots of/a lot of, pile of, some of, any of, all of, percent of, fraction of, a majority of etc. can either be singular (in case of uncountable nouns) or plural (in case of countable nouns) and the verb to be used after the noun depends on the type of noun.

Example: Some of my acquaintances are bureaucrats. (A plural verb has been used because the subject 'my acquaintances' is plural.)

- (ii) Any amount of money, length of distance, and duration of time are singular and demand the use of singular verb.

Examples: Five thousand rupees is not enough to buy a Chanel dress. ('Five thousand rupees' is a singular subject.)

- (iii) If the sentence has a positive and a negative subject, where one is plural and the other singular, then the verb follows the positive subject.

Example: It's not the teachers, but the chairman, who ultimately takes all the decisions. ('Chairman' is the positive subject.)

- (iv) The number of verb(singular/plural) used after adjective phrase is determined by the noun/pronoun used before the phrase.

Example: The girl wearing mask and carrying books has got top rank in PCS exam. ('Girl' is the actual subject and is singular so, the singular verb 'has' has been used.)

2. Pay heed to the concept of Modifiers.

A word/phrase/clause which modifies other words in a sentence is called a modifier. A modifier can be either an adjective or an adverb. An adjective modifies a noun, and an adverb modifies a verb, an adjective, or other adverbs.

ARTICLES

Rules of Articles:

- (i) No article is used before the names of languages, but when definite article (The) is used, it expresses nationality.

Example: The native speakers of English use different accent from non-native speakers. The English defeated the French.

- (ii) When an adjective is preceded by a definite article (The), it behaves as a plural noun and denotes a community.

Example: The young, the old, the poor, the rich, all loved Gandhi ji.

- (iii) Only definite article (the) is used before the

name of body parts in case of injury.

Example: Phillip Hugs died in hospital, two days after being hit on the neck by a ball during a match in Sydney.

- (iv) Definite article (the) is used before the names of public places such as school, college, university, church, court, prison/jail, market etc. when the purpose of visiting these places is secondary rather than primary.

Examples: I go to college at 10 am. (for the purpose of study) [Primary purpose]
Some terrorists were hidden in the mosque. [Secondary purpose]

ADJECTIVES

Rules of Adjectives:

- (i) Adjectives used between certain correlative conjunctions must be in positive degree. These correlative conjunctions are: So + Adj./Adv. (positive degree) + that

So + Adj./Adv. (positive degree) + as

As + Adj./Adv. (positive degree) + as

Too + Adj./Adv. (positive degree) + to

- (ii) Adjectives preceded by adverbs such as relatively, comparatively, less, quite, must be in positive degree.

- (iii) Whole is used before singular common nouns and whole of is used before proper nouns.

PRONOUNS

Rules of Pronouns

- (i) Following indefinite pronouns can be used both in their singular or plural forms, depending on their usage. These are Some, Any, None, All, More/Most. The useful mnemonic, SANAM can be used to keep these pronouns in mind.

Example: A bus full of passengers met with an accident leaving many passengers injured but some didn't even get a scratch.

- (ii) Pronouns used as apposition of a subject of a sentence should be in nominative case.

Example: The two of us, he and I did not agree to her proposal. (Correct)

- (iii) Pronouns preceded by a preposition or a verb should be in objective case.

Example: There lacks amity between you and me. (Correct)

- (iv) Pronouns used in a sentence should be in accordance with the number, person and gender of its antecedent.

Example: These companies have been asked to furnish their financial details and information about their board members.

NON-FINITE VERBS

Non-finite verb is the verb which is not controlled by its subject in terms of number and person. Non-finite verbs do not have any tense and there is no agreement

between the subject and the verb. It can be categorised as infinitive (to + V1), gerund (V1 + ing), participle progressive (V1 + ing) and perfect (having + V3).

Examples: They refused to leave. (Infinitive)

She avoids eating too much. (Gerund)

She is busy playing chess. (Participle)

Rules of Non-finite Verbs

Simple Infinitives

- (i) Verbs used after need/dare should use simple infinitive (to + V1) form of verbs.

Example: They need to arrive on time to enter the venue.

- (ii) Verbs used after 'nothing/anything/everything but' or 'except' preceded by verbs want/like etc. rather than 'do/does/did' should use simple infinitive form of verbs.

Examples: He wants nothing but to earn money.

Bare infinitives (Infinitive without 'to') a. Verbs used after need not/dare not should be used in bare infinitive (V1) form.

Example: She was in such a temper that I dared not ask her any questions.

- (iii) Verbs used after 'nothing/anything/everything but' or 'except' preceded by 'do/does/did' should be bare infinitives.

Example: George does nothing but play even during exam.

- (iv) Verbs used after the phrases had better, had sooner, had rather, would rather, would sooner, sooner than, rather should be in the bare infinitive (infinitive without 'to') form.

Example: It's getting dark, we had better take a taxi.

- (v) Verbs used after possessive verbs (have/has/had) followed by a noun or pronoun should be used in their bare infinitive forms.

Example: I will have Ram do the work.

PARALLELISM

Parallelism means using the same pattern of words to show that two or more ideas have the same level of importance. It adds refinement to what is being written and helps avoid repetition.

Rules of Parallelism

- (i) Parallel structure should be used when two or more clauses or phrases are connected with a coordinating conjunction (for, and, nor, but, or, etc.).

Example: Our PM accepts proposals from other countries that are worthy and of benefit for both the nations. (Incorrect)

Our PM accepts proposals from other countries that are worthy and beneficial for both the nations. (Correct)

- (ii) Two clauses or phrases connected with a correlative conjunction (not only, but also, either...or, neither...nor, if... then, etc.) use parallel structure.

Example: Mr. Rohit visits the meditation centre not only for attaining physical health but also mental peace. (Incorrect)

Mr. Rohit visits the meditation centre for attain-

ing not only physical health but also mental peace. (Correct)

- (iii) Parallel structure of sentences should be observed while comparing elements.

Example: I like watching movies as much as to play cricket. (Incorrect)

I like watching movies as much as playing cricket. (Correct)

- (iv) More than one element given in a list observes parallel structure.

Example: The qualities of a good PM are to be honest, integrity, and quick decision making. (Incorrect)

The qualities of a good PM are honesty, integrity and quick decision making. (Correct)

CONDITIONALS

Conditionals are statements which refer to the result of a certain condition. It comprises two clauses – the 'if clause' (tells about the condition) and 'the main clause' (tells about the result).

The order of these clauses does not alter the meaning.

Example: You will pass exams with flying colours, if you study hard. (Incorrect)

If you study hard, you will pass exams with flying colours. (Correct)

Rules of conditionals:

- (i) **Zero rule:** real condition + certain result. The structure is: if/when + present simple/present simple

Example: If we heat water to 100 degrees, it will boil. (Incorrect)

If we heat water to 100 degrees, it boils. (Correct)

- (ii) **Rule one:** possible condition + possible result. The structure is: if/when + present simple/present form of modal (can/may/will/shall, etc.) + V1 + Rest of the words

Example: You will pass exams with flying colours, if you will study hard. (Incorrect)

If you study hard, you will pass exams with flying colours. (Correct)

- (iii) **Rule two:** imaginary condition + possible result. The structure is: if/when + past simple/corresponding past of modal (could/might/would/should, etc.) + V1 + Rest of the words

Examples: If I have a car, I would go for a long drive. (Incorrect)

If I had a car, I would go for a long drive. (Correct)

- (iv) **Rule three:** unreal condition + imaginary outcome. The structure is: if/when + past perfect tense/corresponding past of modal (could/might/would/should, etc.) + have + V3 + Rest of the words

Example: If you were have gone to Agra, you would have seen the Taj Mahal. (Incorrect)

If you had gone to Agra, you would have seen the Taj Mahal. (Correct)

Mixed Conditionals:

- (i) We use a third second mixed conditional when we imagine present actions or situations that are the result of an imaginary or hypothetical

situation in the past.

Example: If I had completed my work on time, I would have a lot of free time now. (Correct)

- (ii) We use a second third mixed conditional when we imagine a situation in the past that is the result of an imaginary or hypothetical situation that is still true.

Example: If it wasn't so cold in this country, we would have gone out yesterday. (Incorrect)

If it wasn't so cold in this country, we could have gone out yesterday. (Correct)

REPETITION/SUPERFLUOUSNESS

- Sometimes, there is idleness in a sentence. In other words, the similar entity is mentioned twice in a single statement which seems redundant/superfluous.

perfluous. This type of question is asked in both 'spotting the error' and 'sentence improvement'.

- Here is a list of superfluous expressions with their corrections.

Incorrect	Correct
Final conclusion	Conclusion
During the period of war	During the war
Funeral service	Funeral
Flee away	Flee
Suddenly come across	Come across
With bag and baggage	Bag and baggage

EXERCISE

Directions: Look at the underlined part of each sentence. Below each sentence three possible substitutions are given for the underlined part. If one of them, i.e., (a), (b) or (c) is better than the underlined part, indicate your response on the Answer Sheet against the corresponding letter.

- Would you mind closing the window?
(a) to close (b) in closing
(c) for closing (d) No improvement
- I can be grateful if you could write to me about it.
(a) would be (b) should be
(c) must be (d) No improvement
- A majority of the passengers could have been saved if the driver had applied the brakes in time.
(a) had been saved
(b) may have been saved
(c) could have saved
(d) No improvement
- We ought for stand for what is right.
(a) stand at (b) stand on
(c) stand up (d) No improvement
- He enjoys to tell stories to children.
(a) how to tell stories (b) telling stories
(c) to narrate stories (d) No improvement
- It is raining heavily all through this week.
(a) has rained (b) rains
(c) rained (d) No improvement
- The sparrow took no notice about the bread.
(a) notice of (b) notice from
(c) notice to (d) No improvement
- The more they earn, more they spend.
(a) More they earn, more they spend.
(b) More they earn, the more they spend.
(c) The more they earn, the more they spend.
(d) No improvement
- But in all these cases conversion from one scale to another is easy because scales have well-formulated.
(a) can be well-formulated (b) are well-formulated
(c) well-formulated (d) No improvement
- Five years ago on this date, I am sitting in a small Japanese car, driving across Poland towards Berlin.
(a) was sitting (b) sat
(c) have been sitting (d) No improvement
- The old man felled some trees in the garden with hardly no effort at all.
(a) hard effort (b) hardly any effort
(c) a hardly any effort (d) No improvement
- She says she's already paid me back, but I can't remember, so I'll have to take her word.
(a) to take her word true
(b) to take her at her word
(c) to take her word for it
(d) No improvement
- The workers are hell bent at getting what is due to them.
(a) hell bent on getting (b) hell bent for getting
(c) hell bent to getting (d) No improvement
- You are warned against committing the same mistake again.
(a) to commit (b) for committing
(c) against to commit (d) No improvement
- While we would like that all Indian children to go to school, we need to ponder why they do not.
(a) that all the Indian children
(b) if all the children of India
(c) all Indian children
(d) No improvement
- Due to these reasons we are all in favour of universal compulsory education.
(a) Out of these reasons (b) For these reasons
(c) By these reasons (d) No improvement
- When it was feared that the serfs might go too far and gain their freedom from serfdom, the Protestant leaders joined the princes at crushing them.
(a) into crushing (b) in crushing
(c) without crushing (d) No improvement
- In India, today many of our intellectuals still talk in terms of the French revolution and the Rights of Man, not appreciating that much has happened since then.
(a) much has been happening
(b) much had happened
(c) much might happen
(d) No improvement
- Taxpayers are to be conscious of their privileges.
(a) have to (b) need
(c) ought to (d) No improvement

20. I would have waited for you at the station if I knew that you would come.
 (a) had known (b) was knowing
 (c) have known (d) No improvement
21. No one could explain how a calm and balanced person like him could penetrate such a mindless act on his friends.
 (a) perpetuate (b) perpetrate
 (c) precipitate (d) No improvement
22. She is tender-hearted by nature and I have found her eyes moist many a times over the sad face of any child.
 (a) so many a times (b) No improvement
 (c) many a time (d) many time
23. As long to funding is not made available, our construction dreams have been deferred.
 (1) As high as (2) In as much as
 (3) So long as (4) No improvement
24. If I have reached there by this time tomorrow, I will write to you immediately.
 (a) reached (b) reach
 (c) am reaching (d) No improvement
25. If he got his enemy in his hand, he would have crushed him to death.
 (a) has got (b) was getting
 (c) had got (d) No improvement

Answer Key

1. (d)	2. (a)	3. (d)	4. (d)	5. (b)	6. (c)	7. (a)	8. (c)	9. (a)	10. (a)
11. (b)	12. (d)	13. (a)	14. (d)	15. (c)	16. (b)	17. (b)	18. (d)	19. (a)	20. (a)
21. (b)	22. (c)	23. (a)	24. (b)	25. (c)					

Answers with Explanations

- Option (d) is correct.**
Explanation: The sentence is grammatically correct.
- Option (a) is correct.**
Explanation: 'Can be; will be replaced with 'would be'. 'Would' is used to make requests, so, it is the correct option to be used here.
- Option (d) is correct.**
Explanation: The sentence requires no improvement as it is grammatically correct.
- Option (d) is correct.**
Explanation: The correct idiomatic expression 'to stand' means to be in support of something. 'Stand at' means to reach a particular level. 'Stand on' means to continue on the same course. 'Stand up' means to remain valid after close scrutiny.
- Option (b) is correct.**
Explanation: The verb 'enjoy' takes 'ing' form of the verb and not the infinitive form. So 'telling stories' is opted here.
- Option (c) is correct.**
Explanation: In the sentence, 'through this week' means 'from the beginning to the end of the week.' So, the action of raining is a past event. The simple past tense of rain i.e., rained will be used here. The present perfect 'has rained' is incorrect because the action is over and is not continuing in the present.
- Option (a) is correct.**
Explanation: 'Notice' does not take the preposition about'. 'Notice of' is the correct usage and it means to give attention to something. 'Notice from' someone means to get any information from someone else. 'Notice to' someone means to give information to someone else.
- Option (c) is correct.**
Explanation: The construction of the sentence is with 'the' in the first clause. This construction should be maintained in the second clause also, so it should be 'the more they spend.'
- Option (a) is correct.**
Explanation: The helping verb 'can' in option (a) show the ability that scales possess. So 'can' is the right fit than other options.
- Option (a) is correct.**
Explanation: 'Five years ago' indicates that the sentence is in past tense. Also, in the second half of the sentence, 'driving' is in the continuous tense, so 'was sitting' is the best option.
- Option (b) is correct.**
Explanation: 'Hardly' means 'no' or 'nothing'. So, 'no effort' is incorrect after saying 'hardly'. Therefore, 'hardly any effort' fits in.
- Option (d) is correct.**
Explanation: The correct idiomatic expression is 'to take her word', meaning to trust her, therefore option (d) should be chosen.
- Option (a) is correct.**
Explanation: 'Hell bent on' is the correct idiomatic expression meaning 'determined to do something even if there are problems or obstacles.'
- Option (d) is correct.**
Explanation: The sentence correctly uses the preposition 'against' with 'warned' as a warning is always issued against something. Also, the continuous tense 'committing' is correct than 'to commit' because of the use of 'against'. We say 'against committing something' in standard English.
- Option (c) is correct.**
Explanation: 'Like' does not take 'that' after it.
- Option (b) is correct.**
Explanation: 'Due to' and 'reasons' convey the same meaning and so it is unnecessary to use them together. Also, in formal writing we do not start with 'due to'.
- Option (b) is correct.**
Explanation: The gerund 'crushing' in this sentence means 'defeating'. Defeating takes the preposition 'in'.

18. Option (d) is correct.

Explanation: The sentence is correct in terms of construction and subject-verb agreement. Also, the use of present perfect tense 'has happened' with the word 'since' is correct.

19. Option (a) is correct.

Explanation: In the sentence, the taxpayers compulsorily or necessarily should know about their privileges. This meaning is best conveyed with the word 'have to'. 'Are to' expresses a lower sense of conveying that this is always the case. Also 'ought to', is used for moral obligation/duty.

20. Option (a) is correct.

Explanation: The sentence tells about a condition in the past conveyed with the word 'if'. It uses the third conditional which is formed by: would have + past participle + if + past perfect. So 'know' should be replaced with 'had known'.

21. Option (b) is correct.

Explanation: 'Penetrate' means to 'go into' or 'deep inside' whereas we want a word which means 'to do' or 'to carry out'. So, 'perpetrate' will be the right option.

22. Option (c) is correct.

Explanation: 'So long as' in an informal use means 'provided that or on condition that', so the correct answer would be 'so long as'.

23. Option (a) is correct.

Explanation: The activity in the sentence is taking place in the past continuous form, so the correct

24. Option (b) is correct.

Explanation: This is a conditional sentence. When we speculate future in conditional sentences, if the 'if clause' is in simple present tense and the other clause should be in simple future tense.

25. Option (c) is correct.

Explanation: The sentence describes hypothetical condition of past in which the second event would have taken place if the first event had happened. The structure of the sentence should be: 'if clause + past perfect tense + second clause (sub + would/could + have + third form of verb)'.

○○○

DETERMINERS: USE OF ADJECTIVE COMPOUNDS, USE OF A, THE, ANY, ETC

Definition: An adjective is a word, which is used to describe a noun or a pronoun. It can also be understood as a Describing Word.

How to use Adjectives: Adjectives are used in two ways:

- (1) **Attributively:** Adjectives are used attributively when **they are placed before nouns.**

Example: He is a **good** boy.

(The adjective 'good' has been placed before the noun 'boy'.)

- (2) **Predicatively:** Adjectives are used predicatively when **they are placed after the main verb of the sentence.**

Example: He is **good** at English speaking.

(The adjective 'good' has been placed after the verb 'is'.)

- (3) There is a sequence to be followed while using adjectives in a sentence. This sequence can be learnt from the acronym OSASCOMP which refer to Opinion, Size, Age, Shape, Colour, Origin, Material, Purpose.

Example: My uncle gifted me a beautiful oval antique brown Italian leather handbag.

The above sentence has followed the OSASCOMP rule.

Degrees of Comparison

There are three degrees of comparison for adjectives to compare different things with each other.

- (1) **Positive degree:** In positive degree, there is no comparison.

Examples: They are **kind** people.

Manav is a **brave** boy.

- (2) **Comparative degree:** In comparative degree, comparison is made between two people or things.

Examples: They are **kinder** than their neighbours.

Manav is **braver** than his classmates.

- (3) **Superlative degree:** In superlative degree, comparison is made between more than two people or things.

Examples: They are the **kindest** people I have ever known.

Manav is the **bravest** boy in the entire school.

Rule 1 :

- (i) With countable nouns which indicate any specific number or order of nouns, numeral adjectives are used. The word 'fewer' is used with countable nouns.

Example :

- Last four items of the menu have been selected by us.
- Fewer than twenty people booked tickets for the show, so organisers had to cancel it.

- (ii) With uncountable nouns the adjectives of quantity

like- some, all, little, much, any, enough, few, sufficient, huge, a lot, a little bit are used.

Example :

- Many Americans come to visit India every year.
- Nancy has few books so she cannot research well.

Rule 2 : Some adjectives which do not have any comparative or superlative form, are called absolute adjectives, such as Empty, Unique, Round, Perfect, Wet, Pretty, Funny and Cold.

Example :

- That room is completely empty.
- You need to develop some unique qualities in yourself.

Rule 3 : When two qualities of a person or thing are compared in the same sentence, 'er' should not be used in such a case. 'More' should be used instead.

Example :

- She is more simple than humble.
- This test is more easy than convenient.

Rule 4 : Adjectives like Inferior, Superior, Junior, Senior, Prior, Posterior, Anterior, Prefer, etc. should be followed by 'to', 'instead of', 'then'.

Example :

- Naman is junior to Sandy.
- Bella prefers to take milk instead of coffee.

Rule 5 : In the case of many adjectives used in a sentence, adjectives should be used in the order of - Size, Shape, Age, Colour, Nationality, Material.

Example :

- Sachin was a short height sixteen year old Indian boy, when he started his cricket career.

Rule 6 : Double comparatives and double superlatives should not be used together in the same sentence. Article 'the' is used before superlative adjectives.

Example :

- Andy is the brightest student in the class.
- Maharana Pratap was the most powerful of all kings of his time.

Rule 7 : In comparative degree, the comparison should be made between two similar things, qualities, quantities, etc. and for this comparative degree is followed by 'than'.

Example :

- According to the US Air Quality Index, Lahore is more polluted city than Delhi. But there are some exceptions such as Prior, Superior, Inferior, Junior, Senior, Elder, Younger, Prefer, which are followed by 'to'.
- ISRO project COROSAT-3 Mission was accomplished prior to Amazonia-1.

Rule 8 : Two adjectives should be mentioned in the same degree in a sentence, if they qualify the

same noun.

Example :

- Dr. Abdul Kalam was the most famous and inspiring President of India.

Rule 9 : If a gradual or sequential process is mentioned in a sentence, it is conveyed with two comparatives.

Example :

- Indian economy is getting better and better every day after the pandemic.

Rule 10 : Exclamatory adjectives are used to express a strong feeling or opinion in 'How' and 'What' forms.

Example :

- How pretty this little bird is!
- What a powerful car you have!

Demonstratives– They point out which person or thing is meant.

- This boy is stronger than Hari.
- These mangoes are sour.
- Those rascals must be punished.

Quantifiers

Little/A little/The little

We use little with uncountable nouns.

Little – not much (i.e., hardly any). Thus, the adjective little has a negative meaning. There is little hope of his recovery, i.e., he is not likely to recover.

A little – some though not much. 'A little' has a positive meaning. There is a little hope of his recovery, i.e., he may possibly recover. A little tact would have saved the situation.

The little – not much, but all there is.

The little knowledge of carpentry that he possessed stood him in good stead.

Few/A Few/The Few

They are used with countable nouns.

Few – not many, hardly any, 'Few' has a negative meaning.

Few persons can keep a secret.

Few people are so hopeless as drunkards.

A few – some. 'A few' has a positive meaning, and is opposed to 'none'.

A few words spoken in earnest will convince him.

The few – not many, but all there are. The few remarks that he made were very suggestive.

Distributive determiners– They refer to each one of a number.

- Each boy must take his turn.
- India expects every man to do his duty.
- On either side is a narrow lane.

Possessive determiners – They are the ownership denoting adjectives.

- This is your pen.
- That is my car.
- These are our chairs.

ARTICLES

The Indefinite Article is used–

1. In its original numerical sense of one;
 - Twelve inches make a foot.
 - Not a word was said.
 - A word to the wise is sufficient.
2. In the vague sense of a certain; for example,

- A Kishore Kumar (= a certain person named Kishore Kumar) is suspected by the police.
- One evening a beggar came to my door.

3. In the sense of any, to single out an individual as the representative of a class; for example,

- A pupil should obey his teacher.
- A cow is a useful animal.

4. To make a common noun of a proper noun; for example,

- A Daniel comes to judgement! (A Daniel = a very wise man)

5. Before names of professions

- He is an MP/MLA/MLC/SDO/FO.
- She is a nurse/teacher/ dentist.
- He is a manager/carpenter/cobbler.
- Mr. Sharma is an engineer/architect.

6. A is used before A hundred, A thousand, A million, A dozen, A couple

- She has to walk two and a quarter kilometres daily.
- I have a hundred pens.
- He has a thousand rupees.

7. In the structure: Many/rather/quite/such + singular noun

- Many a woman would welcome such a chance.
- It is rather a pity, I say.
- It was quite an impossible task.

8. When an adjective precedes **so, as, too, how, quite:**

- It is too heavy a parcel for me to carry.
- She is as beautiful a girl as you are ever likely to meet.
- Ram is not so big a fool as you think.

9. If a proper noun is used as a common noun:

- He is a Kalidas.

10. If two adjectives are joined by "and," and wants to signify a singular object:

- A black and white cow. (resembles 1 cow)
- An honest and intelligent teacher.

11. If two adjectives are joined by 'and', and used before a singular countable noun and it denotes two different persons or things

- A red and a black shirt.
- A tall and a pretty girl.

USE OF DEFINITE ARTICLE

1. When a singular noun is meant to represent a whole class;

- The horse is a noble animal.
- The cat loves comfort.
- The rose is the sweetest of all flowers.

2. Before some proper names, viz., these kinds of place-names:

- (a) oceans and seas, e.g., the Pacific, the Black Sea
- (b) rivers, e.g., the Ganga, the Nile
- (c) canals, e.g., the Suez Canal
- (d) deserts, e.g., the Sahara
- (e) groups of islands, e.g., the West Indies
- (f) mountain-ranges, e.g., the Himalayas, the Alps
- (g) a very few names of countries, which include words like republic and kingdom (e.g., the Irish Republic, the United Kingdom) also: the Ukraine, the Netherlands (and its seat of government the Hague)

3. Before the names of certain books;

- The Vedas, the Puranas, the Iliad, the Ramayana.
 - But we say – Homer's Iliad, Valmiki's Ramayana.
4. Before names of things unique of their kind; for example,
 - The Sun, the sky, the ocean, the sea, the Earth.
 5. Before a Proper noun when it is qualified by an adjective or a defining adjectival clause; for example,
 - The great Caesar, the immortal Shakespeare.

OMISSION OF ARTICLES

1. School, College, University, Bed, Church, Court, Hospital, Prison, Market – DO NOT USE 'THE'

NOTE: The is used with these words when we refer to them as a definite place, building or object rather than to the normal activity that goes on there; as,

2. When words like **appoint, made, nominate, elect, declare, select**, are followed by a noun in the sentence, they do not take any article:

3. Before names of meals (used in a general sense); as,

NOTE: We use "a" when there is an adjective before breakfast, lunch, dinner, etc. We use the when we specify. I had a late lunch today. The dinner we had at the Tourist Hotel was very nice.

4. Before languages;
5. Before names of relations, like father, mother, aunt, uncle, and also cook and nurse, meaning 'our cook', 'our nurse', such as,
6. Before predicative nouns denoting a unique position, i.e., a position that is normally held at one time by one person only; such as,
7. Before most proper nouns (except those referred to earlier), namely, names of people (e.g., Gopal, Rahim), names of continents, countries, cities, etc. (e.g., Europe, Pakistan, Nagpur), names of individual mountains
When 'most' is used in the sense of very in superlative degree, THE is not used.
8. Before plural countable nouns used in a general sense;
9. Before names of substances and abstract nouns (i.e. uncountable nouns) used in a general sense;
10. Before diseases
11. Before names of subjects
12. Before buildings and institutions, where one of the words is that of a person or a place

EXERCISE

Directions: In the following questions, one part of the sentence may have an error. Find out which part of the sentence has an error and click the button corresponding to it. If the sentence is free from error, click the "No error" option.

1. (a) After a carefully investigation
(b) we discovered
(c) that the house was infested with termites.
(d) No error.
2. (a) He was (b) very kind enough
(c) to invite me (d) No Error
3. (a) The trek is difficult (b) but it is far worth
(c) the endeavour (d) No Error
4. (a) The Statesman has the (b) larger circulation
(c) of all English dailies (d) No Error
5. (a) The author said during the press conference
(b) that there were
(c) two farther volumes to be published
(d) No error
6. (a) He was not able to concentrate
(b) because of the continual music
(c) being played next door.
(d) No Error
7. (a) Everybody knows
(b) that Bhutan is the most peaceful
(c) of all other countries of the world.
(d) No error.
8. (a) The woodcutter felled (b) some trees
(c) with hardly (d) many effort at all
9. (a) Many multinational companies
(b) have not been as
(c) successful in India
(d) than we expected.
(e) No error
10. (a) Workers with less
(b) personal problems are
(c) likely to be

- (d) more productive in their work.

- (e) No error.

A. Directions: Below are given alternatives to the bold sentence/part of the sentence at (A), (B) and (C) which may improve the sentence. Choose the correct alternative. In case no improvement is needed, your answer is (D).

11. The plant that opened in a small Pennsylvania village is today the **most largest** chocolate factory in the world.
(a) larger
(b) large
(c) largest
(d) No improvement required
12. I like both tea and coffee but prefer the **later**.
(a) least
(b) latter
(c) last
(d) No improvement required
13. When it started raining, I ran to the **nearer house** for shelter.
(a) the nearest house
(b) a near house
(c) the near house
(d) No improvement required
14. He found a **wooden chair that has broken** in the room.
(a) wooden and broken chair
(b) broken wooden chair
(c) broken and wooden chair
(d) No improvement required

B. In the sentence identify the part that contains the grammatical error.

15. Cyclone Idai is regarded as one of the worst tropical cyclone on record to affect Africa and the Southern Hemisphere as a whole.
(a) Cyclone Idai is regarded

- (b) the worst tropical cyclone
(c) as a whole
(d) to affect Africa
16. The modern man is busy acquiring more and more wealth and designing ways to invest it in more sense pleasures.
(a) modern man is busy
(b) acquiring more and more wealth
(c) in more sense pleasures
(d) designing ways to invest it
17. No least than fifty participants were present for the singing competition.
(a) No least than
(b) fifty participants were
(c) present for
(d) the singing competition
18. From all accounts I learn that Vivek is the best and honest member of the new Cabinet.
(a) From all accounts
(b) I learn that Vivek is
(c) the best and honest member
(d) of the new Cabinet.
19. I asked two persons the way to the station, but none knew it.
(a) I asked twopersons (b) the way to
(c) the station, (d) but none knew it
20. I am sure that all my monthly expenses would exceed the income if I do not economise.
(a) I am sure that all my
(b) monthly expenses would
(c) exceed the income
(d) if I do not economise.

Answer Key

1. (a)	2. (b)	3. (b)	4. (b)	5. (c)	6. (b)	7. (c)	8. (d)	9. (c)	10. (a)
11. (c)	12. (b)	13. (a)	14. (c)	15. (b)	16. (c)	17. (a)	18. (c)	19. (d)	20. (a)

Answers with Explanations

1. **Option (a) is correct.**
Explanation: The investigation is described as a 'careful' one, 'carefully' is an adverb, which cannot modify the noun 'investigation'. Hence, the error is in part (a).
2. **Option (b) is correct.**
Explanation: The adjective 'kind' is already modified by 'enough', so the addition of 'very' makes it repetitive. Hence, it is not proper to use 'very' and 'enough' together. It will be either 'very kind' or 'kind enough'. Hence, the error is in part (b).
3. **Option (b) is correct.**
Explanation: Replace 'far worth' with 'well worth', because we need a positive expression here. 'Well worth' implies 'certainly worth'. The trek is said to be worth the effort, hence 'well worth'. Hence, the error is in part (b).
4. **Option (b) is correct.**
Explanation: The statement will take the superlative degree because the Statesman is said to be the most circulated newspaper of all English dailies. Thus, 'the largest' should replace 'the larger'. Hence, the error is in part (b).
5. **Option (c) is correct.**
Explanation: Further = additional to what already exists. Farther = at to or by a great distance. Hence, two further (additional) volumes to be published..... should be used here. Hence, the error is in part (c).
6. **Option (b) is correct.**
Explanation: 'Continual' is incorrect in the given context. 'Continuous' is the correct adjective that describes music. Hence, the error is in part (b).
7. **Option (c) is correct.**
Explanation: Bhutan is said to be the most peaceful of all other countries 'in the world', not 'of the world'. Hence, the error is in part (c).
8. **Option (d) is correct.**
Explanation: The sentence implies that the woodcutter hardly put in efforts to fell trees. Thus, 'any' is the best alternative because it is used in negative sentences. Thus, 'some trees with...any effort at all' is correct. Hence, the error is in part (d).
9. **Option (c) is correct.**
Explanation: The sentence is in positive degree, thus the correct usage is 'as...as', instead of 'as successful...than'. 'Than' needs to be replaced by 'as'. Hence, the error is in part (c).
10. **Option (a) is correct.**
Explanation: According to the usage rules, 'less' is used with singular mass nouns like salt, money, love, etc. Here, personal problems are not singular mass nouns; they can be counted. So 'fewer' is correct. Part A of the sentence has an error. Hence, the error is in part (a).
11. **Option (c) is correct.**
Explanation: The sentence establishes that the plant that opened in a small Pennsylvania village to be the largest chocolate factory in the world. Moreover, double superlatives are not used in a sentence. Hence, correct answer is (c).
12. **Option (b) is correct.**
Explanation: 'Latter' denotes the second item out of the two (tea and coffee). While 'later' refers to time. Hence, correct answer is (b).
13. **Option (a) is correct.**
Explanation: The sentence applies incorrect comparative adjective, since use of a comparative adjective implies that only two houses were present there. But the sentence suggests that the person ran to the nearest house available. Comparative degree is used to compare two objects. Thus, superlative degree of adjective is appropriate ('nearest'). Hence, correct answer is (a).

14. Option (c) is correct.

Explanation: The general order of Adjectives is as follows:

Opinion	Colour Origin
Appearance Age	Material
↓	↓
<u>broken</u>	<u>wooden</u>

Hence, broken wooden chair is the right usage.

Hence, correct answer is (c).

15. Option (b) is correct.

Explanation: The noun or pronoun placed after 'one of' should be in plural. Thus, 'cyclones' is correct, not 'cyclone'. Hence, correct answer is (b).

16. Option (c) is correct.

Explanation: Pleasures, being an abstract noun can be described by an adjective 'sensory'. Hence, correct answer is (c).

17. Option (a) is correct.

Explanation: The superlative degree 'least' does not work here as no comparison is being made here. The

correct degree is positive degree. Since 'less' is used with uncountable nouns, 'fewer' is the correct option, because of the use of 'than' and the noun being countable (**fifty participants**). Hence, correct answer is (a).

18. Option (c) is correct.

Explanation: The adjectives must be in superlative degree – thus, 'the best' and 'the most honest member' suits the need of the sentence.

Hence, correct answer is (c).

19. Option (d) is correct.

Explanation: 'None' is used for more than two persons, while 'neither' is used in the sense of 'not one nor the other of the two things or people', and it is used for two persons.

Hence, correct answer is (d).

20. Option (a) is correct.

Explanation: All will not be used because all kinds of expenditure have been included in 'monthly expenses'. So the adjective 'all' needs to be deleted.

Hence, correct answer is (a)

○○○

Definition:

Direct speech is a word-to-word repetition of what the speaker or writer conveys. In other words, it is the way in which you report the exact words of the speaker. Indirect Speech is also called indirect statement and is used to report what someone may have said, and hence, it is in the past tense.

We may report the words of a speaker in two ways:-

- (i) We may quote his actual words. This is called Direct Speech.
- (ii) We may report what he said without quoting his exact words. This is called Indirect (or Reported) Speech.

Direct. – Rama said, "I am very busy now."

Indirect. – Rama said that he was very busy then.

DIRECT SPEECH	INDIRECT SPEECH
use inverted commas to convey the exact words of the speaker.	Does not use inverted commas to convey the exact words of the speaker.
use present and future tense.	use only past tense.

Rules for changing Direct Speech into Indirect Speech

- A. When the reporting or principal verb is in the Past Tense, all Present tenses of the Direct speech are changed into the corresponding Past Tenses.

(a) A simple present becomes a simple past.

Direct. – He said, "I am unwell." Indirect. – He said (that) he was unwell.

(b) A present continuous becomes a past continuous.

Direct. – He said, "My master is writing letters." Indirect. – He said (that) his master was writing letters.

(c) A present perfect becomes a past perfect.

Direct. – He said, "I have passed the examination." Indirect. – He said (that) he had passed the examination.

(d) The simple past in the Direct Speech becomes the past perfect in the Indirect Speech.

Direct. – He said, "The horse died in the night." Indirect. – He said that the horse had died in the night.

- B. The tenses may not change if the statement is still relevant or if it is a universal truth. We can often choose whether to keep the original tenses or to change them.

Direct. – "I know her address," said Gopi.

Indirect. – Gopi said he knows/knew her address.

Direct. – The teacher said, "The earth goes round the sun."

Indirect. – The teacher said the earth goes/went round the sun.

She said, "Two and two is four."

She said that two and two is four.

He said, "London is the capital of U.K."

He said that London is the capital of U.K.

- C. **If the reporting verb is in the Present Tense, the tenses of the Direct Speech do not change.**

Direct – He says, "I am unwell."

Indirect – He says (that) he is unwell.

Direct – He says, "I have passed the examination."

Indirect – He says he has passed the examination.

- D. **The pronouns of the Direct Speech are changed,** where necessary, so that their relations with the reporter and his hearer, rather than with the original speaker, are indicated.

Direct – He said to me, "I don't believe you."

Indirect – He said he didn't believe me.

Direct – She said to him, "I don't believe you."

Indirect – She said she didn't believe him.

I will change into **he, she, you, or it.**

We will change into **you or they.**

Singular **you** will change into **I, she, he, or it.**

Plural **you** will change into **we or they.**

He, she and it will change into **I only.**

They will change into **you or we.**

Me will change into **him, her, you, or it.**

Us will change into **you or them.**

Singular **you** used as object will change into **me, him, her, or it.**

Plural **you** used as object will change into **us or them.**

Him, her or it will change into **me or you.**

Them will change into **us or you.**

My will change into **your, his, her, or its.**

Our will change into **your or their.**

His, her and its will change into **my or your.**

Their will change into **our or your.**

E.

Direct Speech	Indirect Speech
This	That
These	Those
Here	There
Hence	Thence
Hither	Thither
Now	Then
Ago	Before
Thus	So
Today	That day
Tomorrow	The next day/the following day
Yesterday	The previous day
Last week/month/year	The previous week/month/year
The last fortnight	The previous fortnight
The day before yesterday	The day before the previous day

F. Tense of Reported Speech

DIRECT SPEECH	INDIRECT SPEECH
Present Indefinite	Past Indefinite
Present Continuous	Past Continuous
Present Perfect	Past Perfect
Present Perfect Continuous	Past Perfect Continuous
Past Indefinite	Past Perfect
Past Continuous	Past Perfect Continuous
Past Perfect	No change of tense
Past Perfect Continuous	No change of tense

Change in Future Sentences (Reported Speech)

DIRECT SPEECH	INDIRECT SPEECH
Will/shall	Would/should
Can	Could
May	Might
Could/should/would/might	No change

Change of speech with types of sentences

Interrogative Sentences

1. Yes/no questions

2. Wh- questions

- When the question is not introduced by an interrogative word, the reporting verb is followed by whether or if.
- 'That' is never used.

- The interrogative sentence changes to an assertive sentence.
- There are changes in terms of pronoun and tense.
- The question mark changes to a full stop.
- Reporting verb - say, says, said changes to ask, asks, asked.
- Often Indirect Speech is introduced by verbs such as asked, inquired, etc.

EXAMPLES:

- Direct - He said to me, "What are you doing?"
Indirect - He asked me what I was doing.
- Direct - "Where do you live?" asked the stranger.
Indirect - The stranger enquired where I lived.
- Direct - He said, "Will you listen to such a man?"
Indirect - He asked them whether they would listen to such a man.
[Or] Would they, he asked, listen to such a man?
- Direct - "Do you suppose you know better than your own father?" jeered his angry mother.
Indirect - His angry mother jeered and asked whether he supposed that he knew better than his own father.
- She said to me, "Will you help me?"
She asked me if I would help her.
- She said to me, "Can you solve this question?"
"No", I said.
She asked me if I could solve that question, I said, I couldn't or I replied in negative.
- He said to me, "Do you know me?" "I said, "Yes".
He asked me if I knew him, I said, I did or I replied in affirmative.

Speculation:

- Direct:** "Shall I ever see her again?", he said.
Speculation
Indirect: He wondered if he would ever see her again.
- Direct:** "When shall I know the result of the election?" Ram asked.
Indirect: Ram asked when he would know the result of the election.
- Direct:** "What shall I say father?" he said.
Indirect: He asked his father what he should say.

Will you/would you/could you

- Direct:** He said, "Will you be there on Monday?"
(Ordinary question)
Indirect: He asked if he would be there on Monday.
- Direct:** He shouted, "Will you keep silence?"
Indirect: He shouted at me to keep silence.

Imperative Sentences

In reporting commands and requests, the Indirect Speech is introduced by some verb expressing command or request, and the imperative mood is changed into the

Infinitive.

1. 'That' is replaced by 'to'.
2. 'Don't' changes to 'not to'.

Examples:

1. Direct – Rama said to Arjun, "Go away."
Indirect – Rama ordered Arjun to go away.
2. Direct – He said to him, "Please wait here till I return."
Indirect – He requested him to wait there till he returned.
3. Direct – "Call the first witness," said the judge.
Indirect – The judge commanded them to call the first witness.
4. Direct – He said, "Be quiet and listen to my words."
Indirect – He urged them to be quiet and listen to his words.
5. Direct – I said to Hari, "Don't pluck the flowers".
Indirect – I ordered Hari not to pluck the flowers.
6. Direct – The teacher said to Hari, "Don't write on the table."
Indirect – The teacher forbade Hari to write on the table.
Or
The teacher instructed Hari not to write on the table.

Use of 'Do'

1. He said to me, "Do have a cup of coffee, please".
He requested me to have a cup of coffee.
2. She said to me, "Do come again."
She requested me to come again.

Imperative and Tag Question

1. He said to me, "Open the door, will you?"
He asked me to open the door.

Sentences with 'LET'

A. LET as proposal

- (1) She said to me, "Let us go to market".
She **suggested** to me to go to market.
Or
She suggested to me that we should go to market.
- (2) He said to me, "Let us purchase this house".
He proposed to me to purchase that house.
Or
He **proposed** to me that we should purchase that house.

B. LET as wish

- (1) The student said, "Let me take food".
The student **wished** that he should take food.
- (2) The lady said, "Let me have an ice cream pack".
The lady **wished** that she should have an ice cream pack.

C. LET as order/request

- (1) The boy said to the teacher, "Let me go home now."
The boy **requested** the teacher to allow him to go home then.
- (2) The Principal said to the peon, "Let the students come in".
The Principal **ordered** the peon to allow the students come in.

OPTATIVE SENTENCES

- (1) My mother said to my friend, "May you live long!"
My mother wished my friend that he might live long.
- (2) She said to Sita, "May God bless you with a son!"
She prayed that God might bless Sita with a son.
- (3) She said, "God grant you success!"
She prayed that God might grant me success.
- (4) He said to the teacher, "Good morning."
He wished the teacher good morning.

EXCLAMATORY SENTENCES

1. In indirect speech, the exclamatory sentences have: exclaimed with joy, exclaimed with sorrow, exclaimed with surprise, exclaimed with applause, exclaimed with anger, exclaimed with contempt.
2. Conjunction 'that' is used in indirect speech.
3. The words 'Alas, Bravo, Oh, Wow, Wah, Hurrah' are eliminated in indirect speech.
4. When the sentence start with 'If I were a bird, If I were the P.M.', the reporting verb is 'wished'.

Examples:

- (1) She said, "If I were a bird!"
She wished that she would be a bird.
- (2) We said, "What a nice place it is!"
We exclaimed with surprise that it was a nice place.
- (3) Ram said, "Alas! I have been ruined."
Ram exclaimed with sorrow that he had been ruined.
- (4) The lady said, "Oh! my dog is dead."
The lady exclaimed with sorrow that her dog was dead.

EXERCISE

1. I told him:
 - (a) what the homework was.
 - (b) what was the homework.

- (c) what was to be the homework.
- (d) what is the homework.

2. He tells me
 - (a) that he would go to Mumbai the next day.
 - (b) that he would go to Mumbai tomorrow.
 - (c) that he will go to Mumbai tomorrow.
 - (d) that he will go to Mumbai the next day.
3. He suggested
 - (a) me that we should go to the hospital.
 - (b) to me that we should go to the hospital.
 - (c) to me to we will go to the hospital.
 - (d) me that we go to the hospital.
4. Rahul asked him
 - (a) if he will go to Jaipur.
 - (b) that if he would go to Jaipur.
 - (c) weather he would go to Jaipur.
 - (d) if he would go to Jaipur.
5. Maneesh said that
 - (a) he might go to Delhi tomorrow.
 - (b) he may go to Delhi the next day.
 - (c) he might go to Delhi the next day.
 - (d) he may go to Delhi tomorrow.
6. The old man asked his son
 - (a) what he (the son) has learned at school.
 - (b) what he (the son) had learnt at school.
 - (c) what had he (the son) learnt at school.
 - (d) what had he (the son) learn at school.
7. The teacher asked Ganesh
 - (a) why was he not taking the examination.
 - (b) that why he was not taking the examination.
 - (c) why he was not taking the examination.
 - (d) why had he not taking the examination.
8. Please tell me:
 - (a) where was the bus stop.
 - (b) where the bus stop be.
 - (c) where stops the bus.
 - (d) where the bus stop was.
9. He told us:
 - (a) that he enjoy the movie.
 - (b) he enjoyed the movie.
 - (c) he be enjoying the movie.
 - (d) that enjoyed the movie.
10. I asked her
 - (a) which school did she attend.
 - (b) which school she had attend.
 - (c) which school she had attended.
 - (d) which school she attend.

Direction: Choose the most appropriate option to change the narration (direct/indirect) of the given sentence.

11. "Everything is going to be alright," said the doctor.
 - (a) The doctor said that everything are going to be alright.
 - (b) The doctor said that everything will be alright.
 - (c) The doctor said that everything was going to be alright.
 - (d) The doctor said that everything is going to be alright.

12. "Is he alright now?" I asked my neighbour's wife about her husband, Mr. Gulyani.
 - (a) I asked Mrs. Gulyani, my neighbour's wife, if her husband was alright then.
 - (b) I asked Mrs. Gulyani, my neighbour's wife, if your husband will be alright now.
 - (c) I asked Mrs. Gulyani, my neighbour's wife, if her husband had become alright then.
 - (d) I asked Mrs. Gulyani, my neighbour's wife, if her husband is alright now.
13. "Whom did you see at the shopping mall today?" I asked my daughter.
 - (a) I asked my daughter that whom did she saw at the shopping mall that day.
 - (b) I asked my daughter who she has seen at the shopping mall today.
 - (c) I asked my daughter that whom she saw at the shopping mall on that day.
 - (d) I asked my daughter whom she had seen at the shopping mall that day.
14. "Is the door closed?" Alka asked Tarun.
 - (a) Alka asked Tarun is the door close.
 - (b) Alka asked Tarun if the door is closed.
 - (c) Alka asked Tarun whether she closed the door.
 - (d) Alka asked Tarun whether the door was closed.
15. He said to the hotel receptionist, "Can you tell me the tariff of rooms?"
 - (a) He asked the hotel receptionist that if he can tell him the tariff of rooms.
 - (b) He asked the hotel receptionist to tell him the tariff of rooms.
 - (c) He enquired the hotel receptionist if he can tell him the tariff of rooms.
 - (d) He asked the hotel receptionist if he could tell him the tariff of rooms.
16. He said to me, "What are you doing?"
 - (a) He said that what I was doing.
 - (b) He said what I had been doing.
 - (c) He asked me what I was doing.
 - (d) He asked me that what was I doing.
17. She said to me, "What time is your flight tomorrow?"
 - (a) She asked me what time is my flight the next day.
 - (b) She asked me that what time was my flight tomorrow.
 - (c) She asked me what time my flight was the following day.
 - (d) She asked me what time my flight will be the next day.
18. The teacher commanded the students not to shout.
 - (a) The teacher said to the students, "Don't shout."
 - (b) The teacher said to the student, "You should not shout."
 - (c) The teacher told to the students, "You must not shout."
 - (d) The teacher says to the students, "Do not shout."

19. The teacher said to me, "You have not submitted the assignment."
 - (a) The teacher said me that I had not submitted the assignment.
 - (b) The teacher told me that I had not submitted the assignment.
 - (c) The teacher said to me that I have not submitted the assignment.
 - (d) The teacher told to me that I have not submitted that assignment.
20. The lawyer said to me, "There is no proof of your involvement in this case."
 - (a) The lawyer told that there is no proof of my involvement in that case.
 - (b) The lawyer told me that there is no proof of your involvement in this case.
 - (c) The lawyer told me that there was no proof of my involvement in that case.
 - (d) The lawyer said me that there was no proof of 21. what is good idea!", Seema remarked.
21. "What a good idea!", Seema remarked.
 - (a) Seema said what a good idea it is.
 - (b) Seema told what an idea!
 - (c) Seema exclaimed that it was a very good idea.
 - (d) Seema exclaimed that the idea is good.
22. "Hello", he said to his friend. "What can I do for you?"
 - (a) He asked and greeted his friend that what he can do for him.
 - (b) He greeted his friend and asked what he could do for him.
 - (c) He told hello and asked his friend what he could do for him.
 - (d) He said hello and asked his friend what can do for him.
23. He said to me, "What time do the banks open and close?"
 - (a) He asked me what time the banks opened and closed.
 - (b) He said me what time did the banks open and close.
 - (c) He wanted to know what time the banks open and close.
 - (d) He asked me what time do the banks open and close.
24. "What a rare flower!" she said.
 - (a) She asked if it was a rare flower.
 - (b) She exclaimed that it was a very rare flower.
 - (c) She exclaimed what a rare flower it is.
 - (d) She told that what a rare flower it was.
25. Mahesh said to Rita, "Don't play in the sun."
 - (a) Mahesh advised Rita not to play in the sun.
 - (b) Mahesh told to Rita to not play in the sun.
 - (c) Mahesh requested Rita that don't play in the sun.
 - (d) Mahesh ordered Rita that not play in the sun.
26. She said, "We visited Uttarakhand in the summer vacation."
 - (a) She said that they had visited Uttarakhand in the summer vacation.
 - (b) She said that we visited Uttarakhand in the summer vacation.
 - (c) She said that they will visit to Uttarakhand in the summer vacation.
 - (d) She said that they have visited Uttarakhand in the summer vacation.
27. The traveller said to a passerby, "Can you help me find my way, please?"
 - (a) The traveller asked the passerby could you kindly help me find my way?
 - (b) The traveller asked the passerby that he can kindly help him find his way.
 - (c) The traveller requested to the passerby if he can help him find his way.
 - (d) The traveller asked the passerby if he could kindly help him find his way.
28. The guard said to the visitors, "Do not take photographs inside the museum."
 - (a) The guard forbade the visitors to take photographs inside the museum.
 - (b) The guard told the visitors that do not take photographs inside the museum.
 - (c) The guard warned the visitors not to be taking photographs inside the museum.
 - (d) The guard requested the visitors to not took photographs inside the museum.
29. The Principal has said, "Mr. Surinder will coach the football team."
 - (a) The Principal has said that Mr. Surinder would coach the football team.
 - (b) The Principal said that Mr. Surinder would coach the football team.
 - (c) The Principal has asked that will Mr. Surinder coach the football team.
 - (d) The Principal has said that Mr. Surinder will coach the football team.
30. Rahul said to his mother, "Can I go out and play now?"
 - (a) Rahul asked his mother if he could go out and play then.
 - (b) Rahul asked to his mother could he go out and play now.
 - (c) Rahul asked his mother that could he go out and play then.
 - (d) Rahul asked his mother if can he go out and play now.
31. "What are you going to call the puppy, Jane?" the boy asked.
 - (a) The boy wondered what Jane were going to call the puppy.
 - (b) The boy told Jane what she was going to call her puppy.
 - (c) The boy asked Jane what she was going to call the puppy.
 - (d) Jane asked the boy what is he going to call the puppy.

32. Johnny's mother said, "Johnny, run down to the grocer's and get a loaf of French bread and a pound of cheddar cheese."
 (a) Johnny's mother pleaded with John to run down to the grocer's and get her a loaf of French bread and a pound of cheddar cheese.
 (b) Johnny asked his mother to run down to the grocer's and get a loaf of French bread and a pound of cheddar cheese.
 (c) Johnny's mother asked him to run down to the grocer's and get a loaf of French bread and a pound of cheddar cheese.
 (d) Johnny commanded his mother to go down to the grocer and buy a loaf of French bread and a pound of cheddar cheese.
33. Malini said that she would go for the meeting.
 (a) Malini said, "I go for the meeting".
 (b) Malini said, "I am going for the meeting."
 (c) Malini said, "I will go for the meeting."
 (d) Malini said, "I went for the meeting"
34. "Please don't cry", he said.
 (a) He begged that I should not cry.
 (b) He begged me not to cry.
 (c) He said to please him and not cry.
 (d) He said to me to not to cry.
35. The doctor said, "Well. what can I do for you?"
 (a) The doctor wanted to know what he could do for her.
 (b) The doctor said that he couldn't do anything for her.
 (c) The doctor asked what he couldn't do for her.
 (d) The doctor wondered what he could do for her.
36. You will say, "Ganesh works hard."
 (a) You will say that Ganesh works hard.
 (b) You say that Ganesh works hard.
 (c) You said that Ganesh worked hard.
 (d) You will say that Ganesh has worked hard.
37. She said, "I was sleeping when Julie called."
 (a) She said that she had been sleeping when Julie called.
 (b) She said that was sleeping when Julie called.
 (c) She said that she was sleeping when Julie had called.
 (d) She said that she had been sleeping when Julie had called.
38. She said that they had gone to the cinema and then to a Chinese restaurant.
 (a) She said, "We went to the cinema and then to a Chinese restaurant."
 (b) She said, "We have gone to the cinema and then to a Chinese restaurant."
 (c) She said, "We had gone to the cinema and then to a Chinese restaurant."
 (d) She said, "We go to the cinema and then to a Chinese restaurant."
39. "Could you tell me where the post office is?"
 (a) She asked where the post office was.
 (b) She asked me to tell her where the post office was.
 (c) She asked me that where the post office was.
 (d) She asked me to tell her where was the post office.
40. He said, 'I must leave early today.'
 (a) He said that he must leave early today.
 (b) He said that he had to leave early that day.
 (c) He said that he must leave early that day.
 (d) He said that he might leave early that day.

Answer Key

1. (a)	2. (c)	3. (b)	4. (d)	5. (c)	6. (b)	7. (c)	8. (d)	9. (b)	10. (c)
11. (c)	12. (a)	13. (d)	14. (d)	15. (d)	16. (c)	17. (c)	18. (a)	19. (b)	20. (c)
21. (c)	22. (b)	23. (a)	24. (b)	25. (a)	26. (a)	27. (d)	28. (a)	29. (d)	30. (a)
31. (c)	32. (c)	33. (c)	34. (d)	35. (a)	36. (a)	37. (d)	38. (c)	39. (b)	40. (b)

Answers with Explanations

1. **Option (a) is correct.**

Explanation: The structure of assertive sentences in reported speech is: wh-word + subject + verb. So only (a) fits the criterion.

2. **Option (c) is correct.**

Explanation: Since the reporting verb is in present tense, there will be no change in the transition to reporting speech. Hence, 'will' and 'tomorrow' are in tact.

3. **Option (b) is correct.**

Explanation: 'Suggested' is followed by 'to', and also need the past tense.

4. **Option (d) is correct.**

Explanation: The structure of interrogative sentence in reporting speech is: if + noun/pronoun + verb and the incorporation of past tense.

5. **Option (c) is correct.**

Explanation: In reported speech, 'tomorrow' changes to 'the next day' and 'may' changes to 'might'.

6. **Option (b) is correct.**

Explanation: The structure of interrogative sentence in reported speech is: wh-word + noun/pronoun + verb. Moreover 'had' takes the participle form of learn (learnt).

7. **Option (c) is correct.**

Explanation: The structure of interrogative sentence in reported speech is: wh-word + noun/pronoun + verb. The tense will be in past continuous (was not taking).

8. **Option (d) is correct.**

Explanation: The structure of interrogative sentence in reported speech is: wh-word + noun/pronoun + verb.

9. **Option (b) is correct.**

Explanation: Since the reporting verb is in past tense, the sentence will be in past 'enjoyed'.

10. **Option (c) is correct.**

Explanation: We do not use auxiliary 'do' in indirect speech, so 'did' changes to past perfect 'had'.

11. **Option (c) is correct.**

Explanation: In reported speech, Present continuous changes to Past continuous. So 'is going to be' changes to 'was going to be'.

12. **Option (a) is correct.**

Explanation: In reported speech, in yes/no questions, 'if' is applied and present indefinite changes to Past indefinite and 'now' changes to 'then'.

13. **Option (d) is correct.**

Explanation: In reported speech, with 'wh' questions, the structure is: wh word + noun/pronoun + verb and 'today' changes to 'that day'.

14. **Option (d) is correct.**

Explanation: In reported speech, in yes/no questions, 'if/whether' is applied and present indefinite changes (is) to Past indefinite (was).

15. **Option (d) is correct.**

Explanation: In reported speech, in yes/no questions, 'if' is applied. 'Can' changes to 'could'.

16. **Option (c) is correct.**

Explanation: In reported speech, with 'wh' questions, the structure is: wh word + noun/pronoun+ verb and present continuous changes to Past continuous.

17. **Option (c) is correct.**

Explanation: In reported speech, with 'wh' questions, the structure is: wh word + noun/pronoun+ verb and present indefinite changes to Past indefinite and 'tomorrow' changes to 'the following day'.

18. **Option (a) is correct.**

Explanation: In imperative sentences, the verb 'commanded' will take 'said to', whereas 'told to' and 'says to' are incorrect, 'not to' changes to 'Do not'.

19. **Option (b) is correct.**

Explanation: In reported speech, 'said to me' is written as 'told me'. Present perfect (have) changes to Past perfect (had).

20. **Option (c) is correct.**

Explanation: In reported speech, 'said to me' is written as 'told me'. Moreover, present indefinite (is) changes to Past indefinite (was).

21. **Option (c) is correct.**

Explanation: 'Remarked' will be changed to 'exclaimed' according to the sense of the sentence. Inverted commas will be removed and conjunction 'that' will be used. As the reported speech starts with 'what', it will be changed to 'very' in the indirect speech.

22. **Option (b) is correct.**

Explanation: 'Hello' changes to 'greeted' in reported speech.

23. **Option (a) is correct.**

Explanation: In reported speech, with 'wh' questions, the structure is: wh word + noun/pronoun+ verb and present indefinite (do) changes to Past indefinite (opened and closed).

24. **Option (b) is correct.**

Explanation: The original sentence contains an exclamation point, thus the reported speech should incorporate this. Option C uses present tense but reported speech should be in the past tense, thus (C) cannot be the answer.

25. **Option (a) is correct.**

Explanation: 'Said' changes to 'advised'. The imperative form is changed to the infinitive form (to+V1). 'Don't' changes to 'not to'.

26. **Option (a) is correct.**

Explanation: In reported speech, simple past (visited) change to past perfect (had visited).

27. **Option (d) is correct.**

Explanation: The personal pronoun 'you' changes to 'he' and 'my' changes to 'him'. 'please' changes to 'kindly'. The structure is: if + noun/pronoun+ verb.

28. **Option (a) is correct.**

Explanation: 'Do not take' will change to 'forbade/told/warned', and the sentence uses 'to' form of the verb.

29. **Option (d) is correct.**

Explanation: The reporting verb is in present tense so it will remain unchanged.

30. **Option (a) is correct.**

Explanation: 'Said to' changes to 'asked' in reporting speech. The structure is: if + noun/pronoun+ verb.

31. **Option (c) is correct.**

Explanation: The structure is: wh word + noun/pronoun+ verb. Present continuous (are you going) will change to (was going).

32. **Option (c) is correct.**

Explanation: An imperative sentence takes 'to+verb', and the present tense.

33. **Option (c) is correct.**

Explanation: 'Will' changes to 'would' in reporting speech.

34. Option (d) is correct.

Explanation: An imperative sentence takes 'to+verb', and the present tense. As the given Reported speech (Direct speech) is negative; thus 'not' before 'to-infinitive' is added in the Indirect Speech.

35. Option (a) is correct.

Explanation: The structure is: wh word + noun/pronoun+ verb 'Can' changes to 'could'.

36. Option (a) is correct.

Explanation: Since the reported verb is in future tense, the verb does not change in reported speech, neither does the verb within the inverted commas.

37. Option (d) is correct.

Explanation: In reported speech, Past Continuous (was sleeping) changes to Past Perfect Continuous (had been sleeping). (D) makes an unnecessary change – Julie had called.

38. Option (c) is correct.

Explanation: Past Perfect in direct speech does not change in reported speech.

39. Option (b) is correct.

Explanation: The structure is: wh word + noun/pronoun+ verb 'that' is never used in interrogative sentences.

40. Option (b) is correct.

Explanation: 'Must' changes to 'had to' in indirect speech.

○○○

Chapter 11

PUNCTUATION

Definition:

The art of separating composition by means of the marks into clauses and parts of sentences is called Punctuation. The punctuation has following marks.

- (1) Full Stop or Period (.)
- (2) Colon (:)
- (3) Semi Colon (;)
- (4) Comma (,)
- (5) Mark of Interrogation (?)
- (6) Mark of Exclamation (!)
- (7) Dash (-)
- (8) Inverted Commas ("")
- (9) Parenthesis [()]
- (10) Hyphen (-)
- (11) Apostrophe (')
- (12) Capital Letters (A to Z)

Full Stop or Period (.)

A full stop, also known as a period, is chiefly used to end a sentence. It is mostly used at the end of declarative sentences and imperative sentences. A full stop marks a longer pause than a comma and a semicolon. Furthermore, it marks the end of a thought and the beginning of another.

Rules	Examples
It is used at the end of a sentence.	A cynic is a man who knows the price of everything but the value of nothing.
It is used with many abbreviations.	P.M., etc., i.e., Mon., Jan., M.P., M.D., D.S.R., F.D.R.
It is used before academic degrees when they are abbreviated.	M.B.B.S., Ph.D., L.L.B

Colon:

A colon is a punctuation mark that's used to connect sentences, put emphasis on a word or phrase, or introduce a quote or explanation. Whatever follows a colon should be connected to what came before it.

A colon should be used to separate two independent clauses when the second clause is directly related to the first, or when the emphasis is on the second clause.

Rules	Examples
It is used to introduce a quotation.	He ended with the immortal words of Neil Young: "Rock and Roll can never die."

It is used to introduce formal lists enumerations.	Taylor Swift has written a few songs about John Mayer: "Dear John," "Paper Doll," and "Would've, Could've, Should've."
It is used in reference to time to separate hours from minutes.	10:15, 12:30, 2:45, 6:50
It is used following the situation in a formal business letter.	Dear Ms. Fouzia: Dear Sir, Gentlemen
It is used between the sentences grammatically independent but closely connected.	Study is to acquire a habit of thinking; no study is more important.

THE SEMICOLON

The semicolon is used to separate independent clauses in specific situations. It also separates a series of items which contain internal punctuation.

Rules	Examples
It is used to separate the main clauses of a sentence when they are not joined by coordinating conjunctions.	I do not like to eat chocolate; ice cream is my food of choice.
It is used to separate the clauses in a compound sentence.	He was a brave, large hearted man; and we all honoured him.
It separates main clauses of a sentence that are joined by conjunctive adverbs.	I do not like to ride the bus; however, I ride it when I visit my sister.
It is used to separate the items in a series when parts of the items are already separated by commas.	The winners are Shazia, first; Hasan, second; Beena, third.
It is used to separate clauses to emphasise by providing a longer pause between them.	The coconut tree gives us food; it gives us drink; it gives us roofing material; indeed, with nearly all the necessities.

PARENTHESES

Parentheses are used to explain the statement or provide explanatory information in the sentence.

Rules	Examples
Enclose definitions.	Medicare (Australia's universal health insurance scheme) guarantees all Australians access to a wide range of health and hospital services.

Parentheses are used to separate a portion of a sentence from the rest of the sentence.	She gained from heaven (It was all she wished) a son.
Brackets are used to enclose a correction or explanatory comment in quoted material.	She [Mrs. Azra Touffique] used to counsel Jamil at great length.

COMMA (,)

Rules	Examples
It is used to separate a set of words in the same construction of a sentence.	He has lost his lands, money, reputation and friends.
It is used to separate each pair of words connected by and, or.	High and low, rich and poor, wise and foolish, all must die.
It is used to separate the two nouns in a sentence which mean the same person or thing.	Paul, the apostle, was beheaded in the reign of Nero.
It is used to separate two or more coordinate adjectives.	John is a short, stocky, powerful wrestler.
It is used after adverbs and other introductory words.	Excuse me, should I come with you? Actually, that's an interesting point.
It is used to separate three or more verbs in a sentence.	He is writing, directing, and producing new TV-serial.
It normally follows participial phrases that introduce a sentence.	Grabbing her umbrella, Kate raced out of the house.
It is used to mark off noun clauses or adjective clauses.	I didn't know when he did it, and why he did it?
It is used with a question tag.	These willow trees are beautiful, aren't they?
It is used to set off non-essential clauses.	The business report, which the manager had edited, explained the agency's strategy.
Use commas to set off expressions that interrupt the sentence flow (<i>nevertheless, after all, by the way, on the other hand, however, etc.</i>).	I am, by the way, very nervous about this.
Use a comma before the word 'but' if it is joining two independent clauses.	Cleo is a good singer, but she's an even better dancer.
Use commas to enclose degrees or titles used with names.	Al Mooney, M.D., is here.
It is used to mark off the words that are used for addressing.	Lord of the universe, shield us and guide us.

MARK OF INTERROGATION (?)

Rules	Examples
A question mark is used after every direct question.	Are you dressed for the wedding?
A mark of interrogation is not used after an indirect question.	He asked what time it was.
A question mark is enclosed in parenthesis after a fact which is doubtful.	America was first visited by a white man in A.D. 1000 (?)
A mark of interrogation is used after each separate question in a series of questions.	If you poison us, do we not die? If you wrong us, shall we not revenge?

MARK OF EXCLAMATION (!)

The **exclamation mark** (!), known informally as a *bang* or a *shriek*, is used at the end of a sentence or a short phrase which expresses very strong feeling.

Rules	Examples
It is used to add a forceful emphasis to a declarative word, expression or sentence.	"Get out!" She screamed.
It is used after an expression of strong emotion.	How the mighty be fallen!
An interjection is a word that expresses emotion. It may be strong enough on its own merit not to require an exclamation mark. Thus, a mark of exclamation merely helps to strengthen it.	Oh, what a beautiful day!
The exclamation mark is placed either immediately after an interjection that begins a sentence, or at the end of the sentence introduced by an interjection.	Hark! Someone is singing a sweet song.

DASH (-)

A dash is a little horizontal line that floats in the middle of a line of text (not at the bottom: that's an underscore). It's longer than a hyphen and is commonly used to indicate a range or a pause. Dashes are used to separate groups of words, not to separate parts of words like a hyphen does.

Rules	Examples
It is used to indicate a sudden break or change of thought.	Are you — do you feel all right?
The dash is used to follow a direct quotation to indicate an interruption in discourse.	"Really, now you ask me," said Alice, very much confused, "I don't think"
It is used sometimes in the place of commas.	His clothes – dirty, shabby, torn – belied his circumstances.

The dash may be used as a substitute for the expressions that is, in other words, and namely.	He admits that there are two sides of every question – his own and the wrong side.
The dash may be used to set off a word or group of words to add an element of surprise or to slow an unexpected turn of thought.	A pun is the lowest form of humour – when you don't think of it first.
It is used before a summarising such as, all such, these and all these.	Barrymore, Gielgud, Evans – these were Hamlets in their time.

Inverted Commas (" "): Inverted commas are punctuation marks that are used in writing to show where speech or a quotation begins and ends.

Rules	Examples
Inverted commas or quotation marks are used to indicate and question.	But what is the use of saying, "Call not man happy till he dies."
A single quotation mark is used to enclose a quotation within a quotation.	"You might as well say," added the March Hare, "that 'I like what I get' is the same thing as 'I get what I like.'"

THE HYPHEN

Rules	Examples
The Hyphen is used to connect the parts of a compound word.	Passer-by, man-of-war, jack-of-all-trades.

EXERCISE

Choose the part of the sentence (a/b/c/d) that has an error.

- "There's no room for error" (a)/ said the engineer, (b)/ "so we have to double check every calculation." (c). No error. (d)
- In last week's New Yorker, (a)/ one of my favourite magazines, (b)/ I enjoyed reading Leland's article (c)/ How Not to Go Camping. (d)
- The following are (a)/ the primary colors: (b)/ red, blue and yellow. (c). No error. (d)
- "Whoever thought," said Helen, (a)/ "that Jack would be (b)/ elected class president" (c). No error. (d)
- "Whoever is elected secretary of the club (a)/-Ashley, or Chandra, or Aisha (b)/ must be prepared to do a great deal of work," (c)/ said Junita, the previous secretary. (d)
- It is a fine idea, (a)/ let us hope that (b) it is going to work. (c). No error. (d)
- In the words of Murphy's Law: (a)/ 'Anything that can go (b)/ wrong will go wrong.' (c). No error. (d)
- We will be arriving on Monday morning (a)/ – at least (b) I think so. (c). No error. (d)
- "Your cat is an exceptional animal," (a)/ the veterinarian said (b)/ "he has overcome tremendous odds." (c). No error. (d)
- I have to admit that I'm a Star Trek fan; (a)/ my favourite episode is (b) The Trouble with Tribbles. (c). No error. (d)

Choose the sentence which has the correct use of punctuation.

- (a) This assignment is extra credit only, however, we still need to hand it in.
(b) John finished all his homework; but Kathleen did not finish hers.
(c) Chocolate, my favourite treat, always makes me feel better after a bad day.
(d) The winning tenderer (a local company), signed the contract on Tuesday.

- (a) Although they were shaking and sweating, the firefighters, were relieved to feel the first drops of a downpour.
(b) The weather, I was happy to see was beginning to clear.
(c) Edgar Allan Poe's poem, *The Raven*, is a classic.
(d) "Pass me that thesaurus," said Matthew.
- (a) Geoff's cooking skills if you can call them skills left something to be desired.
(b) This is the guiding principle for our workplace: collaboration.
(c) Cleo's favourite activities are singing on stage, and relaxing in the sunshine.
(d) I still have to buy a gift, pack the suitcases and arrange for someone to water the plants while we're at the wedding.
- (a) Three of the people in my class – Tom, Dick and Harry – refused to join the demonstration.
(b) "Wretch" said the king. "What harm did I do thee that thou shouldst seek to take my life with your own hand?"
(c) I met a beautiful, European woman.
(d) Jeremy yelled, "We're over here."
- (a) A bouquet of flowers may be created using more than one type of flower- I'll have an, apple.
(b) A, bouquet of flowers may be created, using more than one type of flower. I'll have an, apple.
(c) A bouquet of flowers may be created using more than one type of flowers. I'll have an apple.
(d) A bouquet of flowers may be created, using more than one type of flower; I'll have- an apple.
- (a) The actor, knowing it was late, hurried home.
(b) The actor knowing it was late, hurried home.
(c) The actor, knowing, it was late; hurried home.
(d) The actor knowing it was late, hurried home.

17. (a) Our planning meetings will take place on Friday November 13 and Thursday December 10.
 (b) Our planning meetings will take place on Friday, November 13 and Thursday, December 10.
 (c) Our planning meetings will take place on Friday, November 13, and Thursday, December 10.
 (d) Our planning meetings will take place on: Friday, November 13 and Thursday, December 10.
18. (a) Toni Morrison wrote, "If there's a book that you want to read; but it hasn't been written yet, then you must write it."
 (b) Toni Morrison wrote, "If there is a book that you want to read, but it hasn't been written yet; then you must write it."
 (c) Toni Morrison wrote, "If there's a book that you want to read, but it hasn't been written yet, then you must write it."
 (d) Toni Morrison wrote- "If there's a book that you want to read, but it hasn't been written, yet, then you must write it."
19. (a) The gentleman, who is standing by the fireplace is a well-known composer.
 (b) The gentleman who is standing by the fireplace is a well-known composer.
 (c) The gentleman, who is standing by the fireplace, is a well-known composer.
 (d) The gentleman who is standing by the fireplace, is a well-known composer.
20. (a) Why do fairy tales always end with the sentence, "And they lived happily ever after"?
 (b) Why do fairy tales always end with the sentence? "And they lived happily ever after"?
 (c) Why do fairy tales always end with the sentence? "And they lived happily ever after."
 (d) Why do fairy tales always end with the sentence, "And they lived happily ever after."?
21. (a) Who said, "Why do fools fall in love"?
 (b) Who said, "Why do fools fall in love?"?
 (c) Who said, "Why do fools fall in love?"
 (d) Who said- "Why do fools fall in love?"
22. (a). "Do you really want to swim out to the raft,?" he asked "Why would you take such a risk?"
 (b) "Do you really want to swim out to the raft?" he asked? "Why would you take such a risk?"
 (c) "Do you really want to swim out to the raft?", he asked. "Why would you take such a risk?"
 (d) "Do you really want to swim out to the raft," he asked, "Why would you take such a risk?"
23. (a) Lucy- the girl sitting over there eating an ice cream cone is my best friend.
 (b) Lucy- the girl sitting over there eating an ice cream cone- is my best friend.
 (c) Lucy the girl- sitting over there eating an ice cream cone is my best friend.
 (d) Lucy the girl sitting over there eating an ice cream cone- is my best friend.
24. (a) I was late for school this morning the bus was full and I had to walk.
 (b) I was late for school this morning: the bus was full and I had to walk.
 (c) I was late for school this morning; the bus was full and I had to walk.
 (d) I was late for school this morning, the bus was full and I had to walk.
25. (a) We have to buy these things, a table, three chairs and two stools
 (b) We have to buy these things; a table, three chairs and two stools.
 (c) We have to buy these things: a table, three chairs and two stools.
 (d) We have to buy these things, 'A table, three chairs and two stools.'

Answer Key

1. (a)	2. (d)	3. (c)	4. (c)	5. (b)	6. (a)	7. (d)	8. (b)	9. (b)	10. (c)
11 (c)	12. (d)	13. (b)	14. (a)	15. (c)	16. (a)	17. (b)	18. (c)	19. (b)	20. (a)
21 (c)	22. (c)	23. (b)	24. (b)	25. (c)					

Answers with Explanations

1. **Option (a) is Correct.**
Explanation: The correct statement is: "There's no room for error," said the engineer, "so we have to double check every calculation." A comma is placed before 'said'.
2. **Option (d) is Correct.**
Explanation: The correct statement is: In last week's New Yorker, one of my favorite magazines, I enjoyed reading Leland's article "How Not to Go Camping." Hence (D) has the error.
3. **Option (c) is Correct.**
Explanation: The correct statement is: The following are the primary colors: red, blue, and yellow. Hence (C) has the error.
4. **Option (c) is Correct.**
Explanation: The correct statement is: "Whoever thought," said Helen, "that Jack would be elected class president?"
5. **Option (b) is Correct.**
Explanation: The correct statement is: "Whoever is elected secretary of the club--Ashley, or Chandra, or Aisha--must be prepared to do a great deal of work," said Jumita, the previous secretary.
6. **Option (a) is Correct.**
Explanation: The correct sentence is: It is a fine idea; let us hope that it is going to work. The sentence has the error of comma splice.
7. **Option (d) is Correct.**
Explanation: This is the correct sentence.

8. **Option (b) is Correct.**

Explanation: The correct sentence is: We will be arriving on Monday morning – at least, I think so.

9. **Option (b) is Correct.**

Explanation: The correct sentence is: “Your cat is an exceptional animal,” the veterinarian said, “he has overcome tremendous odds.”

10. **Option (c) is Correct.**

Explanation: The correct sentence is: I have to admit that I’m a Star Trek fan; my favourite episode is “The Trouble with Tribbles.”

11. **Option (c) is Correct.**

Explanation:

(a) This assignment is extra credit only; however, we still need to hand it in.

(b) John finished all his homework, but Kathleen did not finish hers.
(semicolon is used to separate the main clauses of a sentence when they are not joined by coordinating conjunctions.)

(c) CORRECT. (If you could remove the appositive without changing the meaning of the sentence, it is said to be *nonessential* and should be set off with commas.)

(d) The winning tenderer (a local company) signed the contract on Tuesday.
(A comma follows a closing bracket only if you would have used a comma if there were no brackets.)

12. **Option (d) is Correct.**

Explanation:

(a) Although they were shaking and sweating, the firefighters were relieved to feel the first drops of a downpour.

(b) The weather, I was happy to see, was beginning to clear.
(Commas are used to set off non-essential clauses)

(c) Edgar Allan Poe’s poem *The Raven* is a classic.
(The Raven is an essential appositive, which cannot be separated by commas).

(d) CORRECT.

13. **Option (b) is Correct.**

Explanation:

(a) Geoff’s cooking skills, if you can call them skills, left something to be desired.

(Commas are used to set off non-essential clauses. Both interrupters and parenthetical elements should be set off with commas.)

(b) CORRECT

(c) Cleo’s favourite activities are singing on stage and relaxing in the sunshine.

(d) I still have to buy a gift, pack the suitcases, and arrange for someone to water the plants while we’re at the wedding.
(When you are listing three or more items, commas should separate each element of the list.)

14. **Option (a) is Correct.**

Explanation:

(a) CORRECT.

(b) “Wretch!” said the king. “What harm did I do thee that thou shouldst seek to take my life with your own hand?”

(c) I met a beautiful European woman.

(d) Jeremy yelled, “We’re over here!”

15. **Option (c) is Correct.**

Explanation: (c) is correct. Comma cannot be used between an article and a noun. So after ‘a’ no comma is to be used.

16. **Option (a) is Correct.**

Explanation: (a) is correct. We use commas to set off nonessential words, clauses, and phrases; here the nonessential phrase is knowing it was late, because even after deleting them, the sentence makes sense.

17. **Option (b) is Correct.**

Explanation: (b) is correct. We use a comma to separate the day of the month from the year, and one after the year, also.

18. **Option (c) is Correct.**

Explanation: (c) is correct. Use a comma before the word ‘but’ if it is joining two independent clauses ; ‘theirs, is correct, as in ‘there is’.

19. **Option (b) is Correct.**

Explanation: (b) is correct. Comma are used to enclose clauses not essential to the meaning of a sentence, but the one in (b) – who is standing by the fireplace is an essential information to refer to the person who is a well-known composer. Hence, it is a restrictive clause.

20. **Option (a) is Correct.**

Explanation: (a) is correct. (b) and (c) seem that the answer to the first part of the question is: “And they lived happily ever after”. (d) wrong places full stop and question mark together.

21. **Option (c) is Correct.**

Explanation: (c) is correct. In reported speech, period, marks of exclamation and interrogation are followed by quotation marks. Moreover, the reporting verb is also followed by a comma. Only (c) fulfils the criterion.

22. **Option (c) is Correct.**

Explanation: (c) is correct. Only (c) fulfils the rule:. In reported speech, period, marks of exclamation and interrogation are followed by quotation marks and the reporting verb is also followed/preceded by a comma.

23. **Option (b) is Correct.**

Explanation: (b) is correct. Dash is used to set off appositives that contain commas. (An appositive is a noun or noun phrase that immediately follows and renames a noun or pronoun and is usually surrounded by commas.)

24. **Option (b) is Correct.**

Explanation: The second sentence explains the first sentence and hence colon to be used to connect these sentences.

25. **Option (c) is Correct.**

Explanation: Colon is used to introduce items in a long list if the introductory clause can stand by itself.

Chapter 12

ACTIVE AND PASSIVE VOICE

Definition:

The voice of a verb expresses whether the subject in the sentence has performed or received the action. For example,

- The watchman opens the door.
- The door is opened by the watchman.

1. **ACTIVE VOICE:** When an action performed by the subject is expressed by the verb, it is active voice. Active voice is used when more straightforward relation and clarity is required between the subject and the verb.

For example; *Hens lay eggs.*

Birds build nests.

Rama helps Hari.

This form of the verb shows that the person denoted by the subject does something. Rama (the person denoted as the subject) does something.

2. **PASSIVE VOICE:** When the action expressed by the verb is received by the subject, it is passive voice. Passive voice is used when the doer of the action is not known and the focus of the sentence is on the action and not the subject.

For example; *Eggs are laid by hens.*

Nests are built by birds.

Hari is helped by Rama.

This form of the verb shows that something is done to the person denoted by the Subject. Something is done to Hari (the person denoted by the Subject.)

Examples of Voice change:

Active Voice

1. Sita loves Savitri.
2. The mason is building the wall.
3. The peon opened the gate.
4. Some boys were helping the wounded man.
5. He will finish the work in a fortnight.
6. Who did this?
7. Why did your brother write such a letter?

Passive Voice

1. Savitri is loved by Sita.
2. The wall is being built by the mason.
3. The gate was opened by the peon.
4. The wounded man was being helped by some boys.
5. The work will be finished by him in a fortnight.
6. By whom was this done?
7. Why was such a letter written by your brother?

It will be noticed that when the verb is changed from the Active Voice to the Passive Voice, the object of the transitive verb in the active voice becomes the subject of

the verb in the passive voice. [Thus in sentence 1, /savitri which is the object of 'loves' in the active voice, becomes the subject of 'is loved' in the Passive Voice.] Since the object of a verb in the active voice becomes the subject of the passive form, it follows that only transitive verbs can be used in the passive voice, because an intransitive verb has no object.

Conversion of Active and Passive voice examples:

- Rita wrote a letter. (Subject + Verb + Object)
- A letter was written by Rita. (Object) + (Auxiliary Verb) + (Past Participle) + (by Subject).
- She cooks food. (Subject + Verb + Object)
- The food is cooked by her. (Object) + (Auxiliary Verb) + (Past Participle) + (by Subject)

Forms of Active and Passive Voice for All Tenses

Simple Present Tense

The structure of the sentence from active voice to passive voice is:

Subject + Verb-I + Object (ACTIVE VOICE)

He writes a letter.

Passive Voice:

A letter is written by him.

Subject + is/are + V₃ + by + Object

Active Sentences	Passive Sentences
He writes an essay.	An essay is written by him.
Sheena does the housework.	The housework is done by Sheena.
She cares for the rabbit.	The rabbit is being cared for by her.
Jacob always plays the guitar.	The guitar is always played by Jacob.

Present Continuous Tense

The structure of the sentence from active voice to passive voice is:

Active voice: He is writing a letter. Subject + is / am / are + verb + ing + Object

Passive Voice:

A letter is being written by him.

Subject + is/am/are + being + V₃ + by Object. (Subject of Active)

Active Sentences	Passive Sentences
They are eating bananas.	The bananas are being eaten by them.
Bob is drawing a diagram.	A diagram is being drawn by Bob.

Samta is playing the piano.	A piano is being played by Samta.
She is waiting for Reema.	Reema is being waited for by her.

Present Perfect Tense

The structure of the sentence from active voice to passive voice is:

Active voice: I have written a letter.

Subject + have / has + V₃ + Object

Passive voice:

A letter has been written by me.

Subject + has/have + been + V₃ + by + Object.

• Active Sentences	• Passive Sentences
• Has he done the work?	• Has the work been done by him?
• Have they left the apartment?	• Has the apartment been left by them?
• He has created this masterpiece.	• This masterpiece has been created by him.
• He has read the newspaper.	• The newspaper has been read by him.

Simple Past Tense

The structure of the sentence from active voice to passive voice is:

Active voice: I wrote a letter.

Subject + V₂ + Object.

Passive voice:

A letter was written by him.

Subject + was/were + V₃ + by + Object. (Subject of Active)

• Active Sentences	• Passive Sentences
• Ria paid the bills.	• The bills were paid by Ria.
• The teacher called the student.	• The student was called by the teacher.
• She did not buy the fruits.	• The fruits were not bought by her.

Past Continuous Tense

The structure of the sentence from active voice to passive voice is:

Active voice: Subject + was / were + (verb + ing) + Object

He was writing a letter.

Passive voice:

A letter was being written by him.

Subject + was + being + V₃ + by + Object

Active Sentences	Passive Sentences
They were waiting for him.	He was being waited for by them.

Astha was learning French.	French was being learnt by Astha.
She was playing kabaddi.	Kabaddi was being played by her.

Past Perfect Tense

The structure of the sentence from active voice to passive voice is:

Active Voice: Subject + had + V₃ + Object

He had written a letter.

Passive Voice:

A letter had been written by him.

Subject + had been + V₃ + by + (Agent) Object

Active Sentences	Passive Sentences
She had won the match.	The match had been won by her.
I had finished her work.	Her work had been finished by me.
He had missed the last metro.	The last metro had been missed by him.

Simple Future Tense

The structure of the sentence from active voice to passive voice is:

Active Voice: Subject + shall / will + V₁ + Object

I shall write a letter.

Passive Voice:

A letter will be written by me.

Subject + will be + V₃ + by + Object (Subject of Active)

Subject + will / shall be + V₃ + by + (Agent) Object

Active Sentences	Passive Sentences
He will write a letter.	A letter will be written by him.
He will repair her cycle.	Her cycle will be repaired by him.
He shall start the meeting	The meeting will be started by him.

Future Perfect Tense

The structure of the sentence from active voice to passive voice is:

Active Voice: Subject + will / shall have + V₃ + Object.

I shall have bought a pen.

Passive Voice:

A pen will have been bought by me.

Subject + will / shall have been + V₃ + by + Agent (Object)

I. INETERROGATIVE SENTENCES:

There are two types of interrogative sentences:

(1) Yes / No questions. (2) Wh-questions.

A. Yes / No questions:

(a) **Is/am/are.** The structure is:

Is / am / are / + subject + being + V₃ + by + agent?

Active	Passive
(1) Is he writing a letter?	Is a letter being written by him?
(2) Are they ploughing the field?	Is the field being ploughed by them?
(3) Am I catching the thief?	Is the thief being caught by me?

(b) **Do does.** The structure is:

Is / am / are / + subject + V₃ + by + agent?

Active	Passive
(1) Does he need a pen?	Is a pen needed by him?
(2) Do you like the rooms?	Are the rooms liked by you?
(3) Do you write a lesson?	Is a lesson written by you?

(c) **Did questions:**

Was / Were + subject + V₃ + by + agent?

Active	Passive
(1) Did he write a letter?	Was a letter written by him?
(2) Did she help you?	Were you helped by her?
(3) Did he phone you?	Were you phoned by him?

(d) **Was/were questions:**

Was/Were + subject + being + V₃ + by + agent?

Active	Passive
(1) Was she cooking food?	Was food being cooked by her?
(2) Were they playing chess?	Was the chess being played by them?
(3) Was he washing the clothes?	Were the clothes being washed by him?

(e) **Have / has / had questions:**

Has/Have/Had + subject + been + V₃ + by + agent?

Active	Passive
(1) Has he written a book?	Has a book been written by him?
(2) Have they played a match?	Has a match been played by them?
(3) Had she written a letter?	Had a letter been written by her?

(f) **Modal Auxiliary verbs questions:**

Modal Auxiliary + Subject + be + V₃ + by + agent?

Active	Passive
(1) Can she control the situation?	Can the situation be controlled by her?
(2) May he cross the river?	May the river be crossed by him?
(3) Should he cut the wood?	Should the wood be cut by him?

B. **'Wh' questions:**

'wh' words

Active	Passive
(1) Why have you broken the mirror?	Why has the mirror been broken by you?
(2) Why do you write a letter?	Why is a letter written by you?
(3) When will you return my pen?	When will my pen be returned by you?
(4) Why are you laughing at her?	Why is she being laughed at by you?
(5) Where did he put the things?	Where were the things put by him?

Who words

Active	Passive
(1) Who broke this beautiful glass?	By whom was this beautiful glass broken?
(2) Who solved that problem?	By whom was that problem solved?
(3) Who can break this stick?	By whom can this stick be broken?

Whom

Active	Passive
(1) Whom have you invited?	Who has been invited by you?
(2) Whom has he abused?	Who has been abused by him?
(3) Whom has she reported?	Who has been reported by her?

Sentences with Modals

Active	Passive
(1) Sita could help him.	He could be helped by Sita.
(2) I will write a letter.	A letter will be written by me.
(3) You must write a book.	A book must be written by you.

II. IMPERATIVE SENTENCES:Order-

Active	Passive
(1) Shut the window.	Let the window be shut.
(2) Bring a glass of water.	Let a glass of water be brought.
(3) Finish this work.	Let this work be finished.

Request -

Active	Passive
(1) Please open the door.	You are requested to open the door.
(2) Please help that old lady.	You are requested to help that old lady.
(3) Take this seat, please.	You are requested to take this seat.

Suggestion -

Active	Passive
(1) Help the disabled.	The disabled should be helped.
(2) Listen to me.	I should be listened to.
(3) Help the needy and poor.	The needy and poor should be helped.

Advice -

Active	Passive
(1) Work hard.	You are advised to work hard.
(2) Get up early.	You are advised to get up early.
(3) Do not smoke.	You are advised not to smoke.

Miscellaneous Structures**(A) It is time to + V-I + noun**

It is time + for + noun + to be + V₃ Form

Active	Passive
(1) It is time to close the office.	It is time for the office to be closed.
(2) It is time to supply food.	It is time for food to be supplied.
(3) It is time to stop writing.	It is time for the writing to be stopped.

(B) There:

There + be form of verb + noun + infinitive
Active structure

There + be form of verb + noun + to be + V₃.

Active	Passive
(1) There is a lot of work to complete.	There is a lot of work to be completed.
(2) There are six letters to write.	There are six letters to be written.
(3) There is no time to lose.	There is no time to be lost.

(c) Causative Verbs:

Active	Passive
(1) I made him polish my shoes.	He was made to polish my shoes.
(2) He made me sing a song.	I was made to sing a song.

(d) Double Passive:

(1) They looted the shop and took away the money. *Active*

The shop was looted and the money was taken away. *Passive*

(2) She wrote the book and got published. *Active*
The book was written and got published by her. *Passive*

(e) With phrases like They believed, People think, People say, We know, Everyone knows the start in the passive voice is: (1) It was/is believed that..... (2) He is known / said / believed to be.....

Active	Passive
(1) They believed that Mr. Shukla was very rich. Mr. Shukla was believed to be very rich	It was believed that Mr. Shukla was very rich.
(2) We know that some children go wise early. Some children are known to go wise early.	It is known that some children go wise early.

(F) Passive of Verbs Showing, State of Mind or Feelings:

Active	Passive
(1) I know her.	She is known to me. (Prep.—'to')
(2) He annoyed me.	I was annoyed with him. (Prep.—'with')
(3) He satisfied everybody.	Everybody was satisfied with him. (Prep.—'with')

(4) This magazine has greatly interested me.	I have been greatly interested in this magazine. (Prep.—'in')
(5) Her behaviour annoyed him.	He was annoyed at her behaviour. (Prep.—'at')

Other words: Known to, amazed at, annoyed with (some person), annoyed at (something), startled at, vexed at, decorated with, filled with, surprised at, embodied in, etc.

EXERCISE

- The management is planning a new project with my company.
 - A new project was being planned with my company by the committee.
 - A new project is being planned with my company by the committee.
 - A new project had been planned with my company by the committee.
 - A new project is planned with my company by the management.
- The melody has been composed wonderfully by A.R. Rahman.
 - A.R. Rahman had composed the melody wonderfully.
 - A.R. Rahman wonderfully composed the melody.
 - A.R. Rahman has been composing the melody wonderfully.
 - A.R. Rahman has composed the melody wonderfully.
- The army was helping the people to reach their homes.
 - The people had been helped to reach their homes.
 - The people were helped by the army to reach their homes.
 - The people were being helped by the army to reach their homes.
 - The people was helped to reach their homes.
- The housekeeper arranged the furniture in order.
 - The furniture were to be arranged in order by the housekeeper.
 - The furniture was arranged in order by the housekeeper.
 - The furniture were being arranged in order by the housekeeper.
 - The furniture had been arranged in order by the housekeeper.
- Rishaani will speak to her parents tomorrow.
 - Her parents will be spoken to by Rishaani tomorrow.
 - Her parents will speak to Rishaani tomorrow.
 - Her parents will be spoken by Rishaani tomorrow.
 - Her parents will be speaking by Rishaani tomorrow.
- Old workers are being replaced by skilled people in this factory.
 - Skilled people are replacing old workers in this factory.
 - Skilled people were replacing old workers in this factory.
 - Skilled people have replaced old workers in this factory.
 - Skilled people is being replaced old workers in this factory.
- Weren't you told about the new policies of resigning?
 - Was anyone telling you about the new policies of resigning?
 - Wasn't anyone telling you about the new policies of resigning?
 - Didn't anyone tell you about the new policies of resigning?
 - Hadn't anyone told you about the new policies of resigning?
- The government is worrying about the severe natural calamity in the northern region.
 - The severe natural calamity in the northern region is being worried about by the government.
 - The severe natural calamity in the northern region is worrying about the government.
 - The severe natural calamity in the northern region has been worried about by the government.
 - The severe natural calamity in the northern region is been worried about by the government.
- Does he purchase books from the store?
 - Did he purchase the books from the store?
 - Had he purchased the books from the store?
 - Are books purchased by him from the store?
 - Did he purchased the books from the store?
- I paid fifty thousand for the payment of tax.
 - Fifty thousand has paid by me for the payment of tax.
 - Fifty thousand had paid by me for the payment of tax.
 - Fifty thousand is paid by me for the payment of tax.
 - Fifty thousand was paid by me for the payment of tax.

11. The invigilator is advising the students not to carry calculators into the examination hall.
 - (a) The students are being advised by the invigilator not to carry calculators into the examination hall.
 - (b) The students have been advised not to carry calculators into the examination hall by the invigilator.
 - (c) The students were advised not to carry calculators into the examination hall.
 - (d) The students were advised the invigilator not to carry calculators into the examination hall.
12. The sunrays burned my face.
 - (a) My face is burned by the sunrays.
 - (b) My face has been burned by the sunrays.
 - (c) My face is being burned by the sunrays.
 - (d) My face was burned by the sunrays.
13. Help the needy people.
 - (a) The needy people are helping.
 - (b) Let the needy people be helped.
 - (c) The needy people are been helped.
 - (d) Let the needy people being helped.
14. Someone stole his traveller's cheques when he was travelling in Europe.
 - (a) His traveller's cheques was stole when he was travelling in Europe.
 - (b) Someone had been stole his traveller's cheques when he was travelling in Europe.
 - (c) His traveller's cheques were stolen when he was travelling in Europe.
 - (d) His traveller's cheques are stole when he was travelling in Europe.
15. The manager keeps the work pending.
 - (a) The work has been kept pending by the manager
 - (b) The work was kept pending by the manager.
 - (c) The work is kept pending by the manager.
 - (d) The work are being kept pending by the manager.
16. Martin drew a picture of the snow-capped mountains.
 - (a) A picture of the snow-capped mountains is being drawn by Martin.
 - (b) A picture of the snow-capped mountains is drawn by Martin.
 - (c) A picture of the snow-capped mountains was drawn by Martin.
 - (d) A picture of the snow-capped mountains has
17. This beautiful story was written by Maya.
 - (a) Maya is writing this beautiful story.
 - (b) Maya writes this beautiful story.
 - (c) Maya wrote this beautiful story.
 - (d) Maya was writing this beautiful story.
18. Did the problems you had to face discourage you?
 - (a) Are you discouraged by the problems you had to face?
 - (b) Have you been discouraged by the problems you have to face?
 - (c) Are you being discouraged by the problems you had to face?
 - (d) Were you discouraged by the problems you had to face?
19. They offered me a chair.
 - (a) A chair is offered to me by them.
 - (b) A chair was being offered to me.
 - (c) I was offered a chair by them.
 - (d) I offered a chair to them.
20. They are constructing a residential youth hostel.
 - (a) A residential youth hostel has been constructed by them.
 - (b) A residential youth hostel is constructed by them.
 - (c) A residential youth hostel is being constructed by them.
 - (d) A residential youth hostel was constructed by them.
21. People claim to have seen the wild and mad elephant in several cities.
 - (a) The wild and mad elephant is claimed to have been seen in several cities.
 - (b) The wild and mad elephant was claimed to be seen by people in several cities.
 - (c) The wild and mad elephant has been claimed to be seen by the people in several cities.
 - (d) The wild and mad elephant is being claimed to be seen in several cities.
22. The farmer is ploughing the fields.
 - (a) The fields are being ploughed by the farmer.
 - (b) The fields are ploughed by the farmer.
 - (c) The fields were ploughed by the farmer.
 - (d) The fields have been ploughed by the farmer.
23. He was taken to the hospital by his friends.
 - (a) His friends had taken him to the hospital.
 - (b) His friends have taken him to the hospital.
 - (c) His friends take him to the hospital.
 - (d) His friends took him to the hospital.
24. I have written this essay.
 - (a) This essay had been written by me.
 - (b) This essay is written by me.
 - (c) This essay has been written by me.
 - (d) This essay was written by me.
25. Who broke the wall painting?
 - (a) By whom was the wall painting broken?
 - (b) By who was the wall painting broken?
 - (c) By whom had the wall painting been broken?
 - (d) By whom is the wall painting broken?
26. Shalini will submit the work at evening.
 - (a) The work would be submitted by Shalini at evening.
 - (b) The work will have been submitted by Shalini at evening.

- (c) The work will be submitted by Shalini at evening.
(d) The work would have been submitted by Shalini at evening.
27. Nothing can be gained without labour.
(a) We could gain nothing without labour.
(b) We cannot gain nothing without labour.
(c) We should gain nothing without labour.
(d) We can gain nothing without labour.
28. Can we rely on him?
(a) Can he be relied by us?
(b) Can we be relied on by him?
(c) Can he be relied on?
(d) Can we be relied by him?
29. The cabinet approved amendments to the Insurance Law Bill pending in the Rajya Sabha.
(a) Amendments pending in the Rajya Sabha, to the Insurance Law Bill was approved by the cabinet.
(b) Amendments to the insurance Law bill pending in the Rajya Sabha was approved.
(c) Amendments to the Insurance Law Bill was approved by the Cabinet pending in the Rajya Sabha.
(d) Amendments to the Insurance Law Bill pending in the Rajya Sabha, were approved by the Cabinet.
30. Mind your own business.
(a) You ordered to mind your own business.
(b) You are to mind your own business.
(c) You are asked to mind your own business.
(d) You should to mind your own business.
31. Please read out this letter to me.
(a) You are requested to read out this letter to me.
(b) You are ordered to read out this letter to me.
(c) You read out this letter to me.
(d) Please read out this letter to me.
32. He is not a man whom you should invite.
(a) He is should not be invited by you.
(b) This man who should not be invited by you.
(c) He is not the man who should be invited by you.
(d) He is not the man who is be invited by you.
33. A child could understand his theory.
(a) His theory was understood by a child.
(b) His theory could be understood by a child.
(c) His theory can be understood by a child.
(d) His theory is being understood by a child.
34. Why do you spend so much money?
(a) Why is so much money being spent by you?
(b) Why is so much money spent by you?
(c) Why was so much money spent by you?
(d) Why has so much money been spent by you?
35. The children sang the National Anthem with great enthusiasm.
(a) The National Anthem was sung with great enthusiasm by the children.
(b) The National Anthem is sung with great enthusiasm by the children.
(c) The National Anthem has been sung with great enthusiasm by the children.
(d) The National Anthem was being sung with great enthusiasm by the children.
36. He incurred heavy losses in business.
(a) Heavy losses are being incurred in business by him.
(b) Heavy losses are incurred in business by him.
(c) Heavy losses were incurred in business by him.
(d) Heavy losses was incurred in business by him.

Answer Key

1. (b)	2. (d)	3. (c)	4. (b)	5. (a)	6. (a)	7. (c)	8. (a)	9. (c)	10. (d)
11. (a)	12. (d)	13. (b)	14. (c)	15. (c)	16. (c)	17. (c)	18. (d)	19. (c)	20. (c)
21. (a)	22. (a)	23. (d)	24. (c)	25. (a)	26. (c)	27. (d)	28. (c)	29. (d)	30. (c)
31. (a)	32. (c)	33. (b)	34. (b)	35. (a)	36. (c)				

Answers with Explanations

1. Option (b) is Correct.

Explanation: The structures for active/passive voices in the specific tense are:

Active: Subject + is/are/am + verb (ing) + object...

Passive: Object + is/are/am + being + verb (IIIrd form) + by + subject...

So, with the help of the above structures, we can convert the given sentence into passive voice as:

A new project is being planned with my company by the management.

2. Option (d) is Correct.

Explanation: The structures for passive/active voices are:

Passive: Object + has/have + been + verb (V_3 form) + by + subject.

Active: Subject + has/have + verb (V_3 form) + object.

3. Option (c) is Correct.

Explanation: To convert the sentence into passive voice:

Object + Helping Verb + Verb V₃ Form + By + Subject

The object 'people' should be written before the subject 'the army.' The tense is past continuous. Thus, the rule to be followed is:

Active voice: Subject + was/were + V₁ + ing + Object

Passive Voice: Object + was/were + being + V₃ + by + Subject

4. Option (b) is Correct.

Explanation: The object 'furniture' should be written before the subject 'housekeeper.' The tense is simple past; thus the rule to be followed is:

Active voice: Subject + V₂ + Object

Passive Voice: Object + was/were + V₃ + by + Subject

5. Option (a) is Correct.

Explanation: The formula for changing a future tense sentence to passive form is:

Object + will + be + past participle + by + Subject

Here 'Rishaani' is the subject, 'spoken to' is a verb and 'parents' is the object.

6. Option (a) is Correct.

Explanation: The structures for active/passive voices are:

Active Voice: Subject + is/am/are + Verb (V₁ + ing form) + Object

Passive Voice: Object + is/am/are + being + Verb (V₃ form) + by + Subject

7. Option (c) is Correct.

Explanation: The structures for active/passive voices are:

Active Voice: Did + Subject + Main Verb + Object?

Passive Voice: Helping Verb (was/were as question word) + Object + Verb (V₃ form) + by + Subject?

8. Option (a) is Correct.

Explanation: The structures for active/passive voices are:

Active Voice: Subject + is/ am/ are + Verb (V₁ + ing form) + Object

Passive Voice: Object + is/am/are + being + Verb (V₃ form) + by + Subject

9. Option (c) is Correct.

Explanation: Structure of active/passive voice is:-

Active voice = Does + Subject + V₁ + Object?

Passive voice = Is/am/are + Object + V₃ + by + subject.

10. Option (d) is Correct.

Explanation: Structure of active/passive voice is:

Active voice: Subject + V₂ + object.

Passive voice: Object + was/were + V₃ + by + subject.

11. Option (a) is Correct.

Explanation: When we convert this sentence into passive voice, the subject 'The invigilator' of the

active voice becomes the object, the object 'the students' becomes the subject.

12. Option (d) is Correct.

Explanation: The structure of the given sentence is as follows:

Subject + V₂ + Object. (Active Voice). Subject (objective case) + was/were + V₃ + Object (subjective case). (Passive Voice).

13. Option (b) is Correct.

Explanation: The imperative sentence in the passive voice takes the following form: Let + object + be + past participle. Thus, the object of the active voice 'the needy people' will come after 'let' which will further be followed by 'be' and past participle form of help, i.e., helped.

14. Option (c) is Correct.

Explanation: The subject in active voice becomes object in passive voice. The structure is: is/ are/ am / were being + verb 3rd form.

15. Option (c) is Correct.

Explanation: Use an appropriate helping verb in passive form according to the tense of the active form. (Present indefinite - is/am/are + V₃). The active voice is in present tense, so the passive voice will take 'is'.

16. Option (c) is Correct.

Explanation: The structure of the sentence from active voice to passive voice is:

Active voice: Subject + V₂ + Object.

Passive voice: Subject + was + V₃ + by + Object. (Subject of Active)

17. Option (c) is Correct.

Explanation: The structure of the sentence from active voice to passive voice in simple past tense is:

Active voice: Subject + V₂ + Object.

Passive voice: Subject + was + V₃ + by + Object. (Subject of Active)

18. Option (d) is Correct.

Explanation: In Did questions the correct pattern is: Was / Were + subject + V₃ + by + agent?

19. Option (c) is Correct.

Explanation: The structure of the sentence from active voice to passive voice in simple past tense is:

Active voice: Subject + V₂ + Object.

Passive voice: Subject + was + V₃ + by + Object. (Subject of Active)

20. Option (c) is Correct.

Explanation: The structure of the sentence from active voice to passive voice is:

Active voice: Subject + is / were + (verb + ing) + Object

Passive voice: Subject + is + being + V₃ + by + Object

21. Option (a) is Correct.

Explanation: The structure of the sentence from active voice to passive voice is:

Active voice: Subject + have / has + V₃ + Object

Passive voice: Subject + has been + V₃ + by + Object.

22. Option (a) is Correct.

Explanation: The structure of the sentence from active voice to passive voice is:

Active voice: Subject + is/are + (verb + ing) + Object

Passive voice: Subject + is/are + being + V₃ + by + Object

23. Option (d) is Correct.

Explanation: The structure of the sentence from active voice to passive voice in simple past tense is:

Active voice: Subject + V-II + Object.

Passive voice: Subject + was + V₃ + by + Object. (Subject of Active)

24. Option (c) is Correct.

Explanation: The structure of the sentence from active voice to passive voice is:

Active voice: Subject + have / has + V₃ + Object

Passive voice: Subject + has been + V₃ + by + Object.

25. Option (a) is Correct.

Explanation: In simple past tense, 'broke' changes to 'was...broken'.

26. Option (c) is Correct.

Explanation: The structure in simple future tense is: Subject + will be + V₃ + by + Object (Subject of Active)

27. Option (d) is Correct.

Explanation: In active voice, modal 'can be' changes to 'can'.

28. Option (c) is Correct.

Explanation: In active voice, modal 'can be' changes to 'can'. Modal Auxiliary verbs questions:

Modal Auxiliary + Subject + be + V₃ + by + agent.

29. Option (d) is Correct.

Explanation: The structure of the sentence from active voice to passive voice in simple past tense is:

Active voice: Subject + V₂ + Object.

Passive voice: Subject + was/were + V₃ + by + Object. (Subject of Active)

30. Option (c) is Correct.

Explanation: The sentence type is imperative, that involves asking someone to do something, so it should include 'asked'.

31. Option (a) is Correct.

Explanation: In imperative sentence, a request, should start with 'you are requested...'

32. Option (c) is Correct.

Explanation: The correct form of modal is 'should be' in passive voice.

33. Option (b) is Correct.

Explanation: The subject 'child' changes to be the object, and the object (his theory) takes the role of the subject. 'Could' remains intact.

34. Option (b) is Correct.

Explanation: In simple past tense, 'do' changes to 'is/are'. The structure is: why + is/are + V-III + by (agent)?

35. Option (a) is Correct.

Explanation: The structure of the sentence from active voice to passive voice in simple past tense is:

Active voice: Subject + V-II + Object.

Passive voice: Subject + was/were + V-III + by + Object. (Subject of Active)

36. Option (c) is Correct.

Explanation: The structure of the sentence from active voice to passive voice in simple past tense is:

Active voice: Subject + V-II + Object.

Passive voice: Subject + was/were + V-III + by + Object. (Subject of Active)

RELATIONS AND FUNCTIONS

- **Cartesian product:** A Cartesian product of two sets A and B , written as $A \times B$, is the set containing **ordered** pairs from A and B . That is, if $C = A \times B$, then each element of C is of the form (x, y) , where $x \in A$ and $y \in B$.

$$A \times B = \{(x, y) : x \in A, \text{ and } y \in B\}$$

- (i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal, i.e., $(x, y) = (u, v)$ if and only if $x = u, y = v$.
- (ii) If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = p \times q$.
- (iii) $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.
- $$R = \{(x, y) : x \in A \text{ and } y \in B \text{ such that } R \subseteq A \times B\}$$

- **Definition:** A relation R from a non-empty set A to a non empty set B is a subset of the Cartesian product set $A \times B$. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$.

The set of all first elements in a relation R , is called the domain of the relation R , and the set of all second elements called images, is called the range of R . A relation may be represented either by the Roster form or by the set builder form, or by an arrow diagram which is a visual representation of a relation.

- (i) If $n(A) = p, n(B) = q$; then the $n(A \times B) = pq$ and the total number of possible relations from the set A to set $B = 2^{pq}$.
- **Types of relation:**
- (i) **Reflexive:** A relation R in the set A is said to be reflexive relation if, $a \in A, aRa \Rightarrow (a, a) \in R \forall a \in A$
- (ii) **Symmetric:** A relation R in the set A is said to be symmetric relation if $a, b \in A, aRb$ and $bRa \Rightarrow (a, b) \in R$ then $(b, a) \in R$ for some $a, b \in A$
- (iii) **Transitive:** A relation R in the set A is said to be transitive relation if $a, b, c \in A, aRb$ and $bRc \Rightarrow aRc$ for some $a, b, c \in A$
- (iv) **Equivalence relation:** A relation R in the set A is said to be an equivalence relation if R is reflexive, symmetric and transitive in set A .

➤ **Fundamentals:**

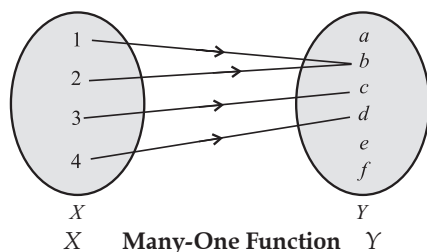
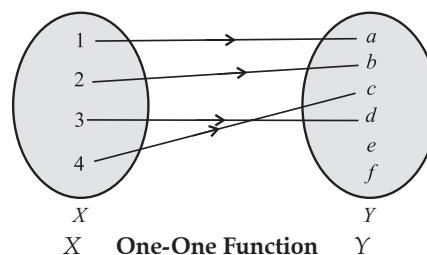
- A relation R from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.
- The set of all the first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R .
- The set of all second elements in a relation R from a set A to a set B is called the range of the relation R .
- The whole set B is called the co-domain of the relation R .

➤ **Types of Relation:**

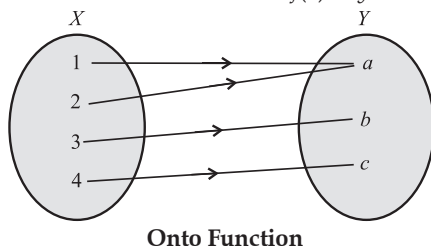
- **Empty Relation :** A relation R in a set A is called empty relation, if no element of A is related to any element of A , i.e., $R = \emptyset \subset A \times A$.
- **Universal Relation :** A relation R in a set A is called universal relation, if each element of A is related to every element of A , i.e., $R = A \times A$.
- A relation R in a set A is called,
 - (i) reflexive, if $(a, a) \in R$, for every $a \in A$.
 - (ii) symmetric, if $(a_1, a_2) \in R \Rightarrow (a_2, a_1) \in R, \forall a_1, a_2 \in A$.
 - (iii) transitive, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R, \forall a_1, a_2, a_3 \in A$.
- **Equivalence Relation :** A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

➤ **Types of Function**

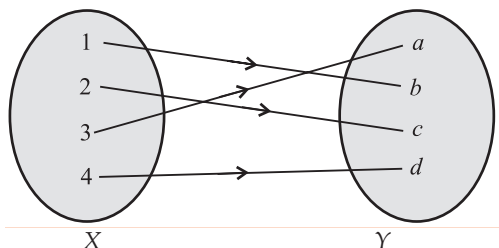
- **One-One (Injective) and Many-One :** A function $f : X \rightarrow Y$ is defined to be one-one (or injective), if the images of distinct elements of X have distinct images in Y under f , i.e., $\forall x_1, x_2 \in X, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Otherwise, f is called many-one.



- **Onto (Surjective) :** A function $f : X \rightarrow Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under f , i.e., $\forall y \in Y, \exists$ an element x in X such that $f(x) = y$.



- **One-One and Onto (Bijective) :** A function $f: X \rightarrow Y$ is said to be one-one and onto (or bijective), if f is both one-one and onto.



Bijective Function

If f is a function from a set A to Set B , then we write,

$$f: A \rightarrow B \text{ or } A \xrightarrow{f} B$$

and it is read as f is a function from A to B .

If $(a, b) \in f \Rightarrow f(a) = b$.

Here, b is the **image** of a under f and a is called the **pre-image** of b under f .

Few Properties to be known :

- (i) For any real number x , we have

$$\sqrt{x^2} = |x|$$

- (ii) If n is an integer and x is a real number between n and $n + 1$, then

(a) $[-n] = -[n]$

(b) $[x + k] = [x] + k$ for any integer

(c) $[-x] = -[x] - 1$

(d) $[-x] = -[x] + 1$, where $x \in \mathbb{R} - \mathbb{Z}$

- (iii) For any real function,

$f: D \rightarrow \mathbb{R}$ and $n \in \mathbb{N}$, we define

$$\underbrace{(f \circ f \circ \dots \circ f)}_{n \text{ times}}(x) = \underbrace{f(x) \circ f(x) \circ \dots \circ f(x)}_{n \text{ times}} = \{f(x)\}^n, \forall x \in D$$

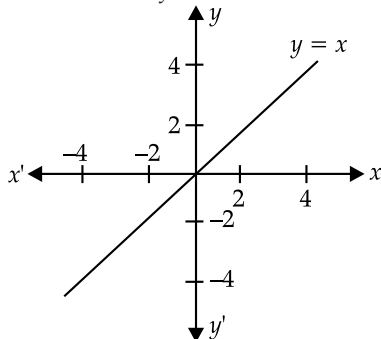
Thus, if a function f is expressed as the set of ordered pairs, then the domain of f is the **set of all first elements** of ordered pairs and the range of f is the **set of second elements** of ordered pairs of f i.e.,

$$D_f = \text{Domain of } f = \{a : (a, b) \in f\}$$

$$R_f = \text{Range of } f = \{b : (a, b) \in f\}$$

Let A and B be two non-empty finite sets such that $n(A) = p$, and $n(B) = q$, then number of functions from A to $B = q^p$.

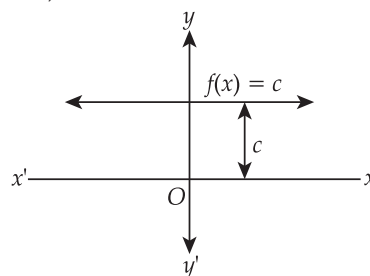
Identity function: Let R be the set of real numbers. A real valued function f is defined as $f: R \rightarrow R$ by $y = f(x) = x$ for each value of $x \in R$. Such a function is called the identity function.



Domain = $\mathbb{R}$ and Range = $\mathbb{R}$

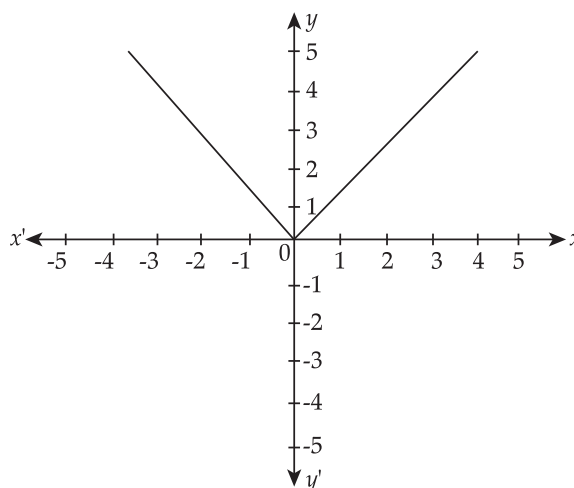
Constant function:

The function $f: R \rightarrow R$ defined by $f(x) = C$ for each $x \in R$ is called constant function. (where C is a constant)



Domain = R and Range = $\{C\}$

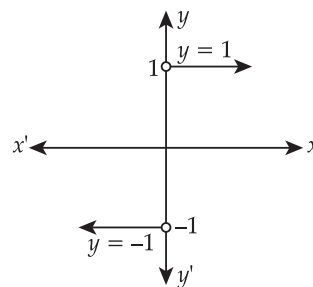
Modulus function: The function $f: R \rightarrow R$ defined by $f(x) = |x|$ for each $x \in R$ is called modulus function or absolute valued function.



$$\text{i. e., } f(x) = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Domain = $\mathbb{R}$ and Range = $\mathbb{R}^+ \cup \{0\} = \{x : x \in \mathbb{R}; x \geq 0\}$

Signum function : Let R be the set of real numbers, then the function $f: R \rightarrow R$ defined by

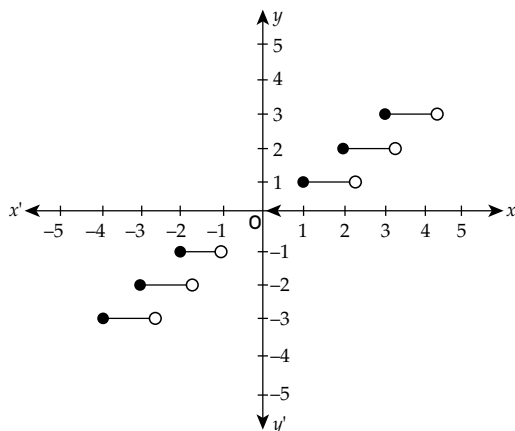


$$f(x) = \frac{|x|}{x} = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

is known as signum function.

Domain = $\mathbb{R}$ and Range = $\{-1, 0, 1\}$

Greatest integer function : The function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = [x]$, $x \in \mathbb{R}$ assumes the value of the greatest integer, less than or equal to x , such a function is called the greatest integer function.



From the definition of $[x]$, we have

$$[x] = -1, \text{ for } -1 \leq x < 0$$

$$= 0, \text{ for } 0 \leq x < 1$$

$$= 1, \text{ for } 1 \leq x < 2$$

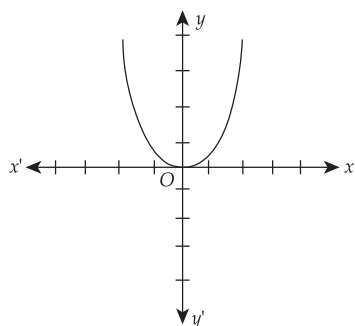
$$= 2, \text{ for } 2 \leq x < 3$$

Domain = $\mathbb{R}$ and Range = $\mathbb{Z}$

Polynomial Functions:

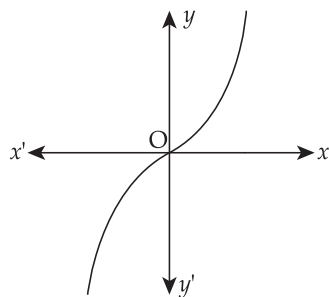
Graphs of some other important functions

- $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^2$



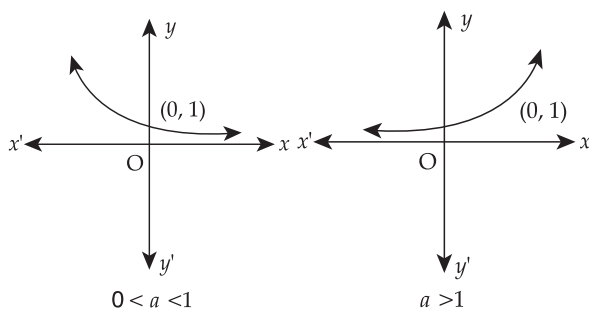
Domain = $\mathbb{R}$ and Range = $[0, \infty)$

- $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3$



Domain = $\mathbb{R}$ and Range = $\mathbb{R}$

- Exponential function,** $f: \mathbb{R} \rightarrow \mathbb{R}, f(x) = a^x, a > 0, a \neq 1$

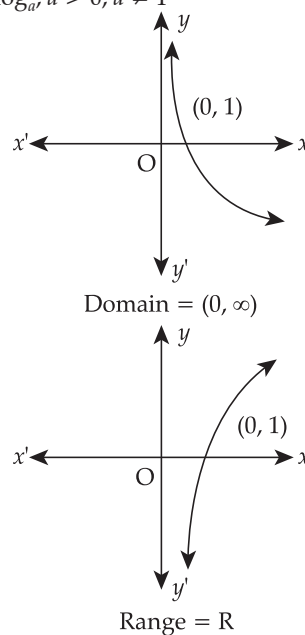


Domain = $\mathbb{R}$ and Range = $(0, \infty)$

- Natural exponential function, $f(x) = e^x$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \infty, 2 < e < 3$$

- Logarithmic function,** $f: (0, \infty) \rightarrow \mathbb{R}; f(x) = \log_a x, a > 0, a \neq 1$



Domain = $(0, \infty)$

Range = $\mathbb{R}$

Domain = $(0, \infty)$ and Range = $\mathbb{R}$

- Natural logarithmic function $f(x) = \log e^x$ or $\ln x$

Algebra of Real Functions:

Let $f: D_1 \rightarrow \mathbb{R}$ and $g: D_2 \rightarrow \mathbb{R}$ be two real functions with domain D_1 and D_2 , respectively. Then, algebraic operations such as addition, subtraction, multiplication and division of two real functions are given below.

- (i) **Addition of two real functions :** The sum function $(f + g)$ is defined by

$$(f + g)(x) = f(x) + g(x), \forall x \in D_1 \cap D_2$$

The domain of $(f + g)$ is $D_1 \cap D_2$.

- (ii) **Subtraction of two real functions :** The difference function $(f - g)$ is defined by

$$(f - g)(x) = f(x) - g(x), \forall x \in D_1 \cap D_2$$

The domain of $(f - g)$ is $D_1 \cap D_2$.

- (iii) **Multiplication of two real functions :** The product function (fg) is defined by

$$(fg)(x) = f(x) \cdot g(x), \forall x \in D_1 \cap D_2$$

The domain of (fg) is $D_1 \cap D_2$.

- (iv) **Quotient of two real functions** : The quotient function is defined by

$$\frac{f}{g}(x) = \frac{f(x)}{g(x)}, \forall x \in D_1 \cap D_2 - [x : g(x) \neq 0]$$

The domain of $\left(\frac{f}{g}\right)$ is $D_1 \cap D_2 - [x : g(x) \neq 0]$

- (v) **Multiplication of a real function by a scalar** : The scalar multiple function cf is defined by

$$(cf)(x) = c \cdot f(x), \forall x \in D_1$$

where, c is scalar (real number).

The domain of cf is D_1 .

EXERCISE

- Consider the following statements in respect of the relation R in the set N of natural numbers defined by xRy if $x^2 - 5xy + 4y^2 = 0$:
 - R is reflexive
 - R is symmetric
 - R is transitive
 Which of the above statements is/are correct?
 - 1 only
 - 2 only
 - 1 and 2 only
 - 1, 2 and 3
- Consider the following statements in respect of any relation R on a set A :
 - R is reflexive, then R^{-1} is also reflexive
 - If R is symmetric, then R^{-1} is also symmetric
 - If R is transitive, then R^{-1} is also transitive
 Which of the above statements are correct?
 - 1 and 2 only.
 - 2 and 3 only.
 - 1 and 3 only.
 - 1, 2 and 3.
- Let R be a relation from N to N defined by $R = \{(x, y) : x, y \in N \text{ and } x^2 = y^3\}$. Which of the following are not correct?
 - $(x, x) \in R$ for all $x \in N$
 - $(x, y) \in R \Rightarrow (y, x) \in R$
 - $(x, y) \in R \text{ and } (y, z) \in R \Rightarrow (x, z) \in R$
 Select the correct answer using the code given below:
 - 1 and 2 only
 - 2 and 3 only
 - 1 and 3 only
 - 1, 2 and 3
- Let R be relation defined as xRy if and only if $2x + 3y = 20$, where $x, y \in N$. How many elements of the form (x, y) are there in R ?
 - 2
 - 3
 - 4
 - 6
- Consider the following statements in respect of relations and functions :
 - All relations are functions but all functions are not relations.
 - A relation from A to B is a subset of Cartesian product $A \times B$.
 - A relation in A is a subset of Cartesian product $A \times A$.
 Which of the above statements are correct?
 - 1 and 2 only
 - 2 and 3 only
 - 1 and 3 only
 - 1, 2 and 3
- Let $S = \{1, 2, 3, \dots\}$. A relation R on $S \times S$ is defined by xRy if $\log_a x > \log_a y$ when $a = \frac{1}{2}$. Then the relation is
 - reflexive only
 - symmetric only
 - transitive only
 - both symmetric and transitive
- Suppose $X = \{1, 2, 3, 4\}$ and R is a relation on X . If $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$, then which one of the following is correct?
 - R is reflexive and symmetric, but not transitive
 - R is symmetric and transitive, but not reflexive
 - R is reflexive and transitive, but not symmetric
 - R is neither reflexive nor transitive, but symmetric
- A relation R is defined on the set N of natural numbers as $xRy \Rightarrow x^2 - 4xy + 3y^2 = 0$. Then which one of the following is correct?
 - R is reflexive and symmetric, but not transitive.
 - R is reflexive and transitive, but not symmetric.
 - R is reflexive, symmetric and transitive.
 - R is reflexive, but neither symmetric nor transitive.
- Let X be the set of all citizens of India. Elements x, y in X are said to be related if the difference of their age is 5 years.
 Which one of the following is correct?
 - The relation is an equivalence relation on X
 - The relation is symmetric but neither reflexive nor transitive.
 - The relation is reflexive but neither symmetric nor transitive.
 - None of the above
- Consider the following relations from A to B where $A = \{u, v, w, x, y, z\}$ and $B = \{p, q, r, s\}$.
 - $\{(u, p), (v, p), (w, p), (x, q), (y, q), (z, q)\}$
 - $\{(u, p), (v, q), (w, r), (z, s)\}$
 - $\{(u, s), (v, r), (w, q), (u, p), (v, q), (z, q)\}$
 - $\{(u, q), (v, p), (w, s), (x, r), (y, q), (z, s)\}$
 Which of the above relations are not functions?
 - 1 and 2
 - 1 and 4
 - 2 and 3
 - 3 and 4
- Let S denote set of all integers. Define a relation R on S as ' aRb if $ab \geq 0$ where $a, b \in S$ '. Then R is :

- (a) Reflexive but neither symmetric nor transitive relation.
 (b) Reflexive, symmetric but not transitive relation.
 (c) An equivalence relation.
 (d) Symmetric but neither reflexive nor transitive relation.
12. If $f(x) = (a-x)^{\frac{1}{n}}$, $a > 0$ and $n \in \mathbb{N}$, then the value of $f(f(x))$ is—
 (a) $\frac{1}{n}$ (b) x (c) x^2 (d) $\frac{1}{x^2}$
13. Express the function $f: A \rightarrow \mathbb{R}$, $f(x) = x^2 - 1$, where $A = \{-4, 0, 1, 4\}$ as a set of ordered pairs.
 (a) $\{(-4, 15), (0, -1), (1, 0), (4, 15)\}$
 (b) $\{(-4, -15), (0, -1), (1, 0), (4, 15)\}$
 (c) $\{(4, 15), (0, -1), (1, 0), (4, 15)\}$
 (d) $\{(-4, 15), (0, -1), (1, 0)\}$
14. Let R be the relation in the set $\mathbb{N}$ given by $R = \{(a, b): a = b - 2, b > 6\}$. Choose the correct answer
 (a) $(2, 4) \in R$ (b) $(3, 8) \in R$
 (c) $(6, 8) \in R$ (d) $(8, 7) \in R$
15. If set A and B have 3 and 4 elements respectively then the number of subsets of set $(A \times B)$ is:
 (a) 1024 (b) 2048
 (c) 512 (d) 4096
16. Let the sets be A, B, C, D then $(A \times B) \times (C \times D)$ is equivalent to—
 (a) $(A \times C) \times (B \times D)$ (b) $(A \times D) \times (B \times C)$
 (c) $(A \times C) \times (B \times D)$ (d) None of the above
17. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is
 (a) m^n (b) $n^m - 1$
 (c) $mn - 1$ (d) $2^{mn} - 1$
18. If $[x]^2 - 5[x] + 6 = 0$, where $[.]$ denote the greatest integer function, then
 (a) $x \in [3, 4]$ (b) $x \in (2, 3]$
 (c) $x \in [2, 3]$ (d) $x \in [2, 4]$
19. Range of $f(x) = \frac{1}{1 - 2\cos x}$ is
 (a) $\left[\frac{1}{3}, 1\right]$ (b) $\left[-1, \frac{1}{3}\right]$
 (c) $(-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$ (d) $\left[-\frac{1}{3}, 1\right]$
20. Let $f(x) = \sqrt{1+x^2}$, then
 (a) $f(xy) = f(x) \cdot f(y)$ (b) $f(xy) \geq f(x) \cdot f(y)$
 (c) $f(xy) \leq f(x) \cdot f(y)$ (d) None of these
21. Domain of $\sqrt{a^2 - x^2}$ ($a > 0$) is
 (a) $(-a, a)$ (b) $[-a, a]$
 (c) $[0, a]$ (d) $(-a, 0]$
22. If $f(x) = ax + b$, where a and b are integers, $f(-1) = -5$ and $f(3) = 3$, then a and b are equal to
 (a) $a = -3, b = -1$ (b) $a = 2, b = -3$
 (c) $a = 0, b = 2$ (d) $a = 2, b = 3$
23. Let set $X = \{1, 2, 3\}$ and a relation R is defined in X as:
 $R = \{(1, 3), (2, 2), (3, 2)\}$, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are
 (a) $\{(1, 1), (2, 3), (1, 2)\}$
 (b) $\{(3, 3), (3, 1), (1, 2)\}$
 (c) $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$
 (d) $\{(1, 1), (3, 3), (3, 1), (1, 2)\}$
24. If $R = \{(x, y): x, y \in \mathbb{Z}, x^2 + y^2 \leq 4\}$ is a relation in set $\mathbb{Z}$, then domain of R is
 (a) $\{0, 1, 2\}$ (b) $\{-2, -1, 0, 1, 2\}$
 (c) $\{0, -1, -2\}$ (d) $\{-1, 0, 1\}$
25. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A ?
 (a) $(1, 1)$ (b) $(1, 2)$
 (c) $(2, 2)$ (d) $(3, 3)$
26. Let the relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$. Then $[1]$, the equivalence class containing 1, is :
 (a) $\{1, 5, 9\}$ (b) $\{0, 1, 2, 5\}$
 (c) ϕ (d) A
27. Let R be the relation in the set $\mathbb{N}$ given by $R = \{(a, b) : a = b - 2, b > 6\}$, then:
 (a) $(2, 4) \in R$ (b) $(3, 8) \in R$
 (c) $(6, 8) \in R$ (d) $(8, 7) \in R$
28. Let $X = \{x^2 : x \in \mathbb{N}\}$ and the relation $f: \mathbb{N} \rightarrow X$ is defined by $f(x) = x^2, x \in \mathbb{N}$. Then, this function is
 (a) injective only (b) not bijective
 (c) surjective only (d) bijective
29. A function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 2 + x^2$ is
 (a) not one-one
 (b) one-one
 (c) not onto
 (d) neither one-one nor onto
30. Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and let $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B . Based on the given information, f is best defined as :
 (a) Surjective function (b) Injective function
 (c) Bijective function (d) None of these
31. If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is
 (a) 720 (b) 120
 (c) 0 (d) None of these
32. Let $A = \{1, 2, 3, \dots, n\}$ and $B = \{a, b\}$. Then the number of surjections from A into B is
 (a) ${}^nP_2$ (b) $2^n - 2$
 (c) $2^n - 1$ (d) None of these

Answer Key									
1. (a)	2. (d)	3. (d)	4. (b)	5. (b)	6. (c)	7. (a)	8. (d)	9. (b)	10. (c)
11. (c)	12. (b)	13. (a)	14. (c)	15. (d)	16. (a)	17. (d)	18. (c)	19. (b)	20. (c)
21. (b)	22. (b)	23. (c)	24. (b)	25. (b)	26. (a)	27. (c)	28. (c)	29. (d)	30. (b)
31. (c)	32. (b)								

Answers with Explanations

1. Option (a) is correct.

Explanation:

For reflexive,

Put, $y = x$

$$x^2 - 5x^2 + 4x^2 = 0$$

$$\therefore x R x$$

$\therefore R$ is reflexive.

So, statement 1 is correct.

For symmetric

Let, $x R y$

$$\Rightarrow x^2 - 4xy + 4y^2 = 0$$

$$\Rightarrow x = y \text{ or } x = 4y$$

If $x \neq y$ then $y \neq 4x$

$$\Rightarrow (y, x) \notin R$$

$\therefore R$ is not symmetric.

So, statement 2 is not correct.

For transitive

$$\text{Let, } x R y \Rightarrow x = y \text{ or } x = 4y \quad \dots(i)$$

$$\text{and } y R z \Rightarrow y = z \text{ or } y = 4z \quad \dots(ii)$$

From, (i) and (ii)

$$\text{If } x = 4y \text{ and } y = 4z$$

$$\text{Then, } x = 16z$$

$$\therefore (x, 2) \notin R$$

$\therefore R$ is not transitive.

So, statement 3 is not correct.

Hints:

- If $x R x$, then R is reflexive.
- If $x R y$ then $y R x$, so R is symmetric.
- If $x R y$ and $y R z$ then $x R z$, so R is transitive.

2. Option (d) is correct.

Explanation:

We, know that If $(x, y) \in R \Leftrightarrow (y, x) \in R^{-1}$

Since, R is reflexive.

$$\therefore (x, x) \in R \Rightarrow (x, x) \in R^{-1}$$

So, R^{-1} is reflexive.

Since, R is symmetric.

$$\text{So, If } (x, y) \in R \Rightarrow (y, x) \in R$$

$$\Rightarrow \text{If } (y, x) \in R^{-1} \text{ then } (x, y) \in R^{-1}$$

So, R^{-1} is symmetric.

Since, R is transitive.

If $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$

$$\Rightarrow (y, x) \in R^{-1}, (z, y) \in R^{-1} \text{ and } (z, x) \in R^{-1}$$

So, R^{-1} is transitive.

3. Option (d) is correct.

Explanation:

$$R = \{(x, y) : x, y \in \mathbb{N} \text{ and } x^2 = y^3\}$$

$$\text{For } x = y, x^2 \neq x^3$$

$$\Rightarrow (x, x) \notin R$$

Now, $(x, y) \in R$

$$\Rightarrow x^2 = y^3$$

$$\Rightarrow x^3 = (x^2)(x)$$

$$\Rightarrow x^3 = y^3 x$$

$$\Rightarrow x^3 \neq y^2$$

$$\Rightarrow (y, x) \notin R$$

Now, $(x, y) \in R$ and $(y, z) \in R$

$$\Rightarrow x^2 = y^3 \text{ and } y^2 = z^3$$

$$\Rightarrow x^2 = y(z^3)$$

$$\Rightarrow x^2 \neq z^2$$

$$\Rightarrow (x, z) \notin R$$

Hint: Let $x = 2 \Rightarrow 2^2 \neq 2^3$

$$\Rightarrow (x, x) \notin R$$

$$\text{Also, } (8, 4) \in R \text{ as } 8^2 = 4^3$$

$$\text{but } 4^2 \neq 8^3$$

$$\Rightarrow (4, 8) \notin R$$

4. Option (b) is correct.

Explanation:

$$R = \{(x, y) : 2x + 3y = 20; x, y \in \mathbb{N}\}$$

$$\therefore 2x + 3y = 20$$

$$\Rightarrow 2x = 20 - 3y$$

$$\Rightarrow x = \frac{20 - 3y}{2}$$

$$\therefore x, y \in \mathbb{N}$$

So, possible pairs of (x, y) are $(7, 2), (4, 4), (1, 6)$.

$\therefore$ Number of elements in R is 3.

Hint:

$$\bullet \text{ Use } x = \frac{20 - 3y}{2}.$$

- Find the values of x by putting value of y considering $x, y \in \mathbb{N}$.

5. Option (b) is correct.

Explanation: Statement 1: All relations are not functions because in function need only one image and in relation more than one image possible. So, false.

Statement 2: It is correct. Relation is subset of $A \times B$.

Statement 3: It is correct. When relation $A \times A$ is possible. So, elements is subset.

Hints:

- All relation not function.
- Use definition of relation.

6. Option (c) is correct.

Explanation:

$$S = \{1, 2, 3, \dots\}$$

$$\text{Given: } xRy \Rightarrow \log_a x > \log_a y, \text{ where } a = \frac{1}{2}$$

Check reflexivity:

$$xRx \Rightarrow \log_a x > \log_a x$$

which is not possible, Hence the given relation is not reflexive.

Check symmetricity:

Let $xRy \in \text{Relation}$,

$$\text{Which means } \log_a x > \log_a y \quad \dots(i)$$

$$\text{Now } yRx \Rightarrow \log_a y > \log_a x$$

Which is not possible, Hence the given relation is not symmetric.

Check Transitivity:

$$\text{Let } xRy \in \text{Relation} \Rightarrow \log_a x > \log_a y \quad \dots(i)$$

$$\text{and } yRz \in \text{Relation} \Rightarrow \log_a y > \log_a z \quad \dots(ii)$$

From equation (i) and (ii),

$$\log_a x > \log_a z$$

which means, $xRz \in \text{Relation}$.

Hence, given relation is transitive.

7. Option (a) is correct.

Explanation: Given: $X = \{1, 2, 3, 4\}$

and R is a relation on X

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$$

As we know, relation R is symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$ where A is the set on which relation is defined. Here $(1, 2)$ and $(2, 1) \in R$ & $(2, 3)$ and $(3, 2) \in R$

$\therefore$ Given relation is symmetric.

As we know, relation R is reflexive if $(a, a) \in R$ for all $a \in A$, where A is the set on which relation is defined.

$$\text{Here } (1, 1), (2, 2), (3, 3) \in R$$

$\therefore$ Given relation is reflexive,

Relation R is transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

$$\text{Here } (1, 2) \in R \text{ and } (2, 3) \in R \text{ but } (1, 3) \notin R$$

$\therefore$ Given relation is not transitive.

Shortcut:

$$(1, 2) \text{ and } (2, 1) \in R \text{ and } (2, 3) \text{ and } (3, 2) \in R$$

$\therefore R$ is symmetric relation.

And, $(a, a) \notin R$, so R is not reflexive relation.

Also, $(1, 2) \in R$ and $(2, 3) \in R$ but $(1, 3) \notin R$
 $\therefore R$ is not transitive relation.

Hints:

- Relation R is symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$
- Relation R is reflexive if $(a, a) \in R$ for all $a \in A$ where A is the set on which relation is defined.
- Relation R is transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$

8. Option (d) is correct.

Explanation:

$$\text{Given: } R = (x, y) : x^2 - 4xy + 3y^2 = 0; x, y \in \mathbb{N}$$

$$\text{Let } a \in \mathbb{N}, a^2 - 4(a)(a) + 3a^2 = 0$$

$$\Rightarrow a^2 - 4a^2 + 3a^2 = 0$$

$$\Rightarrow 0 = 0$$

$$\therefore (a, a) \in R$$

So, R is a reflexive relation.

As we know, a relation is symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$

Let us assume, $x = 3$ and $y = 1$.

$$\Rightarrow x^2 - 4xy + 3y^2 = 0$$

$$\Rightarrow (3)^2 - 4(3)(1) + 3(1)^2 = 0$$

$$\Rightarrow 9 - 12 + 3 = 0$$

$$\Rightarrow 0 = 0$$

$$\therefore (3, 1) \in R$$

Let us check whether $(1, 3) \in R$ or not.

$$\Rightarrow (1)^2 - 4(1)(3) + 3(3)^2 = 0$$

$$\Rightarrow 1 - 12 + 27 = 0$$

$$\Rightarrow 16 \neq 0$$

$$\therefore (1, 3) \notin R$$

So, R is not a symmetric relation.

Also we know that a relation is transitive relation if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

Let us check for $(9, 3)$,

$$\Rightarrow (9)^2 - 4(9)(3) + 3(3)^2 = 0$$

$$\Rightarrow 81 - 108 + 27 = 0$$

$$\Rightarrow 0 = 0$$

$$\therefore (9, 3) \in R$$

$$\text{also } (3, 1) \in R$$

Now, let us check whether $(9, 1)$ belongs to R or not.

$$\Rightarrow (9)^2 - 4(9)(1) + 3(1)^2 = 0$$

$$\Rightarrow 81 - 36 + 3 = 0$$

$$\Rightarrow 48 \neq 0$$

$$\therefore (9, 1) \notin R$$

$$\Rightarrow (9, 3) \in R \text{ and } (3, 1) \in R \text{ but } (9, 1) \notin R$$

$\therefore R$ is not a transitive relation.

Shortcut:

$$a \in \mathbb{N}, (a, a) \in R$$

$\therefore R$ is a reflexive relation.

$$(3, 1) \in R \text{ but } (1, 3) \notin R$$

$\therefore R$ is not a symmetric relation.

$$(9, 3) \in R \text{ and } (3, 1) \in R \text{ but } (9, 1) \notin R$$

$\therefore R$ is not a transitive relation.

Hints:

- Relation R is symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$
- Relation R is reflexive if $(a, a) \in R$ for all $a \in A$ where A is the set on which relation is defined.
- Relation R is transitive if $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$.

9. Option (b) is correct.**Explanation:**

Given, $X = \{\text{Set of all citizens of India}\}$

And $x, y \in X$

Hence, $R = \{(x, y) : x, y \in X \text{ and } |x - y| = 5\}$

where, R represent the relation.

(i) Reflexivity :

For Reflexivity, xRx should belong to Relation.

which means

$$\Rightarrow xRx = |x - x|$$

$$= 0$$

$$\neq 5.$$

again,

$$yRy = |y - y|$$

$$= 0$$

Hence, given relation is not reflexive.

(ii) Symmetricity :

For Symmetricity, if xRy belong to relation then yRx should also belong to relation.

Here,

$$xRy = |x - y|$$

$$= 5$$

[given]

Now,

$$yRx = |y - x|$$

$$= |-(x - y)|$$

$$= |x - y|$$

$$[\because | -x| = |x|]$$

$$\Rightarrow xRy = yRx.$$

Hence, given relation is symmetric.

(iii) Transitivity :

For transitivity, if xRy and yRz belong to the relation then xRz should also belong to the relation.

Then,

$$xRy = |x - y| = 5 \quad \dots (i)$$

and

$$yRz = |y - z| = 5 \quad \dots (ii)$$

From equation (i) and (ii), we can say that $|x - z| \neq 5$

Hence, the given relation is not transitive.

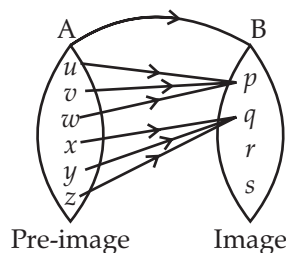
10. Option (c) is correct.**Explanation:**

Given, $A = \{u, v, w, x, y, z\}$

And $B = \{p, q, r, s\}$

Now, mapping of all the parts given in the question.

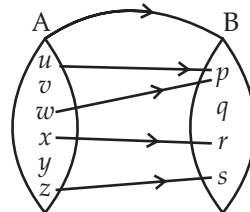
$$(1) \{(u, p), (v, p), (w, p), (x, q), (y, q), (z, q)\}$$



Here, all the elements of set A has some images in B and no element in set A has more than one image in B .

Hence, given relation is a function.

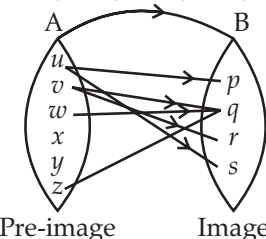
$$(2) \{(u, p), (v, p), (w, r), (z, s)\}$$



Here, few elements of set A (x, y) has no image in B . This relation does not show the right condition of function.

Hence, this is not a function.

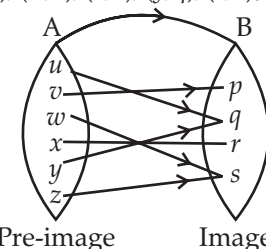
$$(3) \{(u, s), (v, r), (w, q), (u, p), (v, q), (z, q)\}$$



Here, few elements of set A have no image in set B . Also, some elements have more than one image in set B . which contradicts the condition for a relation to be a function.

Hence, this is not a function.

$$(4) \{(u, q), (v, p), (w, s), (x, r), (y, q), (z, s)\}$$



This case shows all the right condition for a relation to be a function. Hence this is a function.

11. Option (c) is correct.**Explanation:**

Given, $S = \{\text{Set of all integers}\}$

$$R = \{(a, b) : a, b \in S \text{ and } ab \geq 0\}$$

(1) Reflexivity :

For Reflexivity, aRa and bRb should belong to the relation.

$$\begin{aligned} aRa &= a \cdot a \\ &= a^2 \\ &\geq 0. \end{aligned}$$

Similarly,

$$\begin{aligned} bRb &= b \cdot b \\ &= b^2 \\ &\geq 0. \end{aligned}$$

Hence, given relation is reflexive.

(2) Symmetricity:

For Symmetricity, if aRb belongs to the relation then bRa should also belong to the relation. Here,

$$\begin{aligned} aRb &= a \cdot b \\ &\geq 0 \end{aligned} \quad \dots (i)$$

Similarly,

$$\begin{aligned} bRa &= b \cdot a \\ &= a \cdot b \quad [\because b \cdot a = a \cdot b] \dots (ii) \\ \Rightarrow aRb &= bRa \quad [\text{From (i) and (ii)}] \end{aligned}$$

Hence, this relation is symmetric.

(3) Transitivity :

For Transitivity, if aRb and bRc belong to the relation then aRc should also belong to the relation.

Here

$$aRb = a \cdot b \geq 0 \quad \dots (i)$$

and

$$bRc = b \cdot c \geq 0 \quad \dots (ii)$$

From equation (i) & (ii),

$$\begin{aligned} aRc &= a \cdot c \\ &\geq 0 \end{aligned}$$

So, R is transitive as well.

Hence, given relation is an equivalence relation.

12. Option (b) is correct.

Explanation: Given, $f(x) = (a - x)^{\frac{1}{n}}$

Now, $f(f(x)) = [(a - f(x))^{\frac{1}{n}}]^{\frac{1}{n}}$

$$\Rightarrow f(f(x)) = [(a - \{(a - x^n)^{\frac{1}{n}}\}^n)^{\frac{1}{n}}]^{\frac{1}{n}}$$

$$\Rightarrow f(f(x)) = [a - (a - x^n)^{\frac{1}{n}}]^{\frac{1}{n}}$$

$$\Rightarrow f(f(x)) = [a - a + x^n]^{\frac{1}{n}}$$

$$\Rightarrow f(f(x)) = (x^n)^{\frac{1}{n}}$$

$$\Rightarrow f(f(x)) = x$$

13. Option (a) is correct.

Explanation: Given,

$$A = \{-4, 0, 1, 4\} \text{ and } f(x) = x^2 - 1$$

$$\Rightarrow f(-4) = (-4)^2 - 1 = 16 - 1 = 15$$

$$\Rightarrow f(0) = (0)^2 - 1 = -1$$

$$\Rightarrow f(1) = (1)^2 - 1 = 0$$

$$\Rightarrow f(4) = (4)^2 - 1 = 16 - 1 = 15$$

Therefore, the set of ordered pairs

$$= \{(-4, 15), (0, -1), (1, 0), (4, 15)\}$$

14. Option (c) is correct.

Explanation: $R = \{(a, b) : a = b - 2, b > 6\}$

$$\Rightarrow (a, b) \in R$$

$$\Rightarrow a = b - 2 \text{ where } b > 6$$

$$\Rightarrow (6, 8) \in R \text{ as } 6 = 8 - 2 \text{ where } b = 8 > 6$$

15. Option (d) is correct.

Explanation: number of elements in $(A \times B) = 3 \times 4 = 12$

$$\text{Number of subsets of } (A \times B) = 2^{12} = 4096$$

16. Option (a) is correct.

Explanation: $(A \times B) \times (C \times D) = (A \times C) \times (B \times D)$

17. Option (d) is correct.

Explanation: Given,

$$n(A) = m \text{ and } n(B) = n$$

$$\therefore n(A \times B) = n(A) \cdot n(B) = mn$$

Now, total no. of relations from A to B

$$= 2^{n(A \times B)} - 1$$

$$= 2^{mn} - 1$$

18. Option (c) is correct.

Explanation: We have,

$$[x]^2 - 5[x] + 6 = 0$$

$$[x]^2 - 3[x] - 2[x] + 6 = 0$$

$$[x]\{[x] - 3\} - 2\{[x] - 3\} = 0$$

$$\{[x] - 3\}\{[x] - 2\} = 0$$

$$[x] = 3, 2$$

19. Option (b) is correct.

Explanation: We have,

$$f(x) = \frac{1}{1 - 2\cos x}$$

Since,

$$-1 \leq \cos x \leq 1$$

$\Rightarrow$

$$1 \geq \cos x \geq -1$$

$\Rightarrow$

$$-2 \leq -2\cos x \leq 2$$

$\Rightarrow$

$$-2 + 1 \leq 1 - 2\cos x \leq 2 + 1$$

$\Rightarrow$

$$-1 \leq 1 - 2\cos x \leq 3$$

$\Rightarrow$

$$-1 \leq \frac{1}{1 - 2\cos x} \leq \frac{1}{3}$$

$\therefore$

$$-1 \leq f(x) \leq \frac{1}{3}$$

or

$$\text{Range of } f(x) = \left[-1, \frac{1}{3}\right]$$

20. Option (c) is correct.

Explanation: We have,

$$f(x) = \sqrt{1 + x^2}$$

Similarly,

$$f(y) = \sqrt{1 + y^2}$$

$$f(xy) = \sqrt{1 + x^2 y^2} \quad \dots (i)$$

Now,

$$f(x) \cdot f(y) = \sqrt{1 + x^2} \cdot \sqrt{1 + y^2}$$

or,

$$f(x) \cdot f(y) = \sqrt{(1 + x^2)(1 + y^2)}$$

or,

$$f(x) \cdot f(y) = \sqrt{1 + x^2 + y^2 + x^2 y^2} \quad \dots (ii)$$

$$\text{Clearly, } \sqrt{1 + x^2 y^2} \leq \sqrt{1 + x^2 + y^2 + x^2 y^2}$$

$$\Rightarrow f(xy) \leq f(x) \cdot f(y)$$

21. Option (b) is correct.

Explanation: Let, $f(x) = \sqrt{a^2 - x^2}$

From domain of $f(x)$,

$$f(x) \geq 0$$

i.e., $\sqrt{a^2 - x^2} \geq 0$

or, $x^2 - a^2 \leq 0$

or, $x^2 \leq a^2$

or, $x \leq \pm a$

or, $-a \leq x \leq a$

Therefore, Domain of $\sqrt{a^2 - x^2} = [-a, a]$

22. Option (b) is correct.

Explanation: Given,

$$f(x) = ax + b \quad \dots(i)$$

$$f(-1) = -5$$

and $f(3) = 3$

Also, from (i)

$$f(-1) = -a + b$$

and $f(3) = 3a + b$

$$\Rightarrow -5 = -a + b \quad \dots(ii)$$

and $3 = 3a + b \quad \dots(iii)$

On solving eqs. (ii) and (iii), we get

$$a = 2, b = -3$$

23. Option (c) is correct.

Explanation:

(i) R is reflexive if it contains $\{(1, 1), (2, 2) \text{ and } (3, 3)\}$.

Since, $(2, 2) \in R$. So, we need to add $(1, 1)$ and $(3, 3)$ to make R reflexive.

(ii) R is symmetric if it contains $\{(2, 2), (1, 3), (3, 1), (3, 2), (2, 3)\}$.

Since, $\{(2, 2), (1, 3), (3, 2)\} \in R$. So, we need to add $(3, 1)$ and $(2, 3)$.

Thus, minimum ordered pairs which should be added in relation R to make it reflexive and symmetric are $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$.

24. Option (b) is correct.

Explanation: Given, $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + y^2 \leq 4\}$

Let $y = 0$, then $x^2 \leq 4 \Rightarrow x = 0, \pm 1, \pm 2$

Thus, domain of $R = \{-2, -1, 0, 1, 2\}$

25. Option (b) is correct.

Explanation: By removing $(1, 2)$, we get identity relation. We know that identity relation is always an equivalence relation.

26. Option (a) is correct.

Explanation: Equivalence class $[1]$ is the set of elements related to $1 = \{1, 5, 9\}$

27. Option (c) is correct.

Explanation: $6 = 8 - 2$

$(6, 8)$ is an element of R.

28. Option (a) is correct.

Explanation: Let $x_1, x_2 \in \mathbb{N}$

$$f(x_1) = f(x_2)$$

$$x_1^2 = x_2^2$$

$$\Rightarrow$$

$$x_1^2 - x_2^2 = 0$$

$$\Rightarrow$$

$$(x_1 + x_2)(x_1 - x_2) = 0$$

$$\Rightarrow$$

$$x_1 = x_2$$

$$\{x_1 + x_1 \neq 0 \text{ as } x_1, x_2 \in \mathbb{N}\}$$

Hence, $f(x)$ is injective.

Also, the elements like 2 and 3 have no pre-image in $\mathbb{N}$. Thus, $f(x)$ is not surjective.

29. Option (d) is correct.

Explanation: $f(x) = 2 + x^2$

For one-one, $f(x_1) = f(x_2)$

$$\Rightarrow 2 + x_1^2 = 2 + x_2^2$$

$$\Rightarrow x_1^2 = x_2^2$$

$$\Rightarrow x_1 = \pm x_2$$

$$\Rightarrow x_1 = x_2$$

or $x_1 = -x_2$

Thus, $f(x)$ is not one-one.

For onto

Let $f(x) = y$ such that $y \in \mathbb{R}$

$$\therefore x^2 = y - 2$$

$$\Rightarrow x = \pm\sqrt{y-2}$$

Put $y = -3$, we get

$$\Rightarrow x = \pm\sqrt{-3-2} = \pm\sqrt{-5}$$

30. Option (b) is correct.

Explanation: f is injective since every element in set B has atmost one pre-image in set A.

31. Option (c) is correct.

Explanation: We know that, if A and B are two non-empty finite sets containing m and n elements, respectively, then the number of one-one and onto mapping from A to B is

$$n! \text{ if } m = n$$

$$0, \text{ if } m \neq n$$

Given that, $m = 5$ and $n = 6$

$$\therefore m \neq n$$

Number of one-one and onto mapping = 0

32. Option (b) is correct.

Explanation: Total number of functions from A to B = 2^n

Number of into functions = 2

Number of surjections from A to B = $2^n - 2$

- Let us denote $\sqrt{-1}$ by the symbol i . Then, we have $i^2 = -1$. This means that i is a solution of the equation $x^2 + 1 = 0$. A number of the form $a + ib$, where a and b are real numbers, is defined to be a complex number.
- For example, $2 + 3i$, $(-1) + 3i$ are complex numbers. For the complex number $z = a + ib$, a is called the real part, denoted by $\text{Re } z$ and b is called the imaginary part denoted by $\text{Im } z$ of the complex number z .

Algebra of Complex Numbers:

Addition of two complex numbers:

Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the sum $z_1 + z_2$ is defined as follows:

The addition of complex numbers satisfy the following properties:

- (i) Closure law ;The sum of two complex numbers is a complex number, i.e., $z_1 + z_2$ is a complex number for all complex numbers z_1 and z_2 .
- (ii) Commutative law ;For any two complex numbers z_1 and z_2 , $z_1 + z_2 = z_2 + z_1$
- (iii) Associative law ;For any three complex numbers z_1, z_2 , and z_3 , $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.
- (iv) Existence of additive identity ;There exists the complex number $0 + i0$ (denoted as 0), called the additive identity or the zero complex number, such that, for every complex number z , $z + 0 = z$.
- (v) Existence of additive inverse ;To every complex number $z = a + ib$, we have the complex number $-a + i(-b)$ (denoted as $-z$), called the additive inverse or negative of z . We observe that $z + (-z) = 0$ (the additive identity).

Difference of two complex numbers:

- Given any two complex numbers z_1 and z_2 , the difference $z_1 - z_2$ is defined as follows: $z_1 - z_2 = z_1 + (-z_2)$.

For example, $(6 + 3i) - (2 - i) = (6 + 3i) + (-2 + i) = 4 + 4i$ and $(2 - i) - (6 + 3i) = (2 - i) + (-6 - 3i) = -4 - 4i$.

Multiplication of two complex numbers:

- Let $z_1 = a + ib$ and $z_2 = c + id$ be any two complex numbers. Then, the product $z_1 z_2$ is defined as follows: $z_1 z_2 = (ac - bd) + i(ad + bc)$ For example, $(3 + i5)(2 + i6) = (3 \times 2 - 5 \times 6) + i(3 \times 6 + 5 \times 2) = -24 + i28$. The multiplication of complex numbers possesses the following properties, which we state without proofs.
- (i) Closure law; The product of two complex numbers is a complex number, the product $z_1 z_2$ is a complex number for all complex numbers z_1

and z_2 .

- (ii) Commutative law; For any two complex numbers z_1 and z_2 , $z_1 z_2 = z_2 z_1$.
- (iii) Associative law; For any three complex numbers z_1, z_2 and z_3 , $(z_1 z_2) z_3 = z_1 (z_2 z_3)$.
- (iv) Existence of multiplicative identity; There exists the complex number $1 + i0$ (denoted as 1), called the multiplicative identity such that $z \cdot 1 = z$, for every complex number z .
- (v) Existence of multiplicative inverse; For every non-zero complex number $z = a + ib$ or $a + bi$ ($a \neq 0, b \neq 0$), we have the complex number $\frac{a}{a^2 + b^2} + i \frac{-b}{a^2 + b^2}$ (denoted by $1/z$ or z^{-1}), called the multiplicative inverse of z such that $z \cdot \frac{1}{z} = 1$ (the multiplicative identity).
- (vi) Distributive law; For any three complex numbers z_1, z_2, z_3 ,
 - (a) $z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$
 - (b) $(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$

Division of two complex numbers: Given any two complex numbers z_1 and z_2 , where $z_2 \neq 0$, the quotient

$$\frac{z_1}{z_2} = z_1 \frac{1}{z_2}$$

Conjugate of a complex number, Modulus of Complex Numbers, Argument of Complex Numbers:

- Let $z = x + iy$ be any complex number

$$\text{Then } |z| = \sqrt{(x^2 + y^2)}$$

$$\text{And } \arg z = \tan^{-1} \left(\frac{y}{x} \right)$$

De Moivre's Theorem, Roots of Unity, Geometry of Complex Numbers

- Let $Z = r(\cos(\theta) + i \sin(\theta))$ be a complex number. Then for any positive integer n , $Z^n = r^n (\cos(n\theta) + i \sin(n\theta))$

$$\text{Where } r^2 = (x^2 + y^2) \text{ and } \theta = \tan^{-1} y/x$$

- **Properties of Dube Roots of Unity:**

- Sum of all cube roots of unity is zero.
- Product of all cube roots of unity is 1.
- Each complex cube root of unity is square of the other.
 - (i) $1 + \omega + \omega^2 = 0$
 - (ii) $\omega^3 = 1$
 - (iii) $1 + \omega^n + \omega^{2n} = 0$

EXERCISE

- The smallest positive integer n for which $\left(\frac{1-i}{1+i}\right)^{n^2} = 1$ where $i = \sqrt{-1}$ is
(a) 2 (b) 4 (c) 6 (d) 8
- What is $\sum_{n=1}^{8x+7} i^n$ equal to, where $i = \sqrt{-1}$?
(a) -1 (b) 1 (c) i (d) $-i$
- Let α and β be real numbers and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct non-real roots with $\operatorname{Re}(z) = 1$, then it is necessary that:
(a) $\beta \in (-1, 0)$ (b) $|\beta| = 1$
(c) $\beta \in (1, \infty)$ (d) $\beta \in (0, 1)$
- What is $i^{1000} + i^{1001} + i^{1002} + i^{1003}$ equal to (where $i = \sqrt{-1}$)?
(a) 0 (b) i (c) $-i$ (d) 1
- The smallest positive integer n for which $\left(\frac{1-i}{1+i}\right)^{n^2} = 1$, is
(a) 1 (b) 4 (c) 8 (d) 16
- What is the square root of i , where $i = \sqrt{-1}$?
(a) $\frac{1+i}{2}$ (b) $\frac{1-i}{2}$
(c) $\frac{1+i}{\sqrt{2}}$ (d) None of these
- What is the modulus of $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)^{200}$?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2^{200}
- If $Z = 1 + i$, where $i = \sqrt{-1}$ then what is the modulus of $\sqrt{3+1}$?
(a) 1 (b) 2 (c) 3 (d) 4
- Consider the following in respect of a complex number Z :
(1) $(\overline{Z^{-1}}) = (\overline{Z})^{-1}$ (2) $ZZ^{-1} = |Z|^2$
Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2
- What is the modulus of the complex number $i2n^{+1}(-i)^{2n-1}$, where $n \in \mathbb{N}$ and $i = \sqrt{-1}$?
(a) -1 (b) 1 (c) $\sqrt{2}$ (d) 2
- If $z = x + iy$, where $i = \frac{1}{6}$, then what does the equation $i = \sqrt{-1} = 0$ represent?
(a) Straight line (b) Parabola
(c) Circle (d) Pair of straight lines
- What is the argument of the complex number $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$ Where $i = \sqrt{-1}$?
(a) 240° (b) 210° (c) 120° (d) 60°
- What is the principal argument of $(-1 - i)$, where $i = \sqrt{-1}$?
(a) $\frac{\pi}{4}$ (b) $-\frac{\pi}{4}$ (c) $-\frac{3\pi}{4}$ (d) $\frac{3\pi}{4}$
- If α and β are different complex numbers with $|\alpha| = 1$, then what is $\left|\frac{\alpha-\beta}{1-\alpha\beta}\right|$ equal to?
(a) $|\beta|$ (b) 2 (c) 1 (d) 0
For the next two (2) items that follow :
Let z be a complex number satisfying
 $\left|\frac{Z-4}{Z-8}\right| = 1$ and $\left|\frac{Z}{Z-2}\right| = \frac{3}{2}$
- What is $|z|$ equal to?
(a) 6 (b) 12 (c) 18 (d) 36
- What is $\left|\frac{z-6}{z+6}\right|$ equal to?
(a) 3 (b) 2 (c) 1 (d) 0
- What is the argument of the complex number $\frac{1}{6}$ where $i = \sqrt{-1}$?
(a) 0 (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{4}$ (d) $\frac{\pi}{2}$
- The modulus-amplitude form of $\sqrt{3+1}$, where $i = \sqrt{-1}$ is
(a) $2\left(\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right)$ (b) $2\left(\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\right)$
(c) $4\left(\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right)$ (d) $4\left(\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\right)$
- What is the value of $\left(\frac{-1+i\sqrt{3}}{2}\right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2}\right)^{3n}$, where $i = \sqrt{-1}$?
(a) 3 (b) 2 (c) 1 (d) 0
- If $z = x + iy = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{-25}$, where $i = \sqrt{-1}$, then what is the fundamental amplitude of $\frac{z-\sqrt{2}}{z-i\sqrt{2}}$?
(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$
- What is $\omega^{100} (+) \omega^{200} (+) \omega^{300}$ equal to, where ω is the cube root of unity?
(a) 1 (b) 3ω (c) $3\omega^2$ (d) 0

22. If $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{107}$, then what is the imaginary part of z equal to?

- (a) 0 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

23. What is $\sqrt{\frac{1+\omega^2}{1+\omega}}$ equal to, where ω is the cube root

of unity?

- (a) 1 (b) ω

- (c) ω^2 (d) $i\omega$, where $i = \sqrt{-1}$

24. If the point $z_1 = 1 + i$, where $i = \sqrt{-1}$, is the reflection of a point $z_2 = x + iy$ in the line $\bar{z} - iz = 5$ then the point z_2 is:

- (a) $1 + 4i$ (b) $4 + i$ (c) $1 - i$ (d) $-1 - i$

25. What is $\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^6$ equal to, where $i = \sqrt{-1}$?

- (a) 1 (b) $\frac{1}{6}$ (c) 6 (d) 2

Answer Key

1. (a)	2. (a)	3. (c)	4. (a)	5. (b)	6. (c)	7. (c)	8. (b)	9. (a)	10. (b)
11. (a)	12. (c)	13. (c)	14. (c)	15. (a)	16. (d)	17. (d)	18. (b)	19. (b)	20. (a)
21. (d)	22. (a)	23. (b)	24. (a)	25. (a)					

Answers with Explanations

1. Option (a) is correct.

Explanation:

Given:

$$\left(\frac{1-i}{1+i}\right)^{n^2} = 1$$

$$\Rightarrow \left(\frac{1-i}{1+i} \times \frac{1-i}{1-i}\right)^{n^2} = 1$$

$$\Rightarrow \left(\frac{1+(i)^2-2i}{1^2-(i)^2}\right)^{n^2} = 1$$

$$\Rightarrow \left(\frac{1-1-2i}{1-(-1)}\right)^{n^2} = 1 \quad \{\because i^2 = -1\}$$

$$\Rightarrow \left(\frac{-2i}{2}\right)^{n^2} = 1$$

$$\Rightarrow (-i)^{n^2} = 1$$

$$\Rightarrow n^2 = 4$$

$$\Rightarrow n = 2$$

Hint:

- Rationalize the $\frac{1}{2}$ and solve further using $i^2 = -1$

2. Option (a) is correct.

Explanation:

Consider, Given $\sum_{n=1}^{8x+7} i^n$

Now, we get by expansion, let

$$I = i^1 + i^2 + i^3 + \dots + i^{8n+7}$$

We know, if consider

$i^1 + i^2 + i^3 + i^4$, we get:

$$\Rightarrow i - 1 - i + 1 = 0$$

$$[\text{value of } i^1 = i, i^2 = -1, i^3 = -i, i^4 = +1]$$

Similarly, in expansion consecutive 4 terms get cancel out

So, in $(8n + 7)$ terms, we are left with only 3 terms.

$\{(8n + 7) \text{ divisible by } 4 \text{ gives remainder } 3\}$

$$\begin{aligned} \text{Hence, } I &= i^1 + i^2 + i^3 \\ &= i + (-1) + (-i) \\ &= -1 \end{aligned}$$

Hints:

- Consecutive 4 terms cancel out.
- Remainder 3 terms.

3. Option (c) is correct.

Explanation:

Let $z = x + iy$

Given, $z^2 + \alpha z + \beta = 0$

$$\Rightarrow (x + iy)^2 + \alpha(x + iy) + \beta = 0$$

$$\Rightarrow x^2 + i^2 y^2 + 2ixy + \alpha x + i \cdot \alpha y + \beta = 0$$

$$\Rightarrow x^2 - y^2 + 2xy \cdot i + \alpha x + \alpha y \cdot i + \beta = 0$$

$$\Rightarrow (x^2 - y^2 + \alpha x + \beta) + (\alpha y + 2xy)i = 0$$

It is given, $\text{Re}(z) = 1$

Which means $x = 1$, so

$$\Rightarrow (1^2 - y^2 + \alpha \times 1 + \beta) + y(\alpha + 2 \times 1)i = 0$$

$$\Rightarrow (1 - y^2 + \alpha + \beta) + y(\alpha + 2)i = 0$$

$$\Rightarrow (1 - y^2 + \alpha + \beta) + y(\alpha + 2)i = 0 + 0 \cdot i$$

On comparing

$$\alpha + 2 = 0 \text{ and } 1 - y^2 + \alpha + \beta = 0$$

$$\Rightarrow \alpha = -2 \text{ and } 1 - y^2 - 2 + \beta = 0$$

$$\Rightarrow \beta = 1 + y^2$$

Given, $y \in \text{Real number} - \{0\}$

as both roots are non real distinct so $y \neq 0$

So,

$$\beta = 1 + (\text{Real number})^2$$

So, $\beta \in \text{Real number greater than 1}$

Hence, $\beta \in (1, \infty)$

4. Option (a) is correct.

Explanation:

$$i^{1000} + i^{1001} + i^{1002} + i^{1003} = 0$$

$$[\because i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0]$$

If we put $n = 1000$, we will get the given expression.

5. Option (b) is correct.

Explanation:

$$\left(\frac{1+i}{1-i} \right)^n = 1$$

$$\Rightarrow \left\{ \left(\frac{1+i}{1-i} \right) \left(\frac{1+i}{1+i} \right) \right\}^n = 1$$

$$\Rightarrow \left\{ \frac{1+i^2+2i}{1-i^2} \right\}^n = 1$$

$$\Rightarrow \left(\frac{1-1+2i}{1+1} \right)^n = 1 \quad [\because i^2 = -1]$$

$$\Rightarrow \left(\frac{2i}{2} \right)^n = 1$$

$$\Rightarrow i^n = 1$$

$$\Rightarrow i^n = i^4 \quad [\because i^4 = 1]$$

On comparing, $n = 4$

6. Option (c) is correct

Explanation:

$$\text{Let } z = \sqrt{i}$$

$$z = \sqrt{0+i \cdot 1}$$

We know that

$$\sqrt{a+ib} = \left\{ \sqrt{\frac{\sqrt{a^2+b^2}+a}{2}} + \sqrt{\frac{\sqrt{a^2+b^2}-a}{2}} \cdot i \right\}$$

So,

$$\sqrt{0+i \cdot 1} = \left\{ \sqrt{\frac{\sqrt{0^2+1^2}+0}{2}} + \sqrt{\frac{\sqrt{0^2+1^2}-0}{2}} \cdot i \right\}$$

$$z = \left\{ \sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}} i \right\}$$

$$z = \frac{1+i}{\sqrt{2}}$$

$$\text{So, } \sqrt{i} = \frac{1+i}{\sqrt{2}}$$

7. Option (c) is correct.

Explanation:

$$\begin{aligned} \left(\frac{\sqrt{-3}}{2} - \frac{1}{2} \right)^{200} &= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right)^{200} \quad [\because \sqrt{-1} = i] \\ \left| \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i \right)^{200} \right| &= \left| -\frac{1}{2} + \frac{\sqrt{3}}{2} i \right|^{200} = \left(\sqrt{\frac{1}{4} + \frac{3}{4}} \right)^{200} \\ &= 1^{200} = 1 \end{aligned}$$

Hints:

- If $z = x + iy$ then $|z| = \sqrt{x^2 + y^2}$
- Use $|z^n| = |z|^n$

8. Option (b) is correct.

Explanation:

$$\text{Given: } Z = 1 + i$$

$$\text{Now, } Z + \frac{2}{Z} = 1 + i + \frac{2}{1+i}$$

$$\Rightarrow Z + \frac{2}{Z} = 1 + i + \frac{2}{1+i} \times \frac{1-i}{1-i}$$

$$\Rightarrow Z + \frac{2}{Z} = 1 + i + \frac{2(1-i)}{1-i^2}$$

$$\Rightarrow Z + \frac{2}{Z} = 1 + i + \frac{2(1-i)}{1-(-1)} \quad [\because i^2 = -1]$$

$$\Rightarrow Z + \frac{2}{Z} = 1 + i + 1 - i$$

$$\Rightarrow Z + \frac{2}{Z} = 2 + 0i$$

$$\therefore \left| Z + \frac{2}{Z} \right| = \sqrt{2^2 + 0^2} = 2$$

Hint:

- Use if $Z = a + ib$, then $|Z| = \sqrt{a^2 + b^2}$

9. Option (a) is correct.

Explanation:

Given: Z is a complex number.

Statement-1:

$$\overline{(Z^{-1})} = (\overline{Z})^{-1}$$

$$\text{Let } Z = a + ib$$

$$\overline{Z} = a - ib$$

Also we know,

$$Z^{-1} = \frac{\overline{Z}}{|Z|^2}$$

$$\therefore \overline{(Z^{-1})} = \overline{\left(\frac{\overline{Z}}{|Z|^2} \right)} = \overline{\left(\frac{a-ib}{(\sqrt{a^2+b^2})^2} \right)}$$

$$= \left(\frac{a-ib}{a^2+b^2} \right)$$

$$\Rightarrow (\overline{Z^{-1}}) = \frac{a+ib}{a^2+b^2} \quad \dots(i)$$

Also, $(\overline{Z})^{-1} = (a-ib)^{-1}$

$$= \frac{a-ib}{(\sqrt{a^2+b^2})^2}$$

$$\Rightarrow (\overline{Z})^{-1} = \frac{a+ib}{a^2+b^2} \quad \dots(ii)$$

By (i) and (ii), $(\overline{Z^{-1}}) = (\overline{Z})^{-1}$

So, statement (1) is correct.

Statement-2:

$$ZZ^{-1} = |z|^2$$

Let $Z = a + ib$

$$\Rightarrow Z^{-1} = \frac{\overline{Z}}{|Z|^2} = \frac{a-ib}{a^2+b^2}$$

$$\therefore ZZ^{-1} = (a+ib) \frac{(a-ib)}{a^2+b^2}$$

$$= \frac{a^2+b^2}{a^2+b^2}$$

$$\Rightarrow ZZ^{-1} = 1$$

and $|Z|^2 = (a^2+b^2)$

$$\therefore ZZ^{-1} \neq |Z|^2$$

So, statement (2) is incorrect.

Hint:

- Use $Z^{-1} = \frac{\overline{Z}}{|Z|^2}$ to simplify both the expressions.
- Assume $Z = a + ib$ and then solve.

10. Option (b) is correct.

Explanation:

Given: A complex number $i^{2n+1}(-i)^{2n-1}$

$$\text{Let } Z = i^{2n+1}(-i)^{2n-1}$$

As we know, $i^2 = -1$, $i^3 = -i$

$$\Rightarrow Z = (i)^{2n+1} (i^3)^{2n-1}$$

$$\Rightarrow Z = (i)^{2n+1} (i)^{6n-3}$$

$$\Rightarrow Z = i^{8n-2}$$

$$\Rightarrow Z = (i^2)^{4n-1}$$

$$\Rightarrow Z = (-1)^{4n-1}$$

$\therefore n \in N$, $4n-1$ is an odd number.

$$\Rightarrow Z = -1$$

So, modulus of given complex number = 1

Hint:

- Use $i^2 = -1$ and $i^3 = -i$ and simplify the given number.

11. Option (a) is correct.

Explanation:

$$z = \frac{1-i\sqrt{3}}{1+i\sqrt{3}}$$

$$\Rightarrow \arg(z) = \arg\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)$$

$$= \arg(1-i\sqrt{3}) - \arg(1+i\sqrt{3})$$

$$= -\tan^{-1}\left|\frac{\sqrt{3}}{1}\right| - \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$$

$$= -\frac{\pi}{3} - \frac{\pi}{3}$$

$$= -\frac{2\pi}{3}$$

Now, required argument will be $= 2\pi - \frac{2\pi}{3}$

$$= \frac{6\pi - 2\pi}{3}$$

$$= \frac{4\pi}{3}$$

$$= 4 \times 60^\circ$$

$$= 240^\circ$$

12. Option (c) is correct.

Explanation:

Given: $z = x + iy$... (i)

We know, conjugate $\overline{z} = x - iy$... (ii)

Now, consider

$$z\overline{z} + |z|^2 + 4(z + \overline{z}) - 48 = 0 - 48 = 0$$

We know,

$|z|$ = Modulus of complex number

$$|z| = \sqrt{x^2 + y^2} \quad \dots(iii)$$

Put (i), (ii), (iii) in equation

$$\Rightarrow (x+iy)(x-iy) + (\sqrt{x^2+y^2})^2$$

$$+ 4(x+iy+x-iy) - 48 = 0$$

$$\Rightarrow x^2 - (iy)^2 + (x^2+y^2) + 4(2x) - 48 = 0 \quad \{i^2 = -1\}$$

$$\Rightarrow x^2 + y^2 + x^2 + y^2 + 8x - 48 = 0$$

$$\Rightarrow 2x^2 + 2y^2 + 8x - 48 = 0 \quad \dots(iv)$$

Compare (iv) with general equation of circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Hence, it is equation of circle.

Hints:

- (1) Use conjugate, modulus
- (2) Compare with circle

13. Option (c) is correct.

Explanation:

Let $z = -1 - i$

Now, $\arg(z) = -\pi + \tan^{-1} \left| \frac{-1}{-1} \right|$

$$= -\pi + \tan^{-1} 1$$

$$\begin{aligned}
 &= -\pi + \frac{\pi}{4} \\
 &= \frac{-4\pi + \pi}{4} = \frac{-3\pi}{4}
 \end{aligned}$$

14. Option (c) is correct.

Explanation:

α and β are two complex numbers, such that $|\alpha| = 1$.

$$\begin{aligned}
 \text{Now, } \left| \frac{\alpha - \beta}{1 - \alpha\bar{\beta}} \right|^2 &= \left(\frac{\alpha - \beta}{1 - \alpha\bar{\beta}} \right) \cdot \left(\frac{\overline{\alpha - \beta}}{1 - \overline{\alpha\bar{\beta}}} \right) \\
 &= \left[\because |z|^2 = z \cdot \bar{z} \right] \\
 &= \left(\frac{\alpha - \beta}{1 - \alpha\bar{\beta}} \right) \left(\frac{\bar{\alpha} - \bar{\beta}}{1 - \overline{\alpha\bar{\beta}}} \right) \\
 &= \left[\because \left(\frac{z_1}{z_2} \right) = \frac{\bar{z}_1}{\bar{z}_2} \right] \\
 &= \frac{(\alpha \cdot \bar{\alpha} - \alpha\bar{\beta} - \beta\bar{\alpha} + \beta\bar{\beta})}{(1 - \alpha\bar{\beta})(1 - \bar{\alpha}\beta)} \\
 &= \left[\because \overline{(\bar{z})} = z \right] \\
 &= \frac{(|\alpha|^2 - \alpha\bar{\beta} - \beta\bar{\alpha} + |\beta|^2)}{(1 - \bar{\alpha}\beta - \alpha\bar{\beta} + \alpha\bar{\alpha} \cdot \beta\bar{\beta})} \\
 &= \frac{(1 - \alpha\bar{\beta} - \beta\bar{\alpha} + |\beta|^2)}{(1 - \bar{\alpha}\beta - \alpha\bar{\beta} + |\alpha|^2 \cdot |\beta|^2)} \\
 &= \frac{(1 - \alpha\bar{\beta} - \beta\bar{\alpha} + |\beta|^2)}{(1 - \bar{\alpha}\beta - \alpha\bar{\beta} + (1)^2 \times |\beta|^2)} \\
 &= \frac{1 - \alpha\bar{\beta} - \beta\bar{\alpha} + |\beta|^2}{1 - \alpha\bar{\beta} - \bar{\alpha}\beta + |\beta|^2}
 \end{aligned}$$

$$\Rightarrow \left| \frac{\alpha - \beta}{1 - \alpha\bar{\beta}} \right|^2 = 1$$

Taking square root of both sides

$$\begin{aligned}
 \Rightarrow \left| \frac{\alpha - \beta}{1 - \alpha\bar{\beta}} \right| &= \sqrt{1} \\
 &= 1
 \end{aligned}$$

15. Option (a) is correct

Explanation:

Given : Let z be a complex number satisfying

$$\left| \frac{z-4}{z-8} \right| = 1 \text{ and } \left| \frac{z}{z-2} \right| = \frac{3}{2}$$

$$\left| \frac{z-4}{z-8} \right| = 1$$

$$\Rightarrow \left| \frac{z-4}{z-8} \right| = 1 \quad \left(\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right)$$

$$\Rightarrow |z-4| = |z-8|$$

$$\text{Put } z = x + iy$$

$$|x + iy - 4| = |x + iy - 8|$$

$$\Rightarrow |x - 4 + iy| = |x - 8 + iy| \quad \dots(1)$$

$$\text{If } z = a + ib, R_e(z) = a, I_m(z) = b$$

$$\text{and } |z| = \sqrt{(R_e(z))^2 + (I_m(z))^2}$$

$$\therefore |x - 4 + iy| = \sqrt{(x-4)^2 + y^2}$$

$$|x - 8 + iy| = \sqrt{(x-8)^2 + y^2}$$

So, (1) becomes

$$\sqrt{(x-4)^2 + y^2} = \sqrt{(x-8)^2 + y^2}$$

Squaring both sides, we get

$$(x-4)^2 + y^2 = (x-8)^2 + y^2$$

$$\Rightarrow (x-4)^2 = (x-8)^2$$

$$\Rightarrow x-4 = \pm(x-8)$$

$$\Rightarrow x-4 = x-8 \text{ or } (x-4) = x+8$$

$$(\text{Not possible}) \Rightarrow 2x = 12$$

$$\Rightarrow \boxed{x=6} \quad \dots(2)$$

$$\left| \frac{z}{z-2} \right| = \frac{3}{2}$$

$$\Rightarrow \left| \frac{z}{z-2} \right| = \frac{3}{2} \quad \left(\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right)$$

$$\Rightarrow 2|z| = 3|z-2|$$

$$\text{Put } z = x + iy$$

$$\therefore 2|x + iy| = 3|x + iy - 2|$$

$$\Rightarrow 2\sqrt{x^2 + y^2} = 3\sqrt{(x-2)^2 + y^2}$$

Squaring both sides, we get

$$4(x^2 + y^2) = 9[(x-2)^2 + y^2]$$

$$\Rightarrow 4x^2 + 4y^2 = 9(x^2 + 4 - 4x + y^2)$$

$$\Rightarrow 5x^2 + 5y^2 - 36x + 36 = 0 \quad \dots(3)$$

Solve equation (2) and (3)

$$5 \times 6^2 + 5y^2 - 36 \times 6 + 36 = 0$$

$$\Rightarrow 180 + 5y^2 - 216 + 36 = 0$$

$$\Rightarrow 5y^2 = 0$$

$$\Rightarrow \boxed{y=0} \quad \dots(4)$$

$$z = x + iy$$

$$\Rightarrow z = 6 + 0i \quad (\text{From (3) and (4)})$$

$$\therefore \boxed{|z|=6}$$

Hints :

(i) If $z = a + ib$, then $|z| = \sqrt{a^2 + b^2}$

(ii) $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$

16. Option (d) is correct.*Explanation :***Given :** z is a complex number satisfying

$$\left| \frac{z-4}{z-8} \right| = 1 \text{ and } \left| \frac{z}{z-2} \right| = \frac{3}{2}$$

$$\left| \frac{z-4}{z-8} \right| = 1$$

$$\Rightarrow \left| \frac{z-4}{z-8} \right| = 1 \quad \left(\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right)$$

$$\Rightarrow |z-4| = |z-8|$$

Let $z = x + iy$

$$|x + iy - 4| = |x + iy - 8|$$

$$\Rightarrow \sqrt{(x-4)^2 + y^2} = \sqrt{(x-8)^2 + y^2}$$

$$= \left(\because |z| = \sqrt{(R_e(z))^2 + (I_m(z))^2} \right)$$

Squaring both sides

$$\Rightarrow (x-4)^2 + y^2 = (x-8)^2 + y^2$$

$$\Rightarrow (x-4)^2 = (x-8)^2$$

$$\Rightarrow x-4 = \pm(x-8)$$

$$\Rightarrow x-4 = -x-8 \quad (\because -4=8 \text{ (Not possible)})$$

$$\Rightarrow 2x = 12$$

$$\Rightarrow \boxed{x=6} \quad \dots(1)$$

$$\left| \frac{z}{z-2} \right| = \frac{3}{2}$$

$$\Rightarrow \left| \frac{z}{z-2} \right| = \frac{3}{2} \quad \left(\because \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \right)$$

$$\Rightarrow 2|z| = 3|z-2|$$

Put $z = x + iy$

$$\Rightarrow 2|x + iy| = 3|x + iy - 2|$$

$$\Rightarrow 2\sqrt{x^2 + y^2} = 3\sqrt{(x-2)^2 + y^2}$$

$$\left(\because |z| = \sqrt{(R_e(z))^2 + (I_m(z))^2} \right)$$

Squaring both sides

$$4(x^2 + y^2) = 9[(x-2)^2 + y^2]$$

$$\Rightarrow 4(x^2 + y^2) = 9(x^2 + 4 - 4x + y^2)$$

$$\Rightarrow 5x^2 + 5y^2 - 36x + 36 = 0 \quad \dots(2)$$

Solve (1) & (2)

$$5(6)^2 + 5y^2 - 36 \times 6 + 36 = 0$$

$$\Rightarrow 180 + 5y^2 - 216 + 36 = 0$$

$$\Rightarrow 5y^2 = 0$$

$$\Rightarrow \boxed{y=0} \quad \dots(3)$$

$$\therefore z = x + iy$$

$$\Rightarrow = 6 + 0i \quad (\text{From (1) \& (3)})$$

$$\frac{z-6}{z+6} = \frac{6+0i-6}{6+0i+6}$$

$$= 0 + 0i$$

$$\left| \frac{z-6}{z+6} \right| = 0$$

Hints :

(i) $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$

(ii) If $z = a + ib$, then $|z| = \sqrt{a^2 + b^2}$

17. Option (d) is correct.*Explanation :*

Given, $z = \frac{(1+i)(2+i)}{(3-i)}$

$$= \frac{(1+i)(2+i)(3+i)}{(3-i)(3+i)}$$

$$= \frac{(2+i+2i+i^2)(3+i)}{(3)^2 - (i)^2}$$

$$= \frac{(2+3i-1)(3+i)}{9+1} \quad [\because i^2 = -1]$$

$$= \frac{(1+3i)(3+i)}{10}$$

$$= \frac{3+i+9i+3i^2}{10}$$

$$= \frac{3+10i-3}{10} \quad [\because i^2 = -1]$$

$$= \frac{10i}{10}$$

$$z = 0 + i$$

$$\Rightarrow \arg(z) = \arg(0 + i)$$

$$= \tan^{-1} \left[\frac{1}{0} \right]$$

$$[\text{If } z = a + ib \text{ then } \arg(z) = \tan^{-1} \left| \frac{b}{a} \right|]$$

$$= \tan^{-1}(\alpha)$$

$$= \frac{\pi}{2}$$

18. Option (b) is correct.*Explanation :*

Let $z = \frac{-1-i\sqrt{3}}{2} + i$

$$\Rightarrow |z| = \left| \sqrt{3} + i \right|$$

$$\Rightarrow |z| = \sqrt{(\sqrt{3})^2 + (1)^2}$$

$$\Rightarrow |z| = \sqrt{3+1}$$

$$\Rightarrow |z| = \frac{-q}{p}$$

$$\Rightarrow |z| = 2$$

$$\text{Now, } \arg(z) = \tan^{-1} \left| \frac{1}{\sqrt{3}} \right|$$

$$\Rightarrow \arg(z) = \frac{\pi}{6}$$

Now, Modulus-amplitude form

$$= 2 \left[\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right]$$

19. Option (b) is correct.

Explanation:

We know that,

$$\text{Since, } \frac{-1+i\sqrt{3}}{2} = \omega,$$

$$\frac{-1-i\sqrt{3}}{2} = \omega^2$$

where ω & ω^2 are the cube roots of unity.

Now,

$$\begin{aligned} &= \left(\frac{-1+i\sqrt{3}}{2} \right)^{3n} + \left(\frac{-1-i\sqrt{3}}{2} \right)^{3n} \\ &= (\omega)^{3n} + (\omega^2)^{3n} \\ &= (\omega^3)^n + (\omega^3)^n \\ &= (1)^n + (1)^n, \quad [\because \omega^3 = 1] \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

20. Option (a) is correct

Explanation:

$$\text{Given : } z = x + iy = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)^{-25} \text{ where}$$

$$i = \sqrt{-1}$$

If $z = a + ib$, $a, b \in \mathbb{R}$

z lies in first quadrant, if a, b are positive real numbers.

z lies in second quadrant if a is negative real number and b is positive real number.

z lies in third quadrant if both a and b are negative real numbers.

z lies in fourth quadrant if a is positive real number and b is negative real number.

$$\text{Let } \theta = \tan^{-1} \left(\frac{|b|}{|a|} \right)$$

Polar form of $z = a + ib$ is $|z| (\cos \phi + i \sin \phi)$

If z is in 1st quadrant,

$$\phi = \theta \text{ and } z = |z| (\cos \phi + i \sin \phi)$$

If z is in 2nd quadrant,

$$\phi = \pi - \theta \text{ and } z = |z| (\cos \phi + i \sin \phi)$$

If z is in 3rd quadrant,

$$\phi = -(\pi - \theta) \text{ and } z = |z| (\cos \phi + i \sin \phi)$$

If z is in 4th quadrant,

$$\phi = -\theta \text{ and } z = |z| (\cos \phi + i \sin \phi)$$

$\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)$ lie in 4th quadrant.

$$\therefore \theta = -\tan^{-1} \left(\frac{\left| -\frac{1}{\sqrt{2}} \right|}{\left| \frac{1}{\sqrt{2}} \right|} \right)$$

$$\Rightarrow \theta = -\tan^{-1}(1)$$

$$\Rightarrow \theta = -\frac{\pi}{4}$$

$$\therefore \phi = -\frac{\pi}{4}$$

$$|z| = \sqrt{(\operatorname{Re}(z))^2 + (\operatorname{Im}(z))^2}$$

$$= \sqrt{\left(\frac{1}{\sqrt{2}} \right)^2 + \left(-\frac{1}{\sqrt{2}} \right)^2}$$

$$= \sqrt{\frac{1}{2} + \frac{1}{2}} = 1$$

$$|z| = 1$$

$$\therefore z = 1 \left(\cos \left(-\frac{\pi}{4} \right) + i \sin \left(-\frac{\pi}{4} \right) \right)$$

$$= \left(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right)$$

$$(\because \cos(-\theta) = \cos \theta \text{ and } \sin(-\theta) = -\sin \theta)$$

According to de Moivre's theorem,

$(\cos \theta + i \sin \theta)^n = \cos(n\theta) + i \sin(n\theta)$ if n is integer.

$$\therefore \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)^{-25} = \left(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4} \right)^{-25}$$

$$= \left(\cos \left(-\frac{25\pi}{4} \right) - i \sin \left(-\frac{25\pi}{4} \right) \right)$$

$$= \cos \left(\frac{25\pi}{4} \right) + i \sin \left(\frac{25\pi}{4} \right)$$

$$= \cos \left(6\pi + \frac{\pi}{4} \right) + i \sin \left(6\pi + \frac{\pi}{4} \right)$$

$$\cos(2n\pi + \theta) = \cos \theta, \text{ if } n \in \mathbb{I}$$

$$\sin(2n\pi + \theta) = \sin \theta, \text{ if } n \in \mathbb{I}$$

$$\therefore \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)^{-25} = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

$$\therefore z = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)^{-25}$$

$$= \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

$$\begin{aligned}
 \frac{z-\sqrt{2}}{z-i\sqrt{2}} &= \frac{\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) - \sqrt{2}}{\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) - i\sqrt{2}} \\
 &= \frac{\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) - i\sqrt{2}}{\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}} \\
 &= \frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}} - \sqrt{2}\right) \\
 &= \frac{\left(\frac{1-2}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}}{\frac{1}{\sqrt{2}} + i\left(\frac{1-2}{\sqrt{2}}\right)} \\
 &= \frac{\frac{1}{\sqrt{2}}(-1+i)}{\frac{1}{\sqrt{2}}(1-i)} \\
 &= \frac{-1+i}{1-i} \\
 &= -1
 \end{aligned}$$

Since, -1 lies on negative real axis, fundamental amplitude of $\frac{z-\sqrt{2}}{z-i\sqrt{2}}$ is π .

Hints :

- **De Moivre's theorem :**

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

where n is any integer.

- Polar form of $z = a + ib$ is $|z| (\cos \theta + i \sin \theta)$.

21. Option (d) is correct.**Explanation :**

Given : ω is cube root of unity

$$\therefore \omega^3 = 1 \quad \dots(i)$$

$$\text{or } 1 + \omega + \omega^2 = 0 \quad \dots(ii)$$

We have to find the value of $\omega^{100} + \omega^{200} + \omega^{300}$

$$\Rightarrow \omega^{100} + \omega^{200} + \omega^{300}$$

$$\Rightarrow \omega^{99} \times \omega + \omega^{198} \times \omega^2 + (\omega^3)^{100}$$

$$\Rightarrow (\omega^3)^{33} \times \omega + (\omega^3)^{66} \times \omega^2 + (1)^{100} \quad [\because \omega^3 = 1]$$

$$\Rightarrow (1)^{33} \times \omega + (1)^{66} \times \omega^2 + 1$$

$$\Rightarrow \omega + \omega^2 + 1 \quad [\because \omega^2 + \omega + 1 = 0]$$

$$\Rightarrow 0$$

Hints :

- As we know that ω is the cube root of unity $\omega^3 = 1$.
- Add $\omega^{100} + \omega^{200} + \omega^{300}$, Put ω^3, ω^2 in them.
- Hence, the answer will be 0.

22. Option (a) is correct.**Explanation :**

$$\text{Given : } z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{107} + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^{107}$$

$$\therefore z = \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^{107} + \left(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6}\right)^{107}$$

Using De Moivre's theorem, we get

$$z = \left(\cos \frac{107\pi}{6} + i \sin \frac{107\pi}{6}\right) + \left(\cos \frac{(107)11\pi}{6} + i \sin \frac{(107)11\pi}{6}\right)$$

$$z = \left[\cos\left(16\pi + \frac{11\pi}{6}\right) + i \sin\left(16\pi + \frac{11\pi}{6}\right)\right] +$$

$$\left[\cos\left\{\left(16\pi + \frac{11\pi}{6}\right)\right\} + i \sin\left\{\left(16\pi + \frac{11\pi}{6}\right)\right\}\right]$$

$$z = \left[\cos\left(\frac{11\pi}{6}\right) + i \sin\left(\frac{11\pi}{6}\right)\right] +$$

$$\left[\cos\left(176\pi + \frac{121\pi}{6}\right) + i \sin\left(176\pi + \frac{121\pi}{6}\right)\right]$$

$$z = \left[\cos\left(\frac{11\pi}{6}\right) + i \sin\left(\frac{11\pi}{6}\right)\right] +$$

$$\left[\cos\left(\frac{121\pi}{6}\right) + i \sin\left(\frac{121\pi}{6}\right)\right]$$

$$z = \left[\cos\left(\frac{11\pi}{6}\right) + i \sin\left(\frac{11\pi}{6}\right)\right] +$$

$$\left[\cos\left(20\pi + \frac{\pi}{6}\right) + i \sin\left(20\pi + \frac{\pi}{6}\right)\right]$$

$$z = \left[\cos\left(\frac{11\pi}{6}\right) + i \sin\left(\frac{11\pi}{6}\right)\right] + \left[\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right)\right]$$

$$z = \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right) + \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$$

$$z = \sqrt{3}$$

$$\therefore I_m(z) = 0$$

Hints :

- Change into the form of imaginary parts.
 - Use De Moivre's theorem into the question.
- $$z^n = r^n (\cos \theta + i \sin \theta)^n = r^n (\cos n\theta + i \sin n\theta)$$
- Where n will be natural numbers.

23. Option (b) is correct.**Explanation :**

$$\text{Given : } \sqrt{\frac{1+\omega^2}{1+\omega}}$$

$\because \omega$ is the root of unity

$$\Rightarrow 1 + \omega + \omega^2 = 0$$

$$\Rightarrow 1 + \omega^2 = -\omega$$

$$\Rightarrow 1 + \omega = -\omega^2$$

$$\begin{aligned}
 \text{From } \sqrt{\frac{1+\omega^2}{1+\omega}} &= \sqrt{\frac{-\omega}{-\omega^2}} \\
 &= \sqrt{\frac{1}{\omega}}
 \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{\frac{\omega^3}{\omega}} \quad \{\omega \cdot \omega^2 = \omega^3 = 1\} \\
 &= \sqrt{\omega^2} \\
 &= \omega
 \end{aligned}$$

Hints :

- Given that ω is root of unity so use $1 + \omega + \omega^2 = 0$
- Use $\omega^3 = \omega \cdot \omega^2 = 1$

24. Option (a) is correct.**Explanation :**

Given, $z_1 = 1 + i$

$z_2 = x + iy$

Equation of line

$$i\bar{z} - iz = 5$$

Let $\bar{z} = a - ib$

Then, $z = \overline{a - ib}$

$$\bar{z} = a + ib$$

Put these values in the given equation of line

$$\Rightarrow i(a - ib) - i(a + ib) = 5$$

$$\Rightarrow i[a - ib - a - ib] = 5$$

$$\Rightarrow i \times -2ib = 5$$

$$\Rightarrow -2b \times i^2 = 5$$

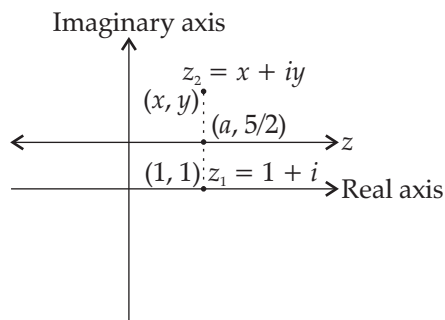
$$\Rightarrow -2b \times -1 = 5$$

$$\Rightarrow 2b = 5$$

$$\Rightarrow b = \frac{5}{2}$$

So, $z = a + \frac{5}{2}i$

Now, $x = 1$, from the graph,



$$\frac{48}{50} = \frac{24}{25}$$

Also, $\frac{y+1}{2} = \frac{5}{2}$ [Using midpoint formula]

$$\Rightarrow y + 1 = 5$$

$$\Rightarrow y = 5 - 1$$

$$\Rightarrow y = 4$$

So, $z_2 = 1 + 4i$

25. Option (a) is correct.**Explanation :**

$$Z = \left(\frac{\sqrt{3} + i}{\sqrt{3} - i} \right)^6$$

$$= \left(\frac{i\sqrt{3} + i^2}{i\sqrt{3} - i^2} \right)^6 \quad \text{[Multiply numerator and denominator by } i]$$

$$= \left(\frac{i\sqrt{3} - 1}{i\sqrt{3} + 1} \right)^6, \quad [\because i^2 = -1]$$

$$= \left[\frac{(-1 + i\sqrt{3}) \div 2}{(1 + i\sqrt{3}) \div 2} \right]^6$$

[divide numerator and denominator by 2]

$$= \left[\frac{\omega}{-\omega^2} \right]^6$$

$$\left[\because \begin{aligned} \frac{-1 + i\sqrt{3}}{2} &= \omega \\ \frac{1 + i\sqrt{3}}{2} &= -\omega^2 \end{aligned} \right]$$

$$= \left[-\frac{1}{\omega} \right]^6$$

$$= \frac{1}{\omega^3 \times \omega^3} = \frac{1}{1 \times 1} = 1 \quad [\because \omega^3 = 1]$$

○○○

- We already know that $ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$, is called a quadratic (or second degree) polynomial in the variable x with real coefficients. Also an equation of the type $ax^2 + bx + c = 0$, $a \neq 0$, is called a **quadratic equation** in the variable x .

The equation $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, is called the **general (or standard) quadratic equation** in the variable x with real coefficients. The equation of the type $ax^2 + c = 0$, $a \neq 0$, is called a pure quadratic equation.

The roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The expression $b^2 - 4ac$ is called the discriminant of the quadratic equation denoted by D or Δ .

- **Solving Pure Quadratic Equation :**

Let the pure quadratic equation be $ax^2 + c = 0$, where $a \neq 0$

$$\Rightarrow ax^2 = -c \Rightarrow x^2 = -\frac{c}{a} \Rightarrow x = \pm \sqrt{-\frac{c}{a}}$$

Hence, the roots are $\sqrt{-\frac{c}{a}}, -\sqrt{-\frac{c}{a}}$.

- **Nature or character of the roots** $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Examining the nature of the roots means to see what type of roots the equation has, i.e., whether they are real or imaginary, rational or irrational, equal or unequal. The nature of the roots depends entirely on the value of the expression $b^2 - 4ac$ which is called the **discriminant** of the equation. Thus if a, b, c are rational,

- 1. The roots are real and unequal if $b^2 - 4ac$ is positive.**

Further,

- (i) The roots are rational if $b^2 - 4ac$ is a perfect square.
 (ii) The roots are irrational if $b^2 - 4ac$ is not a perfect square.
 (iii) The roots are equal if $b^2 - 4ac$ is zero, each root being equal to $-\frac{b}{2a}$. It follows that $ax^2 + bx + c$ is a perfect square if $b^2 - 4ac = 0$.

- 2. The roots are imaginary if $b^2 - 4ac$ is negative i.e., the roots are complex.**

- **Sum and Product of the Roots:**

If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, we have

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\text{i.e., } \alpha + \beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2};$$

$$\text{i.e., } \alpha\beta = \frac{c}{a}$$

Aid to Memory : In a given quadratic equation,

(1) **Sum of the roots** = $-\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2}$

(2) **Product of the roots** = $\frac{\text{Constant term}}{\text{Coeff. of } x^2}$

$$\boxed{\begin{array}{l} \alpha + \beta = \frac{-b}{a} \\ \alpha\beta = \frac{c}{a} \end{array}}$$

- **Signs of the Roots:**

- Positive roots :** Both roots will be positive if $\alpha + \beta$ and $\alpha\beta$ both are positive. i.e., $-\frac{b}{a}$ and $\frac{c}{a}$ are positive. It will be so **when b and a are of opposite sign and c and a are of same sign.**
- Negative roots :** Both roots will be negative if $\alpha + \beta$ is negative and $\alpha\beta$ is positive, i.e., $-\frac{b}{a}$ is negative and $\frac{c}{a}$ is positive. It will be so **when a, b, c all are of same sign.**
- Roots of opposite signs :** It will occur when $\alpha\beta$ is negative, i.e., **c and a are of opposite sign.**
- Roots equal in magnitude but opposite in sign:** It will occur when $\alpha + \beta = 0$, i.e., $-\frac{b}{a} = 0$, i.e., **$b = 0$.** For real solutions the sign of a and c should be opposite.

- **Important Algebraic Results :**

$$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \quad \dots(i)$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta \quad \dots(ii)$$

$$\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$$

$$\begin{aligned}
 &= (\alpha + \beta) \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} \quad \dots(\text{iii}) \\
 \alpha^3 + \beta^3 &= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \quad \dots(\text{iv}) \\
 \alpha^3 - \beta^3 &= (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) \\
 &= (\alpha - \beta)[(\alpha - \beta)^2 + 3\alpha\beta] \\
 &= (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta) \quad \dots(\text{v}) \\
 \alpha^4 + \beta^4 &= (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 \\
 &= [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 \quad \dots(\text{vi}) \\
 \alpha^4 - \beta^4 &= (\alpha^2 - \beta^2)(\alpha^2 + \beta^2) \\
 &= (\alpha - \beta)(\alpha + \beta)(\alpha^2 + \beta^2) \\
 &= \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}(\alpha + \beta)[(\alpha + \beta)^2 - 2\alpha\beta] \quad \dots(\text{vii})
 \end{aligned}$$

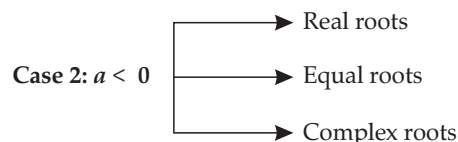
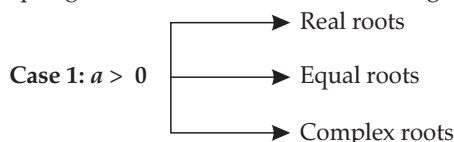
➤ **Quadratic Functions:**

An algebraic expression of the form $ax^2 + bx + c$ is called a *quadratic function of x*.

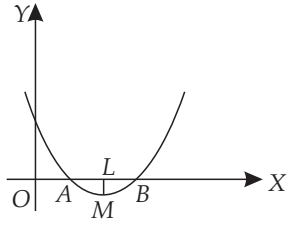
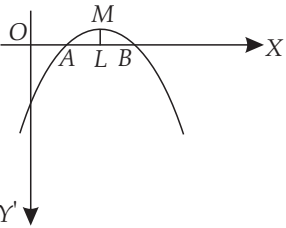
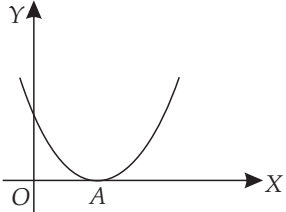
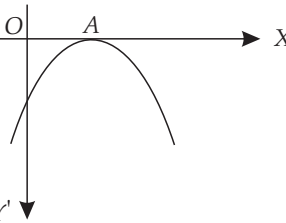
Graph of the Quadratic Function $ax^2 + bx + c$ where $(a \neq 0)$

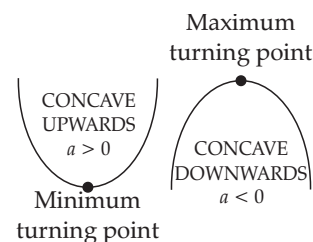
➤ **Studying the nature and Character of the Roots of a Quadratic Equation by Graphing:**

The graphs given below illustrate the following cases.



- (a) If the graph cuts the X-axis, the roots of the equation will be real and unequal. Their values will be given by the abscissae of the points of intersection of the graph and the X-axis.
- (b) If the graph is tangent to the X-axis, the roots are real and equal.
- (c) If the graph has no points in common with the X-axis, the roots of the equation are imaginary and cannot be determined from the graph.

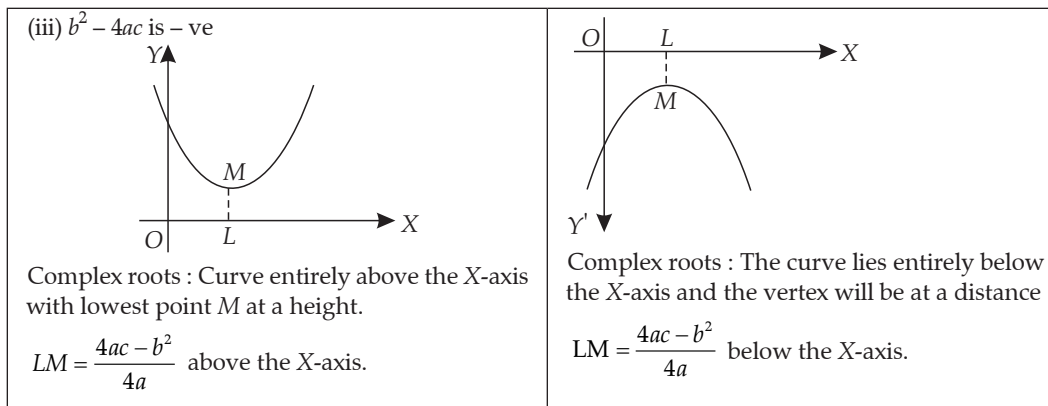
Case 1 : When a is positive. i.e., $a > 0$	Case 2 : When a is negative. i.e., $a < 0$
(i) $b^2 - 4ac$ is +ve.  Real roots OA and OB. Minimum value $LM = \frac{4ac - b^2}{4a}$ at $x = OL = \frac{-b}{2a}$	 Real roots OA and OB. Minimum value $LM = \frac{4ac - b^2}{4a}$ at $x = OL = \frac{-b}{2a}$
(ii) $b^2 - 4ac = 0$  Real and equal roots. Each root = OA	 Real and equal roots. Each root = OA



Let, $f(x) = ax^2 + bx + c$

Procedure:

1. Make a table of corresponding values for x and the function.
 2. Plot these points, as a pair of $x, f(x)$ on the axes.
 3. Draw a smooth curve joining the plotted points.
- The graph of a quadratic function is a **parabola**. The point at which its direction changes is called its **turning point**. It is more commonly called the **vertex** of the parabola.
- The graph of the function is **concave upwards** when $a > 0$ and **concave downwards** when $a < 0$.



EXERCISE

- Find the value of m for which one of the roots of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$.
 (a) 0, 4 (b) 2, -5 (c) 0, -2 (d) 3, 5
- If $x = 2 + \sqrt{3}$ then find the value of $x^4 - 8x^3 + 16x^2$.
 (a) -1 (b) 1 (c) 0 (d) 2
- If $1 + 9r^2 + 81r^4 = 256$ and $1 + 3r + 9r^2 = 32$, then find the value of $1 - 3r + 9r^2$.
 (a) 8 (b) 4 (c) 16 (d) 12
- If the expression $x^2 - 11x + a$ and $x^2 - 14x + 2a$ must have a common factor and $a \neq 0$ then, find the common factor.
 (a) $x - 8$ (b) $x + 2$ (c) $x - 5$ (d) $2x - 1$
- If $x + \frac{1}{x} = \sqrt{3}$, then the value of $x^{18} + x^{12} + x^6 + 1$ is:
 (a) 0 (b) 2 (c) 3 (d) 1
- If $A = \frac{x-1}{x+1}$, then find the value of $A - \frac{1}{A}$ is:
 (a) $\frac{x^2-1}{-4(2x-1)}$ (b) $\frac{-4x}{x^2-1}$
 (c) $\frac{x^2-1}{-4(2x-1)}$ (d) $\frac{-4(2x-1)}{x^2-1}$
- Given that α and β are the roots of the quadratic equation $px^2 + qx + 1 = 0$, find the value of $\alpha^3\beta^2 + \alpha^2\beta^3$.
 (a) $\frac{p}{q}$ (b) $\frac{-q}{p^3}$
 (c) $\frac{p}{q^2}$ (d) None of these
- If $p + \frac{1}{p} = 2$ find the value of $p \times p \times p$.
 (a) 4 (b) 8 (c) 1 (d) 2
- If α and β are the roots of $2x^2 + 5x - 4 = 0$, then find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$.
 (a) $-\frac{41}{8}$ (b) $\frac{21}{5}$ (c) $\frac{31}{8}$ (d) $-\frac{27}{5}$
- If $x + \frac{1}{x} = 8$, then find the value of $\frac{5x}{x^2 + 1 - 6x}$.
 (a) 2.5 (b) 6 (c) 5 (d) 6.5
- The factors of the expression $2x^2 - 5x - 12$ are:
 (a) $(x - 4)$ and $(2x - 3)$ (b) $(x + 4)$ and $(2x + 3)$
 (c) $(x - 4)$ and $(2x + 3)$ (d) $(x + 4)$ and $(2x - 3)$
- Let p, q be the roots of $3x^2 + 6x + 2 = 0$. Form an equation whose roots are $\frac{-p^2}{q}, \frac{-q^2}{p}$.
 (a) $2x^2 + 3x - 5 = 0$ (b) $x^2 + 5x - 6 = 0$
 (c) $3x^2 - 18x + 2 = 0$ (d) $3x^2 - 6x + 7 = 0$
- The area of a square park is $16x^2 + 8x + 1$. What is the length of the park?
 (a) $(4x - 1)$ units (b) $(4x + 1)$ units
 (c) $(4x + 1)^2$ units (d) $4x$ units
- If α and β are two of the quadratic equation $ax^2 + bx + c = 0$ where a, b, c are constants and $a \neq 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is:
 (a) $-\frac{b}{c}$ (b) $\frac{c}{b}$ (c) $\frac{c}{a}$ (d) $\frac{b}{c}$
- If $a^2 + \frac{2}{a^2} = 16$, then find the value of $\frac{72a^2}{a^4 + 2 + 8a^2}$.
 (a) 3 (b) 1 (c) 4 (d) 2
- Find the values of x for the given equation $3x^2 + 5x - 2 = 0$
 (a) -3 and -2 (b) 3 and $-\frac{1}{2}$
 (c) 2 and -3 (d) -2 and $\frac{1}{3}$
- If $a + \frac{1}{a} + 2 = 0$, then the value of $a^{15} - \frac{1}{a^{100}}$ is:
 (a) 0 (b) 2 (c) -2 (d) 1
- If $kx^3 + 4x^2 + 3x - 4$ and $x^3 - 4x + k$ leave the same remainder when divided by $(x - 3)$, then the value of k is:
 (a) 2 (b) -1 (c) 1 (d) 0

19. If $x^3 - 6x^2 + ax + b$ is divisible by $x^2 - 3x + 2$, then the values of a and b are:
 (a) $a = -11$ and $b = 6$ (b) $a = -6$ and $b = -11$
 (c) $a = 11$ and $b = -6$ (d) $a = 6$ and $b = 11$
20. If $4x^2 + y^2 = 40$ and $xy = 6$, ($x > 0, y > 0$) then the value of $2x + y$ is:
 (a) 24 (b) 16 (c) 4 (d) 8
21. The roots of the equation $x^2 - 2\sqrt{2}x + 1 = 0$ are:
 (a) Real and different (b) Imaginary and different
 (c) Real and equal (d) Imaginary and equal
22. The equation with roots are 3 and 4 will be
 (a) $x^2 + 7x + 12 = 0$ (b) $x^2 - 7x + 12 = 0$
 (c) $x^2 - x + 12 = 0$ (d) $x^2 + 7x - 12 = 0$
23. If the roots of the equation $x^2 + 2x + p = 0$ are real, then the value of p is
 (a) $p \leq 2$ (b) $p \leq 1$
 (c) $p \leq 3$ (d) none of these

Answer Key

1. (c)	2. (b)	3. (a)	4. (a)	5. (a)	6. (b)	7. (b)	8. (c)	9. (a)	10. (a)
11. (c)	12. (c)	13. (b)	14. (a)	15. (a)	16. (d)	17. (c)	18. (b)	19. (c)	20. (d)
21. (a)	22. (b)	23. (b)							

Answers with Explanations

1. Option (c) is correct.

Explanation:

Let a be the root of $x^2 - x + m = 0$ and $2a$ be the root of $x^2 - 3x + 2m = 0$. Then, $a^2 - a + m = 0$ and $4a^2 - 6a + 2m = 0$

$$\Rightarrow \frac{a^2}{-m} = \frac{a}{-m}$$

$$= \frac{1}{2}$$

$$\Rightarrow m^2 = -2m$$

$$\Rightarrow m = 0, m = -2$$

2. Option (b) is correct.

Explanation:

Given: $x = 2 + \sqrt{3}$

Now, $x^4 - 8x^3 + 16x^2 = x^2(x^2 - 8x + 16)$
 $= x^2(x - 4)^2$

Consider $x^2(x - 4)^2 = (2 + \sqrt{3})^2 [2 + \sqrt{3} - 4]^2$

$$= (\sqrt{3} + 2)^2 (\sqrt{3} - 2)^2$$

$$= [(\sqrt{3} + 2)(\sqrt{3} - 2)]^2 \quad [\because a^2 b^2 = (ab)^2]$$

$$= [(\sqrt{3})^2 - (2)^2]^2 \quad [\because (a+b)(a-b) = a^2 - b^2]$$

$$= [3 - 4]^2$$

$$= (-1)^2$$

$$= 1$$

3. Option (a) is correct.

Explanation:

Given: $1 + 9r^2 + 81r^4 = 256$ and $1 + 3r + 9r^2 = 32$

We have, $(a+b)(a-b) = a^2 - b^2$

So we can write $(1 + 9r^2 + 3r)(1 + 9r^2 - 3r)$

$$= (1 + 9r^2)^2 - (3r)^2$$

$$= 1 + 81r^4 + 18r^2 - 9r^2$$

$$= 1 + 81r^4 + 9r^2$$

Now, $(32)(1 - 3r + 9r^2) = 256$

$$\Rightarrow 1 - 3r + 9r^2 = \frac{256}{32} = 8$$

4. Option (a) is correct.

Explanation:

Here, let $x - \alpha$ be the common factor then $x = \alpha$ is the root of the corresponding equation.

Therefore, $\alpha^2 - 11\alpha + a = 0$ and $\alpha^2 - 14\alpha + 2a = 0$

Subtracting the above equations:

$$3\alpha - a = 0$$

$$\Rightarrow \alpha = \frac{a}{3}$$

Hence, $\frac{a^2}{9} - 11\frac{a}{3} + a = 0$

$$\Rightarrow a = 0$$

or $a = 24$

but $a \neq 0$

$$\Rightarrow \alpha = 8, \text{ thus the common factor is } x - 8.$$

5. Option (a) is correct.

Explanation:

Given: $x + \frac{1}{x} = \sqrt{3}$

Now, $x^{18} + x^{12} + x^6 + 1$
 $= x^{12}(x^6 + 1) + 1(x^6 + 1)$
 $= (x^6 + 1)(x^{12} + 1)$... (i)

From (i), $x + \frac{1}{x} = \sqrt{3}$

Cubing on both sides,

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x\right)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right)$$

$$\Rightarrow 3\sqrt{3} = x^3 + \frac{1}{x^3} + 3\left(\sqrt{3}\right)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 0$$

$$\Rightarrow x^6 + 1 = 0 \quad \dots (ii)$$

Substitute equation (iii) in (ii),

$$0(x^{12}+1) = 0$$

$$\therefore x^{18} + x^{12} + x^6 + 1 = 0$$

6. Option (b) is correct.

Explanation:

Given: $A = \frac{x-1}{x+1}$, then $\frac{1}{A} = \frac{x+1}{x-1}$

Now,

$$A - \frac{1}{A} = \frac{x-1}{x+1} - \frac{x+1}{x-1}$$

$$= \frac{(x-1)^2 - (x+1)^2}{(x+1)(x-1)}$$

$$= \frac{x^2 + 1 - 2x - x^2 - 1 - 2x}{x^2 - 1}$$

$$= \frac{-4x}{x^2 - 1}$$

7. Option (b) is correct.

Explanation:

Since, α and β are the roots of $px^2 + qx + 1 = 0$.

$$\alpha + \beta = \frac{-q}{p}; \alpha\beta = \frac{1}{p}$$

Now, $\alpha^3\beta^2 + \alpha^2\beta^3 = \alpha^2\beta^2(\alpha + \beta)$

$$= \frac{1}{p^2} \times \left(\frac{-q}{p} \right) = \frac{-q}{p^3}$$

8. Option (c) is correct.

Explanation:

Given: $p - \frac{1}{p} = 2$

$$\Rightarrow p^2 + 1 = 2p$$

$$\Rightarrow p^2 - 2p + 1 = 0$$

$$\Rightarrow p^2 - p - p + 1 = 0$$

$$\Rightarrow p(p-1) - 1(p-1) = 0$$

$$\Rightarrow (p-1)(p-1) = 0$$

$$\Rightarrow (p-1)^2 = 0$$

$$\Rightarrow p-1 = 0$$

$$\Rightarrow p = 1$$

Now, $p \times p \times p \times p = 1 \times 1 \times 1 \times 1 = 1$

9. Option (a) is correct.

Explanation:

Sum of roots, $\alpha + \beta = -\frac{5}{2}$

and product of roots, $\alpha\beta = -\frac{4}{2} = -2$

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$$= \frac{\left(-\frac{5}{2}\right)^2 - 2(-2)}{-2} = \frac{\frac{25}{4} + 4}{-2} = \frac{-41}{8}$$

10. Option (a) is correct.

Explanation:

Given: $x + \frac{1}{x} = 8$

Consider, $\frac{5x}{x^2 + 1 - 6x} = \frac{5x}{x\left[x + \frac{1}{x} - 6\right]}$

$$= \frac{5}{\left[x + \frac{1}{x}\right] - 6} = \frac{5}{8 - 6}$$

$$= \frac{5}{2} = 2.5$$

11. Option (c) is correct.

Explanation:

Consider,

$$2x^2 - 5x - 12$$

$$= 2x^2 - 8x + 3x - 12$$

$$= 2x(x-4) + 3(x-8)$$

$$= (x-4)(2x+3)$$

The factors are $(x-4)$ and $(2x+3)$.

12. Option (c) is correct.

Explanation:

Since, p, q are roots of $3x^2 + 6x + 2 = 0$,

$$p + q = -\frac{6}{3} = -2, pq = \frac{2}{3}$$

Here, sum (S) = $-\frac{p^2}{q} - \frac{q^2}{p} = \frac{-(p^3 + q^3)}{pq}$

$$= \frac{-[(p+q)^3 - 3pq(p+q)]}{pq}$$

$$= \frac{-\left[(-2)^3 - 3\left(\frac{2}{3}\right)(-2)\right]}{\frac{2}{3}} = -(-8+4) \cdot \frac{3}{2} = 6 \text{ and}$$

product (P) = $\left(-\frac{p^2}{q}\right)\left(-\frac{q^2}{p}\right) = pq = \frac{2}{3}$

Required equation is $x^2 - Sx + P = 0$,

i.e., $x^2 - 6x + \frac{2}{3} = 0$

i.e., $3x^2 - 18x + 2 = 0$

13. Option (b) is correct.

Explanation:

We know that the area of a square = $l \times l = l^2$

Where l is the length of the side of square.

$$A = 16x^2 + 8x + 1$$

$$= 16x^2 + 4x + 4x + 1$$

$$= 4x[4x+1] + 1[4x+1]$$

$$\Rightarrow A = [4x+1][4x+1]$$

$$\Rightarrow A = l \times l$$

$\therefore$ Length of the park, $l = 4x+1$ units

14. Option (a) is correct.*Explanation:***Given:** ax^2+bx+c

$$\text{Consider, } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{\alpha + \beta}{\alpha\beta}$$

$$\text{Now, sum of the roots} = \alpha + \beta = \frac{b}{a}$$

$$\text{Product of the roots} = \alpha\beta = \frac{c}{a}$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\frac{-b}{a}}{\frac{c}{a}} = \frac{-b}{c}$$

15. Option (a) is correct.*Explanation:*

$$\text{Given: } a^2 + \frac{2}{a^2} = 16$$

$$\begin{aligned} \text{Consider, } \frac{72a^2}{(a^4 + 2 + 8a^2)} &= \frac{72a^2}{a^2[a^2 + \frac{2}{a^2} + 8]} \\ &= \frac{72}{a^2 + \frac{2}{a^2} + 8} = \frac{72}{16 + 8} = \frac{72}{24} = 3. \end{aligned}$$

16. Option (d) is correct.*Explanation:***Given:**

$$\begin{aligned} &3x^2 + 5x - 2 = 0 \\ \Rightarrow &3x^2 + 6x - x - 2 = 0 \\ \Rightarrow &3x(x+2) - 1(x+2) = 0 \\ \Rightarrow &(3x-1)(x+2) = 0 \\ \Rightarrow &x+2 = 0 \text{ or } 3x-1 = 0 \\ \Rightarrow &x = -2 \text{ or } x = \frac{1}{3} \end{aligned}$$

17. Option (c) is correct.*Explanation:*

$$\text{Given; } a + \frac{1}{a} + 2 = 0,$$

$$\Rightarrow a + \frac{1}{a} + 2 = 0$$

$$\Rightarrow a^2 + 2a + 1 = 0$$

$$\Rightarrow (a+1)^2 = 0$$

$$\Rightarrow a = -1$$

$$\begin{aligned} \text{Now, put } a &= -1 \text{ in } a^{15} - \frac{1}{a^{100}} \\ &= (-1)^{15} - \frac{1}{(-1)^{100}} \\ &= -1 - 1 = -2 \end{aligned}$$

$$\therefore a^{15} - \frac{1}{a^{100}} = -2$$

18. Option (b) is correct.*Explanation:***Given:**

kx^3+4x^2+3x-4 and x^3-4x+k leave same remainder when divided by $x-3$.

Put $x = 3$ in both the equation and equate both.

$$k(3)^3 + 4(3)^2 + 3(3) - 4 = (3)^3 - 4(3) + k$$

$$\Rightarrow 27k + 36 + 9 - 4 = 27 - 12 + k$$

$$\Rightarrow 26k = -26$$

$$\Rightarrow k = -1$$

19. Option (c) is correct.*Explanation:***Given:**

$$x^3 - 6x^2 + 9x + b \text{ is divisible by } x^3 - 3x^2 + 2$$

$$\text{Now, } x^3 - 6x^2 + 9x + b \quad \dots(i)$$

$$x^2 - 3x + 2$$

$$= x^2 - 2x - x + 2$$

$$= (x-2)(x-1)$$

Since, $x-2$ and $x-1$ are the factors of equation (i),

$\therefore x-2$ and $x-1$ will divide $x^3 - 6x^2 + x + ax + b$ and leave remainder 0,

Now, Put $x = 1$ in equation (i)

$$a + b - 5 = 0 \quad \dots(ii)$$

Also, Put $x = 2$ in equation (i)

$$2a + b - 16 = 0 \quad \dots(iii)$$

Solving (iii) and (ii), we get

$$2a + b = 16$$

$$a + b = 5$$

$$\begin{array}{r} (-) \quad (-) \quad (-) \\ \hline a = 11 \end{array}$$

From (ii),

$$11 + b - 5 = 0$$

$$\Rightarrow b = -6$$

$$\therefore a = 11 \text{ and } b = -6$$

20. Option (d) is correct.*Explanation:*

$$\text{Given: } 4x^2 + y^2 = 40 \text{ and } xy = 6, x > 0, y > 0$$

$$\text{Consider, } (2x+y)^2 = (2x)^2 + y^2 + 2(2x)(y)$$

$$\Rightarrow (2x+y)^2 = 4x^2 + y^2 + 4xy$$

$$\Rightarrow (2x+y)^2 = 40 + 4(6)$$

$$\Rightarrow (2x+y)^2 = 64$$

$$\therefore 2x+y=8$$

21. Option (a) is correct.*Explanation:* The discriminant of the equation, $D =$

$$(2\sqrt{2})^2 - 4(1)(1)$$

$$\Rightarrow D = 8 - 4$$

$$\Rightarrow D = 4 > 0$$

So, roots are real and different

22. Option (b) is correct.*Explanation:* The quadratic equation is given by $x^2 -$

$$Sx + P = 0$$

$$\Rightarrow x^2 - (3+4)x + (3 \times 4) = 0$$

$$\Rightarrow x^2 - 7x + 12 = 0$$

23. Option (b) is correct.*Explanation:* Here, $a = 1, b = 2, c = p$

$$b^2 - 4ac > 0$$

$$(2)^2 - 4(1)(p) \geq 0$$

$$4 - 4p \geq 0$$

$$4 \geq 4p$$

$$p \leq 1$$

Arithmetic Progression:

Definition: A sequence of terms in which each term differs from the preceding one by a constant quantity. The constant quantity is called common difference.

AP can be represented as $a, a+d, a+2d, \dots$ etc, where a is called as first term and d is called common difference

Formula:

- $T_n = a + (n-1)d$, where T_n is the n^{th} term.
- $S_n = \frac{n}{2} [2a + (n-1)d]$, where S_n is the sum of first n terms.
- $S_n = \frac{n}{2} [a + l]$, where last term ' l ' is given.

Geometric Progression:

Definition: A sequence of terms in which the ratio between any two successive terms is the same.

G.P. can be represented as $a, ar, ar^2, \dots$ etc.

Where, a is the first term and r is called common ratio.

Formula:

$T_n = ar^{n-1}$, where T_n is the n^{th} term

$S_n = \frac{a(1-r^n)}{1-r}$, where $r < 1$ and S_n is the sum of n term

$S_n = \frac{a(r^n-1)}{r-1}$, where $r > 1$ and S_n is the sum of n terms.

Points to remember:

- Sum of first n natural number $= 1 + 2 + 3 + \dots + n = \sum_{n=1}^n n = \frac{n(n+1)}{2}$
 - Sum of the squares of first n natural numbers $= 1^2 + 2^2 + 3^2 + \dots + n^2 = \sum_{n=1}^n n^2 = \frac{n(n+1)(2n+1)}{6}$
 - Sum of the cubes of first n natural numbers $= 1^3 + 2^3 + 3^3 + \dots + n^3 = \sum_{n=1}^n n^3 = \frac{n^2(n+1)^2}{4}$
- A sequence of non-zero number is said to be G.P., if the ratio of each term, except the first one, by its preceding term is always the same.
- We can say that, a sequence $a_1, a_2, \dots, a_n$ is called geometric progression (geometric sequence), if it follows the relation $\frac{a_{k+1}}{a_k} = r$ (constant).

The constant ratio is called **Common ratio** of the G.P. and it is denoted by r .

In a G.P., we usually denote the first term by a , the

n^{th} term by T_n or a_n .

Thus, G.P. can be written as $a, ar, ar^2, ar^3, \dots$ and so on.

- **General Term of a G.P.:** If a is the first term of a G.P. and its common ratio is r , then general term or n^{th} term,

$T_n = ar^{n-1}$ or $l = ar^{n-1}$, where l is the last term.

m^{th} Term of Finite G.P. from the End: Let a be the first term and r be the common ratio of a G.P. having n terms. Then, m^{th} term from the end is $(n-m+1)^{\text{th}}$ term from the beginning.

Also, m^{th} term from the end $= l \left(\frac{1}{r} \right)^{m-1}$, where l is last term of the finite G.P.

- **Sum of First n Terms of a GP:** If a and r are the first term and common ratio of a G.P. respectively, then sum of n terms of this G.P. is given by

$$S_n = \frac{a(1-r^n)}{1-r}, \text{ where } r < 1$$

$$\text{and } S_n = \frac{a(r^n-1)}{r-1}, \text{ where } r > 1$$

- **Geometric Mean:** Geometric mean between a and b is $\sqrt{ab}$ or If a, b, c are in G.P., then

Geometric mean (G.M.) $b^2 = ac$

Some Important Results:

- Reciprocals of terms in G.P. always form a G.P.
- If $G_1, G_2, G_3, \dots, G_n$ are n numbers inserted between a and b , so that the resulting sequence is G.P., then

$$G_k = a \left(\frac{b}{a} \right)^{\frac{k-1}{n}}, 1 \leq k \leq n$$

- If three terms of G.P. are to be taken, then we choose as $\frac{a}{r}, a, ar$.
- If four terms of G.P. are to be taken, then we choose as $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.
- If five terms of G.P. are to be taken, then we choose as $\frac{a}{r^4}, \frac{a}{r^2}, a, ar, ar^2$.
- If a, b, c are in G.P. then ak, bk, ck are also in G.P. where $k \neq 0$ and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ also in G.P. where $k \neq 0$.
- In a G.P., the product of the terms equidistant from the beginning and from the end is always

same and equal to the product of the first and the last terms.

- If each term of a G.P. be raised to some power then the resulting terms are also in G.P.
- $S_{\infty} = a + ar + ar^2 + ar^3 + \dots \infty$ term if

$$-1 < r < 1 \Rightarrow S_{\infty} = \frac{a}{1-r}.$$

Such that $-1 < r < 1$ or $|r| < 1$

- If a, b, c are A.P. then $2b = a + c$.
- If a, b, c are in G.P. then $b^2 = ac$
- If A and G be A.M. and G.M. of two given positive real number ' a ' and ' b ' respectively then

$$A = \frac{a+b}{2}, \quad G = \sqrt{ab} \quad \text{and} \quad A \geq G.$$

EXERCISE

- Which term of the G.P. $2, 1, \frac{1}{2}, \frac{1}{4}, \dots$ is $\frac{1}{1024}$?
(a) 15 (b) 12 (c) 10 (d) 8
- The sum of 17 consecutive numbers is 289. The sum of another 10 consecutive numbers, whose first term is 5 more than the average of the first set of consecutive numbers is:
(a) 315 (b) 285 (c) 265 (d) 300
- The value of $1+3+5+7+\dots+(2n-1)$ is
(a) $(2n-1) \times (2n-1)$ (b) $\frac{n}{2}$
(c) $n \times n$ (d) $\frac{n(n+1)}{2}$
- If n^{th} term of a G.P. is $2n$, then find the sum of its first 6 terms.
(a) 126 (b) 124 (c) 190 (d) 154
- If the 5th term of a GP is 2, then the product of first 9 terms is
(a) 64 (b) 512 (c) 128 (d) 256
- The first three term of a GP are $m-1, m+1, m+7$. Find the value of m
(a) 7 (b) 5 (c) 6 (d) 3
- If n^{th} term of an AP is $3n-4$, find the 10th term of the AP.
(a) 10 (b) 12 (c) 22 (d) 26
- The sum of AP 2, 5, 8, ... upto 50 terms is
(a) 3775 (b) 3557 (c) 3757 (d) 3575

Answer Key

1.	(b)	2.	(c)	3.	(c)	4.	(a)	5.	(b)	6.	(b)	7.	(d)	8.	(a)		

Answers with Explanations

1. Option (b) is correct.

Explanation:

Given G.P. is $2, 1, \frac{1}{2}, \frac{1}{4}, \dots$

First term $a = 2$

and common ratio $= \frac{1}{2}$

Let n^{th} term of G.P. be $\frac{1}{1024}$

$$\therefore T_n = ar^{n-1}$$

$$\Rightarrow \frac{1}{1024} = 2 \left(\frac{1}{2} \right)^{n-1}$$

$$\Rightarrow \frac{1}{2048} = \left(\frac{1}{2} \right)^{n-1}$$

$$\Rightarrow \left(\frac{1}{2} \right)^{11} = \left(\frac{1}{2} \right)^{n-1}$$

$$\therefore n-1 = 11$$

$$\text{or } n = 12$$

2. Option (c) is correct.

Explanation: The average of first set is,

$$\frac{\text{Sum of 17 terms}}{17} = \frac{289}{17} = 17$$

The first term of new sequence is 5 more than 17 i.e., $17+5=22$

Now, the sum of set of 10 consecutive numbers $22+23+\dots+31$, which is in A.P.

Here, $a=22, d=1$ and $n=10$.

$$\text{We have, } S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_{10} = \frac{10}{2} [2(22) + (10-1)1]$$

$$= 5[44+9]$$

$$= 5 \times 53$$

$$= 265$$

Or

$$S_n = \frac{n}{2} (a+l)$$

(where, l = last term)

$$S_n = \frac{10}{2} (22+31)$$

$$= 5(53) = 265$$

3. Option (c) is correct.

Explanation:

The given series is in A.P.

$$S_n = \frac{n}{2} (a+l) \quad [\text{here, last term } l \text{ is given}]$$

$$\Rightarrow S_n = \frac{n}{2}[1 + (2n - 1)]$$

$$\therefore S_n = n \times n$$

4. Option (a) is correct.

Explanation: Sum of n terms of a GP = $\frac{a(r^n - 1)}{(r - 1)}$

$$\Rightarrow a_n = 2n$$

$$\Rightarrow a_1 = 2$$

$$\Rightarrow a_2 = 4$$

$$\Rightarrow a_3 = 8$$

$$\text{Common ratio } (r) = 4 \div 2$$

$$= 8 \div 4$$

$$= 2$$

$$\text{Sum of 6 terms} = \frac{2(2^6 - 1)}{(2 - 1)}$$

$$\Rightarrow S_6 = 2(64 - 1)$$

$$\Rightarrow S_6 = 2 \times 63$$

$$= 126$$

5. Option (b) is correct.

Explanation: let a and r be the first term and common ratio of the required G.P.

$$\text{Given, } 5^{\text{th}} \text{ term} = 2$$

$$\Rightarrow ar^4 = 2 \quad \text{Eq. (1),}$$

To find: Product of first 9 terms = $a \cdot ar \cdot ar^2 \cdot ar^3 \cdot \dots \cdot ar^8$

$$\Rightarrow a^9 r^{1+2+3+\dots+8} = a^9 r^{\frac{8 \times 9}{2}} = a^9 r^{36}$$

From, Eq. (1),

$$\Rightarrow (ar^4)^9 = 2^9$$

$$= 512$$

6. Option (b) is correct.

Explanation: The first three terms of a geometric progression are $m - 2$, $m + 1$ and $m + 7$.

$$\text{Common ratio} = \frac{a_2}{a_1}$$

$$= \frac{a_3}{a_2}$$

$$\Rightarrow \frac{m+1}{m-2} = \frac{m+7}{m+1}$$

$$\Rightarrow (m+1)^2 = (m+7)(m-2)$$

$$\Rightarrow m^2 + 2m + 1 = m^2 + 5m - 14$$

$$\Rightarrow 3m = 15$$

$$\Rightarrow m = 5$$

7. Option (d) is correct.

Explanation:

$$a_n = 3n - 4$$

$$\Rightarrow a_{10} = 3(10) - 4$$

$$\Rightarrow a_{10} = 30 - 4$$

$$\Rightarrow a_{10} = 26$$

8. Option (a) is correct.

Explanation:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{50} = \frac{50}{2}(2 \times 2 + (50-1)3)$$

$$S_{50} = 25(4 + 49 \times 3)$$

$$S_{50} = 25(4 + 147)$$

$$S_{50} = 25 \times 151$$

$$S_{50} = 3775$$

Chapter 17

TRIGONOMETRY

There are three measures for measuring an angle.

- (i) **Degree Measure :** A right angle is divided into 90 equal parts called degree and 1 degree is denoted as 1° .

One degree is divided into 60 equal parts, called minutes and 1 minute is denoted by $1'$. One minute is divided into 60 equal parts called second.

Thus, $1^\circ = 60'$, $1' = 60''$

- (ii) **Radian Measure :**

In a circle of radius r , an arc of length l subtends an angle ' θ ' radian at the centre, then

$$l = r\theta \Rightarrow \theta = \frac{l}{r}$$

- (iii) **Centesimal System :**

In this system a right angle is divided into 100 equal parts called grades and each grade is subdivided into 100 minutes, and each minute into 100 seconds.

Thus,

1 right angle = 100 grades

1 grade = 100 minutes

1 minute = 100 seconds

Relation between three systems of measurement of an angle is as follows:

$$\frac{D}{90} = \frac{G}{100} = \frac{2R}{\pi}$$

Where, D = Degree measure
 G = Grade measure
 R = Radian measure

A circle subtends at the centre an angle, whose radian measure is 2π and its degree measure is 360° . It follows that,

$$2\pi \text{ radian} = 360^\circ$$

$$(\text{or}) \quad \pi \text{ radian} = 180^\circ$$

$$\therefore 1 \text{ rad} = \frac{180^\circ}{\pi} = 57^\circ 16' 22'' \text{ (approx.)}, \text{ where } \pi = \frac{22}{7} = 3.14159 \text{ (Approx)}$$

Thus,

$$\text{Radian} = \frac{\pi}{180^\circ} \times \text{degree measure.}$$

$$\text{Degree} = \frac{180^\circ}{\pi} \times \text{radian measure.}$$

Note :

- (i) The angle between two consecutive digits in a clock is $30^\circ \left(= \frac{\pi}{6} \text{ radians} \right)$
- (ii) The minute hand rotates through an angle of 6° in one minute.
- (iii) Radian is a constant angle.
- (iv) $1^\circ = \left(\frac{\pi}{180} \right) \text{ rad} = 0.0176 \text{ rad}$

Trigonometric Identities (θ measured in radian)

An equation involving trigonometric functions, which is true for all those angles for which the functions are defined is called trigonometric identity. Some identities are:

$$(i) \sin \theta = \frac{1}{\operatorname{cosec} \theta} \text{ or } \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$(ii) \cos \theta = \frac{1}{\sec \theta} \text{ or } \sec \theta = \frac{1}{\cos \theta}$$

$$(iii) \cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta} \text{ or}$$

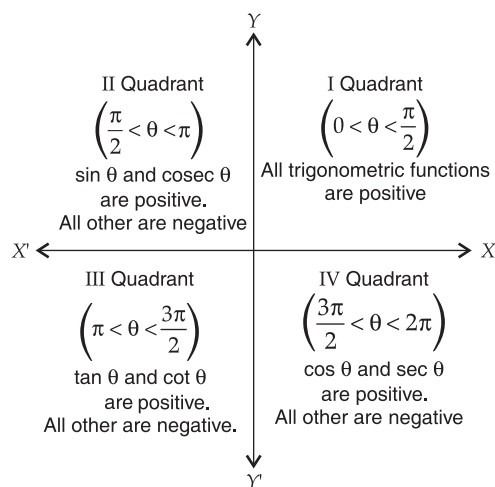
$$\tan \theta = \frac{1}{\cot \theta} = \frac{\sin \theta}{\cos \theta}$$

$$(iv) \cos^2 \theta + \sin^2 \theta = 1 \text{ or } 1 - \cos^2 \theta = \sin^2 \theta \text{ or } 1 - \sin^2 \theta = \cos^2 \theta.$$

$$(v) 1 + \tan^2 \theta = \sec^2 \theta \text{ or } \sec^2 \theta - \tan^2 \theta = 1$$

$$(vi) 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \text{ or } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

Sign of Trigonometric Functions in Different Quadrants:



Trigonometric Ratios of Some Standard Angles

Angle	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$-\infty$
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞

Domain and Range of Trigonometric Functions:

Function	Domain	Range
$\sin x$	$\mathbb{R}$	$[-1, 1]$
$\cos x$	$\mathbb{R}$	$[-1, 1]$
$\tan x$	$\mathbb{R} - \{(2n+1)\left(\frac{\pi}{2}\right); n \in \mathbb{Z}\}$	$\mathbb{R}$
$\operatorname{cosec} x$	$\mathbb{R} - \{n\pi; n \in \mathbb{Z}\}$	$\mathbb{R} - (-1, 1)$
$\sec x$	$\mathbb{R} - \{(2n+1)\left(\frac{\pi}{2}\right); n \in \mathbb{Z}\}$	$\mathbb{R} - (-1, 1)$
$\cot x$	$\mathbb{R} - \{n\pi; n \in \mathbb{Z}\}$	$\mathbb{R}$

(i) Compound Angle Formulae:

- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$
- $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\cot(x+y) = \frac{\cot x \cdot \cot y - 1}{\cot y + \cot x}$
- $\sin(x-y) = \sin x \cos y - \cos x \sin y$
- $\cos(x-y) = \cos x \cos y + \sin x \sin y$

Note:

$$\sin(x+y) \sin(x-y) = \sin^2 x - \sin^2 y = \cos^2 y - \cos^2 x$$

$$\cos(x+y) \cos(x-y) = \cos^2 x - \sin^2 y = \cos^2 y - \sin^2 x$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$\cot(x-y) = \frac{\cot x \cdot \cot y + 1}{\cot y - \cot x}$$

$$\tan(x+y+z) = \frac{[\tan x + \tan y + \tan z - \tan x \tan y \tan z]}{[1 - \tan x \tan y - \tan y \tan z - \tan z \tan x]}$$

(ii) Transformation formulae:

- $2 \sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2 \cos x \sin y = \sin(x+y) - \sin(x-y)$
- $2 \cos x \cos y = \cos(x+y) + \cos(x-y) \quad [x > y]$
- $2 \sin x \sin y = \cos(x-y) - \cos(x+y)$
- $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$
- $\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$
- $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$
- $\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$

(iii) Trigonometric Functions of multiple and sub-multiple angles:

- $\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$
- $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$
- $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$
- $\sin 3x = 3 \sin x - 4 \sin^3 x$
- $\cos 3x = 4 \cos^3 x - 3 \cos x$
- $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$
- $\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}} \begin{cases} +, & \text{if } \frac{A}{2} \text{ lies in quadrants I or II} \\ -, & \text{if } \frac{A}{2} \text{ lies in III or IV quadrants} \end{cases}$
- $\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} \begin{cases} +, & \text{if } \frac{A}{2} \text{ lies in I or IV quadrants} \\ -, & \text{if } \frac{A}{2} \text{ lies in II or III quadrants} \end{cases}$

$$\bullet \tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} \begin{cases} +, \text{ if } \frac{A}{2} \text{ lies in I or III quadrants} \\ -, \text{ if } \frac{A}{2} \text{ lies in II or IV quadrants} \end{cases}$$

Trigonometric functions of an angle of 18°

Let $\theta = 18^\circ$. Then $2\theta = 90^\circ - 3\theta$

Therefore, $\sin 2\theta = \sin(90^\circ - 3\theta) = \cos 3\theta$

or $\sin 2\theta = 4\cos^3 \theta - 3\cos \theta$

Since, $\cos \theta \neq 0$, we get

$$2\sin \theta = 4\cos^2 \theta - 3 = 1 - 4\sin^2 \theta$$

or $4\sin^2 \theta + 2\sin \theta - 1 = 0$.

$$\text{Hence, } \sin \theta = \frac{-2 \pm \sqrt{4 + 16}}{8} = \frac{-1 \pm \sqrt{5}}{4}$$

Since, $\theta = 18^\circ$, $\sin \theta > 0$,

$$\text{therefore, } \sin 18^\circ = \frac{\sqrt{5} - 1}{4}$$

Also,

$$\cos 18^\circ = \sqrt{1 - \sin^2 18^\circ} = \sqrt{1 - \frac{6 - 2\sqrt{5}}{16}} = \sqrt{\frac{10 + 2\sqrt{5}}{4}}$$

Now, we can easily find $\cos 36^\circ$ and $\sin 36^\circ$ as follows:

$$\begin{aligned} \cos 36^\circ &= 1 - 2\sin^2 18^\circ = 1 - \frac{6 - 2\sqrt{5}}{8} = \frac{2 + 2\sqrt{5}}{8} \\ &= \frac{\sqrt{5} + 1}{4} \end{aligned}$$

$$\text{Hence, } \cos 36^\circ = \frac{\sqrt{5} + 1}{4}$$

Also,

$$\sin 36^\circ = \sqrt{1 - \cos^2 36^\circ} = \sqrt{1 - \frac{6 + 2\sqrt{5}}{16}} = \sqrt{\frac{10 - 2\sqrt{5}}{4}}$$

EXERCISE

- If $\sec^2 x - 3 \sec x + 2 = 0$, then the value of $x(0^\circ < x < 90^\circ)$ is
(a) 45° (b) 15° (c) 60° (d) 30°
- $(\sin \theta + \cos \theta)^2 = 2$, $0^\circ < \theta < 90^\circ$, then the value of θ is
(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{4}$
- Find x , if $2 \sin^2 x - 1 = 0$.
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) 0 (d) π
- If $\sin \theta = \frac{4}{5}$, Find the value of $\sin 3\theta$.
(a) $\frac{32}{45}$ (b) $\frac{44}{125}$ (c) $\frac{64}{125}$ (d) $\frac{12}{25}$
- If $a \sin A + b \cos A = c$, then $a \cos A - b \sin A$ is equal to:
(a) $\sqrt{a^2 - b^2 - c^2}$ (b) $\sqrt{a^2 + b^2 - c^2}$
(c) $\sqrt{a^2 + b^2 + c^2}$ (d) $\sqrt{a^2 - b^2 + c^2}$
- If $\sin \theta - \cos \theta = \frac{1}{29}$, find the value of $\sin \theta + \cos \theta$.
(a) $\frac{41}{29}$ (b) $\frac{42}{29}$ (c) $\frac{22}{29}$ (d) $\frac{30}{29}$
- If $\sin \theta = 0.96$, then find $\cot \theta$.
(a) 0.4563 (b) 0.4876 (c) 0.2916 (d) 0.3456
- If $2x = \sin \theta$ and $\frac{2}{x} = \cos \theta$, then the value of $4\left(x^2 + \frac{1}{x^2}\right)$ is:
(a) 2 (b) 0 (c) 4 (d) 1
- If $X = \tan 40^\circ$, then the value of $2 \tan 50^\circ$ will be:
(a) $\frac{1}{2X}$ (b) $2X$ (c) $\frac{1}{X}$ (d) $\frac{2}{X}$
- If $117 \cos^2 A + 129 \sin^2 A = 120$ and $170 \cos^2 B + 158 \sin^2 B = 161$, then the value of $\operatorname{cosec}^2 A \sec^2 B$ is:
(a) 1 (b) 9 (c) 4 (d) 16
- If $\sin x - \cos x = 0$, then the value of $\sin^3 x - \cos^3 x$ is:
(a) 0 (b) 2 (c) 1 (d) 4
- If A and B are acute angles and $\sec A = 3$; $\cot B = 4$, then the value of $\frac{\operatorname{cosec}^2 A + \sin^2 B}{\cot^2 A + \sec^2 B}$ is:
(a) $\frac{25}{261}$ (b) $\frac{322}{323}$ (c) 2 (d) $\frac{1}{261}$
- If $\sin \theta - \cos \theta = \frac{7}{17}$, then find the value of $\sin \theta + \cos \theta$.
(a) $\frac{8}{17}$ (b) $\frac{23}{13}$ (c) $\frac{23}{17}$ (d) $\frac{8}{13}$
- The value of x , if $2 \sin^2 x = 2 - 3 \sin x$, is:
(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$
- If $\frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = 5$ and θ is an acute angle, then the value of $\frac{3\cos^2 \theta + 1}{3\cos^2 \theta - 1}$ is:
(a) 3 (b) 2 (c) 1 (d) 4
- If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, where $A > B > 0^\circ$ and $A + B$ is an acute angle, then the value of A is:
(a) 45° (b) 30° (c) 15° (d) 60°
- If $\alpha + \beta = 90^\circ$ and $\alpha = 2\beta$, then the value of $\cos^2 \alpha + \sin^2 \beta$ is:
(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) 1 (d) $\frac{1}{2}$

18. The elimination of θ from $x \cos \theta - y \sin \theta = 2$ and $x \sin \theta + y \cos \theta = 4$ will give:
 (a) $3x^2 - y^2 = 20$ (b) $x^2 + y^2 = 20$
 (c) $3x^2 + y^2 = 20$ (d) $x^2 - y^2 = 20$
19. $\frac{2 + \tan^2 \theta + \cot^2 \theta}{\sec \theta \operatorname{cosec} \theta}$ is equal to:
 (a) $\cot \theta$ (b) $\cos \theta \sin \theta$
 (c) $\sec \theta \operatorname{cosec} \theta$ (d) $\tan \theta$
20. The value of θ , when $\sqrt{3} \cos \theta + \sin \theta = 1$ ($0^\circ \leq \theta \leq 90^\circ$), is:
 (a) 90° (b) 30° (c) 60° (d) 0°
21. If $\cos^2 \theta - \sin^2 \theta - 3 \cos \theta + 2 = 0$, $0^\circ < \theta < 90^\circ$, then what is the value of $4 \operatorname{cosec} \theta + \cot \theta$?
 (a) $3\sqrt{3}$ (b) 4 (c) $4\sqrt{3}$ (d) 3
22. If $3 \sin \theta = 2 \cos^2 \theta$, $0^\circ < \theta < 90^\circ$, then the value of $(\tan \theta + \cos \theta + \sin \theta)$ is:
- (a) $\frac{5\sqrt{3}}{3}$ (b) $\frac{5\sqrt{3}}{6}$
 (c) $\frac{3+5\sqrt{3}}{6}$ (d) $\frac{3+5\sqrt{3}}{3}$
23. If $A = 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta)$, then the value of 3α such that $\cos \alpha = \sqrt{\frac{3+A}{5+A}}$ is:
 (a) 45° (b) 135° (c) 180° (d) 90°
24. $\frac{\cot x}{1 + \operatorname{cosec} x} + \frac{1 + \operatorname{cosec} x}{\cot x}$ is equal to:
 (a) $2 \sec x$ (b) $2 \cos x$ (c) $2 \operatorname{cosec} x$ (d) $2 \sin x$
25. If $\sin x - \cos x = 0$, $0^\circ < x < 90^\circ$, then the value of $(\sec x + \operatorname{cosec} x)^2$ is:
 (a) 8 (b) 4 (c) 10 (d) 6

Answer Key

1. (c)	2. (d)	3. (a)	4. (b)	5. (b)	6. (a)	7. (c)	8. (d)	9. (d)	10. (d)
11. (a)	12. (b)	13. (c)	14. (a)	15. (d)	16. (a)	17. (d)	18. (b)	19. (c)	20. (a)
21. (a)	22. (c)	23. (b)	24. (a)	25. (a)					

Answers with Explanations

1. Option (c) is correct.

Explanation:

Given: $\sec^2 x - 3 \sec x + 2 = 0$

Put $\sec x = y$, then,

$$y^2 - 3y + 2 = 0$$

$$\Rightarrow y^2 - 2y - y + 2 = 0$$

$$\Rightarrow y(y - 2) - 1(y - 2) = 0$$

$$\Rightarrow (y - 2)(y - 1) = 0$$

$$\Rightarrow y = 2 \text{ or } y = 1$$

$$\Rightarrow \sec x = 2 \text{ or } \sec x = 1 \text{ (not possible)}$$

$$\therefore \sec x = 2$$

$$\Rightarrow \sec x = \sec 60^\circ \quad [\because \sec 60^\circ = 2]$$

$$\therefore x = 60^\circ$$

2. Option (d) is correct.

Explanation:

Given: $(\sin \theta + \cos \theta)^2 = 2$, $0^\circ < \theta < 90^\circ$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2$$

$$\Rightarrow 1 + 2 \sin \theta \cos \theta = 2$$

$$\Rightarrow 2 \sin \theta \cos \theta = 1$$

$$\Rightarrow \sin \theta \cos \theta = \frac{1}{2}$$

Only possibility is $\sin \theta = \frac{1}{\sqrt{2}}$ and $\cos \theta = \frac{1}{\sqrt{2}}$

$$\therefore \theta = \frac{\pi}{4}$$

3. Option (a) is correct.

Explanation:

Given: $2 \sin^2 x - 1 = 0$

$$\Rightarrow 2 \sin^2 x = 1$$

$$\Rightarrow \sin^2 x = \frac{1}{2}$$

$$\Rightarrow \sin x = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \quad [\text{taking (+) value}]$$

$$\Rightarrow \sin x = \sin \frac{\pi}{4}$$

$$\Rightarrow x = \frac{\pi}{4}$$

4. Option (b) is correct.

Explanation:

Given: $\sin \theta = \frac{4}{5}$

We have, $\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$

$$\Rightarrow \sin 3\theta = 3\left(\frac{4}{5}\right) - 4\left(\frac{4}{5}\right)^3$$

$$\Rightarrow \sin 3\theta = \frac{300 - 256}{125} = \frac{44}{125}$$

5. Option (b) is correct.

Explanation:

Given, $a \sin A + b \cos A = c$

Squaring on both the sides,

$$(a \sin A + b \cos A)^2 = c^2$$

$$\Rightarrow a^2 \sin^2 A + b^2 \cos^2 A + 2ab \sin A \cos A = c^2$$

$$\Rightarrow a^2[1 - \cos^2 A] + b^2[1 - \sin^2 A] + 2ab \sin A \cos A = c^2$$

$$\Rightarrow a^2 - a^2 \cos^2 A + b^2 - b^2 \sin^2 A + 2ab \sin A \cos A = c^2$$

$$\Rightarrow a^2 + b^2 - c^2 = a^2 \cos^2 A + b^2 \sin^2 A - 2ab \sin A \cos A$$

$$\Rightarrow a^2 + b^2 - c^2 = (a \cos A - b \sin A)^2$$

$$\therefore (a \cos A - b \sin A) = \sqrt{a^2 + b^2 - c^2}$$

6. Option (a) is correct.

Explanation:

$$\text{Given: } \sin \theta - \cos \theta = \frac{1}{29}$$

Squaring on both sides,

$$(\sin \theta - \cos \theta)^2 = \left(\frac{1}{29}\right)^2$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = \frac{1}{841}$$

$$\Rightarrow 2 \sin \theta \cos \theta = 1 - \frac{1}{841}$$

Now, consider,

$$(\sin \theta + \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$$

$$\Rightarrow (\sin \theta + \cos \theta)^2 = 1 + 1 - \frac{1}{841}$$

$$\Rightarrow (\sin \theta + \cos \theta)^2 = \frac{1681}{841}$$

$$\therefore \sin \theta + \cos \theta = \sqrt{\frac{1681}{841}} = \frac{41}{29}$$

7. Option (c) is correct.

Explanation:

$$\text{Given: } \sin \theta = 0.96$$

$$\text{We have, } \cos^2 \theta = 1 - \sin^2 \theta$$

$$\Rightarrow \cos^2 \theta = 1 - (0.96)^2$$

$$\Rightarrow = 1 - 0.9216 = 0.0784$$

$$\Rightarrow \cos \theta = \sqrt{0.0784}$$

$$\Rightarrow \cos \theta = 0.28$$

$$\therefore \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{0.28}{0.96} = 0.2916$$

8. Option (d) is correct.

Explanation:

$$\text{Given: } 2x = \sin \theta$$

$$\Rightarrow x = \frac{\sin \theta}{2} \quad \dots(i)$$

$$\text{Also, } \frac{2}{x} = \cos \theta \quad (\text{given})$$

$$\Rightarrow \frac{1}{x} = \frac{\cos \theta}{2} \quad \dots(iii)$$

$$\text{Now, } 4 \left(x^2 + \frac{1}{x^2} \right) = 4 \left[\frac{\sin^2 \theta}{4} + \frac{\cos^2 \theta}{4} \right]$$

$$\Rightarrow 4 \left(x^2 + \frac{1}{x^2} \right) = \frac{4}{4} [\sin^2 \theta + \cos^2 \theta]$$

$$\Rightarrow 4 \left(x^2 + \frac{1}{x^2} \right) = 1$$

9. Option (d) is correct.

Explanation:

$$\text{Given: } X = \tan 40^\circ$$

$$\Rightarrow X = \tan(90^\circ - 50^\circ)$$

$$\Rightarrow X = \cot 50^\circ$$

$$\Rightarrow X = \frac{1}{\tan 50^\circ}$$

$$\Rightarrow \tan 50^\circ = \frac{1}{X}$$

$$\Rightarrow 2 \tan 50^\circ = \frac{2}{X}$$

10. Option (d) is correct.

Explanation:

$$\text{Given: } 117 \cos^2 A + 129 \sin^2 A = 120$$

$$170 \cos^2 B + 158 \sin^2 B = 161$$

$$117 \cos^2 A + 117 \sin^2 A + 12 \sin^2 A = 120$$

$$\Rightarrow 117(\cos^2 A + \sin^2 A) + 12 \sin^2 A = 120$$

$$\Rightarrow 117 + 12 \sin^2 A = 120$$

$$\Rightarrow 12 \sin^2 A = 3$$

$$\Rightarrow \sin^2 A = \frac{3}{12} = \frac{1}{4}$$

$$\Rightarrow \sin A = \frac{1}{2}$$

$$\text{Then, } \operatorname{cosec} A = 2$$

Also,

$$170 \cos^2 B + 158 \sin^2 B = 161$$

$$\Rightarrow 158 \sin^2 B + 158 \cos^2 B + 12 \cos^2 B = 161$$

$$\Rightarrow 158[\sin^2 B + \cos^2 B] + 12 \cos^2 B = 161$$

$$\Rightarrow 158(1) + 12 \cos^2 B = 161$$

$$\Rightarrow 12 \cos^2 B = 3$$

$$\Rightarrow \cos^2 B = \frac{3}{12} = \frac{1}{4}$$

$$\Rightarrow \cos B = \frac{1}{2} \quad (\text{taking positive value})$$

$$\therefore \sec B = 2$$

$$\text{Now, } \operatorname{cosec}^2 A \sec^2 B = (2)^2 \cdot (2)^2 = 16$$

11. Option (a) is correct.

Explanation:

$$\text{Given: } \sin x - \cos x = 0$$

$$\text{Using } (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$(\sin x - \cos x)^3 = \sin^3 x - \cos^3 x - 3 \sin x \cos x (\sin x - \cos x)$$

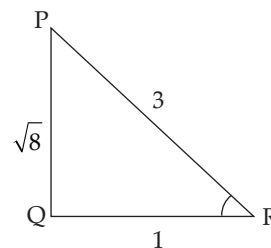
$$\Rightarrow 0 = \sin^3 x - \cos^3 x - 3 \sin x \cos x \cdot (0)$$

$$\Rightarrow \sin^3 x - \cos^3 x = 0$$

12. Option (b) is correct.

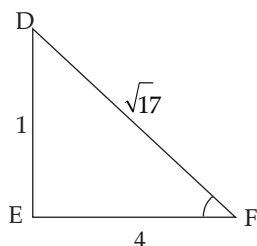
Explanation:

$$\text{Given: } \sec A = \frac{3}{1}$$



$$PQ = \sqrt{9 - 1} = \sqrt{8}$$

$$\operatorname{Cosec} A = \frac{3}{\sqrt{8}}, \cot A = \frac{1}{\sqrt{8}}$$



$$\cot B = \frac{4}{1}$$

$$DF = \sqrt{4^2 + 1^2} = \sqrt{17}$$

$$\sin B = \frac{1}{\sqrt{17}} \text{ and } \sec B = \frac{\sqrt{17}}{4}$$

$$\text{Now, } \frac{\operatorname{cosec}^2 A + \sin^2 B}{\cot^2 A + \sec^2 B}$$

$$= \frac{\left(\frac{3}{\sqrt{8}}\right)^2 + \left(\frac{1}{\sqrt{17}}\right)^2}{\left(\frac{1}{\sqrt{8}}\right)^2 + \left(\frac{\sqrt{17}}{4}\right)^2}$$

$$= \frac{\frac{153}{16} + \frac{8}{136}}{\frac{16}{128} + \frac{136}{152}} = \frac{161}{136} \times \frac{128}{152}$$

$$= \frac{644}{646} = \frac{322}{323}$$

13. Option (c) is correct.

Explanation:

$$\text{Given: } \sin \theta - \cos \theta = \frac{7}{17}$$

Squaring on both sides.

$$(\sin \theta - \cos \theta)^2 = \left(\frac{7}{17}\right)^2$$

$$\Rightarrow \sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta = \frac{49}{289}$$

$$\Rightarrow 1 - 2 \sin \theta \cos \theta = \frac{49}{289}$$

$$\Rightarrow 2 \sin \theta \cos \theta = \frac{240}{289} \quad \dots(i)$$

Now, consider

$$(\sin \theta + \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$$

$$\Rightarrow (\sin \theta + \cos \theta)^2 = 1 + \frac{240}{289}$$

$$\therefore \sin \theta + \cos \theta = \sqrt{\frac{529}{289}} = \frac{23}{17}$$

14. Option (a) is correct.

Explanation:

$$\text{Given: } 2 \sin^2 x = 2 - 3 \sin x$$

$$\text{Put } \sin x = y$$

$$2y^2 = 2 - 3y$$

$$\Rightarrow 2y^2 + 3y - 2 = 0$$

$$\Rightarrow 2y(y + 2) - 1(y + 2) = 0$$

$$\Rightarrow (y + 2)(2y - 1) = 0$$

$$\Rightarrow y = -2 \text{ or } y = \frac{1}{2}$$

$$y = -2 \text{ (not possible)}$$

$$\therefore y = \frac{1}{2}$$

$$\Rightarrow \sin x = \frac{1}{2}$$

$$\Rightarrow \sin x = \sin \frac{\pi}{6}$$

$$\therefore x = \frac{\pi}{6}$$

15. Option (d) is correct.

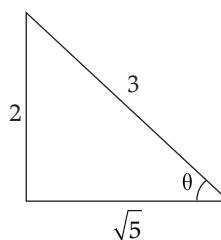
Explanation:

$$\text{Given: } \frac{\sec \theta + \tan \theta}{\sec \theta - \tan \theta} = 5, \theta \text{ is acute angle}$$

$$\Rightarrow \sec \theta + \tan \theta = 5 \sec \theta - 5 \tan \theta$$

$$\Rightarrow 6 \tan \theta = 4 \sec \theta$$

$$\Rightarrow \frac{\frac{\sin \theta}{\cos \theta}}{\frac{1}{\cos \theta}} = \frac{4}{6} = \frac{2}{3}$$



$$\Rightarrow \sin \theta = \frac{2}{3}, \text{ then } \cos \theta = \frac{\sqrt{5}}{3}$$

$$\text{Now, } \frac{3 \cos^2 \theta + 1}{3 \cos^2 \theta - 1} = \frac{3 \left(\frac{\sqrt{5}}{3}\right)^2 + 1}{3 \left(\frac{\sqrt{5}}{3}\right)^2 - 1}$$

$$= \frac{3 \times \frac{5}{9} + 1}{3 \times \frac{5}{9} - 1} = \frac{\frac{5}{3} + 1}{\frac{5}{3} - 1} = \frac{\frac{8}{3}}{\frac{2}{3}} = 4$$

16. Option (a) is correct.

Explanation:

$$\text{Given: } \sin(A - B) = \frac{1}{2}, A > B > 0^\circ$$

$$\Rightarrow \sin(A - B) = \sin 30^\circ$$

$$\Rightarrow A - B = 30^\circ \quad \dots(i)$$

And,

$$\text{Also, } \cos(A + B) = \frac{1}{2} \quad (\text{given})$$

$$\cos(A + B) = \cos 60^\circ$$

$$A + B = 60^\circ \quad \dots(ii)$$

Solving (i) and (ii),

$$\begin{aligned} A + B &= 60^\circ \\ A - B &= 30^\circ \\ \hline 2A &= 90^\circ \\ \Rightarrow A &= \frac{90^\circ}{2} = 45^\circ \end{aligned}$$

$\therefore$ The value of A is 45° .

17. Option (d) is correct.

Explanation:

Given:

$$\alpha + \beta = 90^\circ \text{ and } \alpha = 2\beta$$

$$\begin{aligned} \text{Consider } \alpha + \beta &= 90^\circ \\ \Rightarrow 2\beta + \beta &= 90^\circ & (\because \alpha = 2\beta) \\ \Rightarrow 3\beta &= 90^\circ \\ \Rightarrow \beta &= 30^\circ \\ \therefore \alpha &= 2\beta = 2 \times 30^\circ = 60^\circ \end{aligned}$$

$$\begin{aligned} \text{Now, } \cos^2 \alpha + \sin^2 \beta &= (\cos 60^\circ)^2 + (\sin 30^\circ)^2 \\ &= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \\ &= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \end{aligned}$$

18. Option (b) is correct.

Explanation:

Given

$$\begin{aligned} x \cos \theta - y \sin \theta &= 2 & \dots(i) \\ x \sin \theta + y \cos \theta &= 4 & \dots(ii) \end{aligned}$$

Multiply equation (i) by $\cos \theta$ and multiply equation (ii) by $\sin \theta$,

$$\begin{aligned} x \cos^2 \theta - y \sin \theta \cos \theta &= 2 \cos \theta \\ x \sin^2 \theta + y \sin \theta \cos \theta &= 4 \sin \theta \\ x (\cos^2 \theta + \sin^2 \theta) &= 2 \cos \theta + 4 \sin \theta \\ \Rightarrow x &= 2 \cos \theta + 4 \sin \theta & \dots(iii) \end{aligned}$$

Now multiply equation (i) by $\sin \theta$ and multiply equation (ii) by $\cos \theta$,

$$\begin{aligned} x \cos \theta \sin \theta - y \sin^2 \theta &= 2 \sin \theta \\ x \cos \theta \sin \theta + y \cos^2 \theta &= 4 \cos \theta \\ \hline (-) \quad \quad (-) \quad \quad (-) \end{aligned}$$

$$-y = 2 \sin \theta - 4 \cos \theta$$

$$y = 4 \cos \theta - 2 \sin \theta$$

$$\text{Now, } x^2 + y^2 = 4 \cos^2 \theta + 16 \sin^2 \theta + 16 \cos^2 \theta + 4 \sin^2 \theta + 16 \cos \theta \sin \theta - 16 \cos \theta \sin \theta$$

$$\Rightarrow x^2 + y^2 = 4(\cos^2 \theta + \sin^2 \theta) + 16(\sin^2 \theta + \cos^2 \theta)$$

$$\Rightarrow x^2 + y^2 = 4 + 16$$

$$\Rightarrow x^2 + y^2 = 20$$

19. Option (c) is correct.

Explanation:

$$\begin{aligned} \text{Given: } \frac{2 + \tan^2 \theta + \cot^2 \theta}{\sec \theta \operatorname{cosec} \theta} &= \frac{(1 + \tan^2 \theta) + (1 + \cot^2 \theta)}{\sec \theta \operatorname{cosec} \theta} = \frac{\sec^2 \theta + \operatorname{cosec}^2 \theta}{\sec \theta \operatorname{cosec} \theta} \end{aligned}$$

$$\begin{aligned} &= \frac{\sec \theta}{\operatorname{cosec} \theta} + \frac{\operatorname{cosec} \theta}{\sec \theta} \\ &= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \\ &= \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{1}{\sin \theta \cos \theta} = \sec \theta \operatorname{cosec} \theta \end{aligned}$$

20. Option (a) is correct.

Explanation:

Given:

$$\sqrt{3} \cos \theta + \sin \theta = 1, 0^\circ \leq \theta \leq 90^\circ$$

Trial and error method

$$\text{Put } \theta = 0^\circ,$$

$$\sqrt{3}(1) + 0 = 1$$

$$\sqrt{3} \neq 1$$

$$\therefore \theta \neq 0^\circ$$

$$\text{Put } \theta = 90^\circ$$

$$\sqrt{3}(0) + 1 = 1$$

$$1 = 1$$

$$\therefore \theta = 90^\circ$$

21. Option (a) is correct.

Explanation:

$$\text{Given: } \cos^2 \theta - \sin^2 \theta - 3 \cos \theta + 2 = 0, 0^\circ < \theta < 90^\circ$$

Consider,

$$\begin{aligned} \cos^2 \theta - \sin^2 \theta - 3 \cos \theta + 2 &= 0 \\ \Rightarrow \cos^2 \theta - 3 \cos \theta + 2 &= \sin^2 \theta \\ &[\because \sin^2 \theta = 1 - \cos^2 \theta] \end{aligned}$$

$$\Rightarrow 2 \cos^2 \theta - 3 \cos \theta + 1 = 0$$

$$\text{Put } \cos \theta = x$$

$$2x^2 - 3x + 1 = 0$$

$$\Rightarrow 2x^2 - 2x - x + 1 = 0$$

$$\Rightarrow 2x(x-1) - 1(x-1) = 0$$

$$\Rightarrow (2x-1)(x-1) = 0$$

$$\Rightarrow x = \frac{1}{2} \text{ or } x = 1$$

$$\text{Now, } \cos \theta = \frac{1}{2} \text{ or } \cos \theta = 1$$

$$\therefore \theta = 60^\circ \text{ or } \theta = 0^\circ$$

$$[\theta = 0^\circ, \text{ not possible, given } 0 < \theta < 90^\circ]$$

$$\text{Now, } 4 \operatorname{cosec} \theta + \cot \theta = 4 \operatorname{cosec} 60^\circ + \cot 60^\circ$$

$$= 4 \times \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}$$

$$= \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

22. Option (c) is correct.

Explanation:

$$\text{Given: } 3 \sin \theta = 2 \cos^2 \theta, 0^\circ < \theta < 90^\circ$$

$$\Rightarrow \frac{3}{2} \sin \theta = \cos^2 \theta$$

$$\Rightarrow \frac{3}{2} \sin \theta + \sin^2 \theta = \cos^2 \theta + \sin^2 \theta$$

$$\Rightarrow \sin^2 \theta + \frac{3}{2} \sin \theta - 1 = 0$$

$$\Rightarrow 2 \sin^2 \theta + 3 \sin \theta - 2 = 0$$

Put $\sin \theta = x$ then

$$\Rightarrow 2x^2 + 4x - x - 2 = 0$$

$$\Rightarrow 2x(x+2) - 1(x+2) = 0$$

$$\Rightarrow (x+2)(2x-1) = 0$$

$$\Rightarrow x = -2 (\text{not possible}) \text{ or } x = \frac{1}{2}$$

$$\therefore x = \frac{1}{2}$$

$$\Rightarrow \sin \theta = \frac{1}{2}$$

$$\Rightarrow \sin \theta = \sin 30^\circ$$

$$\Rightarrow \theta = 30^\circ$$

Now,

$$\tan \theta + \cos \theta + \sin \theta = \tan 30^\circ + \cos 30^\circ + \sin 30^\circ$$

$$= \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{2+3+\sqrt{3}}{2\sqrt{3}} = \frac{5+\sqrt{3}}{2\sqrt{3}}$$

$$= \frac{5+\sqrt{3}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{5\sqrt{3}+3}{6}$$

23. Option (b) is correct.

Explanation:

Given:

$$A = 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta)$$

$$\Rightarrow A = 2(1 - 3 \sin^2 \theta \cos^2 \theta) - 3(1 - 2 \sin^2 \theta \cos^2 \theta)$$

$$\Rightarrow A = 2 - 6 \sin^2 \theta \cos^2 \theta - 3 + 6 \sin^2 \theta \cos^2 \theta$$

$$\Rightarrow A = 2 - 3$$

$$\Rightarrow A = -1$$

Now,

$$\cos \alpha = \sqrt{\frac{3+A}{5+A}} = \sqrt{\frac{3-1}{5-1}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos \alpha = \cos 45^\circ$$

$$\Rightarrow \alpha = 45^\circ$$

$$\therefore 3\alpha = 3 \times 45 = 135$$

24. Option (a) is correct.

Explanation:

$$\text{Given: } \frac{\cot x}{1 + \operatorname{cosec} x} + \frac{1 + \operatorname{cosec} x}{\cot x}$$

$$= \frac{\cot^2 x + (1 + \operatorname{cosec} x)^2}{\cot x(1 + \operatorname{cosec} x)}$$

$$= \frac{\cot^2 x + 1 + \operatorname{cosec}^2 x + 2 \operatorname{cosec} x}{\cot x(1 + \operatorname{cosec} x)}$$

$$= \frac{\operatorname{cosec}^2 x + \operatorname{cosec}^2 x + 2 \operatorname{cosec} x}{\cot x(1 + \operatorname{cosec} x)}$$

$$= \frac{2 \operatorname{cosec} x [\operatorname{cosec} x + 1]}{\cot x(1 + \operatorname{cosec} x)}$$

$$= \frac{2 \operatorname{cosec} x}{\cot x}$$

$$= 2 \left[\frac{1}{\sin x} \right] \left[\frac{\cos x}{\sin x} \right] = 2 \left[\frac{1}{\cos x} \right] = 2 \sec x$$

25. Option (a) is correct.

Explanation:

$$\text{Given: } \sin x - \cos x = 0, 0^\circ < \theta < 90^\circ$$

$$= \sin x = \cos x$$

$$= \frac{\sin x}{\cos x} = 1$$

$$= \tan x = 1$$

$$= x = 45^\circ$$

$$\text{Now, } (\sec x + \operatorname{cosec} x)^2$$

$$= [\sec 45^\circ + \operatorname{cosec} 45^\circ]$$

$$= [\sqrt{2} + \sqrt{2}]^2$$

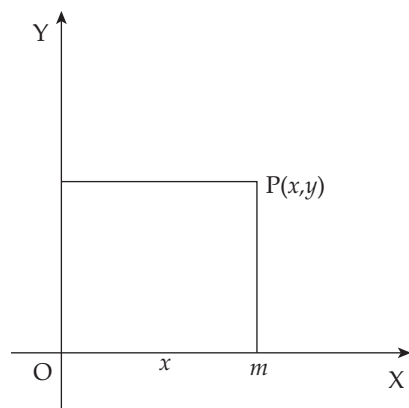
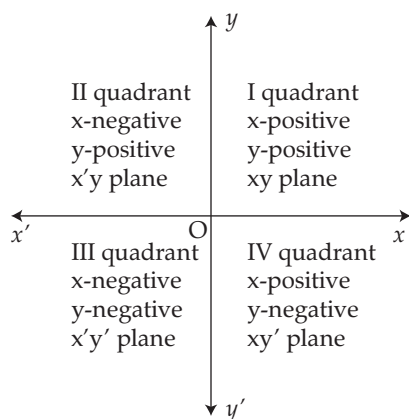
$$= [2\sqrt{2}]^2$$

$$= 4 \times 2 = 8$$

CARTESIAN SYSTEM OF RECTANGULAR COORDINATES STRAIGHT LINE, FAMILY OF STRAIGHT LINE

Cartesian Coordinate System :

- Co-ordinate system is used to locate the position of a Point in a plane using two perpendicg lines. Points is represented in the form of coordinates, (x, y) with respect to X and Y-axis in two dimensions.
- To simplify the calculation, we divide the plane into 4 quadrants using X-axis and Y-axis such that X-axis and Y-axis intersects at origin.
- Let P be any point on the plane. Draw PM perpendicular to X-axis. The lengths OM and MP are called the coordinates the point P.
- OM being X-coordinate or abscissa and MP is the Y-coordinate or ordinate.



We denote OM by x and MP by y and the point P by (x, y)

Points to Remember:

Distance formula: Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points on a plane, then the distance between them is

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of a point $P(x, y)$ from origin is $OP = \sqrt{x^2 + y^2}$

Section formula: Let $P(x, y)$ and $Q(x_2, y_2)$ be two points,

and let $A(x, y)$ be the point which divides PQ in the ratio $m:n$ internally, then

$$A(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

Midpoint: Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points, then the midpoint of PQ:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle, then the co-ordinates of centroid are

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of a triangle, then the area of triangle ABC:

$$\text{Area of } \triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Collinear: If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear points (lie in one line), then

$$\text{Area of } \triangle ABC = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

Also, if $AB + BC = AC$, then the points A, B and C collinear.

Equation of a Straight line

- Point-Slope formula:** To find the equation of a line when one point (x_1, y_1) and slope of the line ' m ' is given

$$(y - y_1) = m(x - x_1)$$

- Two-point formula:** To find the equation of a line when two points (x_1, y_1) and (x_2, y_2) are given:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

$$[\text{Slope of a line, } m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}]$$

Slope-intercepts formula: To find the equation of a line when slope and y -intercept is given

$y = mx + c$, where ' m ' is slope and ' c ' is y -intercept.

Intercept form: To find the equation of a line when x -intercept ' a ' and y -intercept ' b ' are given.

$$\frac{x}{a} + \frac{y}{b} = 1$$

General Equation: General equation of a line is $Ax + By + C = 0$

Equation of a circle

- Equation of circle, whose centre is at origin $(0, 0)$ and radius ' a ' is $x^2 + y^2 = a^2$
- Equation of a circle, whose centre is at (h, k) and radius ' a ' is $(x - h)^2 + (y - k)^2 = a^2$

- Equation of a circle, in general form is $x^2 + y^2 + 2gx + 2fy + c = 0$

Where, center is $(-g, -f)$ and radius is $r = \sqrt{g^2 + f^2 - c}$

Let $P(x_1, y_1)$ be any point on a circle and $O(x, y)$ be the centre of the circle, then radius is OP

$$OP = \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$$

EXERCISE

- The equation of a line that passes through the points $(1, 5)$ and $(2, 3)$ is:
 (a) $2x + y - 7 = 0$ (b) $2x - y - 7 = 0$
 (c) $x + 2y - 7 = 0$ (d) $2x + y + 7 = 0$
- The locus of a point, whose abscissa and ordinate are always equal is
 (a) $x - y = 0$ (b) $x + y = 1$
 (c) $x + y + 1 = 0$ (d) None of these
- The coordinates of the foot of the perpendicular from the point $(2, 3)$ on the line $x + y - 11 = 0$ are:
 (a) $(-6, 5)$ (b) $(5, 6)$ (c) $(-5, 6)$ (d) $(6, 5)$
- The equation of a straight line that passes through the point $(3, 4)$ and perpendicular to the line $3x + 2y + 5 = 0$ is
 (a) $2x - 3y + 6 = 0$ (b) $2x + 3y + 6 = 0$
 (c) $2x - 3y - 6 = 0$ (d) $2x + 3y - 6 = 0$
- What is the distance of $(5, 12)$ from the origin?
 (a) 5 units (b) 8 units
 (c) 12 units (d) 13 units
- Find the distance between the points $(3, -2)$ and $(-1, -1)$.
 (a) $\sqrt{19}$ units (b) 5 units
 (c) $\sqrt{17}$ units (d) 3 units
- In what ratio Y-axis divides the line segment joining the points $(3, 4)$ and $(-2, 1)$?
 (a) 3 : 2 (b) 2 : 3 (c) 3 : 1 (d) 1 : 3
- Co-ordinates of centroid of $\triangle ABC$ are $(1, -1)$. Vertices of $\triangle ABC$ are $A(-5, 3)$, $B(p, -1)$ and $C(6, q)$. Find p and q .
 (a) $p = 3; q = 1$ (b) $p = 2; q = -5$
 (c) $p = -2; q = 2$ (d) $p = 1; q = -1$
- Find the angle between the lines whose slopes are $(2 - \sqrt{3})$ and $(2 + \sqrt{3})$.
 (a) 30° (b) 90° (c) 60° (d) 45°
- Find the slope of a line perpendicular to the line passing through $(-3, 8)$ and $(2, -2)$.
 (a) $\frac{1}{2}$ (b) 1 (c) 2 (d) $-\frac{1}{2}$
- Find the slope of the line $6x + 3y - 5 = 0$.
 (a) -2 (b) 2 (c) 1 (d) 3

Answer Key

1. (a)	2. (a)	3. (b)	4. (a)	5. (d)	6. (c)	7. (a)	8. (b)	9. (c)	10. (a)
11. (a)									

Answers with Explanations

1. Option (a) is correct.

Explanation: We know that the equation of a line passes through two points (x_1, y_1) and (x_2, y_2) is

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(x_1, y_1) = (1, 5)$$

$$(x_2, y_2) = (2, 3)$$

Now, substitute the values in the formula, we get

$$\frac{y - 5}{x - 1} = \frac{3 - 5}{2 - 1}$$

$$\frac{y - 5}{x - 1} = \frac{-2}{1}$$

$$y - 5 = -2(x - 1)$$

$$y - 5 = -2x + 2$$

$$2x + y - 5 - 2 = 0$$

$$2x + y - 7 = 0$$

Therefore, the equation of a line that passes through the points $(1, 5)$ and $(2, 3)$ is $2x + y - 7 = 0$.

2. Option (a) is correct.

Explanation: Let the abscissa and ordinate of a point "P" be (x, y)

Given condition: Abscissa = Ordinate (i.e.) $x = y$

Hence, the locus of a point is $x - y = 0$.

3. Option (b) is correct.

Explanation: Let (h, k) be the coordinates of the foot of the perpendicular from the point $(2, 3)$ on the line $x + y - 11 = 0$. Then, the slope of the perpendicular line is $k - 3h - 2$.

Again, the slope of the given line $x + y - 11 = 0$ is -1 . Using the condition of perpendicularity of lines, we have

$$\frac{k - 3}{h - 2} \times -1 = -1$$

$$\text{or } k - h = 1 \quad \dots(1)$$

Since, (h, k) lies on the given line, we have, $h + k - 11 = 0$ or $h + k = 11$...(2)

Solving (1) and (2), we get $h = 5$ and $k = 6$.

Thus $(5, 6)$ are the required coordinates of the foot of the perpendicular.

4. Option (a) is correct.

Explanation: The equation of a straight line perpendicular to $3x + 2y + 5 = 0$ is $2x - 3y + \lambda = 0$

... (1)

This passes through the point $(3, 4)$.

Now, substitute in equation (1), we get

$$2(2) - 3(4) + \lambda = 0$$

$$4 - 12 + \lambda = 0$$

$$-6 + \lambda = 0$$

$$\lambda = 6$$

Substituting $\lambda = 6$ in (1), we get $2x - 3y + 6 = 0$, which is the required equation.

5. Option (d) is correct.

Explanation: Let the points be $A(0, 0)$ and $B(5, 12)$.

$$A(0, 0) = (x_1, y_1)$$

$$B(5, 12) = (x_2, y_2)$$

The distance between two points,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(5-0)^2 + (12-0)^2}$$

$$AB = \sqrt{5^2 + 12^2}$$

$$AB = \sqrt{25 + 144}$$

$$AB = \sqrt{169}$$

$$AB = 13$$

Hence, the distance of $(5, 12)$ from the origin is 13 units.

6. Option (c) is correct.

Explanation: Let, the points be $A(x_1, y_1) = (3, -2)$

and $B(x_2, y_2) = (-1, -1)$.

Then, the distance

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(3+1)^2 + (-2+1)^2}$$

$$= \sqrt{4^2 + (-1)^2}$$

$$= \sqrt{17} \text{ units.}$$

7. Option (a) is correct.

Explanation: Let the given points be $A(x_1, y_1) = (3, 4)$

and $B(x_2, y_2) = (-2, 1)$. Let $m : n$ be the ratio in which

the line segment is divided by Y-axis (the co-ordinates of the point are $(0, y)$).

$$\therefore (0, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$\Rightarrow (0, y) = \left(\frac{2m + 3n}{m+n}, \frac{m + 4n}{m+n} \right)$$

$$\Rightarrow 0 = \frac{-2m + 3n}{m+n}, \quad y = \frac{m + 4n}{m+n}$$

$$\Rightarrow -2m + 3n = 0$$

$$\Rightarrow 2m = 3n$$

$$\Rightarrow m : n = 3 : 2$$

8. Option (b) is correct.

Explanation: Co-ordinates of centroid of a triangle with vertices (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are given by

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Since, the given vertices are $A(-5, 3)$, $B(p, -1)$ and $C(6, q)$. So the co-ordinates of the centroid of $\triangle ABC$ are

$$\left(\frac{-5+p+6}{3}, \frac{3-1+q}{3} \right) = \left(\frac{p+1}{3}, \frac{2+q}{3} \right)$$

$$\text{Given, } \left(\frac{p+1}{3}, \frac{2+q}{3} \right) = (1, -1)$$

$$\Rightarrow \frac{p+1}{3} = 1 \quad \text{and} \quad \frac{2+q}{3} = -1$$

$$\Rightarrow p = 2 \quad \Rightarrow q = -5$$

9. Option (c) is correct.

Explanation: Let the slopes of the two lines are

$$m_1 = 2 - \sqrt{3} \text{ and } m_2 = (2 + \sqrt{3}).$$

Then, angle ' θ ' between the lines is given by

$$\theta = \tan^{-1} \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \tan^{-1} \frac{(2 + \sqrt{3}) - (2 - \sqrt{3})}{1 + (2 - \sqrt{3})(2 + \sqrt{3})}$$

$$= \tan^{-1} \frac{2\sqrt{3}}{1 + 4 - 3}$$

$$= \tan^{-1}(\sqrt{3})$$

$$= \tan^{-1}(\tan 60^\circ)$$

$$= 60^\circ$$

10. Option (a) is correct.

Explanation: Slope of the line passing through $(-3,$

$$8) \text{ and } (2, -2), m_1 = \frac{8+2}{-3-2} = -2.$$

If m_2 be the slope of the perpendicular line, then

$$m_1 m_2 = -1$$

$$\Rightarrow m_2 = \frac{1}{2}.$$

11. Option (a) is correct.

Explanation: Given line is $6x + 3y - 5 = 0$

$$\Rightarrow 3y = -6x + 5$$

$$\Rightarrow y = -2x + \frac{5}{3}$$

$$\therefore \text{Slope} = -2.$$

Section of a cone

Let be l a fixed vertical line and m be another line intersecting it at a fixed point V and inclined to it at an angle α (Fig 1).

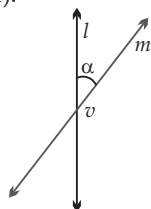


Fig 1

Suppose we rotate the line m around the line l in such a way that the angle α remains constant. Then the surface generated is a double-napped right circular hollow cone herein after referred as Cone and extending indefinitely far in both directions (Fig 2)

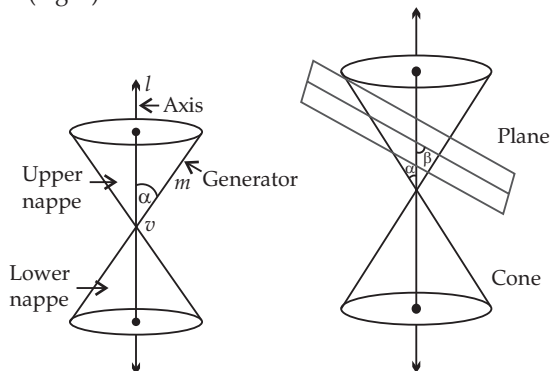


Fig 2

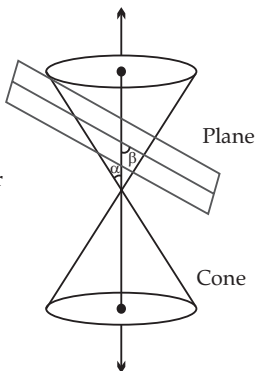


Fig 3

The point V is called the *vertex*; the line l is axis of the cone. The rotating line m is called a *generator* of the cone. The *vertex* separates the cone into two parts called *nappes*.

If We take the intersection of a plane with a cone, the section so obtained is called a *conic section*. Thus, conic sections are the curves obtained by intersecting a right circular cone by a plane.

We obtain different kinds of conic sections on the position of the intersecting plane with respect to the cone and by the angle made by it with the vertical axis of the cone. Let β be the angle made by the intersecting plane with the vertical axis of the cone (Fig 3).

The intersecting of the plane with the cone can take place either at the vertex of the cone or at any other part of the nappe either below or above the vertex.

Circle, parabola and hyperbola

When the plane cuts the nappe (other than the vertex) of the cone, we have the following situations:

- When $\beta = 90^\circ$, the section is a *circle* (Fig 4).
 - When $\alpha < \beta < 90^\circ$, the section is an *ellipse* (Fig 5).
 - When $\beta = \alpha$; the section is a *parabola* (Fig 6).
- (in each of the above three situations, the plane cuts entire across one nappe of the cone)
- When $0 \leq \beta < \alpha$; the plane cuts through both the napped and the curves of intersection is a *hyperbola* (Fig 7).

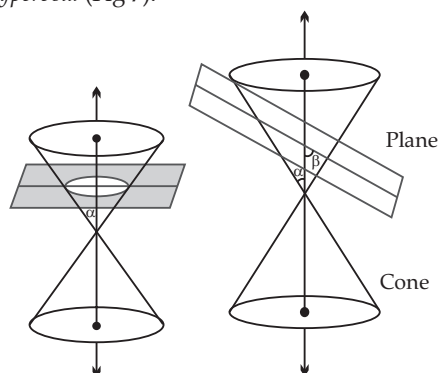


Fig 4

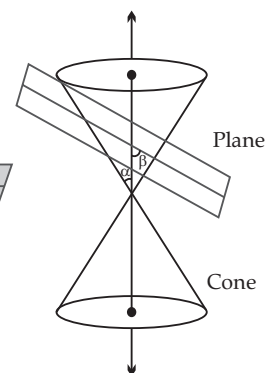


Fig 5

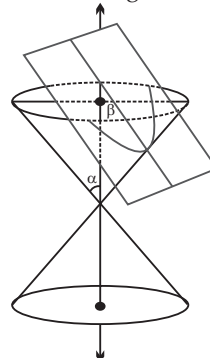


Fig 6

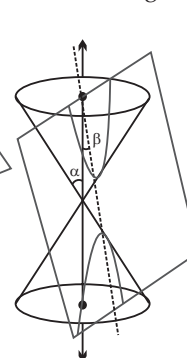


Fig 7

Degenerated conic sections

When the plane cuts at the vertex of the cone, we have the following different cases;

- When $\alpha < \beta \leq 90^\circ$, then the section is a point (Fig 8).
- When $\beta = \alpha$, the plane contains a generator of the cone and the section is a straight line (Fig 9). It is the degenerated case of a parabola.
- When $0 \leq \beta < \alpha$, the section is a pair of intersecting straight lines (Fig 10). It is the degenerated case of a hyperbola.

In the following sections, we shall obtain the equations of each of these conic sections on standard form by defining them based on geometric properties.

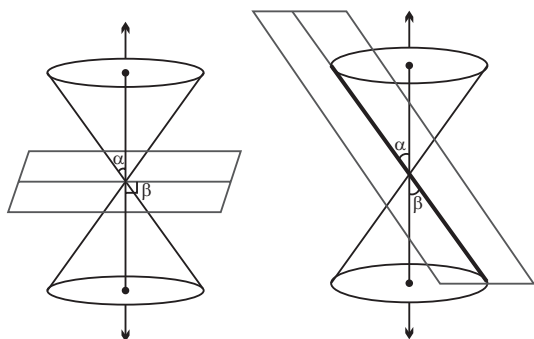


Fig 8

Fig 9

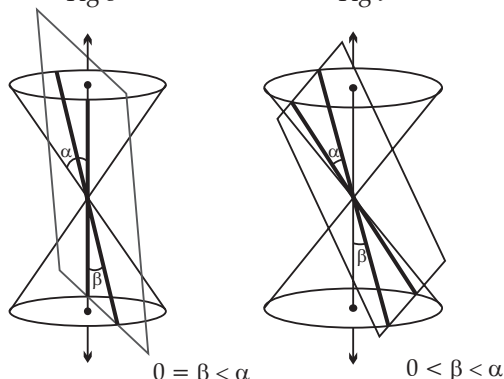


Fig 10

Fig 11

Circle

Definition 1 A circle is the set of all points in a plane that are equidistant from a fixed point in the plane. The fixed point is called the *center of the circle* and the distance from the center to any point on the circle is called the *radius* of the circle (Fig 11).

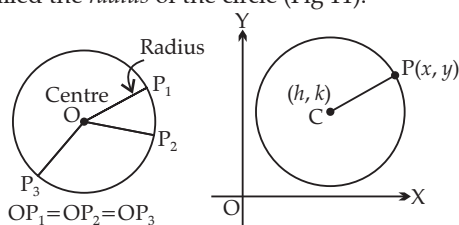


Fig 11

Fig 12

The equation of the circle is simplest if the center of the circle is at the origin.

However, we derive below the equation of the circle with a given center and radius (Fig 12).

Given $C(h, k)$ be the center and r the radius of circle. Let $P(x, y)$ be any point on the circle (Fig 12). Then, by the definition, $CP = r$. By the distance formula, We have

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$\text{i.e. } (x-h)^2 + (y-k)^2 = r^2$$

This is the required equation of the circle with center at (h, k) and radius r .

Parabola

Definition 2 A parabola is the set of all points in a plane that are equidistant from a fixed line and a fixed point (not on the line) in a plane.

The fixed line is called the *directrix* of the parabola and the fixed point F is called the *focus* (Fig 13). ('Para'

means 'for' and 'bola' means 'throwing', i.e., the shape described when you know a ball in the air).

Note If the fixed point lies on the fixed line, then the set of points in the plane, which are equidistant from the fixed point and the fixed line is the straight line through the fixed point and perpendicular to the fixed line. We call this straight line as degenerate case of the parabola.

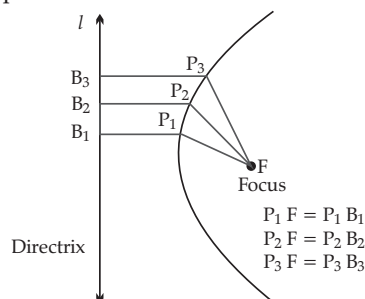


Fig 13

A line through the focus and perpendicular to the *directrix* is called the *axis* of the parabola. The point of intersection of parabola with the axis is called the *vertex* of the parabola (Fig 14).

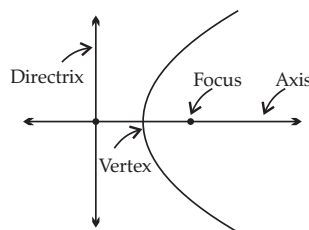


Fig 14

Standard equations of parabola

The equation of a parabola is simplest if the vertex is at the origin and the axis of symmetry is along the x -axis or y -axis. The four possible such orientations of parabola are shown below in Fig (a) to (d).

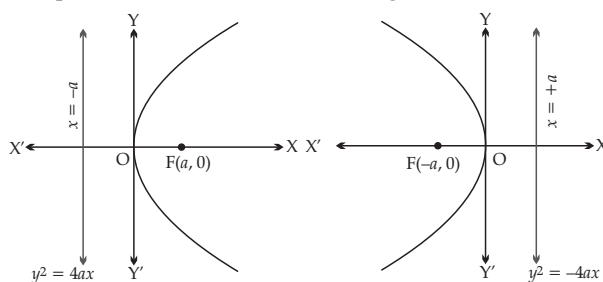


Fig 15(a)

Fig 15(b)

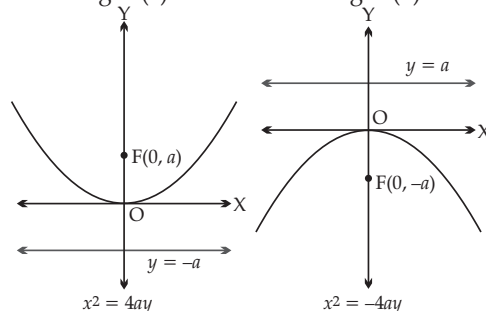


Fig 15(c)

Fig 15(d)

We will derive the equation for the parabola shown above in Fig 15 (a) with focus at $(a, 0)$ $a > 0$; and directrix $x = -a$ as below :

Let F be the focus and l the directrix. Let FM be perpendicular to the directrix and bisect FM at the point O . Produce MO to X . By the definition of parabola, the mid point O is on the parabola and is called the *vertex* of the parabola. Take O as origin, OX the x -axis and OY perpendicular to it as the y -axis. Let the distance from the directrix to the focus be $2a$. Then, the coordinates of the focus are $(a, 0)$, and the equation of the directrix is $x + a = 0$ as in Fig 16.

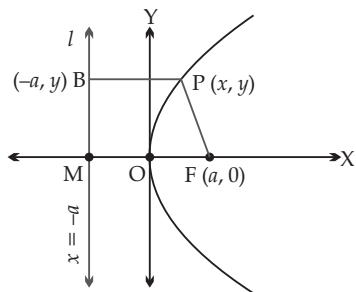


Fig. 16

Let $P(x, y)$ be any point on the parabola such that $PF = PB$, ...(1)

Where PB is perpendicular to l . The coordinates of B are $(-a, y)$. By the distance formula, we have

$$PF = \sqrt{(x-a)^2 + y^2} \text{ and } PB = \sqrt{(x+a)^2}$$

Since $PF = PB$, we have

$$\sqrt{(x-a)^2 + y^2} = \sqrt{(x+a)^2}$$

$$\text{i.e., } (x-a)^2 + y^2 = (x+a)^2$$

$$\text{or } x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$

$$\text{or } y^2 = 4ax \quad (a > 0)$$

Hence, any point on the parabola satisfies

$$y^2 = 4ax \quad \dots(2)$$

Conversely, let $P(x, y)$ satisfy the equation (2)

$$\begin{aligned} PF &= \sqrt{(x-a)^2 + y^2} = \sqrt{(x-a)^2 + 4ax} \\ &= \sqrt{(x+a)^2} = PB \end{aligned}$$

And so $P(x, y)$ lies on the parabola.

Thus, from (2) and (3) we have proved that the equation to the parabola with vertex at the origin, focus at $(a, 0)$ and directrix $x = -a$ is $y^2 = 4ax$.

Discussion in equation (2), since $a > 0$, x can assume any positive value or zero but no negative value and the curve extends indefinitely far into the first and fourth quadrants. The axis of the parabola is the

positive x -axis.

Similarly, we can derive the equations of the parabolas in:

$$\text{Fig 15 (b) as } y^2 = -4ax,$$

$$\text{Fig 15 (c) as } x^2 = 4ay,$$

$$\text{Fig 15 (d) as } x^2 = -4ay,$$

These four equations are known as standard equations of parabolas.

Note The standard equations of parabolas have focus on one of the coordinate axis, vertex at the origin and thereby the directrix is parallel to the other coordinate axis. However, the study of the equations of parabolas with focus at any point and any line as directrix is beyond the scope here.

From the standard equations of the parabolas, Fig. 15, we have the following observations :

1. parabola is symmetric with respect to the axis of the parabola. If the equation has a y^2 term, then the axis of symmetry is along the x -axis and if the equation has an x^2 term, then the axis of symmetry is along the y -axis.
2. When the axis of symmetry is along the x -axis the parabola opens to the
 - (a) right if the coefficient of x is positive.
 - (b) left if the coefficient of x is negative.
3. When the axis of symmetry is along the y -axis the parabola opens
 - (c) upwards if the coefficient of y is positive.
 - (d) downwards if the coefficient of y is negative.

Latus rectum

Definition 3 Latus rectum of a parabola is a line segment perpendicular to the axis of the parabola, through the focus and whose end points lie on the parabola (fig 17)

To find the length of the latus rectum of the parabola $y^2 = 4ax$ (fig 18).

By the definition of the parabola, $AF = AC$.

$$\text{But } AC = FM = 2a$$

$$\text{Hence } AF = 2a$$

And since the parabola is symmetric with respect to x -axis $AF = FB$ and so

$$AB = \text{Length of the latus rectum} = 4a$$

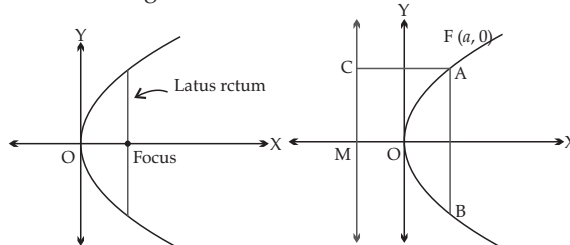


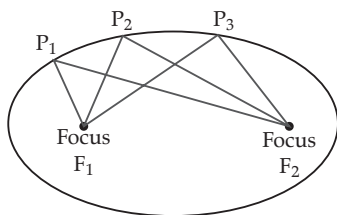
Fig 17

Fig 18

Ellipse

Definition 4 An *ellipse* is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

The two fixed points are called the *foci* (plural of 'focus') of the ellipse (Fig 20).



$P_1F_1 + P_1F_2 = P_2F_1 + P_2F_2 = P_3F_1 + P_3F_2$
Fig. 20

Note The constant which is the sum of the distances of a point on the ellipse from the two fixed points is always greater than the distance between the two fixed points.

The mid point of the line segment joining the foci is called the *centre* of the ellipse. The line segment through the foci of the ellipse is called the *major axis* and the line segment through the centre and perpendicular to the major axis is called the *minor axis*. The end points of the major axis are called the *vertices* of the ellipse (Fig 21).

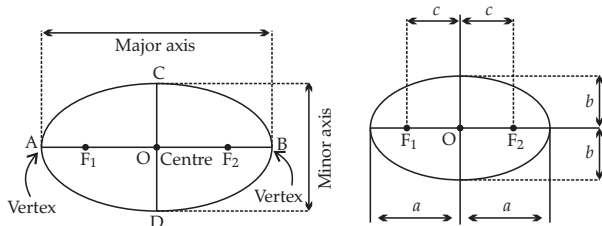


Fig. 21

Fig. 22

We denote the length of the major axis by $2a$, the length of the minor axis by $2b$ and the distance between the foci by $2c$. Thus, the length of the semi major axis is a and semi-minor axis is b (Fig 22)

Relationship between semi-major axis, semi-minor axis and the distance of the focus from the centre of ellipse (Fig 23).

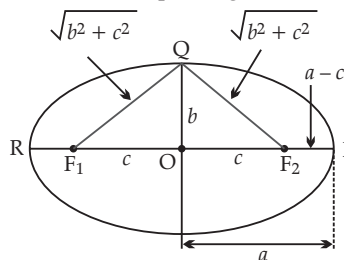


Fig. 23

Take a point P at one end of the major axis.

Sum of the distances of the point P to the foci is $F_1P + F_2P = F_1O + OP + F_2P$

$$\begin{aligned} \text{(Since, } F_1P &= F_1O + OP) \\ &= c + a + a - c = 2a \end{aligned}$$

Take a point Q at one end of the minor axis.

Sum of the distances of the point Q to the foci is

$$F_1Q + F_2Q = \sqrt{b^2 + c^2} + \sqrt{b^2 + c^2} = 2\sqrt{b^2 + c^2}$$

Since both P and Q lies on the ellipse.

By the definition of ellipse, we have

$$2\sqrt{b^2 + c^2} = 2a, \text{ i.e., } a = \sqrt{b^2 + c^2}$$

$$\text{or } a^2 = b^2 + c^2, \text{ i.e., } c = \sqrt{a^2 - b^2}$$

Special cases of an ellipse

In the equation $c^2 = a^2 - b^2$ obtained above, if we keep a fixed and vary c from 0 to a , the resulting ellipse will vary in shape.

Case (i) When $c = 0$, both foci merge together with the centre of the ellipse and $a^2 = b^2$, i.e., $a = b$, and so the ellipse becomes circle (Fig 24). Thus, circle is a special case of an ellipse which is dealt in Section 11.3.

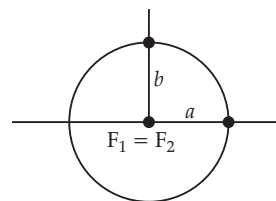


Fig. 24

Case (ii) When $c = a$, then $b = 0$. The ellipse reduces to the line segment F_1F_2 joining the two foci (Fig. 25).



Fig. 25

Eccentricity

Definition 5 The eccentricity of an ellipse is the ratio of the distance from the centre of the ellipse to one of the foci and to one of the vertices of the ellipse

(eccentricity is denoted by e) i.e., $e = \frac{c}{a}$

Then since the focus is at a distance of c from the centre, in terms of the eccentricity the focus is at a distance of ae from the centre.

Standard equations of an ellipse

The equation of an ellipse is simplest if the centre of the ellipse is at the origin and the foci are on the x -axis or y -axis. The two such possible shown above in Fig. 26.

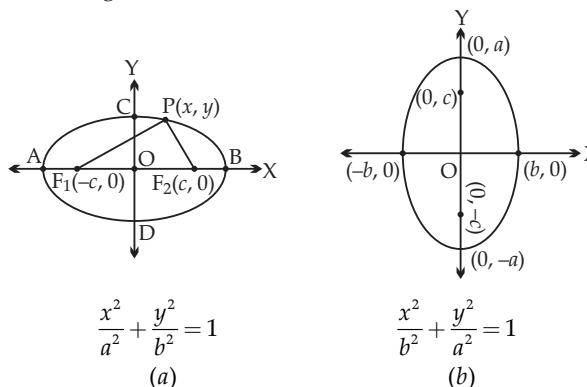


Fig. 26

We will x -axis or y -axis. The two such possible orientations are shown in Fig 26 (a) with foci on the x -axis.

Let F_1 and F_2 be the foci and O be the mid-point of the line segment $F_1 F_2$. Let O be the origin and the line from O through F_2 be the positive x -axis and that

through O perpendicular to the x-axis be the y-axis. Let the coordinates of F_1 be $(-c, 0)$ and F_2 be $(c, 0)$ (Fig. 27).

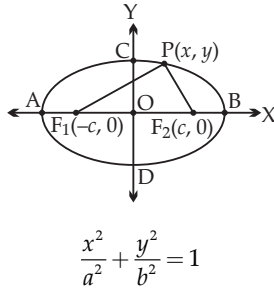


Fig. 27

Let $P(x, y)$ be any point on the ellipse such that the sum of the distance from P to the foci be $2a$ so given

$$PF_1 + PF_2 = 2a. \quad \dots(1)$$

Using the distance formula, we have

$$\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a$$

$$\text{i.e., } \sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$

Squaring both sides we get

$$(x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

Which on simplification gives

$$\sqrt{(x+c)^2 + y^2} = a - \frac{c}{a}x$$

Squaring again and simplifying, we get

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1$$

$$\text{i.e., } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (\text{Since } c^2 = a^2 - b^2)$$

Hence any point on the ellipse satisfies

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \dots(2)$$

Conversely, let $P(x, y)$ satisfy the equation (2) with $0 < c < a$. Then

$$y^2 = b^2 \left(1 - \frac{x^2}{a^2} \right)$$

$$\text{Therefore, } PF_1 = \sqrt{(x+c)^2 + y^2}$$

$$= \sqrt{(x+c)^2 + b^2 \left(\frac{a^2 - x^2}{a^2} \right)}$$

$$= \sqrt{(x+c)^2 + (a^2 - c^2) \left(\frac{a^2 - x^2}{a^2} \right)}$$

$$(\text{Since } b^2 = a^2 - c^2)$$

$$= \sqrt{\left(a - \frac{cx}{a} \right)^2} = a - \frac{c}{a}x$$

$$\text{Similarly } PF_2 = a - \frac{c}{a}x$$

$$\text{Hence } PF_1 - PF_2 = a + \frac{c}{a}x + a - \frac{c}{a}x = 2a \quad \dots(3)$$

So, any point that satisfies $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, satisfies the

geometric condition and so $P(x, y)$ lies on the ellipse.

Hence from (2) and (3), we proved that the equation of an ellipse with center of the origin and major axis along the x-axis is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Discussion From the equation of the ellipse obtained above, it follows that for every point $P(x, y)$ on the ellipse, we have

$$\frac{x^2}{a^2} = 1 - \frac{y^2}{b^2} \leq 1, \text{ i.e., } x^2 \leq a^2, \text{ so } -a \leq x \leq a.$$

Therefore, the ellipse lies between the lines $x = -a$ and $x = a$ and touches these lines.

Similarly, the ellipse lies between the lines $y = -b$ and $y = b$ and touches these lines.

Similarly, we can derive the equation of the ellipse in fig. 26(b) as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

These two equations are known as standard equations of the ellipses.

Note: The standard equations of the ellipses have centre at the origin and the major and minor axis are coordinate axes. However, the study of the ellipses with centre at any other point, and any line through the centre as major and the minor axes passing through the centre and perpendicular to major and the minor axes are beyond the scope here.

From the standard equation of the ellipses (fig. 26), we have the following observations.

1. Ellipse is symmetric with respect to both the coordinate axes since if (x, y) is a point on the ellipse, then $(-x, y)$, $(x, -y)$ and $(-x, -y)$ are also points on the ellipse.
2. The foci always lie on the major axis. The major axis can be determined by finding the intercepts on the axes of symmetry. That is major axis is along the x-axis if the coefficient of y^2 has the larger denominator.

Latus rectum

Definition 6 Latus rectum of an ellipse is a line segment Perpendicular to the major axis through any of the foci and whose end points lie on the ellipse (Fig. 28).

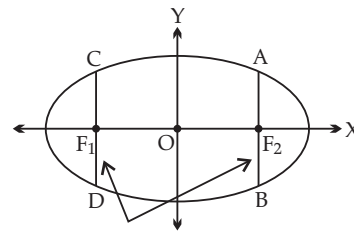


Fig. 28

To find the length of the latus rectum of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Let the length of AF_2 be l .

Then the coordinates of A are (c, l) i.e., (ae, l)

Since A lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have

$$\frac{(ae)^2}{a^2} + \frac{l^2}{b^2} = 1$$

$$\Rightarrow l^2 = b^2(1 - e^2)$$

$$\text{But } e^2 = \frac{c^2}{a^2} = \frac{a^2 - b^2}{a^2} = 1 - \frac{b^2}{a^2}$$

$$\text{Therefore } l^2 = \frac{b^2}{a^2}, \text{ i.e., } l = \frac{b^2}{a^2}$$

Since the ellipse is symmetric with respect to y -axis (of course, it is symmetric w.r.t. both the coordinate axes), $AF_2 = F_2B$ and so length of the latus rectum is $\frac{2b^2}{a^2}$.

Hyperbola

Definition 7 A hyperbola is the set of all points in a plane, the difference of whose distances from two fixed points in the plane is a constant.

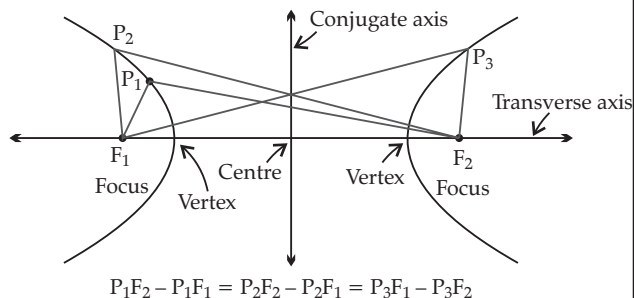


Fig. 29

The term "difference" that is used in the definition means the distance to the farther point minus the distance to the closer point. The two fixed points are called the foci of the hyperbola. The mid-point of the line segment joining the foci is called the *centre of the hyperbola*. The line through the foci is called the *transverse axis* and the line through the centre and perpendicular to the transverse axis is called the *conjugate axis*. The points at which the hyperbola intersects the transverse axis are called the *vertex of the hyperbola* (Fig 29).

We denote the distance between the two foci by $2c$, the distance between two vertex (the length of the transverse axis) by $2a$ and we define the quantity b as

$$b = \sqrt{c^2 - a^2}$$

Also $2b$ is the length of the conjugate axis (Fig 30)

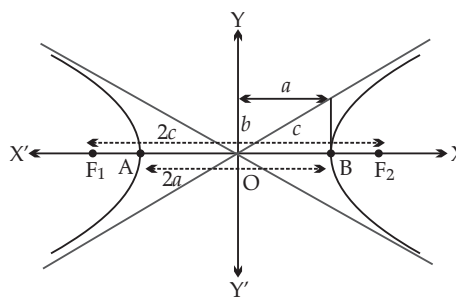


Fig. 30

To find the constant $P_1F_2 - P_1F_1$:

By taking the point P at A and B in the Fig 30, we have

$BF_1 - BF_2 = AF_2 - AF_1$ (by the definition of the hyperbola)

$$BA + AF_1 - BF_2 = AB + BF_2 - AF_1$$

$$\text{i.e., } AF_1 = BF_2$$

So that, $BF_1 - BF_2 = BA + AF_1 - BF_2 = BA = 2a$

Eccentricity

Definition 8 Just like an ellipse, the ratio $e = \frac{c}{a}$ is

called the *eccentricity of the hyperbola*. Since $c \geq a$, the eccentricity is never less than one. In terms of the eccentricity, the foci are at a distance of ae from the centre.

Standard equation of Hyperbola

The equation of a hyperbola is simplest if the centre of the hyperbola is at the origin and the foci are on the x -axis or y -axis. The two such possible orientations are shown in Fig. 31.

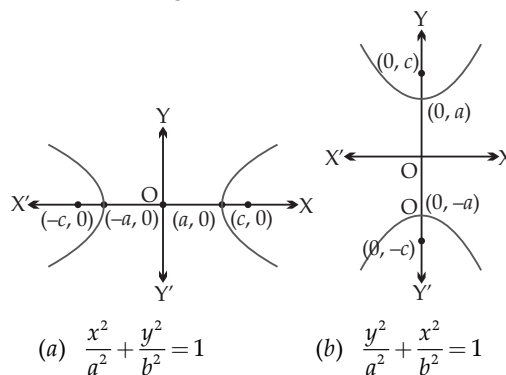


Fig 31

We will derive the equation for the hyperbola shown in Fig 31(a) with foci on the x -axis.

Let F_1 and F_2 be the foci and O be the mid-point of the line segment F_1F_2 . Let O be the origin and the line through F_2 be the positive x -axis and that through F_1 as the negative x -axis. The line through O perpendicular to the x -axis be the y -axis. Let the coordinates of F_1 be $(-c, 0)$ and F_2 be $(c, 0)$ (Fig 32).

Let $P(x, y)$ be any point on the hyperbola such that the difference of the distances from P to the farther point minus the closer point be $2a$. So given, $PF_1 - PF_2 = 2a$

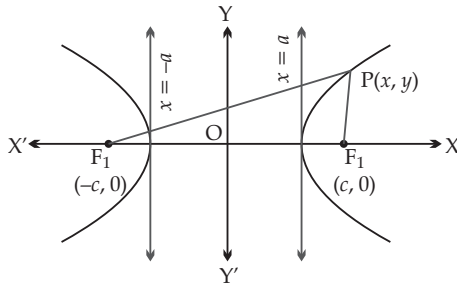


Fig. 32

Using the distance formula, we have

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = 2a$$

$$\text{i.e., } \sqrt{(x+c)^2 + y^2} = 2a + \sqrt{(x-c)^2 + y^2}$$

Squaring both side, we get

$$(x+c)^2 + y^2 = 4a^2 + 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

And on simplifying, we get

$$\frac{cx}{a} - a = \sqrt{(x-c)^2 + y^2}$$

On squaring again and further simplify, we get

$$\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1$$

$$\text{i.e., } \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (\text{Since } c^2 - a^2 = b^2)$$

Hence any point on the hyperbola satisfies $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Conversely, let $P(x, y)$ satisfy the above equation with $0 < a < c$. Then

$$y^2 = b^2 \left(\frac{x^2 - a^2}{a^2} \right)$$

Therefore, $PF_1 = +\sqrt{(x+c)^2 + y^2}$

$$= +\sqrt{(x+c)^2 + b^2 \left(\frac{x^2 - a^2}{a^2} \right)} = a + \frac{c}{a}x$$

Similarly, $PF_2 = a - \frac{c}{a}x$

In hyperbola $c > a$; and since P is to the right of the line $x = a$, $x > a$, $\frac{c}{a}x > a$. Therefore, $a - \frac{c}{a}x$

becomes negative. Thus, $PF_2 = \frac{c}{a}x - a$.

Therefore, $PF_1 - PF_2 = a - \frac{c}{a}x - \frac{cx}{a} + a = 2a$

Also, note that if P is to the left of the line $x = -a$, then

$$PF_1 = \left(a + \frac{c}{a}x \right), PF_2 = a - \frac{c}{a}x$$

In that case $PF_2 - PF_1 = 2a$. So, any point that satisfies

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \text{ lies on the hyperbola.}$$

Thus, we proved that the equation of hyperbola with origin $(0,0)$ and transverse axis along x -axis is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Note- A hyperbola in which $a = b$ is called an equilateral hyperbola.

Discussion From the equation of the hyperbola we have obtained, it follows that, we have for every

point (x, y) on the hyperbola, $\frac{x^2}{a^2} = 1 + \frac{y^2}{b^2} \geq 1$.

i.e., $\left| \frac{x}{a} \right| \geq 1$, i.e., $x \leq -a$ or $x \geq a$. Therefore, no portion

of the curve lies between the lines $x = +a$ and $x = -a$, (i.e., no real intercept on the conjugate axis).

Similarly, we can derive the equation of the

hyperbola in Fig 31 (b) as $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

These two equations are known as the *standard equations of hyperbolas*.

Note: The standard equations of hyperbolas have transverse and conjugate axes as the coordinate axes and the centre at the origin. However, there are hyperbolas with any two perpendicular lines as transverse and conjugate axes, but the study of such cases will be dealt in higher classes.

From the standard equations of hyperbolas (Fig 29), we have the following observations:

Hyperbola is symmetric with respect to both the axes, since if (x, y) is a point on the hyperbola, then $(-x, y)$, (x, y) and $(-x, -y)$ are also points on the hyperbola.

The foci are always on the transverse axis. It is the positive term whose denominator gives the transverse axis. For example, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ has

transverse axis along x -axis of length 6, while $\frac{y^2}{25} - \frac{x^2}{16} = 1$ has transverse axis along y -axis of length 10.

Latus rectum

Definition 9 Latus rectum of hyperbola is a line segment perpendicular to the transverse axis through any of the foci and whose end points lie on the hyperbola.

As in ellipse, it is easy to show that the length of the latus rectum in hyperbola is $\frac{2b^2}{a}$.

EXERCISE

- At what point of the parabola $x^2 = 9y$ is the abscissa three times that of ordinate
 (a) (1, 1) (b) (3, 1)
 (c) (-3, 1) (d) (-3, -3)
- Find an equation of the circle with centre at (0,0) and radius r .
 (a) $x^2 + y^2 = r^2$ (b) $x^2 + y^2 = 1$
 (c) $x^2 - y^2 = r^2$ (d) $x^2 + 2y^2 = r^2$
- Find the equation of the circle with centre (-3, 2) and radius 4.
 (a) $(x + 3)^2 + (y - 2)^2 = 16$
 (b) $(x - 3)^2 + (y + 2)^2 = 8$
 (c) $(x + 3)^2 + (y + 2)^2 = 8$
 (d) $(x + 3)^2 + (y - 2)^2 = 8$
- Find the centre and the radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$
 (a) (3, 4) ; 5 (b) (-4, -5) ; 8
 (c) (4, 5) ; 8 (d) (-4, -5) ; 7
- Find the equation of the circle which passes through the points (2,-2), and (3,4) and whose centre lies on the line $x + y = 2$.
 (a) $(x - 0.5)^2 + (y - 0.3)^2 = 11.28$
 (b) $(x - 0.7)^2 + (y - 0.5)^2 = 18.90$
 (c) $(x - 0.7)^2 + (y - 1.3)^2 = 12.58$
 (d) $(x - 0.6)^2 + (y - 5)^2 = 15.35$
- Find the equation of the directrix of the parabola $y^2 = 8x$.
 (a) $x = -2$ (b) $y = 2$ (c) $x = 1$ (d) $y = 3$
- Find the equation of the parabola with focus (2,0) and directrix $x = -2$.
 (a) $x^2 = 4y$ (c) $y^2 = 8x$
 (c) $y^2 = 4x$ (d) $x^2 = 8y$
- Find the equation of the parabola with vertex at (0, 0) and focus at (0, 2).
 (a) $x^2 = 5y$ (b) $x^2 = 10y$
 (c) $x^2 = 8y$ (d) $x^2 = 6y$
- Find the equation of the parabola which is symmetric about the y -axis, and passes through the point (2,-3).
 (A) $3x^2 = 8y$ (B) $2x^2 = 5y$
 (c) $x^2 = 5y$ (d) $3x^2 = -4y$
- Find the eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.
 (a) $\frac{4}{5}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{4}$
- Find the lengths of minor axes of the ellipse $9x^2 + 4y^2 = 36$.
 (a) 6 units (b) 4 units
 (c) 5 units (d) 8 units
- Find the equation of the ellipse whose vertices are (± 13 , 0) and foci are (± 5 , 0).
 (a) $\frac{x^2}{169} + \frac{y^2}{144} = 1$ (b) $\frac{x^2}{64} + \frac{y^2}{81} = 1$
 (c) $\frac{x^2}{144} + \frac{y^2}{169} = 1$ (d) $\frac{x^2}{25} + \frac{y^2}{64} = 1$
- Find the equation of the ellipse, whose length of the major axis is 20 and foci are (0, ± 5).
 (a) $\frac{x^2}{100} + \frac{y^2}{75} = 1$ (b) $\frac{x^2}{75} + \frac{y^2}{100} = 1$
 (c) $\frac{x^2}{25} + \frac{y^2}{50} = 1$ (d) $\frac{x^2}{69} + \frac{y^2}{60} = 1$
- Find the equation of the ellipse, with major axis along the x -axis and passing through the points (4, 3) and (-1,4).
 (a) $9x^2 + 16y^2 = 247$ (b) $7x^2 - 25y^2 = 147$
 (c) $7x^2 + 15y^2 = 247$ (d) $6x^2 + 9y^2 = 141$
- Find the coordinates of the foci of the hyperbolas:
 (a) (0, ± 4) (b) (± 3 , 0) (c) (0, ± 2) (d) (± 5 , 0)
- Find the equation of the hyperbola with foci (0, ± 3) and vertices (0, $\pm \frac{\sqrt{11}}{2}$).
 (a) $45x^2 - 90y^2 = 180$ (b) $100x^2 + 44y^2 = 175$
 (c) $90y^2 + 50x^2 = 175$ (d) $100y^2 - 44x^2 = 275$
- Find the equation of the hyperbola where foci are (0, ± 12) and the length of the latus rectum is 36.
 (a) $3y^2 - x^2 = 108$ (b) $3x^2 + 2y^2 = 59$
 (c) $2x^2 - y = 51$ (d) $3y^2 + x^2 = 75$
- Find the equation of the ellipse with foci at (± 5 , 0) and $x = \frac{36}{5}$ as one of 5 the directrices.
 (a) $\frac{x^2}{36} + \frac{y^2}{15} = 1$ (b) $\frac{x^2}{36} + \frac{y^2}{11} = 1$
 (c) $\frac{x^2}{15} + \frac{y^2}{26} = 1$ (d) $\frac{x^2}{9} + \frac{y^2}{25} = 1$
- If one end of a diameter of circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the co-ordinate of the other end of the diameter.
 (a) (2, 3) (b) (1, 2) (c) (-1, -2) (d) (2, -1)
- Find the eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.
 (a) $\frac{4}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{1}{4}$
- Find the length of major axis of the ellipse $16x^2 + 25y^2 = 400$.
 (a) 10 (b) 5 (c) 15 (d) 20
- Find the distance between the directrices of the hyperbola $x^2 - y^2 = 8$.
 (a) 2 units (b) 4 units
 (c) 5 units (d) 6 units
- If the parabola $y^2 = 4ax$ passes through the point (3, 2), find the length of its latus rectum.
 (a) $\frac{2}{3}$ units (b) $\frac{3}{5}$ units
 (c) $\frac{4}{3}$ units (d) $\frac{1}{2}$ units

Answer Key

1. (b)	2. (a)	3. (c)	4. (d)	5. (c)	6. (a)	7. (a)	8. (c)	9. (d)	10. (a)
11. (b)	12. (a)	13. (b)	14. (c)	15. (a)	16. (d)	17. (a)	18. (b)	19. (b)	20. (a)
21. (b)	22. (b)	23. (c)							

Answers with Explanations

1. Option (a) is correct.

Explanation: Given, parabola is $x^2 = 9y$
 Let $P(h, k)$ is the point on the parabola such that abscissa is 3 times the ordinate.
 So, $h = 3k$ (i)
 Since, $P(h, k)$ lies on the parabola
 So, $h^2 = 9k$ (ii)
 From equation (i) and (ii),
 we get $(3k)^2 = 9k$
 $\Rightarrow 9k^2 = 9k$
 $\Rightarrow 9k^2 - 9k = 0$
 $\Rightarrow 9k(k - 1) = 0$
 $\Rightarrow k = 0, 1$
 When $k = 0, h = 0$
 So, $k = 1$
 Now, from equation (i), $h = 3 \times 1 = 3$
 So, the point is $(3, 1)$

2. Option (a) is correct.

Explanation: Here $h = k = 0$. Therefore, the equation of the circle is $x^2 + y^2 = r^2$

3. Option (c) is correct.

Explanation: Here $h = -3, k = 2$ and $r = 4$. Therefore, the equation of the required circle is $(x + 3)^2 + (y - 2)^2 = 16$

4. Option (d) is correct.

Explanation: The given equation is $(x^2 + 8x) + (y^2 + 10y) = 8$

Now, completing the squares within the parenthesis, we get

$$(x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$$

$$\text{i.e., } (x + 4)^2 + (y + 5)^2 = 49$$

$$\text{i.e., } \{x - (-4)\}^2 + \{y - (-5)\}^2 = 7^2$$

Therefore, the given circle has centre at $(-4, -5)$ and radius 7.

5. Option (c) is correct.

Explanation: Let the equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through $(2, -2)$ and $(3, 4)$, we have

$$(2 - h)^2 + (-2 - k)^2 = r^2 \quad \dots(1)$$

$$\text{and } (3 - h)^2 + (4 - k)^2 = r^2 \quad \dots(2)$$

Also since the centre lies on the line $x + y = 2$, we have

$$h + k = 2 \quad \dots(3)$$

Solving the equations (1), (2) and (3), we get

$$h = 0.7, k = 1.3 \text{ and } r^2 = 12.58$$

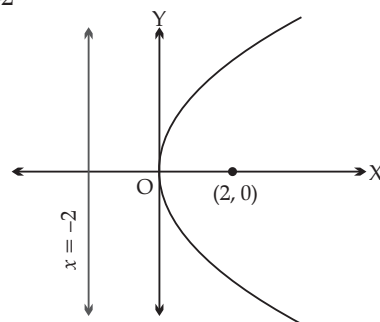
Hence, the equation of the required circle is $(x - 0.7)^2 + (y - 1.3)^2 = 12.58$

6. Option (a) is correct.

Explanation: The given equation involves y^2 , so the axis of symmetry is along the x -axis.

The coefficient of x is positive so the parabola opens to the right. Comparing with the given equation $y^2 = 4ax$, we find that $a = 2$.

Thus, the equation of the directrix of the parabola is $x = -2$



7. Option (a) is correct.

Explanation: Since the focus $(2, 0)$ lies on the x -axis, the x -axis itself is the axis of the parabola. Hence the equation of the parabola is of the form either $y^2 = 4ax$ or $y^2 = -4ax$. Since the directrix is $x = -2$ and the focus is $(2,0)$, the parabola is to be of the form $y^2 = 4ax$ with $a = 2$. Hence the required equation is $y^2 = 4(2)x = 8x$

8. Option (c) is correct.

Explanation: Since the vertex is at $(0,0)$ and the focus is at $(0,2)$ which lies on y -axis, the y -axis is the axis of the parabola. Therefore, equation of the parabola is of the form $x^2 = 4ay$. thus, we have $x^2 = 4(2)y$, i.e., $x^2 = 8y$.

9. Option (d) is correct.

Explanation: Since the parabola is symmetric about y -axis and has its vertex at the origin, the equation is of the form $x^2 = 4ay$ or $x^2 = -4ay$, where the sign depends on whether the parabola opens upwards or downwards. But the parabola passes through $(2, -3)$ which lies in the fourth quadrant, it must open downwards. Thus the equation is of the form $x^2 = -4ay$.

Since the parabola passes through $(2, -3)$, we have

$$2^2 = -4a(-3), \text{ i.e., } a = \frac{1}{3}$$

Therefore, the equation of the parabola is

$$x^2 = -4\left(\frac{1}{3}\right)y, \text{ i.e., } 3x^2 = -4y.$$

10. Option (a) is correct.

Explanation: Since denominator of $\frac{x^2}{25}$ is larger than

the denominator of $\frac{y^2}{9}$, the major axis is along the x -axis. Comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

we get

$$a = 5 \text{ and } b = 3. \text{ Also}$$

$$c = \sqrt{a^2 - b^2} = \sqrt{25 - 9} = 4$$

The eccentricity is $\frac{4}{5}$.

11. Option (b) is correct.

Explanation: The given equation of the ellipse can be written in standard form as $\sqrt{a^2 - b^2} = 1$.

Since the denominator of $\frac{y^2}{9}$ is larger than the denominator of $\frac{x^2}{4}$, the major axis is along the

y -axis. Comparing the given equation with the standard equation

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \text{ we have } b = 2 \text{ and } a = 3.$$

$$\text{Also } c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$$

So, length of the minor axis is 4 units.

12. Option (a) is correct.

Explanation: Since the vertices are on x -axis, the

equation will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.

Given that $a = 13$, $c = \pm 5$.

Therefore, from the relation $c^2 = a^2 - b^2$, we get

$$25 = 169 - b^2, \text{ i.e., } b = 12$$

Hence the equation of the ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

13. Option (b) is correct.

Explanation: Since the foci are on y -axis, the major axis is along the y -axis. So, equation of the ellipse is

$$\text{of the form } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

Given that,

$$a = \text{semi-major axis} = \frac{20}{2} = 10$$

and the relation $c^2 = a^2 - b^2$ gives

$$5^2 = 10^2 - b^2 \text{ i.e., } b^2 = 75$$

Therefore, the equation of the ellipse is $\frac{x^2}{75} + \frac{y^2}{100} = 1$

14. Option (c) is correct.

Explanation: The standard form of the ellipse is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Since the points $(4, 3)$ and $(-1, 4)$ lie on the ellipse, we have

$$\frac{16}{a^2} + \frac{9}{b^2} = 1 \quad \dots(1)$$

$$\text{And } \frac{1}{a^2} + \frac{16}{b^2} = 1 \quad \dots(2)$$

Solving equations (1) and (2), we find that $a^2 = \frac{247}{7}$
= and $b^2 = \frac{247}{15}$

Hence the required equation is

$$\frac{x^2}{\left(\frac{247}{7}\right)} + \frac{y^2}{\frac{247}{15}} = 1, \text{ i.e., } 7x^2 + 15y^2 = 247.$$

15. Option (a) is correct.

Explanation: Comparing the equation $\frac{x^2}{9} + \frac{y^2}{16} = 1$

with the standard equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\text{Here, } a = 3, b = 4 \text{ and } c = \sqrt{a^2 + b^2} = \sqrt{9 + 16} = 5$$

Therefore, the coordinates of the foci are $(\pm 5, 0)$.

16. Option (d) is correct.

Explanation: Since the foci is on y -axis, the equation

of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$$\text{Since vertices are } (0, \pm \frac{\sqrt{11}}{2}), a = \frac{\sqrt{11}}{2}$$

Also, since foci are $(0, \pm 3)$; $c = 3$ and $b^2 = c^2 - a^2 = \frac{25}{4}$

Therefore, the equation of the hyperbola is

$$\frac{y^2}{\left(\frac{11}{4}\right)} - \frac{x^2}{\left(\frac{25}{4}\right)} = 1, \text{ i.e., } 100y^2 - 44x^2 = 275.$$

17. Option (a) is correct.

Explanation: Since foci are $(0, \pm 12)$, it follows that $c = 12$.

$$\text{Length of the latus rectum} = \frac{2b^2}{a} = 36 \text{ or } b^2 = 18a$$

Therefore $c^2 = a^2 + b^2$; gives

$$144 = a^2 + 18a$$

$$\text{i.e., } a^2 + 18a - 144 = 0,$$

$$\text{So } a = -24, 6.$$

Since a cannot be negative, we take $a = 6$ and so $b^2 = 108$.

Therefore, the equation of the required hyperbola is

$$\frac{y^2}{36} - \frac{x^2}{108} = 1, \text{ i.e., } 3y^2 - x^2 = 108$$

18. Option (b) is correct.

Explanation: We have $ae = 5$, $\frac{a}{e} = \frac{36}{5}$ which give $a^2 = 36$ or $a = 6$.

$$\text{Therefore, } e = \frac{5}{6}.$$

$$\text{Now } b = a\sqrt{1 - e^2} = 6\sqrt{1 - \frac{25}{36}} = \sqrt{11}. \text{ Thus, the}$$

$$\text{equation of the ellipse is } \frac{x^2}{36} + \frac{y^2}{11} = 1.$$

19. Option (b) is correct.*Explanation:* Given equation of the circle is

$$x^2 + y^2 - 4x - 6y + 11 = 0$$

$$\Rightarrow (x^2 - 4x + 4) + (y^2 - 6y + 9) - 4 - 9 + 11 = 0$$

$$\Rightarrow (x-2)^2 + (y-3)^2 = (\sqrt{2})^2$$

$\therefore$ Centre of the circle, $(h, k) = (2, 3)$.

Also, one end of the diameter has co-ordinates $(3, 4)$.

Let the other end of the diameter has co-ordinates (a, b) .

Since, the centre is the midpoint of the diameter.

$$\therefore \frac{3+a}{2} = 2 \text{ and } \frac{4+b}{2} = 3$$

$$\Rightarrow a = 1$$

$$\Rightarrow b = 2$$

$\therefore$ The co-ordinates of the other end of the diameter are $(1, 2)$.

20. Option (a) is correct.*Explanation:* Given equation of the ellipse is

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1, a = 5, b = 3.$$

$$\begin{aligned} \therefore e &= \sqrt{1 - \frac{b^2}{a^2}} \\ &= \sqrt{1 - \frac{9}{25}} \\ &= \sqrt{\frac{16}{25}} \\ &= \frac{4}{5}. \end{aligned}$$

21. Option (b) is correct.*Explanation:* Given equation of the ellipse is

$$16x^2 + 25y^2 = 400$$

$$\Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1$$

$$\Rightarrow \frac{x^2}{5^2} + \frac{y^2}{4^2} = 1, a = 5, b = 4.$$

$$\therefore \text{Length of Major axis} = 2a = 10$$

The given hyperbola is

$$\frac{x^2}{5^2} - \frac{y^2}{2^2} = 1, a = 5, b = 2.$$

$$\text{Vertices} = (\pm a, 0) = (\pm 5, 0)$$

$$\text{foci} = (\pm c, 0), \text{ where } c = \sqrt{a^2 + b^2} = \sqrt{29}$$

$$= (\pm \sqrt{29}, 0)$$

$$\text{eccentricity} = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{29}}{5}$$

$$\text{Latus rectum} = 2 \frac{b^2}{a} = \frac{8}{5}.$$

22. Option (b) is correct.*Explanation:* Given equation of the hyperbola is

$$x^2 - y^2 = 8.$$

$$\Rightarrow \frac{x^2}{(2\sqrt{2})^2} - \frac{y^2}{(2\sqrt{2})^2} = 1, a = 2\sqrt{2}, b = 2\sqrt{2}$$

$$\text{eccentricity, } e = \frac{\sqrt{a^2 + b^2}}{a} = \frac{\sqrt{16}}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

$$\begin{aligned} \therefore \text{Distance between the directrices} &= \frac{2a}{e} \\ &= \frac{2 \cdot 2\sqrt{2}}{\sqrt{2}} \\ &= 4 \text{ units.} \end{aligned}$$

23. Option (c) is correct.*Explanation:* Given equation of the parabola is

$$y^2 = 4ax$$

Since, it passes through $(3, 2)$.

$$4 = 4a \cdot 3$$

$$a = \frac{1}{3}$$

$$\begin{aligned} \therefore \text{Length of the latus rectum} &= 4a \\ &= \frac{4}{3} \end{aligned}$$

Chapter 20

PERMUTATIONS AND COMBINATIONS

Permutation and Combination

Suppose an event E can occur in m different ways and associated with each way of occurring of E, another event F can occur in n different ways, then the total number of occurrence of the two events in the given order is $m \times n$.

Addition principle: If an event E can occur in m ways and another event F can occur in n ways, and suppose that both cannot occur together, then E or F can occur in $m + n$ ways.

Permutations

- A permutation is an arrangement of objects in a definite order.
- Permutation of n different objects The number of permutations of n objects taken all at a time, denoted by the symbol ${}^n P_n$ is given by

$${}^n P_n = n!$$

Factorial

$n! = n(n-1)(n-2) \dots$ 3.2.1, read as factorial n , or n factorial.

The number of permutations of n objects taken r at a time, where $0 < r \leq n$, denoted by ${}^n P_r$, is given by

$${}^n P_r = \frac{n!}{(n-r)!}$$

We assume that $0! = 1$

When repetition of objects is allowed The number of permutations of n things taken all at a time, when repetition of objects is allowed is n^n .

The number of permutations of n objects, taken r at a time, when repetition of objects is allowed, is n^r .

Permutations when the objects are not distinct. The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest if any, are of different kinds is:

$$\frac{n!}{p_1! p_2! \dots p_k!}$$

Combinations

On many occasions we are not interested in arranging but only in selecting r objects from given n objects. A combination is a selection of some or all of a number of different objects where the order of selection is immaterial. The number of selections of r objects from the given n objects is denoted by ${}^n C_r$, and is given by:

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Remarks

1. Use permutations if a problem calls for the number of arrangements of objects and different orders are to be counted.
2. Use combinations if a problem calls for the number of ways of selecting objects and the order of selection is not to be counted.

Some important results Let n and r be positive integers such that $r \leq n$. Then :

- (i) ${}^n C_r = {}^n C_{n-r}$
- (ii) ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$
- (iii) $n {}^{n-1} C_{r-1} = (n-r+1) {}^n C_{r-1}$

EXERCISE

1. How many odd numbers between 300 and 400 are there in which none of the digits is repeated?
(a) 32 (b) 36 (c) 40 (d) 45
2. How many four-digit natural numbers are there such that all of the digits are odd?
(a) 625 (b) 400 (c) 196 (d) 120
3. If different permutations of the letters of the word 'MATHEMATICS' are listed as in a dictionary, how many words (with or without meaning) are there in the list before the first word that starts with C?
(a) 302400 (b) 403600
(c) 907200 (d) 1814400
4. With the numbers 2, 4, 6, 8 all the possible determinants with these four different elements are constructed. What is the sum of the values of all such determinants?
(a) 128 (b) 64 (c) 32 (d) 0
5. Consider the digits 3, 5, 7, 9. What is the number of 5-digit numbers formed by these digits in which each of these four digits appears?
(a) 240 (b) 180 (c) 120 (d) 60
6. What is the number of ways in which the letters of the word 'ABLE' can be arranged so that the vowels occupy even places?
(a) 2 (b) 4 (c) 6 (d) 8
7. Let x be the number of integers lying between 2999 and 8001 which have at least two digits equal. Then, x is equal to
(a) 2480 (b) 2481 (c) 2482 (d) 2483
8. The total number of 5-digit numbers that can be composed of distinct digits from 0 to 9 is
(a) 45360 (b) 30240 (c) 27216 (d) 15120
9. What is the sum of all three-digit numbers that can be formed using all the digits 3, 4 and 5, when repetition of digits is not allowed?
(a) 2664 (b) 3882 (c) 4044 (d) 4444
10. What is the number of natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25?
(a) 860 (b) 854 (c) 840 (d) 824

11. What is the number of ways in which 3 holiday travel tickets are to be given to 10 employees of an organization, if each employee is eligible for any one or more of the tickets ?
 (a) 60 (b) 120 (c) 500 (d) 1000
12. What is the number of different messages that can be represented by three 0's and two 1's ?
 (a) 10 (b) 9 (c) 8 (d) 7
13. If different words are formed with all the letters of the word 'AGAIN' and are arranged alphabetically among themselves as in a dictionary, the word at the 50th place will be
 (a) NAAGI (b) NAAIG
 (c) IAAGN (d) IAANG
14. The number of ways in which 3 holiday tickets can be given to 20 employees of an organization if each employee is eligible for any one or more of the tickets, is
 (a) 1140 (b) 3420 (c) 6840 (d) 8000
15. What is the maximum value of n such that 5^n divides $(30! + 35!)$, where n is a natural number?
 (a) 4 (b) 6 (c) 7 (d) 8
16. Consider the following statements:
 (1) $\frac{n!}{3!}$ is divisible by 6, where $n > 3$
 (2) $\frac{n!}{3!} + 3$ is divisible by 7, where $n > 3$
 Which of the above statements is/are correct?
 (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2
17. How many permutations are there of the letters of the word 'TIGER' in which the vowels should not occupy the even positions?
 (a) 72 (b) 36 (c) 18 (d) 12
18. In how many ways can a team of 5 players be selected out of 9 players so as to exclude two particular players?
 (a) 14 (b) 21 (c) 35 (d) 42
19. How many 4-letter words each of two vowels and two consonants with or without meaning, can be formed ?
 (a) 36 (b) 144 (c) 576 (d) 864
20. A box contains 2 white balls, 3 black balls and 4 red balls. What is the number of ways of drawing 3 balls from the box with at least one black ball ?
 (a) 84 (b) 72 (c) 64 (d) 48
- Consider the following for the next three (03) items that follow :
 Consider the word 'QUESTION':
21. If $C(n, 4)$, $C(n, 5)$ and $C(n, 6)$ are in AP, then what is the value of n ?
 (a) 7 (b) 8 (c) 9 (d) 10
22. Suppose 20 distinct points are placed randomly on a circle. Which of the following statements is/are correct?
 1. The number of straight lines that can be drawn by joining any two of these points is 380.
 2. The number of triangles that can be drawn by joining any three of these points is 1140.
 Select the correct answer using the code given below.
 (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2
23. Consider a regular polygon with 10 sides. What is the number of triangles that can be formed by joining the vertices which have no common side with any of the sides of the polygon?
 (a) 25 (b) 50 (c) 75 (d) 100
24. What is the maximum number of points of intersection of 5 non-overlapping circles?
 (a) 10 (b) 15 (c) 20 (d) 25
25. Out of 15 points in a plane, n points are in the same straight line, 445 triangles can be formed by joining these points. What is the value of n ?
 (a) 3 (b) 4 (c) 5 (d) 6

Answer Key

1. (a)	2. (a)	3. (c)	4. (d)	5. (a)	6. (b)	7. (b)	8. (c)	9. (a)	10. (b)
11. (d)	12. (a)	13. (b)	14. (d)	15. (c)	16. (d)	17. (b)	18. (b)	19. (c)	20. (d)
21. (a)	22. (b)	23. (b)	24. (c)	25. (c)					

Answers with Explanations

1. Option (a) is correct.

Explanation:

For odd numbers between 300 and 400. When digits are not repeated.

3	.	.
1	x	8
	x	4

Here one choice 3 for hundred place; 4 choices 8 {1, 5, 7, 9} for unit place and remaining 8 choices 8 for ten's place

$\therefore$ Total odd numbers between 300 and 400, when digits are not repeated = $1 \times 8 \times 4 = 32$.

Hints:

- Use for odd number unit place digits will be 1, 3, 5, 7, 9.
- 3 is fixed at hundred place.
- Use non repetition case.

2. Option (a) is correct.

Explanation:

We have to form 4-digit natural numbers using 1, 3, 5, 7, 9.

So for each digit we have 5 possible numbers.

$\Rightarrow$ Total number of 4-digit numbers possible

$$= 5 \times 5 \times 5 \times 5$$

$$= 625$$

Hint:

Recall Fundamental principle of counting.

3. Option (c) is correct.**Explanation:**

Given alphabets are MATHEMATICS.

In the dictionary the words are arranged alphabetically.

The words in the dictionary before the first word that starts with C will start with A.

So we will fix letter A at first place and arrange 2M, A, 2T, H, E, I, C, S.

$$\Rightarrow \text{Number of required words} = \frac{10!}{2!2!}$$

$$= 907200$$

Hint: The number of permutations of n things taken all at a time when ' p ' of them are of same, ' q ' of them are of same and rest are different is $\frac{n!}{p!q!}$

4. Option (d) is correct.**Explanation:**

Given: 4 elements 2, 4, 6, 8.

Now, total number of determinants possible = $4! = 24$ ways.

$$\text{Let one way be } \begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = (2 \times 4) - (6 \times 8) = -40$$

Now, we will get -40 in three other ways also.

$$\begin{vmatrix} 4 & 8 \\ 6 & 2 \end{vmatrix}, \begin{vmatrix} 2 & 6 \\ 8 & 4 \end{vmatrix}, \begin{vmatrix} 4 & 6 \\ 8 & 2 \end{vmatrix}$$

So in 4 ways we will get determinant = -40

and similarly if we interchange any row or column then determinant gets negative.

$$\Rightarrow \text{As } \begin{vmatrix} 2 & 8 \\ 6 & 4 \end{vmatrix} = -40$$

$$\Rightarrow \begin{vmatrix} 8 & 2 \\ 4 & 6 \end{vmatrix} = (8 \times 6) - (2 \times 4) = +40$$

So, 4 determinants will have $+40$ as its value.

Similarly, for every determinant there exists another determinant with negative sign.

 $\Rightarrow$ Sum of determinants = 0**Hint:**

- Use fundamental principle of counting to find total number of ways.
- Recall the properties of determinants.

5. Option (a) is correct.**Explanation:**

Given: five digits number need to make from 3, 5, 7, 9.

For 5 digit number from 4 digit, one number will get repeat and it is given all 4 digits must appear.

So, we get:

Number of ways for 5 digit number where 2 digit

$$\text{repeat} = \frac{5!}{2!}$$

Now, finally all digits use so multiply by 4.

Hence,

$$\text{Number formed} = \frac{5!}{2!} \times 4$$

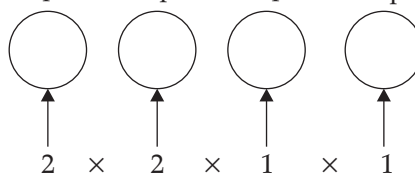
$$= 240$$

Hints:

- 5 digit number but 2 repeat.
- All four used, so multiply.

6. Option (b) is correct.**Explanation:**

Given letters: A, B, L, E

Ist place IInd place IIIrd place IVth place

$$2 \times 2 \times 1 \times 1 = 4 \text{ ways.}$$

Here, IInd place and IVth place can be filled with only A and E.

So, the number of ways to fill these places

$$= 2 \times 1$$

Similarly Ist place and IIIrd place can be filled with only B and L.

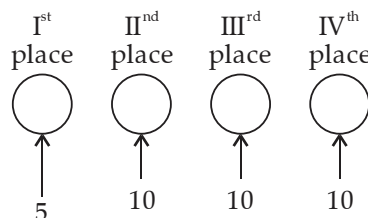
So, the number of ways to fill these places

$$= 2 \times 1$$

So, total number of ways

$$= 2 \times 1 \times 2 \times 1$$

$$= 4 \text{ ways}$$

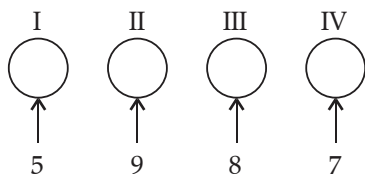
7. Option (b) is correct.**Explanation:**Here, x is a four digit number lying between 2999 and 8001. So, it can start with any digit from 3, 4, 5, 6 and 7.Now, IInd, IIIrd and IVth place can be filled by any digit from 0 to 9.

So, all the numbers between 2999 and 8001 are $5 \times 10 \times 10 \times 10 + 1$, (8000 should be included).

$$= 5000 + 1$$

$$= 5001$$

Now, find all the numbers in which no digit is repeated.



Here, Ist place can be filled by any digit from 3, 4, 5, 6, 7.

IInd place can be filled by 9 ways.

IIIrd place can be filled by 8 ways.

IVth place can be filled by 7 ways.

So, all the numbers which have different digits are $5 \times 9 \times 8 \times 7 = 2520$.

So, total number of integers which have at least two digit repeated = All the numbers between 2999 and 8001 – All the numbers which have different digits

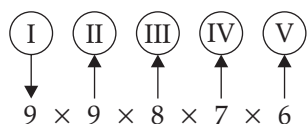
$$= 5001 - 2520$$

$$= 2481$$

8. Option (c) is correct.

Explanation:

We have 10 digits from 0 to 9



First place can be filled by any digit from 1–9.

So, total number of ways to fill first place is 9.

After filling first place II, III, IV and V place can be filled by 9, 8, 7 and 6 ways respectively.

$$\text{So, total number of ways} = 9 \times 9 \times 8 \times 7 \times 6 = 27216$$

9. Option (a) is correct.

Explanation:

$$\text{I}^{\text{st}} \text{ number} = 345$$

$$\text{II}^{\text{nd}} \text{ number} = 354$$

$$\text{III}^{\text{rd}} \text{ number} = 435$$

$$\text{IV}^{\text{th}} \text{ number} = 453$$

$$\text{V}^{\text{th}} \text{ number} = 534$$

$$\text{VI}^{\text{th}} \text{ number} = 543$$

$$\therefore \text{Sum of all numbers} = 345 + 354 + 435 + 453 + 534 + 543 = 2664$$

10. Option (b) is correct.

Explanation:

Given: Natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25.

Numbers divisible by 10

$$= \{10, 20, 30, 40, \dots, 1000\}$$

Numbers of such numbers

$$= 100$$

Numbers divisible by 15

$$= \left[\frac{1000}{15} \right]$$

$$= 66$$

$\therefore$ Numbers of natural numbers less than or equal to n divisible by $k = \left[\frac{n}{k} \right]$ where $[.]$ denotes greatest integer function.

Numbers divisible by 25

$$= \left[\frac{1000}{25} \right] = 40$$

Numbers divisible by both 10 and 15 i.e., by LCM of 10 and 15

$$= \left[\frac{1000}{30} \right] = 30$$

Numbers divisible by 15 and 25

$$= \left[\frac{1000}{\text{LCM}(15, 25)} \right]$$

$$= \left[\frac{1000}{75} \right]$$

$$= 13$$

Numbers divisible by 10 and 25

$$= \left[\frac{1000}{\text{LCM}(10, 25)} \right]$$

$$= \left[\frac{1000}{50} \right]$$

$$= 20$$

Numbers of numbers divisible by 10, 15 and 25

$$= \left[\frac{1000}{\text{LCM}(10, 15, 25)} \right]$$

$$= \left[\frac{1000}{150} \right]$$

$$= 6$$

$\therefore$ Required numbers of numbers

$$= 1000 - (100 + 66 + 40 - 33 - 13 - 20 + 6)$$

$$= 1000 - 146 = 854$$

Hints:

Number of natural numbers less than or equal to n and divisible by k

$$= \left[\frac{n}{k} \right]$$

11. Option (d) is correct

Explanation:

Given: 3 holiday travel tickets are given to 10 employees of an organisation, if each employee is eligible for any one or more of the tickets.

For each travel ticket, there are 10 possibilities.

$$\begin{aligned}\therefore \text{Total number of ways} &= 10 \times 10 \times 10 \\ &= 10^3 \\ &= 1000.\end{aligned}$$

Hints :

r things can be distributed to n persons in nr ways if each person is eligible for any one or more things.

12. Option (a) is correct**Explanation :**

Three 0's and two 1's n things out of which n_1 are alike of one kind and n_2 are alike of second kind can

be arranged in $\frac{n!}{n_1! n_2!}$ ways

$\therefore$ Three 0's and Two 1's can be arranged in $\frac{5!}{3! 2!}$

$$\begin{aligned}&= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2 \times 1} \\ &= 10.\end{aligned}$$

Hints :

- n things out of which n_1 are alike of one kind and n_2 are alike of second kind can be arranged in $\frac{n!}{n_1! n_2!}$ ways
- $n! = n(n-1)(n-2)\dots 1$

13. Option (b) is correct**Explanation :**

We have 2-A, 1-G, 1-I and 1-N

$$\begin{array}{c} \boxed{A} \quad \square \quad \square \quad \square \quad \square \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ \text{Words start with A} = 1 \times 4 \times 3 \times 2 \times 1 \\ = 24 \text{ ways} \end{array}$$

$$\begin{array}{c} \boxed{G} \quad \square \quad \square \quad \square \quad \square \\ \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow \\ \text{Words start with G} = 1 \times 4 \times 3 \times 2 \times 1 \\ = \frac{4 \times 3 \times 2 \times 1}{2} \\ = 12 \text{ ways} \end{array}$$

$$\begin{array}{c} \boxed{I} \quad \square \quad \square \quad \square \quad \square \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ \text{Words start with I} = 1 \times 4 \times 3 \times 2 \times 1 \\ = \frac{4 \times 3 \times 2 \times 1}{2} \\ = 12 \text{ ways} \end{array}$$

Now, total words formed that starts with A, G and I = $24 + 12 + 12$
= 48

Now, 49th word will start with N.

So, 49th word will be $\boxed{N} \boxed{A} \boxed{A} \boxed{G} \boxed{I}$

and 50th word will be $\boxed{N} \boxed{A} \boxed{A} \boxed{I} \boxed{G}$

14. Option (d) is correct.**Explanation :**

Number of employees = 20

Number of holidays = 3

Number of ways in which each employee is eligible for one or more of tickets = $(20)^3$

$$= 20 \times 20 \times 20$$

$$= 8000$$

15. Option (c) is correct.**Explanation :**

$$30! + 35! = 30! + 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30!$$

$$= 30! (1 + 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31)$$

$\therefore (1 + 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31)$ not divisible by 5

$\therefore$ Only 30! is divisible by 5^n

So, maximum value of n such that

5^n divides 30!

$$\begin{aligned}&= \left[\frac{30}{5} \right] + \left[\frac{30}{5^2} \right] + \left[\frac{30}{5^3} \right] \\ &= 6 + 1 + 0 = 7\end{aligned}$$

Maximum value of $n = 7$

Hints:

- If p is prime number then the highest power of p in $n!$ is given by $\left[\frac{n}{p} \right] + \left[\frac{n}{p^2} \right] + \left[\frac{n}{p^3} \right] + \dots$
- $[x]$ is greatest integer less than equal to x e.g. $[2.5] = [2.99] = 2$

16. Option (d) is correct.**Explanation :**

Statement 1

Put $n = 4$

$$\therefore \frac{n!}{3!} = \frac{4!}{3!} = 4 \text{ is not divisible by 6}$$

So, statement 1 is not correct.

Statement 2

Put $n = 5$

$$\frac{n!}{3!} + 3 = \frac{5!}{3!} + 3 = 5 \cdot 4 + 3 = 23 \text{ is not divisible by 7.}$$

So, statement 2 is not correct.

Hints:

- Put $n = 4, 5 \dots$ and check validity
- Use $n! = n(n-1)!$

17. Option (b) is correct.**Explanation :**

Given, Word: TIGER

Vowels : I, E

1 2 3 4 5
 $\square \quad \square \quad \square \quad \square \quad \square$

Since, vowels not occupy the even positions.
So, there are 3 odd places are available of vowels.

$$\therefore \text{Number of words} = {}^3C_2 \times 2! \times 3! = 36$$

Hints:

- First select number of places for vowels.
- Then arrange number of vowels
- Also arrange number of consonant.

18. Option (b) is correct.

Explanation:

Number of way of selecting 5 players out of 9 players as to exclude two particular players

$$= {}^7C_5 = \frac{7!}{5!2!} = 21$$

Hints:

- Number of ways of selecting r things out of n things such that k things excluded ($r > k$) = ${}^{n-k}C_r$.

19. Option (c) is correct.

Explanation:

We have 2 white, 3 black and 4 red balls.
Required ways = Total ways – no. of ways of drawing no black ball.

$$\begin{aligned} &= {}^9C_3 - {}^6C_3 \\ &= \frac{9 \times 8 \times 7}{3 \times 2} - \frac{6 \times 5 \times 4}{3 \times 2} \\ &= 64 \text{ ways.} \end{aligned}$$

Hint: Required ways = Total ways – no. of ways of drawing no black balls.

20. Option (d) is correct.

Explanation:

Given word is QUESTION

Vowels are UEIO and consonants are QSTN.

We have to form 4 letter word using 2 vowels and 2 consonants.

So firstly we will choose 2 vowels and 2 consonants and then arrange them

So required words = ${}^4C_2 \times {}^4C_2 \times 4!$

$$\begin{aligned} &= \frac{4 \times 3}{2} \times \frac{4 \times 3}{2} \times 4 \times 3 \times 2 \\ &= 864 \end{aligned}$$

Hint:

Choose 2 vowels and 2 consonents and then arrange them.

21. Option (a) is correct.

Explanation:

Given: $C(n, 4)$, $C(n, 5)$, $C(n, 6)$ in A.P.

We know, formula for combination

$$C(n, r) = {}^nC_r = \frac{n!}{r!(n-r)!}, \text{ we get}$$

$$\Rightarrow C(n, 4) = \frac{n!}{4!(n-4)!}$$

$$\Rightarrow C(n, 5) = \frac{n!}{5!(n-5)!}$$

$$\Rightarrow C(n, 6) = \frac{n!}{6!(n-6)!}$$

Since, given in A.P.

$$2 \times C(n, 5) = C(n, 4) + C(n, 6)$$

$$2 {}^nC_5 = {}^nC_4 + {}^nC_6$$

$$\frac{2n!}{(n-5)!5!} = \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)!5!} = \frac{1}{(n-4)!4!} + \frac{1}{(n-6)!6!}$$

$$\Rightarrow \frac{2}{(n-5)(n-6)!5!} = \frac{1}{(n-4)(n-5)(n-6)!4!} + \frac{1}{(n-6)!6!}$$

Multiplying $6!(n-6)!$ in all terms

$$\Rightarrow \frac{2 \times 6!}{5! \times (n-5)} = \frac{6!}{(n-4)(n-5)4!} + 1$$

$$\Rightarrow \frac{12}{(n-5)} = \frac{30}{(n-4)(n-5)} + 1$$

$$\Rightarrow \frac{12}{(n-5)} - \frac{30}{(n-4)(n-5)} = 1$$

$$\Rightarrow \frac{12n - 48 - 30}{(n-4)(n-5)} = 1$$

$$\Rightarrow 12n - 48 - 30 = n^2 - 9n + 20$$

$$\Rightarrow n^2 - 21n + 98 = 0$$

$$\Rightarrow (n-7)(n-14) = 0$$

$$\Rightarrow n = 7 \text{ or } n = 14$$

Hence, $n = 7$ is correct answer.

$$\Rightarrow \frac{2 \times n!}{5!(n-5)!} = \frac{n!}{4!(n-4)!} + \frac{n!}{6!(n-6)!}$$

$$\Rightarrow \frac{2}{5!(n-5)!} = \frac{1}{4!(n-4)!} + \frac{1}{6!(n-6)!}$$

$$\Rightarrow \frac{2}{5!(n-5)!} = \frac{6!(n-6)! + 4!(n-4)!}{4!(n-4)! \times 6!(n-6)!}$$

$$\Rightarrow \frac{2 \cdot 4!(n-4)! \times 6!(n-6)!}{5!(n-5)!}$$

$$\begin{aligned}
 &= 6!(n-6)! + 4!(n-4)! \\
 \Rightarrow & \frac{2 \times 4!(n-4)! \times 6 \times 5! \times (n-6) \times (n-5)!}{5!(n-5)!} \\
 &= 6!(n-6)! + 4!(n-4)! \\
 \Rightarrow & 2[4!(n-4)! \times 6 \times (n-6)] \\
 &= 6!(n-6)! + 4!(n-4)! \\
 \Rightarrow & 4!(n-4)! \times 12 \times (n-6) = 6 \times 5 \times 4! \\
 &\quad \times (n-6) \times (n-5)(n-4)! + 4!(n-4)! \\
 \Rightarrow & 4!(n-4)! \times 6 \times (n-6) = 4!(n-4)! \\
 &\quad [6 \times 5(n-6)(n-5) + 1] \\
 \Rightarrow & 12(n-6) = 30(n-6)(n-5) + 1 \\
 \Rightarrow & 12n - 72 = 30[n^2 - 5n - 6n + 30] + 1 \\
 \Rightarrow & 12n - 72 = 30[n^2 - 11n + 30] + 1 \\
 \Rightarrow & 12n - 72 = 30n^2 - 330n + 900 + 1 \\
 \Rightarrow & 12n - 72 = 30n^2 - 330n + 901 \\
 \Rightarrow & 30n^2 - 342n + 973 = 0, \text{ we get} \\
 \Rightarrow & n = 7
 \end{aligned}$$

Hints:

- Use combination formula.
- Solve equation.

22. Option (b) is correct.**Explanation:**

Given: Twenty distinct points on circle.
First, we have join any two points to make line.
Using concept of combination, we get
Number of lines(l) = ${}^{20}C_2$

$$\begin{aligned}
 l &= \frac{20!}{18! \times 2!} \\
 \left[{}^nC_r &= \frac{n!}{r!(n-r)!} \right] \\
 l &= \frac{20 \times 19}{2} \\
 [n! &= n(n-1)(n-2)! \dots] \\
 l &= 190
 \end{aligned}$$

Statement 1 is incorrect.

Now, to make triangle, we have to connect 3 points.

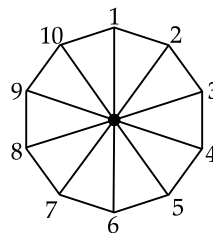
Number of triangles (T) = ${}^{20}C_3$

$$\begin{aligned}
 T &= \frac{20!}{3!(20-3)!} \\
 T &= \frac{20!}{3! \times 17!} \\
 T &= \frac{20 \times 19 \times 18 \times 17!}{3 \times 2 \times 17!} \\
 T &= \frac{20 \times 19 \times 18}{3 \times 2} \\
 T &= 1140
 \end{aligned}$$

Statement 2 is correct.

Hint:

- Use combination concept.

23. Option (b) is correct.**Explanation:**

Number of triangles that can be formed from any 10 vertices = ${}^{10}C_3 = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$

Number of triangles that can be formed with only one side of triangles(1, 2, 9), (2, 3, 5) (2, 3, 10) = $(10-4) \times 10 = 60$

Number of triangles that can be formed with two consecutive sides of polygon.

e.g., {(1, 2, 3), (2, 3, 4) (10, 1, 2)} = 10

∴ Number of triangles that can be formed with no common side with any of the sides of the polygon

$$= 120 - 60 - 10 = 50$$

Hints:

- Find all triangles,
- Find number of triangles with one side common.
- Find number of triangles with two sides common.

24. Option (c) is correct.**Explanation:**

$$\begin{aligned}
 \text{Number of ways} &= 2 \cdot {}^5C_2 \\
 &= 2 \times \frac{5!}{2! \times (5-2)!} \\
 &= 2 \times \frac{5 \times 4 \times 3!}{2 \times 3!} \\
 &= 20
 \end{aligned}$$

So, maximum number of points of intersection = 20.

25. Option (c) is correct.**Explanation:**

Given : Number of triangles that can be formed is equal to the number of ways to select 3 non-collinear points.

Number of ways to select 3 points out of the n collinear points = ${}^nC_3$.

n collinear points = ${}^{15}C_3$

$$\begin{aligned}
 {}^nC_r &= \frac{n!}{r!(n-r)!} \\
 {}^{15}C_3 &= \frac{15!}{3!(15-3)!} \\
 &= \frac{15!}{3! \times 12!} = 455
 \end{aligned}$$

Let n points be collinear.

Number of ways to select 3 points out of the n collinear points = ${}^nC_3$.

Given number of triangles that can be formed = 445

So, number of ways to select 3 non-collinear points

$$\Rightarrow {}^{15}C_3 - {}^nC_3$$

$$\Rightarrow 445 = {}^{15}C_3 - {}^nC_3$$

$$\Rightarrow {}^nC_3 = {}^{15}C_3 - 445$$

$$\Rightarrow {}^nC_3 = 455 - 445$$

$$\Rightarrow {}^nC_3 = 10$$

$$\frac{n!}{3!(n-3)!} = 10$$

$$\therefore \frac{n(n-1)(n-2)}{3 \times 2 \times 1} = 10$$

$$\therefore n(n-1)(n-2) = 60$$

$$\Rightarrow n^2 - n(n-2) = 60$$

$$\Rightarrow n^3 - 2n^2 - n^2 + 2n = 60$$

$$\Rightarrow n^3 - 3n^2 + 2n = 60$$

$$\Rightarrow n = 5$$

Hints :

- By number of ways to select 3 points out of the n collinear points = ${}^nC_3$.
- ${}^nC_r = \frac{n!}{r!(n-r)!}$
- Now, find number of ways to select 3 non-collinear points.



Exponential and Logarithmic Series

1. In the following chapter we are about to obtain an expansion in power of x for the expression a^x , where both a and x are real, and also to obtain an expansion for $\log_e (1 + x)$, where x is real and less than unity, and e stands for a quantity to be defined.

2. To find the value of the quantity $\left(1 + \frac{1}{n}\right)^n$, when n becomes infinitely great and is real.

Since $\frac{1}{n} < 1$, we have, by the Binomial Theorem,

$$\begin{aligned} \left(1 + \frac{1}{n}\right)^n &= 1 + n \cdot \frac{1}{n} + \frac{n(n-1)}{1 \cdot 2} \frac{1}{n^2} \\ &\quad + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \frac{1}{n^3} + \dots \\ &= 1 + 1 + \frac{1 - \frac{1}{n}}{1 \cdot 2} + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)}{1 \cdot 2 \cdot 3} \\ &\quad + \frac{\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\left(1 - \frac{3}{n}\right)}{1 \cdot 2 \cdot 3 \cdot 4} + \dots \end{aligned} \quad \dots(1)$$

This series is true for all values of n , however great. Make then n infinite and the right hand side

$$= 1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \text{ad inf.} \quad \dots(2)$$

Hence the limiting value, when n is infinite, of

$$\left(1 + \frac{1}{n}\right)^n \text{ is the sum of the series.}$$

$$= 1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \text{ad inf.}$$

The sum of the series is always denoted by the quantity e .

Hence we have

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e,$$

Where $\lim_{n \rightarrow \infty}$ stands for "the limit when $n = \infty$ ".

Cor. By putting $n = \frac{1}{m}$, it follows (since m is zero when n is infinity) that

$$\lim_{m \rightarrow \infty} (1 + m)^{\frac{1}{m}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e.$$

3. This quantity e is finite.

For since $\frac{1}{3} < \frac{1}{2 \cdot 2} < \frac{1}{2^2},$

$$\frac{1}{4} < \frac{1}{2 \cdot 2 \cdot 2} < \frac{1}{2^3},$$

.....

We have

$$e < 1 + 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots \text{ad inf.}$$

$$< 1 + \frac{1}{1 - \frac{1}{2}}$$

$$< 1 + 2 \text{ i.e., } < 3.$$

Also clearly $e > 2$

Hence it lies between 2 and 3.

By taking a sufficient number of term in the series, it can be shown that

$$e = 2.7182818285 \dots$$

4. The quantity e is incommensurable.

For, if possible, suppose it to be equal to a fraction $\frac{p}{q}$, where p and q are whole numbers.

We have then

$$\frac{p}{q} = 1 + 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{q} + \frac{1}{q+1} + \frac{1}{q+2} + \dots \quad \dots(1)$$

Multiply this equation by $\frac{q}{q+1}$, so that all the terms of the series (1) become integers except those

commencing with $\frac{q}{q+1}$. Hence we have

$$p \frac{q-1}{q} = \text{whole number} + \frac{q}{q+1} + \frac{q}{q+2} + \frac{q}{q+3} + \dots,$$

i.e. an integer =

$$\frac{1}{q+1} + \frac{1}{(q+1)(q+2)} + \frac{1}{(q+1)(q+2)(q+3)} + \dots \quad \dots(2)$$

But the right-hand side of this equation is $\frac{1}{q+1}$, and

$$< \frac{1}{q+1} + \frac{1}{(q+1)^2} + \frac{1}{(q+1)^3} + \dots,$$

$$\text{i.e., } < \frac{1}{q+1} \div \left(1 - \frac{1}{q+1}\right),$$

$$\text{i.e., } < \frac{1}{q}$$

Hence the right-hand side of (2) lies between $\frac{1}{q+1}$

and $\frac{1}{q}$, and therefore a fraction and so cannot be

equal to the left-hand side.

Hence our supposition that e was commensurable is

incorrect and it therefore must be incommensurable.

5. **Exponential Series:** When x is real, to prove that

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \text{ad inf.},$$

and that

$$a^x = 1 + x \log_e a + \frac{x^2}{2} (\log_e a)^2 + \dots \text{ad inf.},$$

When n is greater than unity, we have

$$\begin{aligned} \left\{1 + \frac{1}{n}\right\}^x &= \left(1 + \frac{1}{n}\right)^{nx} \\ &= 1 + nx \frac{1}{n} + \frac{nx(nx-1)}{1 \cdot 2} \frac{1}{n^2} \\ &\quad + \frac{nx(nx-1)(nx-2)}{1 \cdot 2 \cdot 3} \frac{1}{n^3} + \dots \\ &= 1 + x + \frac{x\left(x - \frac{1}{n}\right)}{1 \cdot 2} + \frac{x\left(x - \frac{1}{n}\right)\left(x - \frac{2}{n}\right)}{1 \cdot 2 \cdot 3} + \dots \end{aligned}$$

In this expression make n infinitely great. The left-hand becomes, as in Art. 2, e^x

$$1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

Hence we have

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \text{ad inf.}, \quad \dots(1)$$

Let $a = e^c$, so that $c = \log_e a$.

$$\therefore a^x = e^{cx} = 1 + cx + \frac{c^2 x^2}{2} + \frac{c^3 x^3}{3} + \dots \text{ad inf.}$$

By substituting cx for x in the series (1).

$$\begin{aligned} \therefore a^x &= 1 + x \log_e a + \frac{x^2}{2} (\log_e a)^2 + \frac{x^3}{3} (\log_e a)^3 + \dots \text{ad inf.} \\ &\dots(2) \end{aligned}$$

6. It can be shown (as in C. Smith's *Algebra*, Art. 278) that the series (1), and therefore (2), of the last article is convergent for all real values of x .

8. **Logarithmic series :** To prove that, when y is real and numerically < 1 , then

$$\log_e(1+y) = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \dots \text{ad inf.}$$

In the equation (2) of Art. 5, put

$$a = 1 + y,$$

and we have

$$(1+y)^x = 1 + x \log_e(1+y) + \frac{x^2}{2} \{\log_e(1+y)\}^2 + \dots \dots(1)$$

But, since y is real and numerically < 1 , we have

$$\begin{aligned} (1+y)^x &= 1 + x \cdot y + \frac{x(x-1)}{1 \cdot 2} y^2 \\ &\quad + \frac{x(x-1)(x-2)}{1 \cdot 2 \cdot 3} y^3 + \dots \end{aligned} \dots(2)$$

The series of right hand side of (1) and (2) are equal to another and both convergent, when y is numerically < 1 . Also it could be shown that the series on the right hand-side of (2) is convergent when it is arranged in power of x . Hence we may equate like powers of x .

Thus we have

$$\begin{aligned} \log_e(1+y) &= y - \frac{y^2}{1 \cdot 2} + \frac{(-1)(-2)}{1 \cdot 2 \cdot 3} y^3 \\ &\quad + \frac{(-1)(-2)(-3)}{1 \cdot 2 \cdot 3 \cdot 4} y^4 + \dots \text{ad inf.}, \end{aligned}$$

$$\text{i.e., } \log_e(1+y) = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \dots \text{ad inf.} \dots(3)$$

9. If $y = 1$, the series (3) of the previous article is equal to

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \text{ad inf.}, \text{ which is known to be convergent,}$$

If $y = -1$, it equals $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \text{ad inf.}$, which is

known to be divergent.

In addition therefore to being true for all values of y between -1 and $+1$, it is true for the value $y = 1$; it is not however true for the value $y = -1$.

10. Calculation of logarithms to base e .

In the logarithm series, if we put $y = 1$, we have

$$\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \text{ad inf.} \dots(1)$$

If we put $y = \frac{1}{2}$,

We have

$$\begin{aligned} \log_e 3 - \log_e 2 &= \log_e \frac{3}{2} = \log_e \left(1 + \frac{1}{2}\right) \\ &= \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2^2} + \frac{1}{3} \cdot \frac{1}{2^3} - \frac{1}{4} \cdot \frac{1}{2^4} + \dots \dots(2) \end{aligned}$$

If we put $y = \frac{1}{3}$.

We have

$$\log_e 4 - \log_e 3 = \log_e \left(1 + \frac{1}{3}\right) = \frac{1}{3} - \frac{1}{2} \cdot \frac{1}{3^2} + \frac{1}{3} \cdot \frac{1}{3^3} + \dots \dots(3)$$

From these equations we could, by taking a sufficient number of terms, calculate $\log_e 2$, $\log_e 3$ and $\log_e 4$.

It would be found that a large number of terms would have to be taken to give the values of these logarithms to the required degree of accuracy. We shall therefore obtain more convenient series.

11. By Art. 8 we have

$$\log_e(1+y) = y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \dots \dots(1)$$

and, by changing the sign of y ,

$$\log_e(1-y) = -y - \frac{1}{2}y^2 - \frac{1}{3}y^3 - \frac{1}{4}y^4 + \dots \dots(2)$$

In order that both these series may be true y must be

numerically less than unity.

By subtraction, we have

$$\log_e(1+y) - \log_e(1-y) = \log_e \frac{1+y}{1-y}$$

$$= 2 \left[y + \frac{1}{3}y^3 + \frac{1}{5}y^5 + \dots \right] \quad \dots(3)$$

Let $y = \frac{m-n}{m+n}$

Where m and n are positive integers and $m > n$, so that

$$\frac{1+y}{1-y} = \frac{m}{n},$$

The equation (3) becomes

$$\log_e \frac{m}{n} = 2 \left[\left(\frac{m-n}{m+n} \right) + \frac{1}{3} \left(\frac{m-n}{m+n} \right)^3 + \frac{1}{5} \left(\frac{m-n}{m+n} \right)^5 + \dots \right] \quad \dots(4)$$

Put $m = 2, n = 1$ in (4) and we get $\log_e 2$.

Put $m = 3, n = 2$ and we get $\log_e 3 - \log_e 2$, and therefore $\log_e 3$.

By proceeding in this way we get the value of the logarithm of any number to base e .

12. Logarithms to base 10. The logarithms of the previous article, to base e , are called Napierian or natural logarithms.

We can convert these logarithms into logarithms to base 10.

For, by Art. 147 (Part I.), we have, if N be any number,

$$\log_e N = \log_{10} N \times \log_e 10.$$

$$\therefore \log_{10} N = \log_e N \times \log_e 10.$$

Now, $\log_e 10$ can be found as in the last article and

then $\frac{1}{\log_e 10}$ is found to be 0.4342944819...,

$$\text{Hence, } \log_{10} N = \log_e N \times 0.43429448 \dots,$$

so that the logarithm of any number to base 10 is found by multiplying its logarithm to base e by the quantity 0.43429448.... This quantity is called the Modulus.

EXERCISE

1. $1 + \frac{1}{3} + \frac{1}{5} + \dots \text{ad inf.} =$

- (a) $\frac{1}{2} \left(e - \frac{1}{e} \right)$ (b) $e + \frac{1}{e}$
(c) $2e - \frac{1}{e}$ (d) None of these

2. Find the sum of series

$$1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \frac{1+2+3+4}{4} + \dots \text{ad inf.}$$

- (a) $2e$ (b) $\frac{3e}{2}$ (c) $\frac{e}{2}$ (d) $e+2$

3. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$, then $x =$

- (a) $\log_e y^2$ (b) $\log_e y$
(c) $\log_e 2y$ (d) None of these

4. $1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + \dots \infty =$

- (a) e (b) $2e$ (c) $3e$ (d) $4e$

5. If $y = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots$, then $x =$

- (a) $\log(1+y)$ (b) $\log \frac{1}{1-y}$
(c) $\log(1-y)$ (d) $\log 2y$

6. $1 - \frac{(\log_e n)^2}{2!} + \frac{(\log_e n)^4}{4!} + \dots =$

- (a) $\frac{n}{2}$ (b) $\frac{n+1}{2}$ (c) $\frac{n+n^{-1}}{2}$ (d) $\frac{n}{3}$

7. The value of $1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 2)^3}{3!} + \dots$ is

- (a) $\frac{3}{4}$ (b) 2 (c) 1 (d) $\frac{1}{2}$

Answer Key

1. (a)	2. (b)	3. (b)	4. (b)	5. (b)	6. (c)	7. (d)			

Answers with Explanations

1. Option (a) is correct.

Explanation: By equation (1) of an Art. 5 we have, by putting x in succession equal to 1 and -1,

$$e = 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \text{ad inf.}$$

$$\text{i.e., } e^{-1} = 1 + \frac{1}{1} - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \text{ad inf.}$$

Hence, by subtraction

$$e - e^{-1} = 2 \left(1 + \frac{1}{3} + \frac{1}{5} + \dots \right),$$

$$\text{i.e., } \frac{1}{2} \left(e - \frac{1}{e} \right) = 1 + \frac{1}{3} + \frac{1}{5} + \dots \text{ad inf.}$$

2. Option (b) is correct.

Explanation:

$$1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \frac{1+2+3+4}{4} + \dots \text{ad inf.}$$

$$\begin{aligned}\text{The } n^{\text{th}} \text{ term} &= \frac{1+2+3+\dots+n}{2} = \frac{1}{2}n(n+1) \\ &= \frac{1}{2} \frac{n+1}{n-1} = \frac{1}{2} \left[\frac{(n-1)+2}{n-1} \right] = \frac{1}{2} \left[\frac{1}{n-2} + \frac{2}{n-1} \right]\end{aligned}$$

Provided that $n > 2$.

Similarly,

$$\text{the } (n-1) = \frac{1}{2} \left[\frac{1}{n-3} + \frac{2}{n-2} \right],$$

$$\dots\dots\dots$$

$$\text{the } 4^{\text{th}} \text{ term} = \frac{1}{2} \left[\frac{1}{2} + \frac{2}{3} \right],$$

$$\text{the } 3^{\text{rd}} \text{ term} = \frac{1}{2} \left[\frac{1}{1} + \frac{2}{2} \right]$$

$$\text{the } 2^{\text{nd}} \text{ term} = \frac{1}{2} \left[1 + \frac{2}{1} \right]$$

$$\text{the } 1^{\text{st}} \text{ term} = \frac{1}{2} \left[\frac{2}{1} \right].$$

Hence by addition, the whole series

$$\begin{aligned}&\frac{1}{2} \left[1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots \text{ad inf.} \right] \\ &\quad + \frac{1}{2} \cdot 2 \left[1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots \text{ad inf.} \right] \\ &= \frac{1}{2} \cdot e + e = \frac{3e}{2}\end{aligned}$$

3. Option (b) is correct.

Explanation:

$$y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = e^x \Rightarrow x = \log_e y.$$

4. Option (b) is correct.

Explanation:

$$\begin{aligned}T_n &= \frac{1+3+5+\dots+(2n-1)}{n!} = \frac{\frac{n}{2}[2.1(n-1)2]}{n!} \\ &= \frac{n-1+1}{(n-1)!} = \frac{1}{(n-2)!} + \frac{1}{(n-1)!}\end{aligned}$$

$$\Rightarrow s = \sum_{n=1}^{\infty} T_n = e + e = 2e.$$

5. Option (b) is correct.

Explanation:

$$\begin{aligned}y &= x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots = 1 - e^{-x} \Rightarrow e^{-x} = 1 - y = -x \\ &= \log_e(1-y) \Rightarrow x = \log \frac{1}{1-y}.\end{aligned}$$

6. Option (c) is correct.

Explanation:

$$\begin{aligned}1 + \frac{(\log_e n)^2}{2!} + \frac{(\log_e n)^4}{4!} + \dots \\ = \frac{e^{\log_e n} + e^{-\log_e n}}{2} = \frac{n + n^{-1}}{2}.\end{aligned}$$

7. Option (d) is correct.

Explanation:

$$\begin{aligned}\text{Given series} &= 1 - \log 2 + \frac{(\log 2)^2}{2!} - \frac{(\log 3)^3}{3!} + \dots \\ &= e^{-\log 2} = e^{\log \left(\frac{1}{2} \right)} = \frac{1}{2}.\end{aligned}$$

➤ **Definition:** A set is a (unordered) collection of objects. These objects are sometimes called elements or members of the set. A set is denoted by capital letter and elements are denoted by small letters.

Examples:

- Set of vowels in the English alphabet $V = \{a, e, i, o, u\}$
- Set of first seven prime numbers $X = \{2, 3, 5, 7, 11, 13, 17\}$

Describing a Set: Roster Method and set builders form

1. Listing (enumerating) the members of the set.
2. Definition by property, using the set builder notation $\{x | x \text{ has property } P\}$.

Example:

- Even integers between 50 and 63. 1) $E = \{52, 54, 56, 58, 60, 62\}$ 2) $E = \{x | 50 < x < 63, x \text{ is an even integer}\}$ If enumeration of the members is hard, then we often use ellipses. Example: a set of integers from 1 and 100
 $A = \{1, 2, 3, \dots, 100\}$

Important sets in discrete math

- Natural numbers: $\mathbb{N} = \{1, 2, 3, \dots\}$
- Integers: $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$
- Positive integers: $\mathbb{Z}^+ = \{1, 2, 3, \dots\}$
- Rational numbers: $\mathbb{Q} = \{p/q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0\}$

Cardinality

➤ **Definition:** Let S be a set. If there are exactly n distinct elements in S , where n is a nonnegative integer, we say S is a finite set and that n is the cardinality of S . The cardinality of S is denoted by $|S|$.

Finite and infinite set

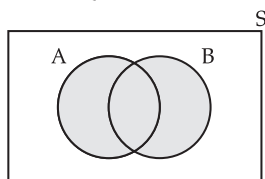
➤ A set is called finite set if it has finite number of elements otherwise it is infinite set.

Example; A set $A = \{x: x < 4, x \in \mathbb{N}\}$ is a finite set. Set $\mathbb{N}$ of all natural numbers, set of integers $\mathbb{I}$, set of even numbers etc. are the examples of infinite set.

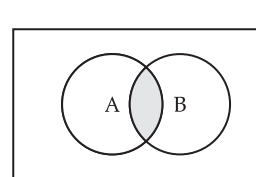
Set operations

➤ **Union of two sets:** Let A and B be two non empty sets then the union of A and B is denoted by $A \cup B$ is a set of all those elements which are belongs to A or B .

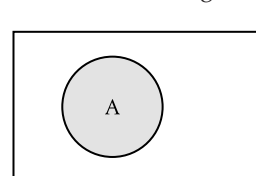
$$A \cup B = \{x: x \in A \text{ or } x \in B\}$$



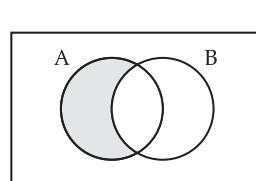
➤ **Intersection of two sets:** The **intersection** of two sets A and B , denoted by $A \cap B$ consists of all elements that are both in A and B . For example, $\{1, 2\} \cap \{2, 3\} = \{2\}$.



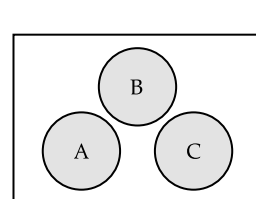
➤ **Complement of a set:** The **complement** of a set A , denoted by A^c or $\bar{A}$ is the set of all elements that are in the universal set S but are not in A . In Figure, $\bar{A}$ is shown by the shaded area using a Venn diagram.



➤ **Difference of two sets:** The **difference (subtraction)** is defined as follows. The set $A - B$ consists of elements that are in A but not in B .



➤ **Disjoint sets:** Two sets A and B are called disjoint sets if $A \cap B = \phi$.



➤ **Universal set** is denoted by U : the set of all objects under the consideration.

➤ **Null set or empty set** A set is called null set if it has no element. it is denoted as or $\{\} \cdot \emptyset$

➤ **Subset:** A set A is said to be a subset of B if and only if B , every element of A is also an element of B . We use $\subseteq$ indicate A is a subset of B .

➤ **Proper subset:** a set A is the proper subset of B if all the elements of A are belongs to the set B

➤ Such that $A \subset B$ and B is called the super set of A ($B \supset A$).

➤ **Power Set:** collection of all the subset of a set A including with is called Power set of A . It is denoted by $P(A)$.

➤ Defined as $P(A) = \{x; x \text{ is a subset of } A \text{ including with } \phi\}$

EXERCISE

1. Consider the following statements in respect of two non-empty sets A and B:

- (1) $x \notin (A \cup B) \Rightarrow x \notin A$ or $x \notin B$
 (2) $x \notin (A \cap B) \Rightarrow x \notin A$ and $x \notin B$

Which of the above statements is/are correct?

- (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2

2. Consider the following :

1. $A \cap B = A \cap C \Rightarrow B = C$
 2. $A \cup B = A \cup C \Rightarrow B = C$

Which of the above is/are correct ?

- (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2

Consider the following for the next three (03) items that follow :

A university awarded medals in basket ball, football and volleyball. Only x students ($x < 6$) got medal in all the three sports and the medals went to a total of $15x$ students. It awarded $5x$ medals in basketball, $(4x+15)$ medals in football and $(x+25)$ medals in volleyball.

3. How many received medals in exactly two of the three sports ?
 (a) $30 - 4x$ (b) $35 - 7x$ (c) $40 - 7x$ (d) $45 - 5x$
4. How many received medals in at least two of three sports ?
 (a) $30 - 6x$ (b) $35 - 6x$ (c) $40 - 5x$ (d) $40 - 6x$
5. How many received medals in exactly one of three sports ?
 (a) $21x - 40$ (b) $21x - 35$
 (c) $20x - 35$ (d) $20x - 25$
6. Consider the following statements in respect of sets :
- The union over intersection of sets is distributive.
 - The complement of union of two sets is equal to intersection of their complements.
 - If the difference of two sets is equal to empty set, then the two sets must be equal.

Which of the above statements are correct?

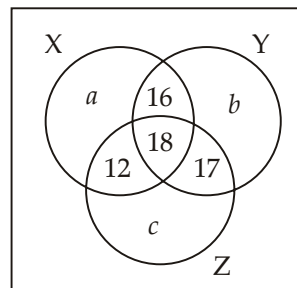
- (a) 1 and 2 only (b) 2 and 3 only
 (c) 1 and 3 only (d) 1, 2 and 3

7. Suppose set A consists of first 250 natural numbers that are multiples of 3 and set B consists of first 200 even natural numbers. How many elements does $A \cup B$ have?

- (a) 324 (b) 364 (c) 384 (d) 400

Directions for the following three (03) items :

Consider the following Venn diagram, where X, Y and Z are three sets. Let the number of elements in Z be denoted by $n(Z)$ which is equal to 90.



8. If the number of elements in Y and Z are in the ratio 4 : 5 then what is the value of b ?

- (a) 18 (b) 19 (c) 21 (d) 23

9. What is the value of

$$n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)?$$

- (a) $a + b + 43$ (b) $a + b + 63$
 (c) $a + b + 96$ (d) $a + b + 106$

10. If the number of elements belonging to neither X, nor Y, nor Z is equal to p , then what is the number of elements in the complement of X?

- (a) $p + b + 60$ (b) $p + b + 40$
 (c) $p + a + 60$ (d) $p + a + 40$

11. Consider the following statements for the two non-empty sets A and B :

- (1) $(A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B) = A \cup B$
 (2) $(A \cup (\bar{A} \cap \bar{B})) = A \cup B$

Which of the above statements is/are correct?

- (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2

12. If A, B and C are subsets of a Universal set, then which one of the following is not correct?

- (a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
 (b) $A' \cup (A \cup B) = (B' \cap A)' \cup A$
 (c) $A' \cup (B \cup C) = (C' \cap B)' \cap A'$
 (d) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

where A' is the complement of A.

Consider the information given below and answer the two (02) items that follow :

A survey was conducted among 300 students. It was found that 125 students like to play cricket. 145 students like to play football and 90 students like to play tennis. 32 students like to play exactly two games out of the three games.

13. How many students like to play all the three games?
 (a) 14 (b) 21 (c) 28 (d) 35
14. How many students like to play exactly one game?
 (a) 196 (b) 228 (c) 254 (d) 268
15. If $S = \{x : x^2 + 1 = 0, x \text{ is real}\}$, then S is
 (a) $\{-1\}$ (b) $\{0\}$
 (c) $\{1\}$ (d) an empty set

16. If $A = \{\{1, 2, 3\}\}$, then how many elements are there in the power set of A?

- (a) 1 (b) 2 (c) 4 (d) 8

17. Consider all the subsets of the set $A = \{1, 2, 3, 4\}$. How many of them are supersets of the set $\{4\}$?

- (a) 6 (b) 7 (c) 8 (d) 9

18. Consider the following statements:

- (1) $A = \{1, 3, 5\}$ and $B = \{2, 4, 7\}$ are equivalent sets.
(2) $A = \{1, 5, 9\}$ and $B = \{1, 5, 5, 9, 9\}$ are equal sets.

Which of the above statements is/are correct?

- (a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

19. Consider three sets X, Y and Z having 6, 5 and 4 elements respectively. All these 15 elements are distinct. Let $S = (X - Y) \cup Z$. How many proper subsets does S have?

- (a) 255 (b) 256 (c) 1023 (d) 1024

20. Consider the proper subsets of $\{1, 2, 3, 4\}$. How many of these proper subsets are superset of the set $\{3\}$?

- (a) 5 (b) 6 (c) 7 (d) 8

21. Let X be a non-empty set and let A, B, C be subsets of X. Consider the following statements:

$$(1) A \subset C \Rightarrow (A \cap B) \subset (C \cap B), (A \cup B) \subset (C \cup B)$$

$$(2) (A \cap B) \subset (C \cap B) \text{ for all sets } B \Rightarrow A \subset C$$

$$(3) (A \cup B) \subset (C \cup B) \text{ for all sets } B \Rightarrow A \subset C$$

Which of the above statements is/are correct?

- (a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

22. If $A = \{\lambda, \{\lambda, \mu\}\}$, then the power set of A is

- (a) $\{\phi, \{\phi\}, \{\lambda\}, \{\lambda, \mu\}\}$
(b) $\{\phi, \{\lambda\}, \{\{\lambda, \mu\}\}, \{\lambda, \{\lambda, \mu\}\}\}$
(c) $\{\phi, \{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$
(d) $\{\{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$

23. Let $S = \{2, 4, 6, 8, \dots, 20\}$, What is the maximum number of subsets does S have?

- (a) 10 (b) 20 (c) 512 (d) 1024

24. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then the number of subsets of A containing two or three elements is

- (a) 45 (b) 120 (c) 165 (d) 330

25. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Then the number of subsets of A containing exactly two elements is

- (a) 20 (b) 40 (c) 45 (d) 90

Answer Key

1. (d)	2. (d)	3. (c)	4. (d)	5. (a)	6. (a)	7. (c)	8. (c)	9. (d)	10. (a)
11. (a)	12. (c)	13. (a)	14. (c)	15. (d)	16. (b)	17. (c)	18. (c)	19. (c)	20. (c)
21. (d)	22. (b)	23. (d)	24. (c)	25. (c)					

Answers with Explanations

1. Option (d) is correct.

Explanation:

$$\text{If } x \in A \cup B$$

$$\Rightarrow x \in A \text{ or } x \in B$$

$$\Rightarrow x \notin A \cup B$$

$$\Rightarrow x \notin A \text{ and } x \notin B$$

So, statement 1 is not correct.

$$\text{Let, } A = \{1, 2\} \text{ and } B = \{2, 3\}$$

$$\therefore A \cap B = \{2\}$$

$$\because 1 \notin A \cap B \text{ but } 1 \in A$$

$$3 \notin A \cap B \text{ but } 3 \in B$$

So, statement 2 is not correct.

Hint:

- Use properties of union and intersection.

2. Option (d) is correct.

Explanation:

$$\text{Let } A = \{2, 4, 6, 8, 10\}$$

$$B = \{2, 4, 12\}$$

$$C = \{2, 4, 14\}$$

$$\Rightarrow A \cap B = \{2, 4\} \text{ and } A \cap C = \{2, 4\}$$

$$\Rightarrow A \cap B = A \cap C$$

$$\text{But } B \neq C$$

So statement 1 is incorrect.

$$\text{Let } A = \{2, 4, 6, 8, 10\}$$

$$B = \{2, 4, 6\}$$

$$C = \{2, 4, 8\}$$

$$\Rightarrow A \cup B = A \text{ and } A \cup C = A$$

$$\Rightarrow A \cup B = A \cup C$$

$$\text{But } B \neq C$$

So statement 2 is also incorrect.

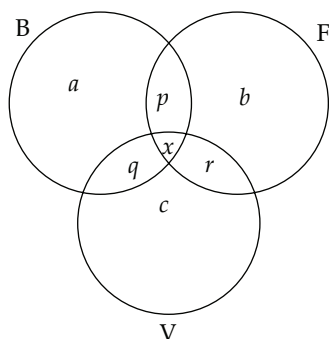
Hints:

- Intersection of two sets contain elements present in both sets.
- Union of two sets contain all elements present in both sets.

3. Option (c) is correct.

Explanation:

Let B, F, V represents Basketball, football and volleyball and a, b, c, p, q, r, x represented in venn diagram.



$$a + b + c + p + q + r + x = 15x \quad \dots(1)$$

$$\Rightarrow a + b + c + p + q + r = 14x$$

$$\& a + p + q + x = 5x \quad \dots(2)$$

$$\& a + p + q = 4x$$

$$\& b + p + r + x = 4x + 15 \quad \dots(3)$$

$$\& b + p + r = 3x + 15$$

$$\& c + q + r + x = x + 25 \quad \dots(4)$$

$$\& c + q + r = 25$$

Adding above (2)+(3)+(4) equations, we get

$$a + b + c + 2(p + q + r) = 7x + 40 \quad (5)$$

$$\text{Eq. (5)} - (1), \text{ we get}$$

$$p + q + r = 7x + 40 - 14x = 40 - 7x$$

Hint: Use venn diagram to solve it.

4. Option (d) is correct.

Explanation:

Using final answer of question No. 5

Medals in at least two sports = $p + q + r + x$

$$= 40 - 7x + x$$

$$= 40 - 6x$$

Hint: Use venn diagram to solve it.

5. Option (a) is correct.

Explanation:

Medals in exactly one of the sport = $a + b + c$.

Putting $p + q + r = 40 - 7x$ in equation (1) of question No. 5. we get.

$$a + b + c = 14x - 40 + 7x$$

$$= 21x - 40$$

Hint: Use venn diagram to solve it.

6. Option (a) is correct.

Explanation:

Statement 1: Union over intersection is distributive correct.

$$\text{Ex. } A \cup (B \cap C) \Rightarrow A \cap (B \cup C)$$

Statement 2: It is property of sets of complement of two sets. So, correct.

$$\text{Ex. } (A \cup B)' = A' \cap B'$$

Statement 3: It is false, prove by example

$$\text{Let } A = \phi, B = \{1, 2, 3\}$$

$$\Rightarrow A - B = \phi$$

So, $A - B$ is void but A and B are not equal.

Hint: Use property of sets.

7. Option (c) is correct.

Explanation: Given: Set A and Set B

Set A is first 250 natural numbers and multiple of 3.

$$\Rightarrow A = \{3, 6, 9, \dots, 750\}$$

Set B is first 200 natural numbers and even.

$$\Rightarrow B = \{2, 4, 8, \dots, 396\}$$

Now, common elements in set A and set B are those that divisible of 6.

$$\Rightarrow A \cap B = \{6, 12, \dots, 396\}$$

Now, we find elements number in $A \cap B$.

This is A.P. = 6, 12, 18, ..., 400

$$\Rightarrow a = 6, d = 6$$

{using formula of n^{th} term}

$$a_n = a + (n - 1)d$$

$$\Rightarrow 396 = 6 + (n - 1)6$$

$$\Rightarrow 390 = (n - 1)6$$

$$\Rightarrow n = 66$$

So, $A \cap B$ have 66 terms.

Now, finally terms in $A \cup B$

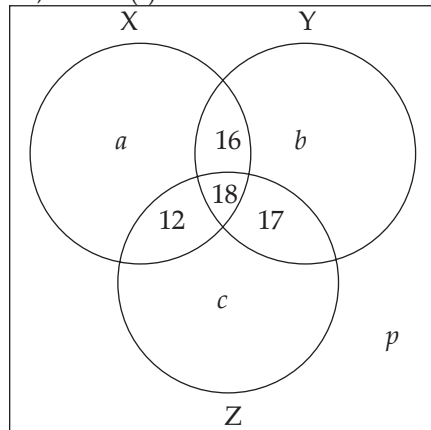
$$\Rightarrow n(A \cup B) = n|A| + n|B| - n(A \cap B)$$

$$= 250 + 200 - 66 = 384$$

8. Option (c) is correct.

Explanation:

$$\text{Given, } n(Z) = 90$$



So,

$$12 + 18 + 17 + c = 90$$

$$\Rightarrow 47 + c = 90$$

$$\Rightarrow c = 90 - 47$$

$$\Rightarrow c = 43$$

....(i)

It is given,

$$\frac{n(Y)}{n(Z)} = \frac{4}{5}$$

$$\Rightarrow \frac{16 + 18 + 17 + b}{90} = \frac{4}{5}$$

$$\Rightarrow 51 + b = 72$$

$$\Rightarrow b = 72 - 51$$

$$\Rightarrow b = 21$$

9. Option (d) is correct.

Explanation:

$$n(X) + n(Y) + n(Z) - n(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$$

$$= n(X \cup Y \cup Z)$$

$$= a + b + c + 16 + 18 + 17 + 12$$

$$= a + b + c + 63$$

$$= a + b + 43 + 63$$

$$[\because c = 43, \text{ from equation (i)}]$$

$$= a + b + 106$$

10. Option (a) is correct.*Explanation:*

$$\begin{aligned} \text{Given, } n(X \cup Y \cup Z)' &= p \\ n(X') &= p + b + 17 + c \\ &\quad [\text{from the graph given}] \\ &= p + b + 17 + 43 \\ &\quad [\because c = 43 \text{ from equation (i)}] \\ &= p + b + 60 \end{aligned}$$

11. Option (a) is correct.*Explanation:*

$$\begin{aligned} (1) (A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B) &= A \cup B \\ \text{L.H.S.} &= (A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B) \\ &= A \cap (B \cup \bar{B}) \cup (\bar{A} \cap B) \\ &\quad [\text{Distributive property}] \\ &= (A \cap S) \cup (\bar{A} \cap B) \\ &\quad [S \text{ is the sample space}] \\ &= A \cup (\bar{A} \cap B) \\ &= (A \cup \bar{A}) \cap (A \cup B) \\ &= S \cap (A \cup B) \\ &= (A \cup B) \\ &= \text{R.H.S.} \\ (2) (A \cup (\bar{A} \cap \bar{B})) &= A \cup B \\ \text{L.H.S.} &= A \cup (\bar{A} \cap \bar{B}) \\ &= (A \cup \bar{A}) \cap (A \cup \bar{B}) \\ &\quad [\text{Using distributive property}] \\ &= S \cap (A \cup \bar{B}) \\ &= A \cup \bar{B} \\ &\neq A \cup B \\ \therefore \text{L.H.S.} &\neq \text{R.H.S.} \end{aligned}$$

Shortcut:

$$\begin{aligned} (1) (A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B) &= A \cap (B \cup \bar{B}) \cup (\bar{A} \cap B) \\ &= A \cup (\bar{A} \cap B) = (A \cup B) \\ (2) A \cup (\bar{A} \cap \bar{B}) &= A \cup \bar{B} \end{aligned}$$

Hint:

- (1) Use the distributive property,
 $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$
 and $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$
- (2) Use the complement law and identity law of sets.
 i.e., $A \cup \bar{A} = S$ and $A \cap S = A$

12. Option (c) is correct.*Explanation:*

Let $U = \{1, 2, 3, 4\}$
 where U is the Universal Set.
 Let $A = \{1\}$, $B = \{2\}$, $C = \{3\}$
 where A , B and C are the subset of U .

(a) Now check option (a)

L.H.S.

$$\begin{aligned} A \cup (B \cap C) &= \{1\} \cup (\{2\} \cap \{3\}) \\ &\quad [\because \text{L.H.S.} = \text{Left Hand Side}] \\ &= \{1\} \cup \{\} \\ &= \{1\} \end{aligned}$$

R.H.S.

$$\begin{aligned} (A \cup B) \cap (A \cup C) &= (\{1\} \cup \{2\}) \cap (\{1\} \cup \{3\}) \\ &\quad [\text{R.H.S.} = \text{Right hand side}] \\ &= (\{1, 2\} \cap \{1, 3\}) \\ &= \{1\} \end{aligned}$$

Hence,

$$\text{L.H.S.} = \text{R.H.S.}$$

(b) Now check option (b).

L.H.S.

$$\begin{aligned} A' \cup (A \cup B) &= (U - A) \cup (A \cup B) \\ &\quad [\because A' = U - A] \\ &= (\{1, 2, 3, 4\} - \{1\}) \cup (\{1\} \cup \{2\}) \\ &= \{2, 3, 4\} \cup \{1, 2\} \\ &= \{1, 2, 3, 4\} \end{aligned}$$

R.H.S.

$$(B' \cap A)' \cup A$$

First calculate

$$\begin{aligned} B' \cap A &= (U - B) \cap A \\ &= (\{1, 2, 3, 4\} - \{2\}) \cap \{1\} \\ &= \{1, 3, 4\} \cap \{1\} \\ &= \{1\} \end{aligned}$$

Now calculate

$$\begin{aligned} (B' \cap A)' &= U - (B' \cap A) \\ &= \{1, 2, 3, 4\} - \{1\} \\ &= \{2, 3, 4\} \end{aligned}$$

Now,

$$\begin{aligned} \text{R.H.S.} &= (B' \cap A)' \cup A \\ &= \{2, 3, 4\} \cup \{1\} \\ &= \{1, 2, 3, 4\} \end{aligned}$$

Here,

$$\text{L.H.S.} = \text{R.H.S.}$$

(c) Now check option (c)

L.H.S.

$$\begin{aligned} &= A' \cup (B \cup C) \\ &= (U - A) \cup (B \cup C) \end{aligned}$$

$$[\because A' = U - A]$$

$$\begin{aligned} &= (\{1, 2, 3, 4\} - \{1\}) \cup (\{2\} \cup \{3\}) \\ &= \{2, 3, 4\} \cup \{2, 3\} \\ &= \{2, 3, 4\} \end{aligned}$$

R.H.S.

$$= (C' \cap B)' \cap A'$$

First calculate

$$\begin{aligned} C' \cap B &= (U - C) \cap B \\ &= (\{1, 2, 3, 4\} - \{3\}) \cap \{2\} \\ &= \{1, 2, 4\} \cap \{2\} = \{2\} \end{aligned}$$

Now calculate

$$\begin{aligned} (C' \cap B)' &= U - (C' \cap B) \\ &= \{1, 2, 3, 4\} - \{2\} \\ &= \{1, 3, 4\} \end{aligned}$$

Now,

$$\begin{aligned} \text{R.H.S.} &= (C' \cap B)' \cap A' \\ &= \{1, 3, 4\} \cap \{2, 3, 4\} \\ &= \{3, 4\} \end{aligned}$$

Here

$$\text{L.H.S.} \neq \text{R.H.S.}$$

(d) Now check option (d)

L.H.S

$$\begin{aligned} A \cap B &= \{1\} \cap \{2\} \\ \Rightarrow A \cap B &= \{ \} \\ (A \cap B) \cup C &= \{ \} \cup \{3\} = \{3\} \end{aligned}$$

R.H.S.

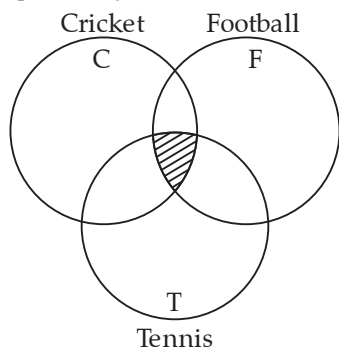
$$\begin{aligned} A \cup C &= \{1\} \cup \{3\} = \{1, 3\} \\ B \cup C &= \{2\} \cup \{3\} = \{2, 3\} \\ (A \cup C) \cap (B \cup C) &= \{1, 3\} \cap \{2, 3\} = \{3\} \end{aligned}$$

Here, L.H.S = R.H.S

13. Option (a) is correct.

Explanation:

We need to find the value of shaded region which can be represented by $n(C \cap F \cap T)$.



Given:

$$n(C) = 125,$$

$$n(F) = 145,$$

$$n(T) = 90$$

$$\begin{aligned} n(C \cap F) + n(F \cap T) + n(T \cap C) \\ - 3n(C \cap F \cap T) = 32 \end{aligned} \quad \dots (i)$$

Now, we know that

$$\begin{aligned} n(C \cup F \cup T) &= n(C) + n(F) + n(T) - n(C \cap F) \\ &\quad - n(F \cap T) - n(T \cap C) + n(C \cap F \cap T) \quad \dots (ii) \\ \Rightarrow n(C \cup F \cup T) &= n(C) + n(F) + n(T) \\ &\quad - [n(C \cap F) + n(F \cap T) + n(T \cap C)] + n(C \cap F \cap T) \\ \Rightarrow n(C \cup F \cup T) &= n(C) + n(F) + n(T) \\ &\quad - [32 + 3n(C \cap F \cap T)] + n(C \cap F \cap T) \\ &= n(C) + n(F) + n(T) - 32 - 3n(C \cap F \cap T) \\ &\quad + n(C \cap F \cap T) \\ \Rightarrow n(C \cup F \cup T) &= n(C) + n(F) + n(T) - 32 \\ &\quad - 2n(C \cap F \cap T) \end{aligned}$$

$$\Rightarrow 300 = 125 + 145 + 90 - 32 - 2n(C \cap F \cap T)$$

$$\Rightarrow 300 = 328 - 2n(C \cap F \cap T)$$

$$\Rightarrow 2n(C \cap F \cap T) = 328 - 300 = 28$$

$$\Rightarrow n(C \cap F \cap T) = 28/2$$

$$\Rightarrow n(C \cap F \cap T) = 14$$

14. Option (c) is correct.

Explanation:

Number of students who play exactly one game = $n(C) + n(F) + n(T) - 2n(C \cap F)$

$$\begin{aligned} &\quad - 2n(F \cap T) - 2n(T \cap C) + 3n(C \cap F \cap T) \\ &= n(C) + n(F) + n(T) - 2[n(C \cap F) + n(F \cap T) \\ &\quad + n(T \cap C)] + 3n(C \cap F \cap T) \\ &= 125 + 145 + 90 - 2[32 + 3n(C \cap F \cap T)] \\ &\quad + 3n(C \cap F \cap T) \\ &= 360 - 2 \times 32 - 6n(C \cap F \cap T) + 3n(C \cap F \cap T) \\ &= 360 - 64 - 3n(C \cap F \cap T) \\ &= 360 - 64 - 3 \times 14 \\ &= 360 - 64 - 42 \\ &= 360 - 106 \\ &= 254 \end{aligned}$$

15. Option (d) is correct

Explanation:

Given,

$$S = \{x : x^2 + 1 = 0, x \in \mathbb{R}\}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$x = \pm\sqrt{-1}$$

$$= \pm i \quad [\because \sqrt{-1} = i]$$

which is an imaginary number

So, $x \notin \mathbb{R}$.

Hence, given set represent an empty set.

16. Option (b) is correct.

Explanation:

Given that, $A = \{\{1, 2, 3\}\}$

$\therefore n(A) = 1$
 Number of subset $= 2^1 = 2$
 So, number of elements in power set of $A = 2$

Hints:

- Use, if $n(A) = n$ then number of element in $p(A) = 2^n$. including with ϕ

17. Option (c) is correct.**Explanation:**

Given set $A = \{1, 2, 3, 4\}$

Super sets of the set $\{4\}$ are $\{4\}, \{4, 3\}, \{4, 2\}, \{4, 1\}, \{4, 3, 2\}, \{4, 3, 1\}, \{4, 2, 1\}, \{4, 3, 2, 1\}$

Total number of supersets $= 8$

Shortcut:

No. of element except 4 $\{1, 2, 3\}$ is 3.
 $\therefore$ Number of supersets of $\{4\} = 2^3 = 8$

Hints:

- Number of subsets of set contain n elements is 2^n .
- If each element of set A belongs to set B, then B is called superset of set A.

18. Option (c) is correct.**Explanation:****Statement-I:**

Given sets $A = \{1, 3, 5\}$ & $B = \{2, 4, 7\}$

As we know equivalent sets are the sets with equal number of elements in them.

$\therefore$ Number of elements in $A =$ Number of elements in $B = 3$

$\therefore A$ & B are equivalent sets.

Statement-II:

Given sets $A = \{1, 5, 9\}$ and $B = \{1, 5, 5, 9, 9\}$

As we know two sets P and Q can be equal only if each element of set P is also the element of the set Q. Also if two sets are the subsets of each other, they are said to be equal.

Here, each element of set A is also the element of the set B and $A \subseteq B$ and $B \subseteq A$.

$\therefore A = B$ (Equal sets)

So, both statements are true.

Hint:

- Two sets A and B are said to be equivalent sets if they have same number of elements.
- Two sets P & Q can be equal only if each element of the set P is also the element of the set Q. Also if two sets are the subset of each other, they are said to be equal.

19. Option (c) is correct.**Explanation:**

Given: X set = 6 elements

Y set = 5 elements

Z set = 4 elements

It is also given that all 15 elements are distinct.

Now, to find elements in S we get step by step.

Elements in $X - Y = X = 6$ elements

[$\because$ No elements common in X, so none get cancel out]

Now,

Elements in $S = (X - Y) \cup Z$

$\Rightarrow S =$ elements in $(X - Y) \cup$ Element in Z

$\Rightarrow S = 6$ elements $\cup$ 4 elements

$\Rightarrow S = 10$ elements

[$\because$ No element common, so, all add]

Now, S have 10 elements, we have to find proper subsets.

Formula of proper subsets $= 2^n - 1$

$\Rightarrow$ Proper subsets $= 2^{10} - 1 = 1024 - 1$
 $= 1023$

Hints:

- No element common, so find total element.
- Use formula of proper subsets.

20. Option (c) is correct.**Explanation:**

$A = \{1, 2, 3, 4\}$

Now, proper subset of $A = \{\phi, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{2, 3, 4\}, \{1, 3, 4\}\}$

Now, superset of $\{3\}$ are $\{3\}, \{1, 3\}, \{2, 3\}, \{3, 4\}, \{1, 2, 3\}, \{2, 3, 4\}, \{1, 3, 4\}$.

So, total number of supersets $= 7$

21. Option (d) is correct.**Explanation:**

Statement I: $A \subset C \Rightarrow (A \cap B) \subset (C \cap B), (A \cup B) \subset (C \cup B)$

Let $p \in A \cap B$

$\Rightarrow p \in A$ and $p \in B$

$\Rightarrow p \in C$ and $p \in B$ [$\because A \subset C$]

$\Rightarrow (A \cap B) \subset (C \cap B)$

$\Rightarrow p \in (C \cap B)$

Let $q \in A \cup B$

$\Rightarrow q \in A$ or $q \in B$

$\Rightarrow q \in C$ or $q \in B$ [$\because A \subset C$]

$\Rightarrow q \in C \cup B$

$\Rightarrow (A \cap B) \subset (C \cap B)$

$\therefore$ Statement I is true.

Statement II: $(A \cap B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$

Let $B = U$ (universal set)

$\Rightarrow A \cap B = A \cap U = A \subset (C \cap U) = C$

$\therefore$ Statement II is true.

Statement III: $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$

Let $B = \phi$ (Empty set)
 $\Rightarrow A \cup B = A \cup \phi = A \subset (C \cup \phi) = C$
 $\therefore$ Statement III is true.

Shortcut:

(1) $A \subset C \Rightarrow (A \cap B) \subset (C \cap B), (A \cup B) \subset (C \cup B)$

Let $p \in A \cap B \Rightarrow p \in A$ and $p \in B$
 $\Rightarrow p \in C$ and $p \in B \Rightarrow p \in (C \cap B)$
 Let $q \in A \cup B \Rightarrow q \in A$ or $q \in B$
 $\Rightarrow q \in C$ or $q \in B \Rightarrow q \in (C \cup B)$

(2) $(A \cap B) \subset (C \cap B)$ for all sets $B \Rightarrow A \subset C$

Let $B = U$ (Universal set)
 $\Rightarrow A \cap B = A \cap U = A \subset (C \cap U) = C$

(3) $(A \cup B) \subset (C \cup B)$ for all sets $B \Rightarrow A \subset C$

Let $B = \phi$ (Empty set)
 $\Rightarrow A \cup B = A \cup \phi = A \subset (C \cup \phi) = C$

Hint:

- For statement I, Let $P \in A \cap B$ and $q \in A \cup B$ and solve further using the definition of union of two set & intersection of two set.
- For statement II, Let $B = U$ (Universal set) and solve further.
- For statement III, Let $B = \phi$ (Empty set) and solve further.

22. Option (b) is correct.**Explanation:**

Given: $A = \{\lambda, \{\lambda, \mu\}\}$

As we know, the power set of a set A is the set of all subsets of A , including the empty set ϕ and set A itself.

$$\Rightarrow P(A) = \{\phi, \{\lambda\}, \{\lambda, \mu\}, \{\lambda, \{\lambda, \mu\}\}\}$$

Hint:

The power set of A is the set of all subsets of A including the empty set ϕ and set A itself.

Shortcut:

$$P(A) = \{\phi, \{\lambda\}, \{\{\lambda, \mu\}\}, \{\lambda, \{\lambda, \mu\}\}\}$$

23. Option (d) is correct.**Explanation:**

Given, $S = \{2, 4, 6, 8, \dots, 20\}$

We know that,

- Cardinality of a set A is defined as the number of elements present in the set A , denoted by $n(A)$.
- Power set: Let A be a non empty set. Then $P(A) = \{B | B \subseteq A\}$ is called as the power set of A .
- For any non-empty set A with $n(A) = x$. Then the total number of subsets of set A is given by 2^x i.e. $n(P(A)) = 2^x$.

The number of elements in set S ,

$$n(S) = 10$$

As we know that for any non-empty set A , with

$$n(A) = x$$

The total number of subsets of set A is given by 2^x

The maximum number of subsets of

$$S = 2^{10} = 1024$$

Shortcut:

Given: $S = \{2, 4, 6, 8, \dots, 20\}$

The number of elements in set S , $n(S) = 10$

As we know that for any non-empty set A with $n(A) = x$.

Then the total number of subset of set A is given by 2^x .

The maximum number of subsets of $S = 2^{10} = 1024$.

Hint:

- Recall the theory and concept of set theory and solve.

24. Option (c) is correct.**Explanation:**

Given,

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$n(A) = 10$$

Number of subsets of A containing two or three elements = ${}^nC_2 + {}^nC_3$

$$= {}^{10}C_2 + {}^{10}C_3$$

$$= \frac{10!}{2! \times 8!} + \frac{10!}{3! \times 7!}$$

$$= \frac{10 \times 9 \times 8}{2 \times 1 \times 8} + \frac{10 \times 9 \times 8 \times 7}{3 \times 2 \times 1 \times 7}$$

$$= 45 + 120$$

$$= 165$$

25. Option (c) is correct.**Explanation:**

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Total elements (n) = 10

Number of subset containing exactly two elements

$$= {}^nC_2$$

$$= {}^{10}C_2$$

$$= \frac{10!}{2! \times 8!}$$

$$= \frac{10 \times 9 \times 8}{2 \times 1 \times 8}$$

$$= 45$$

➤ Data and its types

A group of information that represents the qualitative or quantitative attributes of a variable or set of variables is called data.

There are two types of data. These are :

(i) **Ungrouped data** : In an ungrouped data, data is listed in series e.g., 1, 4, 9, 16, 25, etc., this is also called in divided data.

(ii) **Grouped data-It is of two types :**

(a) **Discrete data** : In this type, data is presented in such a way that exact measurements of items are clearly shown.

(b) **Continuous data** : In this type, data is arranged in groups or classes but they are not exactly measurable, they form a continuous series.

➤ Measures of Central tendency

A certain value that represent the whole data and signifying its characteristics is called measure of central tendency mean, median and mode are the measures of central tendency.

● Mean

Mean of ungrouped data : The mean of n observations $x_1, x_2, x_3, \dots, x_n$ is given by

$$\text{Mean } (\bar{x}) = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}$$

Mean of grouped data : Let $x_1, x_2, \dots, x_n$ be in observations with respective frequencies $f_1, f_2, \dots, f_n$.

Then, Mean

$$\text{Mean } (\bar{x}) = \frac{\sum_{i=1}^n f_i x_i}{n}$$

➤ Limit of the class

The starting and ending values of each class are called lower and upper limits.

➤ Class Interval

The difference between upper and lower boundary of a class is called class interval or size of the class.

➤ Primary and secondary data

The data collected by the investigator himself is known as the primary data, while the data collected by a person, other than the investigator, is known as secondary data.

➤ Frequency

The number of times an observation occurs in the given data, is called the frequency of the observation.

➤ Measure of Dispersion

The measures of central tendency are not sufficient to give complete information about given data. Variability is another factor which is required to be studied under statistics. The single number that describes variability is called measure of dispersion. It is the measure of scattering of the data about some central tendency.

There are following measures of dispersion

1. Range 2. Quartile deviation 3. Mean deviation 4. Standard deviation.

➤ Range

Range is the difference of maximum and minimum value of data

Range = maximum value – minimum value.

for eg given marks of Sameer and Suresh as follows–

Sameer = 79, 62, 40, 5

Suresh = 60, 45, 52, 42

For Sameer, Range = 79 – 5 = 74

For Suresh, Range = 60 – 42 = 18

Thus, range of Sameer > range of Suresh.

So, the scores are scattered or dispersed in case of Sameer while for Suresh, these are close to each other. The range of data gives us a rough idea of Variability or Scatter but does not tell about the dispersion of the data from the measure of central tendency.

➤ Mean Deviation

Mean deviation is an important measure of deviation, which depend upon the deviations of the observations from a central tendency. It is defined as the arithmetic mean of the absolute deviations of all the values taken from a central value or a fixed number a . The mean deviation from ' a ' is denoted by $MD(a)$ and is defined by–

$$M.D.(a) = \frac{\text{Sum of absolute values of deviations from 'a'}}{\text{Number of observations}}$$

➤ Mean deviation for ungrouped data

Let $x, x_2, \dots, x_n$ be n observations. Then, mean deviation about means ($\bar{x}$) or median (M) can be found by the formula

$$\frac{\sum_{i=1}^n |x_i - \bar{x}|}{n} \text{ or } \frac{\sum_{i=1}^n |x_i - M|}{n}, \text{ where } n \text{ is the number of observations}$$

➤ Mean deviation for grouped data

(i) **For discrete frequency distribution** : Let the

data have ' n ' distinct values $x_1, x_2, \dots, x_n$ and their corresponding frequencies are $f_1, f_2, \dots, f_n$ respectively. Then, this data can be represented in the tabular form as

x_i	x_1	x_2	x_3	$\dots$	x_n
f_i	f_1	f_2	f_3	$\dots$	f_n

and is called discrete frequency distribution. There, mean deviation about mean or median is given by

$$\frac{\sum_{i=1}^n f_i |x_i - A|}{N}, \text{ where } N = \sum_{i=1}^n f_i = \text{total frequency and } A = \text{mean or median.}$$

- (ii) **For continuous frequency distribution :** A continuous frequency distribution is a series in which the data is classified into different class intervals without gaps along with their respective frequencies.

Mean deviation about mean ($\bar{x}$), i.e.,

$$MD(\bar{x}) = \frac{1}{N} \sum_{i=1}^n f_i |x_i - \bar{x}|, \text{ where } 'x_i' \text{ s are the}$$

mid-point of the intervals and

$$\text{Also, mean } (\bar{x}) = \frac{1}{N} \sum_{i=1}^n f_i x_i$$

Mean deviation about median (M), i.e.,

$$MD(M) = \frac{1}{N} \sum_{i=1}^n f_i |x_i - M|, \text{ where } x_i' \text{ s are}$$

the mid-points of the intervals and $\sum_{i=1}^n f_i = N$.

$$\text{Also, median } M = l + \frac{\frac{N}{2} - C_f}{f} \cdot h, \text{ where } l, f, h$$

and C_f are lower limit, the frequency, the width of median class and cumulative frequency of class just preceding the median class.

- **Shortcut (Step-deviation) Method for calculating the Mean deviation about Mean :** This method is used to manage large data. In this method, we take an assumed mean, which is in the middle or just close to it, in the data. we denote the new variable by u_i and is defined by

$$u_i = \frac{x_i - a}{h}, \text{ where } a \text{ is the assumed mean and } h$$

is the common factor or length of class interval the mean $\bar{x}$ by step deviation method is given by

$$\bar{x} = a + \frac{\sum_{i=1}^n f_i u_i}{N} \cdot h$$

- (ii) **Mean deviation about median** To find mean deviation about median, we find the median of the given discrete frequency distribution. For this the observations are arranged in ascending order. After this the cumulative frequencies are obtained. Then, we identify the observation whose cumulative frequency is equal to or just greater

$\frac{N}{2}$, where N is the sum of frequencies. This value of the observation lies in the middle of the data, therefore, it is the required median. After finding median, we obtain the mean of the absolute values of the deviations from median. Thus,

$$M.D.(M) = \frac{1}{N} \sum_{i=1}^n f_i |x_i - M|$$

➤ Limitations of Mean Deviation

- If the data is more scattered or the degree of variability is very high, then the median is not a valid representative. Thus, the mean deviation about the median is not fully relied.
- The sum of the deviations from the mean is more than the sum of the deviations from the median. Therefore, the mean deviation about mean is not very scientific.
- The mean deviation is calculated on the basis of absolute values of the deviations and so cannot be subjected to further algebraic treatment. Sometimes, it gives unsatisfactory results.

➤ Extra Information

- The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceding the given class and the frequency of given class.
- Mean deviation may be obtained from any measure of central tendency. However, we study deviations from mean and median in this chapter.

➤ Variance

The mean of squares of deviations from mean is called the variance and it is denoted by the symbol ' σ^2 '.

The variance of ' n ' observations $x_1, x_2, \dots, x_n$ is given by :

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

➤ Significance of deviation

- If the deviation is zero, it means there is no deviation at all and all observations are equal to mean.
- If deviation is small, this indicates that the observations are close to the mean.
- If the deviation is large, there is a high degree of dispersion of the observation from the mean.

➤ **Standard Deviation**

Standard deviation is the square root of the arithmetic mean of the squares of deviations from mean and it is denoted by the symbol σ .

The square root of variance, ^{or} is called standard deviation i.e., $\sqrt{\sigma^2} = \sigma$. It is also known as root mean square deviation.

➤ **Variance and Standard deviation of ungrouped data**

Variance of n observations $x_1, x_2, \dots, x_n$ is given by

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n} = \frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2$$

and, Standard Deviation, $\sigma = \sqrt{\text{Variance}} = \sqrt{\sigma^2}$

$$\begin{aligned} \therefore \sigma &= \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} \quad \text{or} \quad \sqrt{\frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2} \\ &= \sqrt{\frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2} \end{aligned}$$

➤ **Variance and Standard deviation of Grouped data**

(i) For discrete frequency distribution

Let the discrete frequency distribution be $x_1, x_2, x_3, \dots, x_n$ and $f_1, f_2, f_3, \dots, f_n$. Then by direct method :

$$\begin{aligned} \text{Variance } (\sigma^2) &= \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2 \\ &= \frac{1}{N} \sum_{i=1}^n f_i x_i^2 - \left(\frac{\sum_{i=1}^n f_i x_i}{N} \right)^2 \end{aligned}$$

and Standard deviation (σ)

$$\begin{aligned} &= \sqrt{\frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2} \\ &= \frac{1}{N} \sqrt{N \sum_{i=1}^n f_i x_i^2 - \left(\sum_{i=1}^n f_i x_i \right)^2} \end{aligned}$$

where $N = \sum_{i=1}^n f_i$

By short cut method, variance (σ^2)

$$= \frac{1}{N} \sum_{i=1}^n f_i d_i^2 - \left(\frac{\sum_{i=1}^n f_i d_i}{N} \right)^2$$

and standard deviation (σ)

$$= \sqrt{\frac{1}{N} \sum_{i=1}^n f_i d_i^2 - \left(\frac{\sum_{i=1}^n f_i d_i}{N} \right)^2}$$

where $d_i = x_i - a$, deviation from assumed mean and a = assumed mean.

(ii) **For Continuous frequency distribution**

Direct Method : If there is a frequency distribution of n classes and each class defined by its mid-point x_i with corresponding frequency f_i , then the variance and standard deviation are :

$$\text{Variance } (\sigma^2) = \frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2$$

$$\text{and Standard deviation } (\sigma) = \sqrt{\frac{1}{N} \sum_{i=1}^n f_i (x_i - \bar{x})^2}$$

$$\text{or, } \sigma^2 = \frac{1}{N^2} \left[N \sum_{i=1}^n f_i x_i^2 - \left(\sum_{i=1}^n f_i x_i \right)^2 \right]$$

$$\text{and } \sigma = \frac{1}{N^2} \sqrt{N \sum_{i=1}^n f_i x_i^2 - \left(\sum_{i=1}^n f_i x_i \right)^2}$$

Step-Deviation (short-cut method) : Sometimes the values of mid points x_i of different classes in a continuous distribution are large and so the calculation of mean and variance becomes tedious and time consuming. For this, we use the step-deviation method. Here,

$$\text{Variance } (\sigma^2) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{\sum_{i=1}^n f_i u_i}{N} \right)^2 \right]$$

and Standard deviation (σ)

$$= h \sqrt{\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{\sum_{i=1}^n f_i u_i}{N} \right)^2}$$

where, $u_i = \frac{x_i - a}{h}$, a = assumed mean and

h = width of class interval.

➤ **Analysis of frequency distribution**

The mean deviation and standard deviation have the same units in which the data is given. The measures of dispersion are unable to compare the variability of two or more series which are measured in different units. So, we require those measures which are independent of units.

The measures of variability which is independent of units, is called coefficient of variation denoted as CV and it is given by

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

where $\bar{x}$ and σ are respectively the mean and standard deviation of the data. For comparing the variability of two series, we calculate the coefficient of variations for each series.

➤ **Comparison of Two frequency Distributions with same Mean**

Let us consider two frequency distributions with standard deviations σ_1 and σ_2 and having same mean $\bar{x}$, then

$$C.V. (1^{st} \text{ distribution}) = \frac{\sigma_1}{\bar{x}} \times 100$$

$$\text{and } C.V. (2^{nd} \text{ distribution}) = \frac{\sigma_2}{\bar{x}} \times 100$$

$$\frac{C.V. (1^{st} \text{ distribution})}{C.V. (2^{st} \text{ distribution})} = \frac{\frac{\sigma_1}{\bar{x}} \times 100}{\frac{\sigma_2}{\bar{x}} \times 100} = \frac{\sigma_1}{\sigma_2}$$

Thus, two C.V.'s can be compared on the basis of values of σ_1 and σ_2 . Thus, if two series have equal means, then the series with greater standard deviation (or variance is said to be more variable than the other. The other series with lesser value of the standard deviation is said to be more consistent than the other.

➤ **Extra Information**

- A characteristics that varies in magnitude from observation to observation e.g., weight, height, income, age, etc. are variables.
- Due to the limitations of mean deviation, some other method is required for measure of dispersion. Standard deviation is such a measure of dispersion.
- The ratio S.D. (σ) and the A.M. ($\bar{x}$) is called the coefficient of standard deviation $\left(\frac{\sigma}{\bar{x}}\right)$.
- The percentage form of coefficient of S.D. i.e., $\left(\frac{\sigma}{\bar{x}}\right)$ is called coefficient of variation.
- The distribution for which the coefficient of variation is less is called more consistent.
- Standard deviation of first n natural numbers is $\sqrt{\frac{n^2 - 1}{12}}$.
- Standard deviation is independent of change of origin, but it depends of change of scale.

EXERCISE

- The sum of 10 items is 12 and the sum of their squares is 18. The standard deviation is
(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$
- What is the mean deviation from the mean for the following data? 117, 156, 206, 198, 223
(a) 0 (b) 3 (c) 1 (d) $\frac{1}{2}$
- Find the mean deviations about the mean for the following data: 6,7,10,12,13,4,8,12
(a) 2.75 (b) 3.15 (c) 2.90 (d) 3.25
- Find the mean deviation about the mean for the following data: 12,3,18,17,4,9,17,19,20,15,8,17,2,3,16,11,3,1,0,5
(a) 5.8 (b) 7.5 (c) 6.2 (d) 4.9
- Find the mean deviation about the median for the following data: 3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21
(a) 4.15 (b) 4.75 (c) 6.29 (d) 5.27
- Find mean deviation about the mean for the following data:

x_i	2	5	6	8	10	12
f_i	2	8	10	7	8	5

(a) 2.3 (b) 3.1 (c) 2.5 (d) 4.1
- Find the mean deviation about the median for the following data:

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

(a) 6.15 (b) 4.97 (c) 2.91 (d) 5.68
- Find the mean deviation about the mean for the following data:

Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of students	2	3	8	14	8	3	2

- 10
 - 20
 - 15
 - 18
- Calculate the mean deviation about median for the following data :
 v 10. Find the variance of the following data :
 6, 8, 10, 12, 14, 16, 18, 20, 22, 24
 (a) 30 (b) 33 (c) 38 (d) 40
 - Find the variance and standard deviation for the following data :

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

 (a) 45.8, 6.77 (b) 90.5, 9.71
 (c) 48.2, 7.15 (d) 95.9, 9.15
 - Calculate the standard deviation for the following distribution :

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

- 21.50
 - 14.18
 - 15.79
 - 20.91
- Find the standard deviation for the following data :

x_i	3	8	13	18	23
f_i	7	10	15	10	6

 (a) 5.72 (b) 4.82 (c) 5.91 (d) 6.12
 - Calculate variance for the following distribution :

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

- (a) 201 (b) 210 (c) 305 (d) 310
15. Find the mean of following data: 3, 6, 11, 12, 18
(a) 15 (b) 10 (c) 22 (d) 25
16. Find the mean deviation about the mean of the distribution.
- | | | | | | |
|-----------|----|----|----|----|----|
| Size | 20 | 21 | 22 | 23 | 24 |
| Frequency | 6 | 4 | 5 | 1 | 4 |
- (a) 1.25 (b) 8.15 (c) 3.10 (d) 3.25
17. Find the mean deviation about the median of the following distribution.
- | | | | | | |
|-----------------|----|----|----|----|----|
| Marks obtained | 10 | 11 | 12 | 14 | 15 |
| No. of students | 2 | 3 | 5 | 3 | 4 |
- (a) 4.20 (b) 3.10 (c) 1.25 (d) 5.12
18. Calculate the mean deviation from median for the following data: 6, 15, 4, 10, 12, 11, 5, 3, 6
(a) 4 (b) 5 (c) 9 (d) 8

19. Calculate the mean deviation about the mean of the set first n natural numbers when n is an even number
(a) $\frac{x}{2}$ (b) $2x$ (c) $\frac{x}{4}$ (d) $3x$
20. Coefficient of variation of two distributions are 60 and 70, and their standard deviations are 21 and 16 respectively. What are their arithmetic mean?
(a) 35, 22.85 (b) 30, 19.50
(c) 35, 18.95 (d) 40, 20.50
21. The sum of the squares of deviations for 10 observations taken from their mean 50 is 250. Find the coefficient of variation.
(a) 10 (b) 15 (c) 18 (d) 12
22. Find the standard deviation of first 'ten' natural numbers.
(a) 2.45 (b) 3.91 (c) 4.15 (d) 2.87
23. The following relates to sample of size n , $\sum x = 960$ and $\sum x^2 = 18000$, find the variance.
(a) 20 (b) 42 (c) 44 (d) 35

Answer Key

1. (c)	2. (a)	3. (c)	4. (c)	5. (d)	6. (a)	7. (b)	8. (a)	9. (c)	10. (b)
11. (a)	12. (b)	13. (d)	14. (a)	15. (b)	16. (a)	17. (c)	18. (a)	19. (c)	20. (a)
21. (a)	22. (d)	23. (c)							

Answers with Explanations

1. Option (c) is correct.

Explanation: Given, $\sum x = 12$ and $\sum x^2 = 18$

$$\Rightarrow \text{Variance} = \frac{\sum x^2}{n} - \frac{(\sum x)^2}{n^2}$$

$$\Rightarrow \text{Variance} = \frac{18}{10} - \frac{12^2}{10^2}$$

$$\Rightarrow \text{Variance} = \frac{9}{5} - \frac{36}{25}$$

$$\Rightarrow \text{Variance} = \frac{9}{5} - \frac{36}{25}$$

$$\Rightarrow \text{Variance} = \frac{(9 \times 5 - 36)}{25}$$

$$\Rightarrow \text{Variance} = \frac{(45 - 36)}{25}$$

$$\Rightarrow \text{Variance} = \frac{9}{25}$$

$$\Rightarrow \text{Standard deviation} = \sqrt{\frac{9}{25}}$$

$$\Rightarrow \text{Standard deviation} = \frac{3}{5}$$

2. Option (a) is correct.

Explanation:

$$\text{Mean} = \frac{117 + 156 + 206 + 198 + 223}{5} = 180$$

x_i	117	156	206	198	223
$x_i - \text{mean}$	-63	-24	26	18	43

$$\begin{aligned} \text{Mean deviation} &= \frac{\sum |x_i - \bar{x}|}{n} \\ &= \frac{(-63) + (-24) + 26 + 18 + 43}{5} = 0 \end{aligned}$$

3. Option (c) is correct.

Explanation: Solutions we proceed step-wise and get the followings:

Step 1 mean of the given data is

$$\bar{x} = \frac{6 + 7 + 10 + 12 + 13 + 4 + 8 + 12}{8} = \frac{72}{8} = 9$$

Step 2 The deviation of the respective observations from the mean $\bar{x}$, i.e., $x_i - \bar{x}$ are

$$6-9, 7-9, 10-9, 12-9, 13-9, 4-9, 8-9, 12-9$$

$$\text{Or } -3, -2, 1, 3, 4, -5, -1, 3$$

Step 3 The absolute values of the deviations i.e. $|x_i - \bar{x}|$ are

$$3, 2, 1, 3, 4, 5, 1, 3$$

Step 4 The required mean deviation about the mean is

$$\begin{aligned} \text{M.D.}(\bar{x}) &= \frac{\sum |x_i - \bar{x}|}{n} \\ &= \frac{3 + 2 + 1 + 3 + 4 + 5 + 1 + 3}{8} = \frac{22}{8} = 2.75 \end{aligned}$$

4. Option (c) is correct.

Explanation: We have to first find the mean $(\bar{x})$ of the given data

$$\bar{x} = \frac{1}{20} \sum_{i=1}^{20} x_i = \frac{200}{20} = 10$$

The respective absolute values of the deviations from mean, i.e., are 2,7,8,7,6,1,9,10,5,2,7,8,7,6,1,7,9,10,5

$$\text{Therefore } \sum_{i=1}^{20} |x_i - \bar{x}| = 124$$

$$\text{And } (\bar{x}) = \frac{124}{20} = 6.2$$

5. Option (d) is correct.

Explanation: Here the number of observation is 11 which is odd. Arranging the data into ascending order, we have 3,3,4,5,7,9,10,12,18,19,21

$$\text{Now Median} = \left(\frac{11+1}{2} \right)^{\text{th}} \text{ or } 6^{\text{th}} \text{ observation} = 9$$

The absolute values of the respective deviations from the median, i.e., $|x_i - M|$ are 6,6,5,4,2,0,1,3,9,10,12

$$\text{Therefore } \sum_{i=1}^{11} |x_i - M| = 58$$

$$\text{And M.D. (M)} = \frac{1}{11} \sum_{i=1}^{11} |x_i - M| = \frac{1}{11} \times 58 = 5.27$$

6. Option (a) is correct.

Explanation: Let us make a Table of the given data and append other columns after calculations.

x_i	f_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
2	2	4	5.5	11
5	8	40	2.5	20
6	10	60	1.5	15
8	7	56	0.5	3.5
10	8	80	2.5	20
12	5	60	4.5	22.5
	40			92

$$N = \sum_{i=1}^6 f_i = 40, \sum_{i=1}^6 f_i x_i = 300, \sum_{i=1}^6 f_i |x_i - \bar{x}| = 92$$

$$\text{Therefore } \bar{x} = \frac{1}{N} \sum_{i=1}^6 f_i x_i = \frac{1}{40} \times 300 = 7.5$$

$$\text{And } (\bar{x}) = \frac{1}{N} \sum_{i=1}^6 f_i |x_i - \bar{x}| = \frac{1}{40} \times 92 = 2.3$$

7. Option (b) is correct.

Explanation: The given observations are already in ascending order. Adding a row corresponding to cumulative frequencies to the given data, we get (Table).

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3
c.f.	3	7	12	14	18	23	27	30

Now, $N = 30$ which is even.

Median is the mean of the 15th and 16th observations. Both of these observations lie in the cumulative frequency 18, for which the corresponding observation is 13.

Therefore, Median

$$M = \frac{15^{\text{th}} \text{ observation} + 16^{\text{th}} \text{ observation}}{2} = \frac{13 + 13}{2} = 13$$

Now, absolute values of the deviations from median, i.e., $|x_i - M|$ are shown in Table.

$ x_i - M $	10	7	4	1	0	2	8	9
f_i	3	4	5	2	4	5	4	3
$f_i x_i - M $	30	28	20	2	0	10	32	27

$$\text{We have } \sum_{i=1}^8 f_i = 30 \text{ and } \sum_{i=1}^8 f_i |x_i - M| = 149$$

$$\begin{aligned} \text{Therefore M.D (M)} &= \frac{1}{N} \sum_{i=1}^8 f_i |x_i - M| \\ &= \frac{1}{30} \times 149 = 4.97. \end{aligned}$$

8. Option (a) is correct.

Explanation: We make the following Table from the given data :

Marks obtained	Number of students f_i	Mid-points x_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
10-20	2	15	30	30	60
20-30	3	25	75	20	60
30-40	8	35	280	10	80
40-50	14	45	630	0	0
50-60	8	55	440	10	80
60-70	3	65	195	20	60
70-80	2	75	150	30	60
	40		1800		400

$$\begin{aligned} \text{Here } N &= \sum_{i=1}^7 f_i = 40, \sum_{i=1}^7 f_i x_i = 1800, \sum_{i=1}^7 f_i |x_i - \bar{x}| \\ &= 400 \end{aligned}$$

$$\text{Therefore } \bar{x} = \frac{1}{N} \sum_{i=1}^7 f_i x_i = \frac{1800}{40} = 45$$

$$\text{And M.D. } (\bar{x}) = \frac{1}{N} \sum_{i=1}^7 f_i |x_i - \bar{x}| = \frac{1}{40} \times 400 = 10$$

9. Option (c) is correct.

Explanation: From the following Table from the given data :

Class	Frequency	Cumulative frequency f_i	Mid-points (c.f.) x_i	$ x_i - \text{med.} $	$f_i x_i - \text{med.} $
0-10	6	6	5	23	138
10-20	7	13	15	13	91
20-30	15	28	25	3	45
30-40	16	44	35	7	112
40-50	4	48	45	17	68
50-60	2	50	55	27	54
	50				508

The class interval containing $\frac{N}{2}$ or 25th item is 20-30. Therefore, 20-30 is the median class. We know that

$$\text{Median} = l + \frac{\frac{N}{2} - C}{f} \times h$$

Here $l = 20$, $C = 13$, $f = 15$, $h = 10$ and $N = 50$

$$\text{Therefore, Median} = 20 + \frac{25 - 13}{15} \times 10 = 20 + 8 = 28$$

Thus, mean deviation about median is given by

$$\text{M.D.}(M) = \frac{1}{N} \sum_{i=1}^6 f_i |x_i - M| = \frac{1}{50} \times 508 = 10.16$$

10. Option (b) is correct.

Explanation: From the given data we can form the following Table. The mean is calculated by step-deviation method taking 14 as assumed mean. The number of observations is $n = 10$

x_i	$d_i = \frac{x_i - 14}{2}$	Deviations from mean $ x_i - \bar{x} $	$ x_i - \bar{x} $
6	-4	-9	81
8	-3	-7	49
10	-2	-5	25
12	-1	-3	9
14	0	-1	1
16	1	1	1
18	2	3	9
20	3	5	25
22	4	7	49
24	5	9	81
	5		330

$$\text{Therefore, Mean } \bar{x} = \text{assumed mean} + \frac{\sum_{i=1}^n d_i}{n} \times h$$

$$= 14 + \frac{5}{10} \times 2 = 15$$

$$\text{And variance } (\sigma)^2 = \frac{1}{n} \sum_{i=1}^{10} (x_i - \bar{x})^2 = \frac{1}{10} \times 330 = 33$$

11. Option (a) is correct.

Explanation: Presenting the data in tabular form (Table), we get

Table

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
4	3	12	-10	100	300
8	5	40	-6	36	180
11	9	99	-3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
	30	420			1374

$$N = 30, \sum_{i=1}^7 f_i x_i = 420, \sum_{i=1}^7 f_i (x_i - \bar{x})^2 = 1374$$

$$\text{Therefore } \bar{x} = \frac{\sum_{i=1}^7 f_i x_i}{N} = \frac{1}{30} \times 420 = 14$$

$$\text{Hence variance } (\sigma)^2 = \frac{1}{N} \sum_{i=1}^7 f_i (x_i - \bar{x})^2$$

$$= \frac{1}{30} \times 1374 = 45.8$$

And Standard deviation $(\sigma) = \sqrt{45.8} = 6.77$

12. Option (b) is correct.

Explanation: From the given data, we construct the following Table.

Table

Class	Frequency (f_i)	Mid-point (x_i)	$f_i x_i$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
30-40	3	35	105	729	2187
40-50	7	45	315	289	2023
50-60	12	55	660	49	588
60-70	15	65	975	9	135
70-80	8	75	600	169	1352
80-90	3	85	255	529	1587
90-100	2	95	190	1089	2178
	50		3100		10050

$$\text{Thus Mean } (\bar{x}) = \frac{1}{N} \sum_{i=1}^7 f_i x_i = \frac{3100}{50} = 62$$

$$\text{Variance } (\sigma)^2 = \frac{1}{N} \sum_{i=1}^7 f_i (x_i - \bar{x})^2$$

$$= \frac{1}{50} \times 10050 = 201$$

and Standard deviation $(\sigma) = \sqrt{201} = 14.18$

13. Option (d) is correct.*Explanation:* Let us form the following Table:**Table**

x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
3	7	21	9	63
8	10	80	64	640
13	15	195	169	2535
18	10	180	324	3240
23	6	138	529	3174
	48	614		9652

Now, by formula (3), we have

$$\begin{aligned}
 \sigma &= \frac{1}{N} \sqrt{N \sum f_i x_i^2 - (\sum f_i x_i)^2} \\
 &= \frac{1}{48} \sqrt{48 \times 9652 - (614)^2} \\
 &= \frac{1}{48} \sqrt{463296 - 376996} \\
 &= \frac{1}{48} \times 293.77 = 6.12
 \end{aligned}$$

Therefore, Standard deviation (σ) = 6.12**14. Option (a) is correct.***Explanation:* Let the assumed mean $A = 65$, Here $h = 10$

We obtain the following Table from the given data :

Table

Class	Frequency (f_i)	Mid-point x_i	$y_i = \frac{x_i - 65}{10}$	y_i^2	$f_i y_i$	$f_i y_i^2$
30-40	3	35	-3	9	-9	27
40-50	7	45	-2	4	-14	28
50-60	12	55	-1	1	-12	12
60-70	15	65	0	0	0	0
70-80	8	75	1	1	8	8
80-90	3	85	2	4	6	12
90-100	2	95	3	9	6	18
	N = 50				-15	105

$$\begin{aligned}
 \text{Variance } \sigma^2 &= \frac{h^2}{N^2} \left[N \sum f_i y_i^2 - (\sum f_i y_i)^2 \right] \\
 &= \frac{(10)^2}{(50)^2} \left[50 \times 105 - (-15)^2 \right] \\
 &= \frac{1}{25} [5250 - 225] = 201
 \end{aligned}$$

15. Option (b) is correct.*Explanation:* Mean = $\frac{\text{Sum of observations}}{\text{Total number of observations}}$

$$\Rightarrow \text{Mean} = \frac{3 + 6 + 11 + 12 + 18}{5}$$

$$\Rightarrow \text{Mean} = \frac{50}{5} = 10$$

16. Option (a) is correct.*Explanation:*

x_i	f_i	$f_i x_i$	$ x_i - \bar{x} $	$f_i x_i - \bar{x} $
20	6	120	1.65	9.90
21	4	84	0.65	2.60
22	5	110	0.35	1.75
23	1	23	1.35	1.35
24	4	96	2.35	9.40
	20	433		25

$$\therefore \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{433}{20} = 21.65 \quad 1$$

 $\therefore$ Required mean deviation about mean ($\bar{x}$) i.e.,

$$M.D. (\bar{x}) = \frac{\sum f_i |x_i - M|}{\sum f_i} = \frac{25}{20} = 1.25$$

17. Option (c) is correct.*Explanation:*

x_i	f_i	c.f.	$ x_i - M $	$f_i x_i - M $
10	2	2	2	4
11	3	5	1	3
12	8	13	0	0
14	3	16	2	6
15	4	20	3	12
	N = 20			25

Here $N = 20$, even $\therefore$ Median = 12Hence, Mean deviation about Median (M),

$$\begin{aligned}
 M.D. (M) &= \frac{\sum f_i |x_i - M|}{\sum f_i} \\
 &= \frac{25}{20} \\
 &= 1.25
 \end{aligned}$$

18. Option (a) is correct.*Explanation:* Given 6, 15, 4, 10, 12, 11, 5, 3, 6

Ascending order: Ascending order of the given data is: 3, 4, 5, 6, 10, 11, 12, 15, 16

Number of data values = 9

$$\begin{aligned}
 \text{Median} &= \frac{n+1}{2}^{\text{th}} \text{ observation} \\
 &= \frac{9+1}{2} \\
 &= 5^{\text{th}} \text{ observation}
 \end{aligned}$$

$$\text{Median (M)} = 10$$

The absolute values of the respective deviations from the median, i.e., $|x_i - M|$

x_i	$ x_i - M $
3	$ 3 - 10 = 7$
4	$ 4 - 10 = 6$
5	$ 5 - 10 = 5$
6	$ 6 - 10 = 4$
10	$ 10 - 10 = 0$

11	$ 11 - 10 = 1$
12	$ 12 - 10 = 2$
15	$ 15 - 10 = 5$
16	$ 16 - 10 = 6$
	$\Sigma x_i - M = 36$

$$\text{Mean deviation} = \frac{\Sigma |x_i - M|}{n} = \frac{36}{9} = 4$$

19. Option (c) is correct.

Explanation: Let $n = 2k \Rightarrow k = \frac{n}{2}$

$$\begin{aligned} \therefore \bar{x} &= \frac{1+2+3+\dots+n}{n} \\ &= \frac{n(n+1)}{2n} \\ &= \frac{n+1}{2} \\ &= \frac{2k+1}{2} \end{aligned}$$

x_i	$x_i - \bar{x}$	$ x_i - \bar{x} $
1	$1 - \left(\frac{2k+1}{2}\right) = -\left(\frac{2k-1}{2}\right)$	$\frac{2k-1}{2}$
2	$-\left(\frac{2k-3}{2}\right)$	$\frac{2k-3}{2}$
3	$-\left(\frac{2k-5}{2}\right)$	$\frac{2k-5}{2}$
'	'	'
'	'	'
'	'	'
$k-2$	$-\frac{5}{2}$	$\frac{5}{2}$
$k-1$	$-\frac{3}{2}$	$\frac{3}{2}$
k	$-\frac{1}{2}$	$\frac{1}{2}$
$k+1$	$\frac{1}{2}$	$\frac{1}{2}$
$k+2$	$\frac{3}{2}$	$\frac{3}{2}$
'	'	'
'	'	'
'	'	'
$2k-1$	$\frac{2k-3}{2}$	$\frac{2k-3}{2}$
$2k-1$	$\frac{2k-3}{2}$	$\frac{2k-3}{2}$

$$\therefore \Sigma |x - \bar{x}| = \left\{ \frac{2k-1}{2} + \frac{2k-3}{2} + \frac{2k-5}{2} + \dots + \frac{5}{2} + \frac{3}{2} + 1 \right\}$$

$$= 1 + 3 + 5 + \dots + (2k-1)$$

$$= \frac{k}{2} \{1 + (2k-1)\} = k^2 = \left(\frac{n}{2}\right)^2 = \frac{n^2}{4}$$

$\therefore$ Mean deviation about mean, M.D. ($\bar{x}$)

$$= \frac{1}{n} \Sigma |x - \bar{x}| = \frac{1}{n} \cdot \frac{n^2}{4} = \frac{n}{4}$$

20. Option (a) is correct.

Explanation: Given, $CV_1 = 60, CV_2 = 70$

$$\sigma_1 = 21, \sigma_2 = 16$$

$$\text{We know, } CV = \frac{\sigma}{\bar{x}} \times 100$$

$$\Rightarrow \bar{x} = \frac{\sigma}{CV} \times 100$$

$$\therefore \bar{x}_1 = \frac{\sigma_1}{CV_1} \times 100$$

$$= \frac{21}{60} \times 100$$

$$= 35$$

$$\text{and, } \bar{x}_2 = \frac{\sigma_2}{CV_2} \times 100$$

$$= \frac{16}{70} \times 100$$

$$= 22.85$$

$\therefore$ Their arithmetic mean are 35 and 22.85.

21. Option (a) is correct.

Explanation: Here, $N = 10, \bar{x} = 50$

$$\text{and, } \sum_{i=1}^{10} (x_i - 50)^2 = 250$$

$$\Rightarrow \frac{\sum_{i=1}^{10} (x_i - 50)^2}{10} = 25$$

$$\Rightarrow \sigma^2 = 25$$

$$\Rightarrow \sigma = 5$$

$\therefore$ Coefficient of variation,

$$C.V. = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{5}{50} \times 100$$

$$= \frac{500}{50}$$

$$= 10$$

$\therefore$ 10 is the coefficient of variation.

22. Option (d) is correct.

Explanation: Here, the variables are 1, 2, 3, ..., 9, 10

$$n = 10$$

$\therefore$ Standard deviation,

$$\begin{aligned}
 \sigma &= \sqrt{\frac{\sum_{i=1}^{10} x_i^2}{10} - \left(\frac{\sum_{i=1}^{10} x_i}{10}\right)^2} \\
 &= \sqrt{\frac{1}{10} \cdot \frac{10(10+1)(20+1)}{6} - \left(\frac{10(10+1)}{2 \cdot 10}\right)^2} \\
 &= \sqrt{\frac{11 \cdot 21}{6} - \left(\frac{11}{2}\right)^2} \\
 &= \sqrt{\frac{77}{2} - \frac{121}{4}} \\
 &= \frac{1}{2} \sqrt{154 - 121}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \times \sqrt{33} \\
 &= 2.87.
 \end{aligned}$$

23. Option (c) is correct.

Explanation: We know,

$$\begin{aligned}
 \sigma^2 &= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 \\
 &= \frac{18,000}{60} - \left(\frac{960}{60}\right)^2 \\
 &= 300 - (16)^2 \\
 &= 300 - 256 \\
 &= 44
 \end{aligned}$$

$$\therefore \text{Variance} = 44$$

○○○

INTRODUCTION TO THREE-DIMENSIONAL GEOMETRY

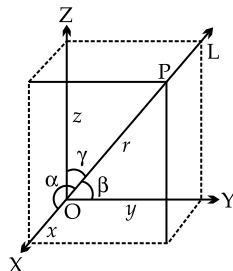
Basics of 3D Geometry

- We will now use vector algebra to three dimensional geometry. The purpose of this approach to 3-dimensional geometry is that it makes the study simple and elegant*. In this chapter, we shall study the direction cosines and direction ratios of a line joining two points and also discuss about the equations of lines and planes in space under different conditions, angle between two lines, two planes, a line and a plane, shortest distance between two skew lines and distance of a point from a plane. Most of the above results are obtained in vector form. Nevertheless, we shall also translate these results in the Cartesian form which, at times, presents a more clearly geometric and analytic picture of the situation.

Direction Cosines and Direction Ratios

- If a directed line L passing through the origin makes angles α , β and γ with x , y and z -axes, respectively, called direction angles, then cosine of these angles, namely, $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are called direction cosines of the directed line L .

If we reverse the direction of L , then the direction angles are replaced by their supplements, i.e., $\pi - \alpha$, $\pi - \beta$, and $\pi - \gamma$. Thus, the signs of the direction cosines are reversed.



Note that a given line in space can be extended in two opposite directions and so it has two sets of direction cosines. In order to have a unique set of direction cosines for a given line in space, we must take the given line as a directed line. These unique direction cosines are denoted by l , m and n .

Remark

- If the given line in space does not pass through the origin, then, in order to find its direction cosines, we draw a line through the origin and parallel to the given line. Now take one of the directed lines from the origin and find its direction cosines as two parallel line have same set of direction cosines. Any three numbers which are proportional to the direction cosines of a line are called the direction ratios of the line. If l , m , n are direction cosines and a , b , c are direction ratios of a line, then $a = \lambda l$, $b = \lambda m$ and $c = \lambda n$, for any nonzero $\lambda \in \mathbb{R}$. Let a , b , c be direction ratios of a line and let l , m and n be the direction cosines (d.c's) of the line. Then

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = k(\text{say}), k \text{ is a constant}$$

Therefore $l = ak$, $m = bk$, $n = ck$

$$\text{But } l^2 + m^2 + n^2 = 1$$

$$\text{Therefore } k^2 (a^2 + b^2 + c^2) = 1 \Rightarrow k^2 = \frac{1}{a^2 + b^2 + c^2}$$

$$\text{Or } k = \pm \sqrt{\frac{1}{a^2 + b^2 + c^2}} = \pm \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

Hence d. c's of the line are

$$l = \pm \frac{a}{\sqrt{a^2 + b^2 + c^2}}, m = \pm \frac{b}{\sqrt{a^2 + b^2 + c^2}}, n = \pm \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Where, depending on the desired sign of k , either a positive or a negative sign is to be taken for l , m and n . For any line, if a , b , c are direction ratios of a line, then ka , kb , kc ; $k \neq 0$ is also a set of direction ratios. So, any two sets of direction ratios of a line are also proportional. Also, for any line there are infinitely many sets of direction ratios.

Direction cosines of a line passing through two points

- Since one and only one line passes through two given points, we can determine the direction cosines of a line passing through the given points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

$$x_2 - x_1, y_2 - y_1, z_2 - z_1$$

Lines in 3D**Equation of a Line in Space**

We have studied equation of lines in two dimensions, we shall now study the vector and Cartesian equations of a line in space. A line is uniquely determined if

- it passes through a given point and has given direction, or
- It passes through two given points.

If l , m , n are the direction cosines of the line, the equation of the line is

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

Angle between Two Lines

Let the angle between two lines L_1 and L_2 is θ

$$\text{Then } \cos \theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$\cos \theta = \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} = L_1$$

$$\cos \theta = \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2} = L_2$$

Interaction between Two Lines in 3D

- If two lines in space intersect at a point, then the shortest distance between them is zero. Also, if two lines in space are parallel, then the shortest distance between them will be the perpendicular distance, i.e. the length of the perpendicular drawn from a point on one line onto the other line. Further, in a space, there are lines which are neither intersecting nor parallel. In fact, such pair of lines are non coplanar and are called skew lines.

We now determine the shortest distance between two lines in the following way: Let L_1 and L_2 be two lines with equations

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$\text{And } \vec{r} = \vec{a}_2 + \lambda \vec{b}_2$$

Hence the required shortest distance

$$d = \frac{|(\vec{b}_1 \times \vec{b}_2)(\vec{a}_1 \times \vec{a}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Distance between parallel lines

If two lines L_1 and L_2 are parallel, then they are coplanar. Let the lines be given by

$$\vec{r} = \vec{a}_1 + \lambda \vec{b} \quad \dots\dots\dots (i)$$

$$\text{And } \vec{r} = \vec{a}_2 + \lambda \vec{b} \quad \dots\dots\dots (ii)$$

Hence, the distance between the given parallel lines is

$$d = \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|}$$

Plane and a Point

- A plane is determined uniquely if any one of the following is known:

- The normal to the plane and its distance from the origin is given, i.e., equation of a plane in normal form.
- It passes through a point and is perpendicular to a given direction.
- It passes through three given non collinear points.

Equation of a plane in normal form

The equation of a plane in normal form is $ax + by + cz = d$. Where a, b, c are the direction ratios of the normal to the plane.

Equation of a plane perpendicular to a given vector and passing through a given point

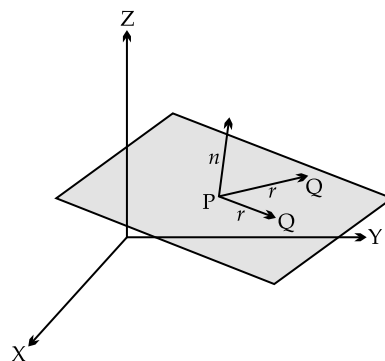
The equation of a plane passing through a given point $P(x_1, y_1, z_1)$ and perpendicular to a given vector is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

Equation of a plane passing through three non co linear points

The equation of a plane passing through three non collinear points (x_1, y_1, z_1) , (x_2, y_2, z_2) and (x_3, y_3, z_3) is:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

**Intercept form of the equation of a plane**

- Let the plane make intercepts a, b, c on x, y and z axes, respectively. Hence, the plane meets x, y and z -axes at $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$, respectively. Then the equation of plane in intercept form is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Plane passing through the intersection of two given planes

- Let $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ be two Cartesian planes then the equation of a plane which passes through these two planes is $(a_1x + b_1y + c_1z - d_1) + \lambda(a_2x + b_2y + c_2z - d_2) = 0$

Intersection between a Line and a Plane

Intersection of a Line and a Plane If we are given the scalar equation of a plane, we are given information about the normal to the plane. If the plane is parallel to the line (or if the line is contained in the plane), the normal of the plane and the direction vector of the line will be perpendicular.

This implies that $\vec{n} \cdot \vec{m} = 0$. If the line is not parallel to the plane, it must intersect at a single point.

A line and a plane may or may not intersect. There are three possibilities:

- The line is contained in the plane (infinite intersections).
- The line is parallel to the plane (no intersections).
- The line intersects the plane (one point of intersection).

Let the L intersect the plane P at the point $Q(a, b, c)$ then the coordinate of this point are

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n} = k(\text{say}) \quad \dots\dots (i)$$

And equation of plane P is

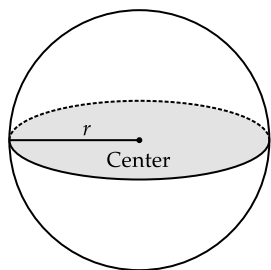
$$Ax + By + Cz = d \quad \dots\dots (ii)$$

From (i) equation $x = kl + x_1$, $y = km + y_1$, $z = kn + z_1$ Which satisfy the given equation of plane P .

Sphere

- In 2-space you know that the set of points that are at a fixed distance d from a fixed point is a circle. A sphere is a generalization of this to 3-space.

Definition: The set of all those points in 3-space which are at a fixed distance d from a point $C(a, b, c)$, is a sphere with centre C and radius d .



- Spheres are, of course, not new to you. A ball and a plum are spherical in shape. However, whenever we talk of a sphere in analytical geometry, we mean the surface of a sphere. Thus, for us a hollow ball is a sphere, and a solid cricket ball is not a sphere. Let us find the equation of a sphere with radius d and centre $C(a, b, c)$ now. If $P(x, y, z)$ is any point on the sphere, then, by the distance formula
- $$(x-a)^2 + (y-b)^2 + (z-c)^2 = d^2 \quad \dots\dots (i)$$
- This is the required equation.

The sphere 'with centre $(0, 0, 0)$ and radius 1 unit is

$$x^2 + y^2 + z^2 = 1 \quad \dots\dots (ii)$$

The general equation of a sphere is

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0 \quad \dots\dots (iii)$$

Its centre $(-u, -v, -w)$ and radius $\sqrt{u^2 + v^2 + w^2 - d}$. Note that the general equation given above will be a real sphere if $u^2 + v^2 + w^2 - d \geq 0$. Otherwise it will be an imaginary sphere, that is, a sphere with no real points on it.

A second degree equation in x, y and z represents a sphere iff

- The coefficient of x^2, y^2 and z^2 are equal.
- The equation has no contain xy, yz and zx .

Equation of sphere in diameter form

Let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be the ends of a diameter of a sphere.

Then the equation of sphere in diameter form is

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) + (z-z_1)(z-z_2) = 0$$

EXERCISE

- The locus of a point $P(x, y, z)$ which moves in such a way that $z = 7$ is a
 - line parallel to x -axis
 - line parallel to y -axis
 - line parallel to z -axis
 - plane parallel to xy -plane
- The xy -plane divides the line segment joining the points $(-1, 3, 4)$ and $(2, -5, 6)$
 - internally in the ratio $2 : 3$
 - internally in the ratio $3 : 2$
 - externally in the ratio $2 : 3$
 - externally in the ratio $2 : 1$
- The points $P(3, 2, 4), Q(4, 5, 2), R(5, 8, 0)$ and $S(2, -1, 6)$ are
 - vertices of a rhombus which is not a square
 - non-coplanar
 - collinear
 - coplanar but not collinear
- Consider the following statements :
 - The direction ratios of y -axis can be $< 0, 4, 0 >$
 - The direction ratios of a line perpendicular to z -axis can be $< 5, 6, 0 >$

Which of the statements given above is/are correct ?

 - 1 only
 - 2 only
 - Both 1 and 2
 - Neither 1 nor 2
- If a line has direction ratios $< a + b, b + c, c + a >$, then what is the sum of the squares of its direction cosines ?
 - $(a + b + c)^2$
 - $2(a + b + c)$
 - 3
 - 1

Directions: (Q. 10 – 11) Let a vector $\vec{r}$ make angles $60^\circ, 30^\circ$ with x and y -axes respectively.
- What angle does $\vec{r}$ make with z -axis?
 - 30°
 - 60°
 - 90°
 - 120°
- What are the direction cosines of $\vec{r}$?
 - $\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$
 - $\left\langle \frac{1}{2}, -\frac{\sqrt{3}}{2}, 0 \right\rangle$
 - $\left\langle \frac{1}{2}, \frac{1}{\sqrt{2}}, 0 \right\rangle$
 - $\left\langle -\frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$
- If l, m, n are the direction cosines of the line $x - 1 = 2(y + 3) = 1 - z$, then what is $l^4 + m^4 + n^4$ equal to?
 - 1
 - $\frac{11}{27}$
 - $\frac{13}{27}$
 - 4
- What is the perpendicular distance from the point $(2, 3, 4)$ to the line $\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0}$?
 - 6 units
 - 5 units
 - 3 units
 - 2 unit
- The line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is given by
 - $x + y + z = 6, x + 2y - 3z = -4$
 - $x + 2y - 2z = -1, 4x + 4y - 5z - 3 = 0$
 - $3x + 2y - 3z = 0, 3x - 6y + 3z = -2$
 - $3x + 2y - 3z = -2, 3x - 6y + 3z = 0$
- A line passes through the points $(6, -7, -1)$ and $(2, -3, 1)$. What are the direction ratios of the line ?
 - $< 4, -4, 2 >$
 - $< 4, 4, 2 >$
 - $< -4, 4, 2 >$
 - $< 2, 1, 1 >$
- What is the angle between the two lines having direction ratios $< 6, 3, 6 >$ and $< 3, 3, 0 >$?
 - $\frac{\pi}{6}$
 - $\frac{\pi}{4}$

- (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
13. If p , q and r are the intercepts made by the plane on the coordinate axes respectively, then what is $(p+q+r)$ equal to?
 (a) 10 (b) 11
 (c) 12 (d) 13
14. A plane cuts intercepts 2, 2, 1 on the coordinate axes. What are the direction cosines of the normal to the plane?
 (a) $\langle 2/3, 2/3, 1/3 \rangle$ (b) $\langle 1/3, 2/3, 2/3 \rangle$
 (c) $\left\langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$ (d) $\left\langle \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle$
15. The foot of the perpendicular drawn from the origin to the plane $x+y+z=3$ is
 (a) (0, 1, 2) (b) (0, 0, 3)
 (c) (1, 1, 1) (d) (-1, 1, 3)
16. What is the equation of the plane which cuts an intercept 5 units on the z -axis and it parallel to xy -plane?
 (a) $x+y=5$ (b) $z=5$
 (c) $z=0$ (d) $x+y+z=5$
17. What is the equation of the plane passing through the points $(-2, 6, -6)$, $(-3, 10, -9)$ and $(-5, 0, -6)$?
 (a) $2x-y-2z=2$ (b) $2x+y+3z=3$
 (c) $x+y+z=6$ (d) $x-y-z=3$
18. A variable plane passes through a fixed point (a, b, c) and cuts the axes in A, B and C respectively. The locus of the centre of the sphere OABC, O being the origin, is
 (a) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ (b) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$
 (c) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$ (d) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 2$
19. The equation of the plane passing through the line of intersection of the planes $x+y+z=1$, $2x+3y+4z=7$, and perpendicular to the plane $x-5y+3z=5$ is given by
 (a) $x+2y+3z-6=0$ (b) $x+2y+3z+6=0$
 (c) $3x+4y+5z-8=0$ (d) $3x+4y+5z+8=0$
20. What is the image of the point (1, -2, 3) in the plane?
 (a) (2, -1, 5) (b) (-1, 2, -3)
 (c) (5, 4, 1) (d) None of these
21. The point of intersection of the line joining the points $(-3, 4, -8)$ and $(5, -6, 4)$ with the XY-plane is
 (a) $\left(\frac{7}{3}, -\frac{8}{3}, 0\right)$ (b) $\left(-\frac{7}{3}, -\frac{8}{3}, 0\right)$
 (c) $\left(-\frac{7}{3}, \frac{8}{3}, 0\right)$ (d) $\left(\frac{7}{3}, \frac{8}{3}, 0\right)$
22. Where does the line cut the plane?
 (a) (0, -4, -1) (b) (0, -4, 1)
 (c) (1, 4, 0) (d) (0, 4, 1)
23. What is the ratio in which the plane divides the line?
 (a) 1 : 1 (b) 2 : 3
 (c) 3 : 4 (d) None of these
24. What is the length of the diameter of the sphere whose centre is at (1, -2, 3) and which touches the plane $6x-3y+2z-4=0$?
 (a) 1 unit (b) 2 units
 (c) 3 units (d) 4 units
25. What is the equation to the sphere whose centre is at $(-2, 3, 4)$ and radius is 6 units?
 (a) $x^2+y^2+z^2+4x-6y-8z=7$
 (b) $x^2+y^2+z^2+6x-4y-8z=7$
 (c) $x^2+y^2+z^2+4x-6y-8z=4$
 (d) $x^2+y^2+z^2+4x+6y+8z=4$
26. The radius of the sphere :
 $3x^2+3y^2+3z^2-8x+4y+8z-15=0$
 (a) 2 (b) 3 (c) 4 (d) 5
27. What is the domain of the function
 $f(x) = \sqrt{1-(x-1)^2}$?
 (a) (0, 1) (b) $[-1, 1]$
 (c) (0, 2) (d) [0, 2]
28. What is $\sum_{x=1}^6 2^x f(x)$ equal to?
 (a) 1365 (b) 2730
 (c) 4024 (d) 5460
29. What is/are the point(s) of intersection of the curves?
 (a) $(-2, 3)$ only
 (b) $(2, 1)$ only
 (c) $(-2, 3)$ and $(2, 1)$
 (d) Neither $(-2, 3)$ nor $(2, 1)$
30. What is $(g \circ f)\left(-\frac{5}{3}\right) - (f \circ g)\left(-\frac{5}{3}\right)$ equal to?
 (a) -1 (b) 0 (c) 1 (d) 2

Answer Key

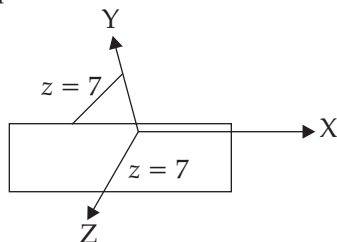
1. (d)	2. (c)	3. (c)	4. (c)	5. (d)	6. (c)	7. (a)	8. (b)	9. (b)	10. (d)
11. (c)	12. (b)	13. (b)	14. (c)	15. (c)	16. (b)	17. (a)	18. (c)	19. (a)	20. (c)
21. (a)	22. (d)	23. (d)	24. (d)	25. (a)	26. (b)	27. (d)	28. (d)	29. (c)	30. (c)

Answers with Explanations

1. Option (d) is correct.

Explanation:

Given, point $z = 7$



It is clear if point moving on Z-axis means it is parallel to XY-plane.

Hint:

- Make diagram for plane.

2. Option (c) is correct.

Explanation:

Given: xy plane divides line segment.

Consider, two points:

$$(x_1, y_1, z_1) = (-1, 3, 4)$$

$$(x_2, y_2, z_2) = (2, -5, 6)$$

Now, xy plane divides by:

$$\text{Ratio} = \frac{-z_1}{z_2} = \frac{-4}{6} = \frac{-2}{3}$$

Since, division in negative.

So, externally division.

Hence, in ratio 2 : 3 externally.

Hint:

- Division by XY-plane. So, only Z-coordinates.

3. Option (c) is correct.

Explanation:

$$PQ = \sqrt{(4-3)^2 + (5-2)^2 + (2-4)^2}$$

$$= \sqrt{(1)^2 + (3)^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}$$

$$QR = \sqrt{(5-4)^2 + (8-5)^2 + (0-2)^2}$$

$$= \sqrt{(1)^2 + (3)^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}$$

$$RS = \sqrt{(2-5)^2 + (-1-8)^2 + (6-0)^2}$$

$$= \sqrt{(-3)^2 + (-9)^2 + (6)^2}$$

$$= \sqrt{9+81+36}$$

$$= \sqrt{126}$$

$$= 3\sqrt{14}$$

$$PS = \sqrt{(2-3)^2 + (-1-2)^2 + (6-4)^2}$$

$$= \sqrt{(-1)^2 + (-3)^2 + (2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}$$

$$\text{Here, } PQ + QR + PS = \sqrt{14} + \sqrt{14} + \sqrt{14}$$

$$= 3\sqrt{14} = RS$$

So, P, Q, R and S are collinear.

4. Option (c) is correct.

Explanation:

As we know that cosines of y -axis

$$= \{\cos 90^\circ, \cos 0^\circ, \cos 90^\circ\}$$

$$= \{0, 1, 0\}$$

$$\therefore \text{Direction ratios of } y\text{-axis} = \{0, k, 0\}; k \in \mathbb{R}$$

So, statement 1 is correct.

Direction cosines of z -axis

$$= \{\cos 90^\circ, \cos 90^\circ, \cos 0^\circ\}$$

$$= \{0, 0, 1\}$$

$$\therefore \text{Direction ratios of } z\text{-axis} = \{0, 0, \lambda\}; \lambda \in \mathbb{R}$$

$$\text{Now, } 5(0) + 6(0) + 0(\lambda) = 0$$

$\therefore$ Line whose direction ratios are $\{5, 6, 0\}$ is perpendicular to z -axis.

So, statement 2 is also correct.

Hints:

- Direction cosines of y -axis = $\{0, 1, 0\}$

- Direction cosines of z -axis = $\{0, 0, 1\}$

- If $a_1a_2 + b_1b_2 + c_1c_2 = 0$, then lines whose direction ratios are $\{a_1, b_1, c_1\}$ and $\{a_2, b_2, c_2\}$, are perpendicular.

5. Option (d) is correct.

Explanation:

$$\text{DR's} \equiv (a+b, b+c, c+a)$$

Let l, m and n are the DC's of the line.

$$\text{We know that } l^2 + m^2 + n^2 = 1$$

6. Option (c) is correct.

Explanation:

Angle made by $\vec{r}$ with x -axis, $\alpha = 60^\circ$

Angle made by $\vec{r}$ with y -axis, $\beta = 30^\circ$

Let angle made by $\vec{r}$ with z -axis (γ) = x°

We know that,

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\Rightarrow \cos^2 60^\circ + \cos^2 30^\circ + \cos^2 x^\circ = 1$$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + \cos^2 x^\circ = 1$$

$$\Rightarrow \left(\frac{1}{4} + \frac{3}{4} \right) + \cos^2 x^\circ = 1$$

$$\Rightarrow 1 + \cos^2 x^\circ = 1$$

$$\Rightarrow \cos^2 x^\circ = 0$$

$$\Rightarrow \cos x^\circ = 0$$

$$\Rightarrow \cos x^\circ = \cos 90^\circ$$

$$\Rightarrow x = 90^\circ$$

7. Option (a) is correct.

Explanation:

Direction cosine of $\vec{r} = \langle \cos 60^\circ, \cos 30^\circ, \cos 90^\circ \rangle$

$$= \left\langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0 \right\rangle$$

Hence, DRJ of the line is $\langle -4, 4, 2 \rangle$

8. Option (b) is correct.

Explanation:

Given line

$$x - 1 = 2(y + 3) = 1 - z$$

$$\Rightarrow \frac{x-1}{2} = \frac{y+3}{1} = \frac{z-1}{-2}$$

$\Rightarrow$ Direction ratios of line = $\{2, 1, -2\}$

[$\therefore$ Direction ratios of line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ is (a, b, c)]

As we know if direction ratios of the line is $\{a, b, c\}$, then direction cosine of the line will be $\{l, m, n\} =$

$$\left\{ \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}} \right\}$$

$\therefore$ Direction cosine of given line is $\{l, m, n\}$

$$= \left\{ \frac{2}{\sqrt{4+1+4}}, \frac{1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} \right\}$$

$$\Rightarrow \{l, m, n\} = \left\{ \frac{2}{3}, \frac{1}{3}, \frac{-2}{3} \right\}$$

$$\begin{aligned} \text{Now, } l^4 + m^4 + n^4 &= \left(\frac{2}{3} \right)^4 + \left(\frac{1}{3} \right)^4 + \left(\frac{-2}{3} \right)^4 \\ &= \frac{16}{81} + \frac{1}{81} + \frac{16}{81} \\ &= \frac{33}{81} = \frac{11}{27} \end{aligned}$$

Hint:

- If a, b & c are the direction ratios of the line then direction cosine of the line is $\{l, m, n\}$

$$= \left\{ \frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}} \right\}$$

9. Option (b) is correct.

Explanation:

$$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} = k \text{ (say)}$$

$$x = k, y = 0, z = 0$$

Here, y -coordinate and z -coordinate of any point lies on the line are zero.

So, distance from $(2, 3, 4)$ to the line

$$\begin{aligned} &= \sqrt{(3)^2 + (4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \text{ units} \end{aligned}$$

10. Option (d) is correct.

Explanation:

Given line

$$L: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

Any point on this line

$$(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$$

$$(a) p_1: x + y + z - 6 = 0$$

$$(2\lambda + 1) + (3\lambda + 2) + (4\lambda + 3) - 6 = 0$$

$$\Rightarrow 9\lambda = 0$$

It is not true for $\lambda \in \mathbb{R}$,

So, L doesn't lie on the plane.

$$(b) p_1: x + 2y - 2z + 1 = 0$$

$$(2\lambda + 1) + 2(3\lambda + 2) - 2(4\lambda + 3) + 1 = 0$$

$$\Rightarrow -2 = 0, \text{ not possible}$$

So, line L doesn't lie on the plane

$$(c) p_1: 3x + 2y - 3z = 0$$

$$3(2\lambda + 1) + 2(3\lambda + 2) - 3(4\lambda + 3) = 0$$

$$\Rightarrow -2 = 0, \text{ not possible}$$

So, L doesn't lie on the plane,

$$(d) p_1: 3x + 2y - 3z = -2$$

$$3(2\lambda + 1) + 2(3\lambda + 2) - 3(4\lambda + 3) = -2$$

$$\Rightarrow -2 = -2$$

$\Rightarrow$ L lies on plane p_1

$$\text{Now, } p_2: 3x - 6y + 3z = 0$$

$$3(2\lambda + 1) - 6(3\lambda + 2) + 3(4\lambda + 3) = 0$$

$$\Rightarrow 0 = 0$$

$\Rightarrow$ L lies on plane p_2

So, line L is given by

$$3x + 2y - 3z = -2, 3x - 6y + 3z = 0$$

11. Option (c) is correct.

Explanation:

We know that if a line passes through the points

(x_1, y_1, z_1) and (x_2, y_2, z_2)

Then, it's equation will be

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

Here, $(x_1, y_1, z_1) \equiv (6, -7, -1)$

and $(x_2, y_2, z_2) \equiv (2, -3, 1)$

So, the equation of the line is

$$\frac{x-6}{2-6} = \frac{y+7}{-3+7} = \frac{z+1}{1+1}$$

$$\Rightarrow \frac{x-6}{-4} = \frac{y+7}{4} = \frac{z+1}{2}$$

Hence, DRS of the line is $\langle -4, 4, 2 \rangle$.

12. Option (b) is correct.

Explanation:

Given: Direction ratios of two line $\{6, 3, 6\}$ and $\{3, 3, 0\}$.

As we know if two lines have direction ratios $\{a_1, b_1, c_1\}$ and $\{a_2, b_2, c_2\}$, then angle between two lines is given by

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, $a_1 = 6, b_1 = 3, c_1 = 6, a_2 = 3, b_2 = 3, c_2 = 0$.

Let angle between two lines be θ .

$$\therefore \cos \theta = \frac{(6)(3) + (3)(3) + (6)(0)}{\sqrt{36 + 9 + 36} \cdot \sqrt{9 + 9 + 0}}$$

$$\Rightarrow \cos \theta = \frac{18 + 9}{(9)(3\sqrt{2})}$$

$$\Rightarrow \cos \theta = \frac{27}{9(3\sqrt{2})}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hint:

- If angle between two lines having direction ratios $\{a_1, b_1, c_1\}$ and $\{a_2, b_2, c_2\}$ be θ , the

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

13. Option (b) is correct.

Explanation:

$$3x + 2y + z = 6$$

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{6} = 1$$

$$\therefore p = 2, q = 3, r = 6$$

$$\text{Now, } p + q + r = 2 + 3 + 6 = 11$$

14. Option (c) is correct.

Explanation:

Given: Plane cuts intercepts 2, 2, 1 on the coordinate axes.

$$\therefore \text{Equation of plane is } \frac{x}{2} + \frac{y}{2} + \frac{z}{1} = 1$$

$$\Rightarrow x + y + 2z = 2$$

So, direction ratios of the normal to plane = $\{1, 1, 2\}$

$$\text{Direction cosines of the normal to plane} = \left\{ \frac{1}{k}, \frac{1}{k}, \frac{2}{k} \right\}$$

$$\text{where } k = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

So, direction cosines of normal to plane

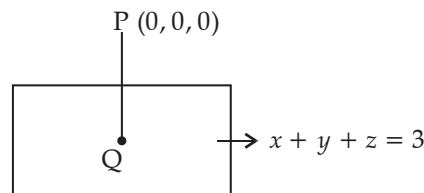
$$= \left\{ \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\}$$

Hints:

- If plane cuts intercepts a, b, c on the coordinate axes, then equation of plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$
- Direction ratios of the normal to plane $Ax + By + Cz + d = 0$ are A, B, C .

15. Option (c) is correct.

Explanation:



Let P be the given point and Q be the foot of the perpendicular.

Given: Equation of plane

$$x + y + z = 3$$

$\therefore$ Direction ratios of the normal of plane are $\{1, 1, 1\}$.

As we know equation of line passing through the point (x_1, y_1, z_1) having direction ratios a, b, c is given

$$\text{by } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\therefore \text{Equation of line PQ is } \frac{x-0}{1} = \frac{y-0}{1} = \frac{z-0}{1}$$

$$\Rightarrow x = y = z = \lambda$$

Let the coordinates of point Q be $(\lambda, \lambda, \lambda)$

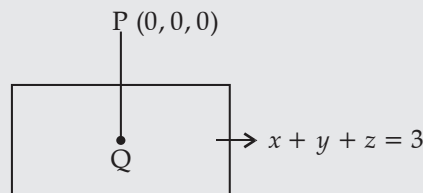
Since, Q lies in the plane $x + y + z = 3$

$$\therefore \lambda + \lambda + \lambda = 3$$

$$\Rightarrow \lambda = 1$$

$\therefore$ Coordinate of Q is $(1, 1, 1)$.

Hint:



- Use equation of line PQ is $x = y = z$. Assume the coordinates of point Q on line and find the value of parametric coefficient by satisfying the point Q in the plane.

16. Option (b) is correct.

Explanation:

If a plane is parallel to $x-y$ plane, then $x = 0$ and $y = 0$.

Here, plane cuts 5 units on z-axis.

Hence, equation of plane is $z = 5$

17. Option (a) is correct.

Explanation :

Let a, b and c are the direction ratios of the plane.

Given, plane is passing through the points $(-2, 6, -6)$, $(-3, 10, -9)$ and $(-5, 0, -6)$.

Let $(x_1, y_1, z_1) \equiv (-2, 6, -6)$

We know that equation of plane is

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$$

$$\Rightarrow a(x + 2) + b(y - 6) + c(z + 6) = 0 \quad \dots(i)$$

Here, equation (i) passes through the points $(-3, 10, -9)$ and $(-5, 0, -6)$. So, put these points in the equation (i).

$$a(-3 + 2) + b(10 - 6) + c(-9 + 6) = 0$$

$$\Rightarrow -a + 4b - 3c = 0 \quad \dots(ii)$$

Again,

$$a(-5 + 2) + b(0 - 6) + c(-6 + 6) = 0$$

$$\Rightarrow -3a - 6b + 0c = 0 \quad \dots(iii)$$

Apply cross-multiplication method in equation (ii) and (iii)

$$\frac{a}{\begin{vmatrix} 4 & -3 \\ -6 & 0 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} -1 & -3 \\ -3 & 0 \end{vmatrix}} = \frac{c}{\begin{vmatrix} -1 & 4 \\ -3 & -6 \end{vmatrix}}$$

$$\Rightarrow \frac{a}{0 - 18} = \frac{-b}{0 - 9} = \frac{c}{6 + 12}$$

$$\Rightarrow \frac{a}{-18} = \frac{b}{9} = \frac{c}{18}$$

$$\Rightarrow \frac{a}{-2} = \frac{b}{1} = \frac{c}{2}$$

Put $a = -2, b = 1, c = 2$, in equation (i)

$$-2(x + 2) + 1(y - 6) + 2(z + 6) = 0$$

$$\Rightarrow -2x - 4 + y - 6 + 2z + 12 = 0$$

$$\Rightarrow -2x + y + 2z + 2 = 0$$

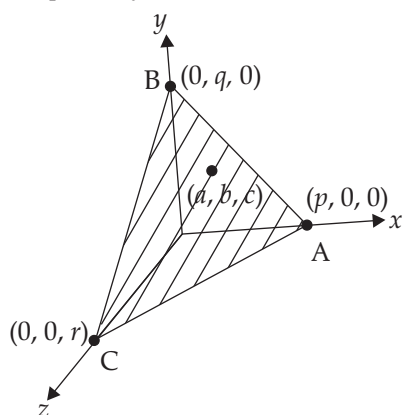
$$\Rightarrow 2x - y - 2z - 2 = 0$$

$$\Rightarrow 2x - y - 2z = 2$$

18. Option (c) is correct.

Explanation :

Let p, q and r be the intercept on x-axis, y-axis and z-axis respectively.



Then equation of plane,

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 1 \quad \dots(i)$$

This plane passes through (a, b, c) ,

put $x = a, y = b$ and $z = c$

$$\frac{a}{p} + \frac{b}{q} + \frac{c}{r} = 1 \quad \dots(ii)$$

Let the equation of the sphere

$$x^2 + y^2 + z^2 + 2gx + 2fy + 2hz + c' = 0$$

This sphere passes through the origin,

Put $x = 0, y = 0, z = 0$

$$0 + 0 + 0 + 0 + 0 + 0 + c' = 0$$

$$\Rightarrow c' = 0$$

$\therefore$ Equation of sphere will be

$$x^2 + y^2 + z^2 + 2gx + 2fy + 2hz = 0 \quad \dots(iii)$$

$\therefore$ Sphere passes through A

$\therefore$ Put $x = p, y = 0, z = 0$

$$\text{Now, } p^2 + 0 + 0 + 2gp + 0 + 0 = 0$$

$$\Rightarrow 2gp = -p^2$$

$$\Rightarrow g = \frac{-p}{2}$$

$\therefore$ Sphere passes through B

$\therefore$ Put $x = 0, y = q, z = 0$ in equation (iii)

$$0 + q^2 + 0 + 0 + 2fq + 0 = 0$$

$$\Rightarrow 2fq = -q^2$$

$$\Rightarrow f = \frac{-q}{2}$$

$\therefore$ Sphere passes through C

$\therefore$ Put $x = 0, y = 0, z = r$ in equation (iii)

$$0 + 0 + r^2 + 0 + 0 + 2hr = 0$$

$$\Rightarrow 2hr = -r^2$$

$$\Rightarrow h = \frac{-r}{2}$$

Now, equation of sphere will be

$$x^2 + y^2 + z^2 - px - qy - rz = 0$$

$$\text{with centre} = \left(\frac{p}{2}, \frac{q}{2}, \frac{r}{2} \right)$$

$$\text{Let } (\alpha, \beta, \gamma) = \left(\frac{p}{2}, \frac{q}{2}, \frac{r}{2} \right)$$

$$\text{So, } \alpha = \frac{p}{2}, \beta = \frac{q}{2}, \gamma = \frac{r}{2}$$

$$\Rightarrow p = 2\alpha \quad q = 2\beta \quad r = 2\gamma$$

Put these values in equation (ii)

$$\frac{a}{2\alpha} + \frac{b}{2\beta} + \frac{c}{2\gamma} = 1$$

$$\Rightarrow \frac{a}{\alpha} + \frac{b}{\beta} + \frac{c}{\gamma} = 2$$

Replace α, β and γ with x, y and z

So, equation $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2,$

will be the locus of the centre of the sphere.

19. Option (a) is correct.

Explanation:

Equation of the plane passing through the intersection of the two given planes will be

$$(x + y + z - 1) + \lambda(2x + 3y + 4z - 7) = 0 \quad \dots(i)$$

$$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z + (-1 - 7\lambda) = 0 \quad \dots(ii)$$

Above plane is perpendicular to

$$x - 5y + 3z = 5$$

So,

$$(1 + 2\lambda)1 + (1 + 3\lambda) \times (-5) + (1 + 4\lambda) \times 3 = 0$$

$$\Rightarrow 2\lambda + 1 - 15\lambda - 5 + 12\lambda + 3 = 0$$

$$\Rightarrow 14\lambda - 15\lambda - 1 = 0$$

$$\Rightarrow -\lambda - 1 = 0$$

$$\Rightarrow \lambda + 1 = 0$$

$$\Rightarrow \lambda = -1$$

Put $\lambda = -1$ in equation (i)

$$(x + y + z - 1) - 1(2x + 3y + 4z - 7) = 0$$

$$\Rightarrow x + y + z - 1 - 2x - 3y - 4z + 7 = 0$$

$$\Rightarrow -x - 2y - 3z + 6 = 0$$

$$\Rightarrow x + 2y + 3z - 6 = 0$$

20. Option (c) is correct.

Explanation:

Let the point of image $\equiv (x_1, y_1, z_1)$

We know that foot of perpendicular (α, β, γ) is the mid point of image and the given point.

Now apply midpoint formula,

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

$$\Rightarrow (3, 1, 2) \equiv \left(\frac{x_1 + 1}{2}, \frac{y_1 - 2}{2}, \frac{z_1 + 3}{2} \right)$$

On comparing the x, y and z -coordinates.

$$\Rightarrow \frac{x_1 + 1}{2} = 3, \quad \frac{y_1 - 2}{2} = 1, \quad \frac{z_1 + 3}{2} = 2$$

$$\Rightarrow x_1 + 1 = 6, \quad y_1 - 2 = 2, \quad z_1 + 3 = 4$$

$$\Rightarrow x_1 = 6 - 1, \quad y_1 = 2 + 2, \quad z_1 = 4 - 3$$

$$\Rightarrow x_1 = 5, \quad \Rightarrow y_1 = 4, \quad \Rightarrow z_1 = 1$$

So, image $\equiv (5, 4, 1)$

21. Option (a) is correct.

Explanation:

$$\begin{array}{ccc} (x_1, y_1, z_1) & & (x_2, y_2, z_2) \\ (-3, 4, -8) & & (5, -6, 4) \end{array}$$

Equation of line passing through two points

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x + 3}{5 + 3} = \frac{y - 4}{-6 - 4} = \frac{z + 8}{4 + 8}$$

$$\Rightarrow \frac{x + 3}{8} = \frac{y - 4}{-10} = \frac{z + 8}{12}$$

$$\Rightarrow \frac{x + 3}{8} = \frac{y - 4}{-10} = \frac{z + 8}{12} = \lambda \text{ (Let)}$$

$$\text{So, } x = 8\lambda - 3, y = -10\lambda + 4, z = 12\lambda - 8$$

Let P be the point of intersection, then

$$P \equiv (8\lambda - 3, -10\lambda + 4, 12\lambda - 8)$$

If the line cuts the xy plane, then z -co-ordinate will be zero.

$$\text{So, } 12\lambda - 8 = 0$$

$$\Rightarrow \lambda = \frac{4}{6}$$

$$\Rightarrow \lambda = \frac{2}{3}$$

Put $\lambda = \frac{2}{3}$ in point P.

$$P = \left(8 \times \frac{2}{3} - 3, -10 \times \frac{2}{3} + 4, 12 \times \frac{2}{3} - 8 \right)$$

$$= \left(\frac{16 - 9}{3}, \frac{-20 + 12}{3}, 8 - 8 \right)$$

$$= \left(\frac{7}{3}, \frac{-8}{3}, 0 \right)$$

22. Option (d) is correct.

Explanation:

Equation of the joining two points

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\begin{array}{ccc} & & (4, -2, 5) \\ & & x_2, y_2, z_2 \\ (2, 1, 3) & & \\ x_1, y_1, z_1 & & \end{array}$$

$$\frac{x - 2}{4 - 2} = \frac{y - 1}{-2 - 1} = \frac{z - 3}{5 - 3}$$

$$\Rightarrow \frac{x - 2}{2} = \frac{y - 1}{-3} = \frac{z - 3}{2} = \lambda \text{ (say)} \quad \dots(i)$$

$$\text{So, } x = 2\lambda + 2, y = -3\lambda + 1, z = 2\lambda + 3$$

If the line cuts the plane $2x + y - z = 3$,

then, put the value of x, y, z in the plane

$$2(2\lambda + 2) + (-3\lambda + 1) - (2\lambda + 3) = 3$$

$$\Rightarrow 4\lambda + 4 - 3\lambda + 1 - 2\lambda - 3 = 3$$

$$\Rightarrow -\lambda + 2 = 3$$

$$\Rightarrow -\lambda = 3 - 2$$

$$\Rightarrow -\lambda = 1$$

$$\Rightarrow \lambda = -1$$

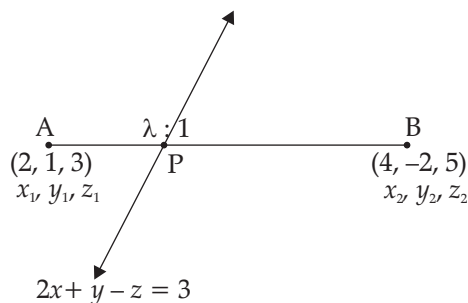
$$\begin{aligned} \text{So, } x &= 2 \times -1 + 2, y = -3 \times -1 + 1, z = 2 \times -1 + 3 \\ \Rightarrow x &= -2 + 2, y = 3 + 1, z = -2 + 3 \\ \Rightarrow x &= 0, y = 4, z = 1 \end{aligned}$$

So, the point will be (0, 4, 1)

23. Option (d) is correct.

Explanation:

Let the plane $2x + y - z = 3$ divides the line AB in $\lambda : 1$.



Now, Use section formula,

$$\begin{aligned} P &= \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}, \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2} \right) \\ &= \left(\frac{\lambda \times 4 + 1 \times 2}{\lambda + 1}, \frac{\lambda \times -2 + 1 \times 1}{\lambda + 1}, \frac{\lambda \times 5 + 1 \times 3}{\lambda + 1} \right) \end{aligned}$$

$$\Rightarrow P = \left(\frac{4\lambda + 2}{\lambda + 1}, \frac{-2\lambda + 1}{\lambda + 1}, \frac{5\lambda + 3}{\lambda + 1} \right)$$

Now, point P will satisfy the equation of plane as it lies on the plane.

$$\begin{aligned} 2 \left(\frac{4\lambda + 2}{\lambda + 1} \right) + \left(\frac{-2\lambda + 1}{\lambda + 1} \right) - \left(\frac{5\lambda + 3}{\lambda + 1} \right) &= 3 \\ \Rightarrow \frac{8\lambda + 4 - 2\lambda + 1 - 5\lambda - 3}{(\lambda + 1)} &= 3 \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{\lambda + 2}{\lambda + 1} &= 3 \\ \Rightarrow \lambda + 2 &= 3\lambda + 3 \\ \Rightarrow 3\lambda - \lambda &= 2 - 3 \\ \Rightarrow 2\lambda &= -1 \\ \Rightarrow \lambda &= -\frac{1}{2} \end{aligned}$$

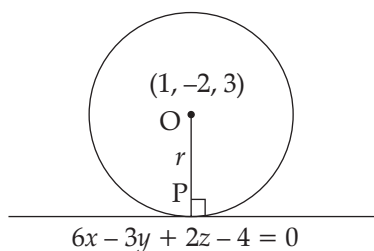
So, the plane divides the line externally in the ratio 1 : 2.

24. Option (d) is correct.

Explanation:

We know that if we join the centre of circle(O) and point of contact (P), we will get the radius of the circle.

Here, $OP \perp$ given plane.



$$r = \frac{|6 \times 1 - 3 \times (-2) + 2(3) - 4|}{\sqrt{(6)^2 + (-3)^2 + (2)^2}}$$

[Using perpendicular distance formula]

$$\Rightarrow r = \frac{|6 + 6 + 6 - 4|}{\sqrt{36 + 9 + 4}}$$

$$\Rightarrow r = \frac{14}{\sqrt{49}}$$

$$\Rightarrow r = \frac{14}{7}$$

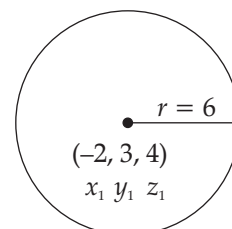
$$\Rightarrow r = 2$$

$$\begin{aligned} \text{Now, diameter of sphere} &= 2 \times \text{radius} \\ &= 2 \times 2 \\ &= 4 \text{ units} \end{aligned}$$

25. Option (a) is correct.

Explanation:

Equation of sphere will be



$$\begin{aligned} (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 &= r^2 \\ \Rightarrow (x + 2)^2 + (y - 3)^2 + (z - 4)^2 &= 6^2 \\ \Rightarrow x^2 + 4 + 4x + y^2 + 9 - 6y + z^2 + 16 - 8z &= 36 \\ \Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z + 29 - 36 &= 0 \\ \Rightarrow x^2 + y^2 + z^2 + 4x - 6y - 8z &= 7 \end{aligned}$$

26. Option (b) is correct

Explanation:

$$3x^2 + 3y^2 + 3z^2 - 8x + 4y - 8z - 15 = 0$$

Divide the equation by (iii)

$$\Rightarrow \frac{3x^2}{3} + \frac{3y^2}{3} + \frac{3z^2}{3} - \frac{8x}{3} + \frac{4y}{3} - \frac{8z}{3} - \frac{15}{3} = 0$$

$$\Rightarrow x^2 + y^2 + z^2 - \frac{8x}{3} + \frac{4y}{3} - \frac{8z}{3} - 5 = 0 \quad \dots(i)$$

$$\Rightarrow x^2 + y^2 + z^2 + 2gx + 2fy + 2hz + c = 0 \quad \dots(ii)$$

Compare equation (i) with equation (ii),

$$2g = \frac{-8}{3}, 2f = \frac{4}{3}, 2h = \frac{-8}{3}, c = -5$$

$$g = \frac{-4}{3}, f = \frac{2}{3}, h = \frac{-4}{3}, c = -5$$

$$\begin{aligned} \text{Radius of sphere} &= \sqrt{g^2 + f^2 + h^2 - c} \\ &= \sqrt{\left(\frac{-4}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{-4}{3}\right)^2 + 5} \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{\frac{16}{9} + \frac{4}{9} + \frac{16}{9} + 5} \\
 &= \sqrt{\frac{16+4+16+45}{9}} \\
 &= \sqrt{\frac{81}{9}} \\
 &= \sqrt{9} \\
 &= 3 \text{ units}
 \end{aligned}$$

27. Option (d) is correct.

Explanation:

$$\begin{aligned}
 f(x) &= \sqrt{1 - (x-1)^2} \\
 \therefore 1 - (x-1)^2 &\geq 0 \\
 \Rightarrow 0 - x^2 + 2x &\geq 0 \\
 \Rightarrow x^2 - 2x &\leq 0 \\
 \Rightarrow x(x-2) &\leq 0
 \end{aligned}$$

$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & 0 & & 2 & & \infty \end{array}$

$\therefore$ Domain = $[0, 2]$

Hints:

- Use $y = \sqrt{f(x)}$ is define when $f(x) \geq 0$
- To solve in equation use wavy curve method

28. Option (d) is correct.

Explanation:

$$\begin{aligned}
 f(x+y) &= f(x)f(y) \text{ and } f(1) = 2 \\
 \Rightarrow f(2) &= f(1)f(1) = 2^2, f(3) = f(2)f(1) = 2^3, f(4) = f(3)f(1) = 2^4 \\
 &= 2^n \\
 \Rightarrow f(n) &= 2^n. \\
 \therefore \sum_{x=1}^6 2^x f(x) &= 2f(1) + 2^2 f(2) + 2^3 f(3) + 2^4 f(4) \\
 &\quad + 2^5 f(5) + 2^6 f(6) \\
 &= 2(2) + 2^2(2^2) + 2^3(2^3) + 2^4(2^4) + 2^5(2^5) + 2^6(2^6) \\
 &= 2^2 + 2^4 + 2^6 + 2^8 + 2^{10} + 2^{12} \\
 &= \frac{2^2(2^{12}-1)}{2^2-1} \quad \left\{ \text{Sum of GP} = \frac{a(r^n-1)}{r-1} \right\} \\
 &= \frac{4 \times 4095}{3} \\
 &= 5460
 \end{aligned}$$

Hint: Use sum of GP = $\frac{a(r^n-1)}{r-1}$, where a = first term and r = common ratio.

29. Option (c) is correct.

Explanation:

Given : Curves are

$$y = |x-1| \text{ and } |x| = 2$$

$$\begin{aligned}
 |x-1| &= \begin{cases} x-1, & x-1 \geq 0 \\ -(x-1), & x-1 < 0 \end{cases} \\
 &= \begin{cases} x-1, & x \geq 1 \\ 1-x, & x < 1 \end{cases} \\
 |x| &= \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \\
 |x| &= 2
 \end{aligned}$$

$$\Rightarrow x = 2 \text{ or } x = -2$$

For point(s) of intersection of the curves put $x = 2$ and $x = -2$ one by one in $y = |x-1|$

For $x = 2$

$$y = |2-1| = 1$$

So, point of intersection is (2, 1)

For $x = -2$

$$y = |-2-1| = |-3| = 3$$

$\therefore$ Point of intersection is (-2, 3)

Hints :

- $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
- $|x-1| = \begin{cases} x-1, & x \geq 1 \\ 1-x, & x < 1 \end{cases}$

30. Option (c) is correct

Explanation:

Given :

$$\begin{aligned}
 f(x) &= [x] \\
 g(x) &= |x| \\
 g \circ f(x) &= |f(x)| \\
 &= |[x]| \\
 f \circ g(x) &= [g(x)] \\
 &= [|x|]
 \end{aligned}$$

$$g \circ f\left(-\frac{5}{3}\right) = \left[\left[-\frac{5}{3}\right]\right] = |-2| = 2$$

$$\begin{aligned}
 f \circ g\left(-\frac{5}{3}\right) &= \left[\left|-\frac{5}{3}\right|\right] \\
 &= \left[\frac{5}{3}\right] = 1
 \end{aligned}$$

$$\begin{aligned}
 \therefore g \circ f\left(-\frac{5}{3}\right) - f \circ g\left(-\frac{5}{3}\right) &= 2 - 1 = 1
 \end{aligned}$$

Shortcut :

$$\begin{aligned}
 g \circ f\left(-\frac{5}{3}\right) - f \circ g\left(-\frac{5}{3}\right) &= \left[\left[-\frac{5}{3}\right]\right] - \left[\left|-\frac{5}{3}\right|\right] \\
 &= 2 - 1 = 1
 \end{aligned}$$

Hints :

- $-2 \leq x < -1 \Rightarrow [x] = -2$
- $1 \leq x < 2 \Rightarrow [x] = 1$

Random experiment - An experiment is random means that the experiment has more than one possible outcome and it is not possible to predict with certainty which outcome that will be. For instance, in an experiment of tossing an ordinary coin, it can be predicted with certainty that the coin will land either heads up or tails up, but it is not known for sure whether heads or tails will occur. If a dice is thrown once, any of the six numbers, i.e., 1, 2, 3, 4, 5, 6 may turn up, not sure which number will come up.

- (i) **Outcome** - A possible result of a random experiment is called its outcome for example if the experiment consists of tossing a coin twice, some of the outcomes are HH, HT etc.
- (ii) **Sample Space** - A sample space is the set of all possible outcomes of an experiment. In fact, it is the universal set S pertinent to a given experiment. The sample space for the experiment of tossing a coin twice is given by

$$S = \{HH, HT, TH, TT\}$$

The sample space for the experiment of drawing a card out of a deck is the set of all cards in the deck.

16.1.2 Event - An event is a subset of a sample space S . For example, the event of drawing an ace from a deck is

$A = \{\text{Ace of Heart, Ace of Club, Ace of Diamond, Ace of Spade}\}.$

16.1.3 Types of events

- (i) **Impossible and Sure Events** The empty set ϕ and the sample space S describe events. In fact ϕ is called an impossible event and S , i.e., the whole sample space is called a sure event.
- (ii) **Simple or Elementary Event** If an event E has only one sample point of a sample space, i.e., a single outcome of an experiment, it is called a simple or elementary event. The sample space of the experiment of tossing two coins is given by

$$S = \{HH, HT, TH, TT\}$$

The event $E_1 = \{HH\}$ containing a single outcome HH of the sample space S is a simple or elementary event. If one card is drawn from a well shuffled deck, any particular card drawn like 'queen of Hearts' is an elementary event.

Compound Event - If an event has more than one sample point it is called a compound event, for example, $S = \{HH, HT\}$ is a compound event.

Complementary event - Given an event A , the complement of A is the event consisting of all sample space outcomes that do not correspond to the occurrence of A .

The complement of A is denoted by A' or $\bar{A}$. It is also called the event 'not A '. Further $P(\bar{A})$ denotes the probability that A will not occur.

$$A' = \bar{A} = S - A = \{\omega : \omega \in S \text{ and } \omega \notin A\}$$

16.1.4 Event 'A or B' - If A and B are two events associated with same sample space, then the event ' A or B ' is same as the event $A \cup B$ and contains all those elements which are either in A or in B or in both. Further more, $P(A \cup B)$ denotes the probability that A or B (or both) will occur.

16.1.5 Event 'A and B' - If A and B are two events associated with a sample space, then the event ' A and B ' is same as the event ' $A \cap B$ ' and contains all those elements which are common to both A and B . Further more, $P(A \cap B)$ denotes the probability that both A and B will simultaneously occur.

16.1.6 The Event A but not B' (Difference A-B) - An event $A - B$ is the set of all those elements of the same space S which are in A but not in B , i.e., $A - B = A \cap B'$.

16.1.7 Mutually exclusive - Two events A and B of a sample space S are **mutually exclusive** if the occurrence of any one of them excludes the occurrence of the other event. Hence, the two events A and B cannot occur simultaneously, and thus $P(A \cap B) = 0$.

Remark - Simple or elementary events of a sample space are always mutually exclusive. For example, the elementary events $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}$ or $\{6\}$ of the experiment of throwing a dice are mutually exclusive.

Note - For a given sample space, there may be two or more mutually exclusive events.

16.1.8 Exhaustive events - If $E_1, E_2, \dots, E_n$ are n events of a sample space S and if

$$E_1 \cup E_2 \cup E_3 \cup \dots \cup E_n = \bigcup_{i=1}^n E_i = S \text{ then } E_1, E_2, \dots, E_n$$

are called exhaustive events.

In other words, events $E_1, E_2, \dots, E_n$ of a sample space S are said to be exhaustive if atleast one of them necessarily occur whenever the experiment is performed.

Consider the example of rolling a dice. We have $S = \{1, 2, 3, 4, 5, 6\}$. Define the two events

A: 'a number less than or equal to 4 appears.'

B: 'a number greater than or equal to 4 appears.'

Now $A = \{1, 2, 3, 4\}, B = \{4, 5, 6\}$

$$A \cup B = \{1, 2, 3, 4, 5, 6\} = S$$

Such events A and B are called exhaustive events.

16.1.9 Mutually exclusive and exhaustive events - If $E_1, E_2, \dots, E_n$ are n events of a sample space S and if $E_i \cap E_j = \phi$ for every $i \neq j$, i.e., E_i and E_j are pairwise

disjoint and $\bigcup_{i=1}^n E_i = S$, then the events $E_1, E_2, \dots, E_n$

are called mutually exclusive and exhaustive events.

Consider the example of rolling a dice.

We have $S = \{1, 2, 3, 4, 5, 6\}$

Let us define the three events as

A = a number which is a perfect square

B = a prime number

C = a number which is greater than or equal to 6

Now $A = \{1, 4\}$, $B = \{2, 3, 5\}$, $C = \{6\}$

Note that $A \cup B \cup C = \{1, 2, 3, 4, 5, 6\} = S$. Therefore, A, B and C are exhaustive events.

Also $A \cap B = B \cap C = C \cap A = \phi$

Hence, the events are pairwise disjoint and thus mutually exclusive.

Classical approach is useful, when all the outcomes of the experiment are equally likely. We can use logic to assign probabilities. To understand the classical method consider the experiment of tossing a fair coin. Here, there are two equally likely outcomes - head (H) and tail (T). When the elementary outcomes are taken as equally likely, we have a uniform probability model. If there are k elementary outcomes in S , each is assigned the probability of $\frac{1}{k}$. Therefore, logic

suggests that the probability of observing a head, denoted by $P(H)$, is $\frac{1}{2} = 0.5$, and that the probability

of observing a tail, denoted $P(T)$, is also $\frac{1}{2} = 0.5$. Notice

that each probability is between 0 and 1. Further H and T are all the outcomes of the experiment and $P(H) + P(T) = 1$.

16.1.10 Classical definition - If all of the outcomes of a sample space are equally likely, then the probability that an event will occur is equal to the ratio:

The number of outcomes favourable to the event
The total number of outcomes of the sample space

Suppose that an event E can happen in h ways out of a total of n possible equally likely ways.

Then the classical probability of occurrence of the event is denoted by

$$P(E) = \frac{h}{n}$$

The probability of non occurrence of the event E is denoted by

$$P(\text{not } E) = \frac{n-h}{n} = 1 - \frac{h}{n} = 1 - P(E)$$

Thus $P(E) + P(\text{not } E) = 1$

The event 'not E' is denoted by $\bar{E}$ or E' (complement of E).

Therefore $P(\bar{E}) = 1 - P(E)$.

Axiomatic approach to probability - Let S be the sample space of a random experiment. The probability P is a real valued function whose domain is the power set of S , i.e., $P(S)$ and range is the interval $[0, 1]$ i.e. $P : P(S) \rightarrow [0, 1]$ satisfying the following axioms.

For any event E , $P(E) \geq 0$.

$P(S) = 1$

If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$.

It follows from (iii) that $P(\phi) = 0$.

Let S be a sample space containing elementary

outcomes $w_1, w_2, \dots, w_n$,

i.e., $S = \{w_1, w_2, \dots, w_n\}$

It follows from the axiomatic definition of probability that

(i) $0 \leq P(w_i) \leq 1$ for each $w_i \in S$

(ii) $P(w_1) + P(w_2) + \dots + P(w_n) = 1$

(iii) $P(A) = P(w_i)$ for any event A containing elementary events w_i

For example, if a fair coin is tossed once

$P(H) = P(T) = \frac{1}{2}$ satisfies the three axioms of probability.

Now suppose the coin is not fair and has double the chances of falling heads up as compared to the tails,

then $P(H) = \frac{2}{3}$ and $P(T) = \frac{1}{3}$.

This assignment of probabilities are also valid for H and these satisfy the axiomatic definitions.

16.1.12 Probabilities of equally likely outcomes - Let a sample space of an experiment be $S = \{w_1, w_2, \dots, w_n\}$ and suppose that all the outcomes are equally likely to occur i.e., the chance of occurrence of each simple event must be the same

i.e., $P(w_i) = p$ for all $w_i \in S$, where $0 \leq p \leq 1$

Since $\sum_{i=1}^n P(w_i) = 1$

i.e., $P + P + P + \dots + P (n \text{ times}) = 1$

$$\Rightarrow nP = 1, \text{ i.e., } p = \frac{1}{n}$$

Let S be the sample space and E be an event, such that $n(S) = n$ and $n(E) = m$. If each outcome is equally likely, then it follows that

$$P(E) = \frac{m}{n} = \frac{\text{Number of outcomes favorable to } E}{\text{Total number of possible outcomes}}$$

Addition rule of probability - If A and B are any two events in a sample space S , then the probability that atleast one of the events A or B will occur is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Similarly, for three events A , B and C , we have

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

16.1.14 Addition rule for mutually exclusive events-

If A and B are disjoint sets, then $P(A \cup B) = P(A) + P(B)$ [since $P(A \cap B) = P(\phi) = 0$, where A and B are disjoint]. The addition rule for mutually exclusive events can be extended to more than two events.

13.1.1 Conditional Probability

If E and F are two events associated with the same sample space of a random experiment, then the conditional probability of the event E under the condition that the event F has occurred, written as $P(E|F)$, is given by

$$P(E|F) = \frac{P(E \cap F)}{P(F)}, \quad P(F) \neq 0$$

13.1.2 Properties of Conditional Probability

Let E and F be events associated with the sample space S of an experiment. Then:

(i) $P(S|F) = P(F|F) = 1$.

(ii) $P[(A \cup B) | F] = P(A|F) + P(B|F) - P[(A \cap B) | F]$,

where A and B are any two events associated with S.
(iii) $P(E'|F) = 1 - P(E|F)$.

13.1.3 Multiplication Theorem on Probability

Let E and F be two events associated with a sample space of an experiment. Then

$$P(E \cap F) = P(E) P(F|E), P(E) \neq 0 \\ = P(F) P(E|F), P(F) \neq 0$$

If E, F and G are three events associated with a sample space, then

$$P(E \cap F \cap G) = P(E) P(F|E) P(G|E \cap F)$$

13.1.4 Independent Events

Let E and F be two events associated with a sample space S. If the probability of occurrence of one of them is not affected by the occurrence of the other, then we say that the two events are independent. Thus, two events E and F will be independent, if

- (a) $P(F|E) = P(F)$, provided $P(E) \neq 0$.
- (b) $P(E|F) = P(E)$, provided $P(F) \neq 0$.

Using the multiplication theorem on probability, we have

$$(c) P(E \cap F) = P(E) P(F).$$

Three events A, B and C are said to be mutually independent if all the following conditions hold:

$$P(A \cap B) = P(A) P(B) \\ P(A \cap C) = P(A) P(C) \\ P(B \cap C) = P(B) P(C)$$

And $P(A \cap B \cap C) = P(A) P(B) P(C)$

13.1.5 Partition of a Sample Space

A set of events $E_1, E_2, \dots, E_n$ is said to represent a partition of a sample space S if

- (a) $E_i \cap E_j = \emptyset, i \neq j = 1, 2, 3, \dots, n$
- (b) $E_1 \cup E_2 \cup \dots \cup E_n = S$, and
- (c) Each $E_i \neq \emptyset$, i.e., $P(E_i) > 0$ for all $i = 1, 2, \dots, n$

13.1.6 Theorem of Total Probability

Let $\{E_1, E_2, \dots, E_n\}$ be a partition of the sample space S. Let A be any event associated with S, then

$$P(A) = \sum_{j=1}^n P(E_j) P(A|E_j)$$

13.1.7 Bayes' Theorem

If $E_1, E_2, \dots, E_n$ are mutually exclusive and exhaustive events associated with a sample space, and A is any event of non zero probability, then

$$P(E_i|A) = \frac{P(E_i)P(A|E_i)}{\sum_{j=1}^n P(E_j)P(A|E_j)}$$

13.1.8 Random Variable and its Probability Distribution

A random variable is a real valued function whose domain is the sample space of a random experiment. The probability distribution of a random variable X is the system of numbers

X:	x_1		x_2		x_n
P(X):	p_1		p_2		p_n

where $p_i > 0, i = 1, 2, \dots, n$. $\sum_{i=1}^n p_i = 1$.

13.1.9 Mean and Variance of a Random variable

Let X be a random variable assuming values $x_1, x_2, \dots, x_n$ with probabilities $p_1, p_2, \dots, p_n$, respectively

such that $p_i \geq 0, \sum_{i=1}^n p_i = 1$. Mean of X, denoted by μ

[or expected value of X denoted by $E(X)$] is defined as

$$\mu = E(X) = \sum_{i=1}^n x_i p_i \text{ and variance, denoted by } \sigma^2, \text{ is defined as}$$

$$\sigma^2 = \sum_{i=1}^n (x_i - \mu)^2 p_i = \sum_{i=1}^n x_i^2 p_i - \mu^2 \text{ or equivalently} \\ \sigma^2 = E(X - \mu)^2$$

Standard deviation of the random variable X is defined as

$$\sigma = \sqrt{\text{variance}(X)} = \sum_{i=1}^n (x_i - \mu)^2 p_i$$

13.1.10 Bernoulli Trials

Trials of a random experiment are called Bernoulli trials, if they satisfy the following conditions:

- (i) There should be a finite number of trials.
- (ii) The trials should be independent.
- (iii) Each trial has exactly two outcomes: success or failure.
- (iv) The probability of success (or failure) remains the same in each trial.

13.1.11 Binomial Distribution

A random variable X taking values 0, 1, 2, ..., n is said to have a binomial distribution with parameters n and p, if its probability distribution is given by

$$P(X = r) = {}^n C_r p^r q^{n-r}$$

where $q = 1 - p$ and $r = 0, 1, 2, \dots, n$.

EXERCISE

- Three identical dice are rolled. What is the probability that the same number will appear on each of them?
(a) $\frac{1}{6}$ (b) $\frac{1}{36}$ (c) $\frac{1}{18}$ (d) $\frac{3}{28}$
- A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected.
(a) 0.25 (b) 0.30 (c) 0.75 (d) 0.50
- The probability of simultaneous occurrence of at least one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then $P(A') + P(B') =$.
(a) 2 - P (b) 2 + P + Q
(c) 2 - 2P + Q (d) P - Q
- 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability of its being defective if it is red.

- (a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
5. Two dice are thrown together. Let A be the event 'getting 6 on the first dice' and B be the event 'getting 2 on the second dice'. So, the events A and B are.
 (a) Dependent (b) Independent
 (c) Can't Say (d) None of these
6. A committee of 4 students is selected at random from a group consisting 8 boys and 4 girls. Given that there is at least one girl on the committee, calculate the probability that there are exactly 2 girls on the committee.
 (a) $\frac{168}{425}$ (b) $\frac{59}{150}$ (c) $\frac{171}{425}$ (d) $\frac{161}{171}$
7. Three machines E1, E2, E3 in a certain factory produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E1 and E2 are defective, and that 5% of those produced on E3 are defective. If one tube is picked up at random from a day's production, calculate the probability that it is defective.
 (a) 0.0512 (b) 0.215
 (c) 0.0425 (d) 0.0545
8. Find the probability that in 10 throws of a fair dice a score which is a multiple of 3 will be obtained in at least 8 of the throws.
 (a) $\frac{201}{3^{10}}$ (b) $\frac{311}{3^9}$
 (c) $\frac{211}{3^{10}}$ (d) None of these
9. A district Random variable x has the following probability distribution:

x	1	2	3	4	5	6	7
P(X)	C	2C	2C	3C	C ²	2C ²	7C ² +C

Find the value of C.

- (a) $\frac{1}{5}$ (b) $\frac{1}{10}$ (c) $\frac{1}{4}$ (d) $\frac{2}{5}$
10. Four balls are to be drawn without replacement from a box containing 8 red and 4 white balls. If X denotes the number of red ball drawn, find the probability distribution of X.

(a)

x	0	1	2	3	4
P(x)	$\frac{1}{215}$	$\frac{32}{215}$	$\frac{64}{215}$	$\frac{39}{215}$	$\frac{41}{215}$

(b)

x	0	1	2	3	4
P(x)	$\frac{1}{495}$	$\frac{32}{495}$	$\frac{168}{495}$	$\frac{224}{495}$	$\frac{70}{495}$

(c)

x	0	1	2	3	4
P(x)	$\frac{1}{315}$	$\frac{18}{315}$	$\frac{41}{315}$	$\frac{53}{315}$	$\frac{79}{315}$

(d) None of these

11. Determine standard deviation of the number of heads in three tosses of a coin
 (a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{1}{\sqrt{5}}$
12. A car manufacturing factory has two plants, X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30%. 80% of the cars at plant X and 90% of the cars at plant Y are rated of standard quality. A car is chosen at random and is found to be of standard quality. What is the probability that it has come from plant X?
 (a) $\frac{2}{81}$ (b) $\frac{1}{50}$
 (c) $\frac{56}{83}$ (d) None of these
13. Let A and B be two events. If $P(A) = 0.2$, $P(B) = 0.4$, $P(A \cup B) = 0.6$, then $P(A|B)$ is equal to.
 (a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{5}$
14. Let A and B be two events such that $P(A) = 0.6$, $P(B) = 0.2$, and $P(A|B) = 0.5$, Calculate $P(A' \cap B')$.
 (a) $\frac{3}{8}$ (b) $\frac{3}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
15. (a) How many two-digit positive integers are multiples of 3?
 (b) What is the probability that a randomly chosen two-digit positive integer is a multiple of 3?
 (a) $30, \frac{1}{3}$ (b) $20, \frac{1}{5}$ (c) $30, \frac{1}{4}$ (d) $15, \frac{1}{3}$
16. A typical PIN (personal identification number) is a sequence of any four symbols chosen from the 26 letters in the alphabet and the ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?
 (a) 0.2152 (b) 0.6131
 (c) 0.1583 (d) 0.2192
17. Probability that a truck stopped at a roadblock will have faulty brakes or badly worn tires are 0.23 and 0.24, respectively. Also, the probability is 0.38 that a truck stopped at the roadblock will have faulty brakes and/or badly working tires. What is the probability that a truck stopped at this roadblock will have faulty breaks as well as badly worn tires?
 (a) 0.15 (b) 0.09 (c) 0.05 (d) 0.78
18. If a person visits his dentist, suppose the probability that he will have his teeth cleaned is 0.48, the probability that he will have a cavity filled is 0.25, the probability that he will have a tooth extracted is 0.20, the probability that he will have a teeth cleaned and a cavity filled is 0.09, the probability that he will have his teeth cleaned and a tooth extracted is 0.12, the probability that he will have a cavity filled and a tooth extracted is 0.07, and the probability that he will have his teeth cleaned, a cavity filled, and a tooth extracted is 0.03. What is the probability that a person visiting his dentist 1 will have atleast one of these things done to him?
 (a) 0.68 (b) 0.32 (c) 0.25 (d) 0.40
19. An urn contains twenty white slips of paper numbered from 1 through 20, ten red slips of paper

numbered from 1 through 10, forty yellow slips of paper numbered from 1 through 40, and ten blue slips of paper numbered from 1 through 10. If these 80 slips of paper are thoroughly shuffled so that each slip has the same probability of being drawn. Find the probabilities of drawing a slip of paper that is.

- (a) $\frac{1}{20}$ (b) $\frac{1}{10}$ (c) $\frac{1}{5}$ (d) $\frac{1}{30}$

20. In a leap year the probability of having 53 Sundays or 53 Mondays is.

- (a) $\frac{3}{7}$ (b) $\frac{2}{7}$ (c) $\frac{1}{7}$ (d) $\frac{1}{5}$

21. Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers. What is the probability that this number has the same digits?

- (a) $\frac{1}{25}$ (b) $\frac{1}{40}$ (c) $\frac{1}{30}$ (d) $\frac{1}{20}$

22. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that $3P(A) = 2P(B) = P(C)$, then $P(A)$ is equal to

- (a) $\frac{3}{11}$ (b) $\frac{1}{11}$ (c) $\frac{2}{11}$ (d) $\frac{3}{15}$

Answer Key

1. (b)	2. (a)	3. (c)	4. (b)	5. (b)	6. (a)	7. (c)	8. (c)	9. (b)	10. (b)
11. (b)	12. (c)	13. (a)	14. (a)	15. (a)	16. (c)	17. (b)	18. (a)	19. (b)	20. (a)
21. (a)	22. (c)								

Answers with Explanations

1. **Option (b) is correct.**

Explanation: Total number of cases = $6^3 = 216$
The same number can appear on each of the dice in the following ways:

$(1,1,1), (2,2,2), \dots, (3,3,3)$

So, favourable number of cases = 6

Hence, required probability = $\bigcup_{i=1}^n E_i$

2. **Option (a) is correct.**

Explanation: Let p be the probability that B gets selected.

$P(\text{Exactly one of A, B is selected}) = 0.6$ (given)

$\Rightarrow P(A \text{ is selected, B is not selected; B is selected, A is not selected}) = 0.6$

$\Rightarrow P(A \cap B') + P(AB) = 0.6$

$\Rightarrow P(A)P(B) + P(A')P(B) = 0.6$

$\Rightarrow (0.7)(1-p) + (0.3)p = 0.6$

$\Rightarrow p = 0.25$

Thus the probability that B get selected is 0.25.

3. **Option (c) is correct.**

Explanation: Since $P(\text{exactly one of A, B occurs}) = q$ (given), We get

$$P(A \cup B) - P(A \cap B) = q$$

$$\Rightarrow p - P(A \cap B) = q$$

$$\Rightarrow P(A \cap B) = p - q$$

$$\Rightarrow 1 - P(A' \cap B') = p - q$$

$$\Rightarrow P(A' \cap B') = 1 - p + q$$

$$\Rightarrow P(A') + P(B') - P(A' \cap B') = 1 - p + q$$

$$\begin{aligned} \Rightarrow P(A') + P(B') &= (1 - p + q) + P(A' \cap B') \\ &= (1 - p + q) + (1 - p + q) \\ &= (1 - p + q) + (1 - p) \\ &= 2 - 2p + q. \end{aligned}$$

4. **Option (b) is correct.**

Explanation: Let A and B be the events that the bulb is red and defective, respectively

$$P(A) = \frac{10}{100} = \frac{1}{10}$$

$$P(A \cap B) = \frac{2}{100} = \frac{1}{50}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{50} \times \frac{10}{1} = \frac{1}{5}$$

Thus the probability of the picked up bulb of its being defective, if it is red is $\frac{1}{5}$.

5. **Option (b) is correct.**

Explanation: $A = \{(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$B = \{(1,2), (2,2), (3,2), (4,2), (5,2), (6,2)\}$

$A \cap B = \{(6,2)\}$

$$P(A) = \frac{6}{36} = \frac{1}{6}, P(B) = \frac{1}{6}, P(A \cap B) = \frac{1}{36}$$

Events A and B will be independent if

$$P(A \cap B) = P(A)P(B)$$

$$\text{i.e., LHS} = P(A \cap B) = \frac{1}{36}, \text{ RHS} = P(A)P(B)$$

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Hence, A and B are independent.

6. **Option (a) is correct.**

Explanation: Let A denote the event that at least one girl will be chosen, and B the event that exactly 2 girls will be chosen.

We require $P(B|A)$.

Since A denote the event that at least one girl will be chosen, A' denotes that no girl is chosen i.e., 4 boys are chosen. Then

$$P(A') = \frac{{}^8C_4}{{}^{12}C_4} = \frac{70}{495} = \frac{14}{99}$$

$$\Rightarrow P(A) = 1 - \frac{14}{99} = \frac{85}{99}$$

$$\text{Now } P(A \cap B) = P(2 \text{ boys and } 2 \text{ girls}) = \frac{{}^8C_2 \cdot {}^4C_2}{{}^{12}C_4}$$

$$= \frac{6 \times 28}{495} = \frac{56}{165}$$

$$\text{Thus } P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{56}{165} \times \frac{99}{85} = \frac{168}{425}$$

7. Option (c) is correct.

Explanation: Let D be the event that the picked up tube is defective

Let A_1, A_2 and A_3 be events that the tube is produced on machines E_1, E_2 and E_3 , respectively.

$$P(D) = P(A_1)P(D|A_1) + P(A_2)P(D|A_2) + P(A_3)P(D|A_3) \quad (1)$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, P(A_2) = \frac{1}{4}, P(A_3) = \frac{1}{4}$$

$$\text{Also } P(D|A_1) = P(D|A_2) = \frac{4}{100} = \frac{1}{25}$$

$$P(D|A_3) = \frac{5}{100} = \frac{1}{20}$$

Putting these values in (1), we get

$$\begin{aligned} P(D) &= \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20} \\ &= \frac{1}{50} + \frac{1}{100} + \frac{1}{80} = \frac{17}{400} = 0.0425 \end{aligned}$$

8. Option (c) is correct.

Explanation: Here success is a score which is a multiple of 3 i.e., 3 or 6.

$$\text{Therefore, } p(3 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$$

The probability of r successes in 10 throws is given by

$$P(r) = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{10-r}$$

$$\text{Now } P(\text{at least 8 successes}) = P(8) + P(9) + P(10)$$

$$\begin{aligned} &= {}^{10}C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2 + {}^{10}C_9 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right)^1 + {}^{10}C_{10} \left(\frac{1}{3}\right)^{10} \\ &= \frac{1}{3^{10}} [45 \times 4 + 10 \times 2 + 1] = \frac{201}{3^{10}} \end{aligned}$$

9. Option (b) is correct.

Explanation: Since $\sum p_i = 1$, we have

$$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$$

$$\text{i.e., } 10C^2 + 9C - 1 = 0$$

$$\text{i.e., } (10C - 1)(C + 1) = 0$$

$$\Rightarrow C = \frac{1}{10}, C = -1$$

$$\text{Therefore, the permissible value of } C = \frac{1}{10}$$

$$= 3.66.$$

10. Option (b) is correct.

Explanation: Since 4 balls have to be drawn, therefore, X can take the values 0, 1, 2, 3, 4

$$P(X = 0) = P(\text{no red ball}) = P(4 \text{ white balls})$$

$$= \frac{{}^4C_4}{{}^{12}C_4} = \frac{1}{495}$$

$$P(X = 1) = P(1 \text{ red ball and } 3 \text{ white balls})$$

$$= \frac{{}^8C_1 \times {}^4C_3}{{}^{12}C_4} = \frac{32}{495}$$

$$P(X = 2) = P(2 \text{ red balls and } 2 \text{ white balls})$$

$$= \frac{{}^8C_2 \times {}^4C_2}{{}^{12}C_4} = \frac{168}{495}$$

$$P(X = 3) = P(3 \text{ red balls and } 1 \text{ white ball})$$

$$= \frac{{}^8C_3 \times {}^4C_1}{{}^{12}C_4} = \frac{224}{495}$$

$$P(X = 4) = P(4 \text{ red balls}) = \frac{{}^8C_4}{{}^{12}C_4} = \frac{70}{495}$$

Thus the following is the required probability distribution of X

X	0	1	2	3	4
P(X)	$\frac{1}{495}$	$\frac{32}{495}$	$\frac{168}{495}$	$\frac{224}{495}$	$\frac{70}{495}$

11. Option (b) is correct.

Explanation: Solution Let X denote the number of heads tossed. So, X can take the values 0, 1, 2, 3. When a coin is tossed three times, we get

Sample space $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$$P(X = 0) = P(\text{no head}) = P(TTT) = \frac{1}{8}$$

$$P(X = 1) = P(\text{one head}) = P(HTT, THT, TTH) = \frac{3}{8}$$

$$P(X = 2) = P(\text{two heads}) = P(HHT, HTH, THH) = \frac{3}{8}$$

$$P(X = 3) = P(\text{three heads}) = P(HHH) = \frac{1}{8}$$

Thus the probability distribution of X is:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$$\text{Variance of } X = \sigma^2 = \sum x_i^2 p_i - \mu^2 \quad (1)$$

Where μ is the mean of X given by

$$\mu = \sum x_i p_i = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = \frac{3}{2} \quad (2)$$

Now

$$\sum x_i^2 p_i = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8} = 3$$

From (1), (2) and (3), we get

$$\sigma^2 = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$$

$$\text{Standard deviation} = \sqrt{\sigma^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

12. Option (c) is correct.

Explanation: Let E be the event that the car is of standard quality. Let B_1 and B_2 be the events that the car is manufactured in plants X and Y, respectively. Now

$$P(B_1) = \frac{70}{100} = \frac{7}{10}, P(B_2) = \frac{30}{100} = \frac{3}{10}$$

$P(E|B_1)$ = Probability that a standard quality car has come from plant X

$$\begin{aligned} &= \frac{P(B_1) \times P(E|B_1)}{P(B_1) \cdot P(E|B_1) + P(B_2) \cdot P(E|B_2)} \\ &= \frac{\frac{7}{10} \times \frac{8}{10}}{\frac{7}{10} \times \frac{8}{10} + \frac{3}{10} \times \frac{9}{10}} = \frac{56}{83} \end{aligned}$$

Hence the required probability is $\frac{56}{83}$.

13. Option (a) is correct.

Explanation: From the given data $P(A) + P(B) = P(A \cap B)$

This shows that $P(A \cap B) = 0$. Thus $P(A|B) = \frac{P(A \cap B)}{P(B)} = 0$.

14. Option (a) is correct.

Explanation: The correct answer is (C). $P(A \cap B) = P(A|B)P(B)$

$$= 0.5 \times 0.2 = 0.1$$

$$\begin{aligned} P(A' \cap B') &= \frac{P(A' \cap B')}{P(B')} = \frac{P[(A \cup B)']}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} \\ &= \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - 0.2} = \frac{3}{8} \end{aligned}$$

15. Option (a) is correct.

Explanation:

(a) 2 digit positive integers which are multiples of 3 are 12, 15, 18, ..., 99. Thus, there are 30 such integers.

(b) 2-digit positive integers are 10, 11, 12, ..., 99. Thus, there are such numbers. Since out of these, 30 numbers are multiple of 3, therefore, the probability that a random chosen positive

2-digit integer is a multiple of 3, is $\frac{30}{90} = \frac{1}{3}$

16. Option (c) is correct.

Explanation: A PIN is a sequence of four symbols selected from 36 (26 letters + 10 digits) Symbols.

By the fundamental principle of counting, there are $36 \times 36 \times 36 \times 36 = 36^4 = 1,679,616$ PINs in all. When repetition is not allowed the multiplication rule can be applied to conclude that there are $36 \times 35 \times 34 \times 33 = 1,413,720$ different PINs

The number of PINs that contain at least one repeated symbol = $1.679 \times 0.616 - 1,413,720 = 2,65,896$

Thus, the probability that a randomly chosen PIN

contains a repeated symbol is

$$\frac{265,896}{1,679,616} = .1583$$

17. Option (b) is correct.

Explanation: Let B be the event that a truck stopped at the roadblock will have faulty brakes and T be the event that it will have badly worn tires. We have $P(B) = 0.23$, $P(T) = 0.24$ and $P(B \cup T) = 0.38$

And $P(B \cup T) = P(B) + P(T) - P(B \cap T)$

So $0.38 = 0.23 + 0.24 - P(B \cap T)$

$\Rightarrow P(B \cap T) = 0.23 + 0.24 - 0.38 = 0.09$

18. Option (a) is correct.

Explanation: Let C be the event that the person will have his teeth cleaned and F and E be, the event of getting cavity filled or tooth extracted, respectively. We are given

$P(C) = 0.48$, $P(F) = 0.25$, $P(E) = 0.20$, $P(C \cap F) = .09$,

$P(C \cap E) = 0.12$, $P(E \cap F) = 0.07$ and $P(C \cap F \cap E) = 0.03$

$$\begin{aligned} \text{Now, } P(C \cup F \cup E) &= P(C) + P(F) + P(E) \\ &\quad - P(C \cap F) - P(C \cap E) - P(F \cap E) + P(C \cap F \cap E) \\ &= 0.48 + 0.25 + 0.20 - 0.09 - 0.12 - 0.07 + 0.03 \\ &= 0.68 \end{aligned}$$

19. Option (b) is correct.

Explanation: P (numbered 5, 15, 25, or 35)

$$= P(5) + P(15) + P(25) + P(35)$$

= P(5 of white, Red, Yellow, Blue) + P(15 of white, yellow) + P(25 of yellow) + P(35 of yellow)

$$= \frac{4}{80} + \frac{2}{80} + \frac{1}{80} + \frac{1}{80} = \frac{8}{80} = \frac{1}{10}$$

20. Option (a) is correct.

Explanation: Since a leap year has 366 days and hence 52 weeks and 2 days. The 2 days can be SM, MT, TW, WTh, ThF, FSt, StS.

Therefore, $P(53 \text{ Sundays or } 53 \text{ Mondays}) = \frac{3}{7}$

21. Option (a) is correct.

Explanation: Since a 3-digit number cannot start with digit 0, the hundredth place can have any of the 4 digits. Now, the tens and units place can have all the 5 digits. Therefore, the total possible 3-digit number are $4 \times 5 \times 5$, i.e., 100.

The total possible 3-digit number with same digits same = 4

Hence, P (3-digit number with same digits

$$= \frac{4}{100} = \frac{1}{25})$$

22. Option (c) is correct.

Explanation: (B) is the correct answer.

Let $3P(A) = 2P(B) = P(C) = p$ which gives $P(A) = \frac{p}{3}$

, $P(B) = \frac{p}{2}$ and $P(C) = p$

Now since A, B, C are mutually exclusive and exhaustive events, we have $P(A) + P(B) + P(C) = 1$

Hence, $P(A) = \frac{p}{3} = \frac{2}{11}$

LIMITS, CONTINUITY APPLICATION OF DERIVATIVES AND DIFFERENTIATION

Basics of Limits

- The development of calculus was stimulated by two geometric problems: finding areas of plane regions and finding tangent lines to curves. Both these problems are related to the concept of "limit". The portion of calculus arising from the tangent problem is called differential calculus and that arising from the area problem is called integral calculus.

Standard limits in calculus

- Notation One-sided limits of $f(x)$ at x_0 : $\lim_{x \rightarrow x_0^-} f(x)$ and $\lim_{x \rightarrow x_0^+} f(x)$ the limit of $f(x)$ as x approaches

x_0 from the left (right). Two-sided limit of $f(x)$ at x_0 : $\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow x_0^-} f(x) = \lim_{x \rightarrow x_0^+} f(x)$ the limit of $f(x)$ as x approaches x_0 . Limits at infinity:

$$\lim_{x \rightarrow \infty} f(x), \lim_{x \rightarrow -\infty} f(x)$$

Computational techniques

- $\lim_{x \rightarrow x_0} k = k, \lim_{x \rightarrow -\infty} k = \lim_{x \rightarrow +\infty} k = k$
- $\lim_{x \rightarrow x_0} x = x_0, \lim_{x \rightarrow -\infty} x = -\infty, \lim_{x \rightarrow +\infty} x = +\infty$
- $\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty, \lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty, \lim_{x \rightarrow +\infty} \frac{1}{x} = 0, \lim_{x \rightarrow -\infty} \frac{1}{x} = 0$
- $\lim_{x \rightarrow a} |f(x) \pm g(x)| = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} |f(x)g(x)| = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$
- $\lim_{x \rightarrow a} |f(x)/g(x)| = \lim_{x \rightarrow a} f(x) / \lim_{x \rightarrow a} g(x)$

Methods of limits

Direct method, factorization method, partial fraction

- $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 1$
- $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$

Continuity and differentiability

- A moving physical object cannot vanish at some point and reappear someplace else to continue its motion. The path of a moving object is a single, unbroken curve without gaps, jumps or holes. Such curves are described as continuous.
- Definition:** A function f is said to be continuous at a point c if the following conditions are satisfied: — $f(c)$ is defined:

$$\lim_{x \rightarrow c^-} f(x) \text{ exists}$$

$$\lim_{x \rightarrow c^-} f(x) = f(c)$$

- A function f is said to be continuous from the right at a point c if the following conditions are satisfied:
- $f(c)$ is defined

$$\lim_{x \rightarrow c^+} f(x) \text{ exists}$$

$$\lim_{x \rightarrow c^+} f(x) = f(c)$$

A function f is said to be continuous on a closed interval $[a, b]$ if the following conditions are satisfied:

- f is continuous on (a, b)
- f is continuous from the right at a .
- f is continuous from the left at b .

Differentiability of a function

- A function is said to be differentiable if the derivative of the function exists at all points in its domain. Particularly, if a function $f(x)$ is differentiable at $x=a$, then $f'(a)$ exists in its domain.
- The converse does not hold.

Differential coefficient

Basics of Differentiation

- Many physical phenomena involve changing quantities- the speed of a rocket, the inflation of a currency, the number of bacteria in a culture, the shock intensity of an earthquake, the voltage of electric signals. A relationship exists between tangent lines and rates of change.

Derivative: The function

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ or } \frac{dy}{dx}$$

is called the derivative with respect to x of the function f . The domain of $f'(x)$ consists of all the points for which the limit exists. Geometric interpretation of the derivative: Slope of the tangent Rate of change interpretation: function whose value at x is the instantaneous rate of change of y with respect to x at the point x .

Existence of derivatives The most commonly encountered points of non differentiability can be classified as corners, vertical tangents, and points of discontinuity.

The Chain Rule

- If g is differentiable at the point x and f is differentiable at the point $g(x)$, then the composition $f \circ g$ is differentiable at the point x . If $y = f(g(x))$ and $u = g(x)$, then $y = f(u)$ and $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

Rules of Differentiation

- If f is a constant function i.e. $f(x) = c$ constant then $\frac{d}{dx} f(x) = \frac{d}{dx} c = 0$
- If n positive integer, then for every real value of $\frac{d}{dx} x^n = nx^{n-1}$

- Let c be a constant. If f is differentiable at x , then so is

$$cf, \text{ and } \frac{d}{dx} cf(x) = c \frac{d}{dx} f(x)$$

- If f and g are differentiable at x , then so is $f \pm g$, and

$$\frac{d}{dx}(f \pm g) = \frac{d}{dx} f(x) \pm \frac{d}{dx} g(x)$$

- If f and g are differentiable at x , then so is $f * g$, and

$$\frac{d}{dx}(f \cdot g) = f(x) \frac{d}{dx} g(x) + g(x) \frac{d}{dx} f(x)$$

- If f and g are differentiable at x , and $g(x) \neq 0$, then so is

$$\frac{f}{g} \text{ and } \frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) \frac{d}{dx} f(x) - f(x) \frac{d}{dx} g(x)}{[g(x)]^2}$$

- The higher derivative of the function $y = f(x)$ is

$$\frac{d^2 y}{dx^2}, \frac{d^3 y}{dx^3}, \dots, \frac{d^n y}{dx^n}$$

Derivatives of Trigonometric functions

- $\frac{d}{dx} \sin x = \cos x$
- $\frac{d}{dx} \cos x = -\sin x$
- $\frac{d}{dx} \tan x = \sec^2 x$
- $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$
- $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x$
- $\frac{d}{dx} \sec x = \sec x \cdot \tan x$
- $\frac{d}{dx} \log x = \frac{1}{x}$
- $\frac{d}{dx} \log u = \frac{1}{u} \frac{du}{dx}$
- $\frac{d}{dx} e^x = e^x$
- $\frac{d}{dx} e^{-x} = -e^{-x}$
- $\frac{d}{dx} a^x = a^x \log a$

Application of derivatives

Derivative as Rate of Change

- Related rates problems. In this kind of problem, one tries to find the rate at which some quantity changes by relating it to other quantities whose rates of change are known.

Examples — If a rocket is rising vertically at 880 ft/sec, when it is at 4000 ft up, how fast is the camera-to-rocket distance changing at that instant? (horizontal distance between camera and rocket is 3000 ft).

— How fast should the camera elevation angle change at that instant to keep the rocket in sight?

Monotonicity

Intervals of increase and decrease; concavity

- Let f be a function that is continuous on a closed interval $[a, b]$ and differentiable on the open interval (a, b) .
- If $f'(x) > 0$ for every value of x in (a, b) , then f is increasing on $[a, b]$.
- If $f'(x) < 0$ for every value of x in (a, b) , then f is decreasing on $[a, b]$.
- If $f'(x) = 0$ for every value of x in (a, b) , then f is constant on $[a, b]$.
- Let f be a function that is differentiable on an interval.
- If $f'(x)$ is increasing on the interval, then f is concave up on the interval. $\Leftrightarrow f''(x) > 0$
- If $f'(x)$ is decreasing on the interval, then f is concave down on the interval. $\Leftrightarrow f''(x) < 0$
- Inflection point of f is the point that f changes direction of its concavity. $f''(x) = 0$

Maxima and minima

Relative Extrema

- A critical point for a function f is any value of x in the domain of f at which $f'(x) = 0$ or at which f is not differentiable; the critical values where $f'(x) = 0$ are called stationary points of f .

First derivative test: The relative extrema of a continuous non constant function f if any occur at those critical points where f' changes sign.

Second derivative test: Suppose f is twice differentiable at a stationary point a .

- If $f''(a) > 0$, then f has a relative minimum at $x = a$.
- If $f''(a) < 0$, then f has a relative maximum at $x = a$.

Optimization Problems

- These are problems concerned with finding the best way to perform a task. A large class of these problems can be reduced to finding the largest and smallest value of a function and determining whether this value occurs.
- Extreme value or absolute extremum is a maximum or minimum in the whole domain of the function.
- Extreme value theorem: if a function f is continuous on a closed interval $[a, b]$, then f has both a maximum and a minimum value on $[a, b]$.
- If a function f has an extreme value (either a maximum and a minimum value on (a, b)), then the extreme value occurs at a critical point of f .

Tangents and Normal

Equation of tangent line of any function $y = f(x)$ at the point $p(x_0, y_0)$

$$(y - y_0) = \left(\frac{dy}{dx} \right)_{(x_0, y_0)} (x - x_0) \quad \dots (i)$$

Equation of normal line to the function $y = f(x)$ at the point $p(x_0, y_0)$

$$(y - y_0) = \frac{-1}{\left(\frac{dy}{dx} \right)_{(x_0, y_0)}} (x - x_0) \quad \dots (ii)$$

Here, $m = \left(\frac{dy}{dx} \right)_{(x_0, y_0)}$ be the slope of the curve.

EXERCISE

1. If a differentiable function $f(x)$ satisfies

$$\lim_{x \rightarrow -1} \frac{f(x) + 1}{x^2 - 1} = -\frac{3}{2}$$

then what is $\lim_{x \rightarrow -1} f(x)$ equal to?

- (a) $-\frac{3}{2}$ (b) -1 (c) 0 (d) 1
2. What is $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1 - \cos 4x}}$ equal to?
- (a) $\frac{1}{2\sqrt{2}}$ (b) $-\frac{1}{2\sqrt{2}}$
(c) $\sqrt{2}$ (d) Limit does not exist
3. What is $\lim_{x \rightarrow 0} \frac{\sin x \log(1-x)}{x^2}$ equal to?
- (a) -1 (b) Zero (c) $-e$ (d) $-\frac{1}{e}$
4. If $\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2}$, where $k \neq 0$, then what is the value of k ?
- (a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) 4
5. What is $\lim_{x \rightarrow \frac{\pi}{6}} \frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$ equal to?
- (a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) -2 (d) -3
6. If the function $f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$ is continuous, then what is the value of $(a + b)$?
- (a) 5 (b) 10 (c) 15 (d) 20
7. Consider the following statements in respect of the function $f(x) = \sin\left(\frac{1}{x^2}\right)$, $x \neq 0$:
1. It is continuous at $x = 0$, if $f(0) = 0$.
 2. It is continuous at $x = \frac{2}{\sqrt{\pi}}$.
- Which of the above statements is/are correct?
- (a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2
8. If $f(x) = \frac{\sin x}{x}$, where $x \in \mathbb{R}$, is to be continuous at $x = 0$, then the value of the function at $x = 0$.
- (a) should be 0 (b) should be 1
(c) should be 2 (d) cannot be determined
9. Let $f(x)$ be defined as follows:

$$f(x) = \begin{cases} 2x + 1, & -3 < x < -2 \\ x - 1, & -2 \leq x < 0 \\ x + 2, & 0 \leq x < 1 \end{cases}$$

Which one of the following statements is correct in respect of the above function?

- (a) It is discontinuous at $x = -2$ but continuous at every other point.
(b) It is continuous only in the interval $(-3, -2)$.
(c) It is discontinuous at $x = 0$ but continuous at every other point.
(d) It is discontinuous at every point.
10. What is the value of a ?
(a) -1 (b) 0 (c) 1 (d) 2
11. What is the value of b ?
(a) -1 (b) 1 (c) 0 (d) 2
12. If $f(x) = e^{|x|}$, which one of the following is correct?
(a) $f'(0) = 1$ (b) $f'(0) = -1$
(c) $f'(0) = 0$ (d) $f'(0)$ does not exist

13. Consider the following statements for

$$f(x) = e^{-|x|}:$$

- (1) The function is continuous at $x = 0$.
(2) The function is differentiable at $x = 0$.

Which of the above statements is/are correct?

- (a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2
14. A function is defined as follows:

$$f(x) = \begin{cases} -\frac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Which one of the following is correct in respect of the above function?

- (a) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$.
(b) $f(x)$ is continuous as well as differentiable at $x = 0$.
(c) $f(x)$ is discontinuous at $x = 0$.
(d) None of the above.

15. Consider the following statements:

- (1) The function $f(x) = |x|$ is not differentiable at $x = 1$.
(2) The function $f(x) = e^x$ is differentiable at $x = 0$.

Which of the above statements is/are correct?

- (a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2
16. If the derivative of the function $f(x) = \frac{m}{x} + 2nx + 1$ vanishes at $x = 2$, then what is the value of $m + 8n$?
- (a) -2
(b) 0
(c) 2
(d) Cannot be determined due to insufficient data

17. If $\frac{d}{dx} \left(\frac{1 + x^4 + x^8}{1 - x^2 + x^4} \right) = ax + bx^3$, then which one of the following is correct?

- (a) $a = b$ (b) $a = 2b$ (c) $a + b = 0$ (d) $2a = b$

18. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is equal to.
- (a) 0 (b) 1 (c) $\frac{x-1}{x+1}$ (d) $\frac{x+1}{x-1}$
19. If $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, then $\frac{dy}{dx}$ is equal to:
- (a) $-\frac{2}{1+x^2}$ for all $|x| < 1$
 (b) $-\frac{2}{1+x^2}$ for all $|x| > 1$
 (c) $\frac{2}{1+x^2}$ for all $|x| < 1$
 (d) None of the above
20. Let $f(x) = x + \frac{1}{x}$, where $x \in (0, 1)$. Then which one of the following is correct?
- (a) $f(x)$ fluctuates in the interval.
 (b) $f(x)$ increases in the interval.
 (c) $f(x)$ decreases in the interval.
 (d) None of the above.
21. Which one of the following, statements is correct ?
- (a) $f(x)$ is increasing in $\left(-\infty, \frac{1}{2}\right)$ and decreasing in $\left(\frac{1}{2}, \infty\right)$
- (b) $f(x)$ is decreasing in $\left(-\infty, \frac{1}{2}\right)$ and increasing in $\left(\frac{1}{2}, \infty\right)$
- (c) $f(x)$ is increasing in $(-\infty, 1)$ and decreasing in $(1, \infty)$
- (d) $f(x)$ is decreasing in $(-\infty, 1)$ and increasing in $(1, \infty)$
22. Consider the following statements in respect of the function $f(x) = x + \frac{1}{x}$:
1. The local maximum value of $f(x)$ is less than its local minimum value.
 2. The local maximum value of $f(x)$ occurs at $x = 1$.
 Which of the above statements is/are correct?
 (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2
23. What is the slope of the tangent of $y = \cos^{-1}(\cos x)$ at $x = -\frac{\pi}{4}$?
- (a) -1 (b) 0 (c) 1 (d) 2
24. What is the slope of the tangent to the curve at $(0, 1)$?
- (a) 0 (b) 1 (c) 2 (d) 4
25. Where does the tangent to the curve at $(0, 1)$ meet the x-axis?
- (a) $(1, 0)$ (b) $(2, 0)$
 (c) $\left(-\frac{1}{2}, 0\right)$ (d) $\left(\frac{1}{2}, 0\right)$

Answer Key

1. (b)	2. (d)	3. (a)	4. (c)	5. (d)	6. (a)	7. (b)	8. (b)	9. (c)	10. (d)
11. (a)	12. (d)	13. (a)	14. (c)	15. (b)	16. (d)	17. (d)	18. (a)	19. (a)	20. (c)
21. (b)	22. (a)	23. (a)	24. (c)	25. (c)					

Answers with Explanations

1. Option (b) is correct.

Explanation:

$$\text{Given: } \lim_{x \rightarrow -1} \frac{f(x)+1}{x^2-1} = -\frac{3}{2}$$

$$\Rightarrow \frac{\lim_{x \rightarrow -1} \{f(x)+1\}}{\lim_{x \rightarrow -1} \{x^2+1\}} = -\frac{3}{2}$$

$$\Rightarrow \lim_{x \rightarrow -1} \{f(x)+1\} = -\frac{3}{2} \left\{ \lim_{x \rightarrow -1} (x^2-1) \right\}$$

$$\Rightarrow \lim_{x \rightarrow -1} f(x)+1 = -\frac{3}{2} \{(-1)^2-1\}$$

$$\Rightarrow \lim_{x \rightarrow -1} f(x)+1 = 0$$

$$\Rightarrow \lim_{x \rightarrow -1} f(x) = -1$$

Hint:

- Simplify the given limit using algebra of limits and solve further.

2. Option (d) is correct.

Explanation:

$$\text{Let } A = \lim_{x \rightarrow 0} \frac{x}{\sqrt{1-\cos 4x}}$$

$$\Rightarrow A = \lim_{x \rightarrow 0} \frac{x}{\sqrt{2} |\sin 2x|} \quad \{\because 1 - \cos^2 x = 2 \sin^2 x\}$$

$$\begin{aligned} \text{Now, } \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2} |\sin 2x|} &= \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2} (-\sin 2x)} \\ &= \lim_{x \rightarrow 0^+} -\left(\frac{2x}{2\sqrt{2} \sin 2x}\right) \end{aligned}$$

$$= -\frac{1}{2\sqrt{2}}$$

$$\begin{aligned}\text{And } \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2} |\sin 2x|} &= \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2} \sin 2x} \\ &= \lim_{x \rightarrow 0^+} \frac{1}{2\sqrt{2}} \left(\frac{2x}{\sin 2x} \right) \\ &= \frac{1}{2\sqrt{2}}\end{aligned}$$

$$\therefore \lim_{x \rightarrow 0^-} \frac{x}{\sqrt{2} |\sin 2x|} \neq \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{2} |\sin 2x|}$$

$$\therefore \lim_{x \rightarrow 0} \frac{x}{\sqrt{1 - \cos 4x}} \text{ does not exist.}$$

Hints:

- Find L.H.L and R.H.L at $x = 0$ and analyse further.
- If $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$, then $\lim_{x \rightarrow a} f(x)$ does not exist.

3. Option (a) is correct.*Explanation:*

$$\lim_{x \rightarrow 0} \frac{\sin x \cdot \log(1-x)}{x^2}$$

We know that,

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \infty \text{ and}$$

$$\log(1-x) = -\left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \infty\right]$$

$$\begin{aligned}&\left\{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \infty\right\} \\ &\cdot \left[-\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \infty\right)\right]\end{aligned}$$

$$\text{Now, } \lim_{x \rightarrow 0} \frac{\left\{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \infty\right\} \cdot \left[-\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \infty\right)\right]}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{x \left\{1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \infty\right\} \cdot \left[-\left(1 + \frac{x}{2} + \frac{x^2}{3} + \dots \infty\right)\right]}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\left[-x \left\{1 + \frac{x}{2} + \frac{x^2}{3} + \dots \infty\right\}\right]}{x^2}$$

$$= (1 - 0 + 0 \dots \infty) \times [-1 + 0 + 0 \dots \infty]$$

$$= -1$$

4. Option (c) is correct.*Explanation:*

$$\lim_{x \rightarrow 1} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{x^3 - k^3}{x^2 - k^2}$$

Apply L'Hospital Rule,

$$\Rightarrow \lim_{x \rightarrow 1} \frac{4x^3 - 0}{1 - 0} = \lim_{x \rightarrow k} \frac{3x^2 - 0}{2x - 0}$$

$$\Rightarrow \lim_{x \rightarrow 1} 4x^3 = \lim_{x \rightarrow k} \frac{3}{2}x$$

$$\Rightarrow 4 \times 1^3 = \frac{3}{2} \times k$$

$$\Rightarrow k = \frac{8}{3}$$

5. Option (d) is correct.*Explanation:*

$$\lim_{x \rightarrow \frac{\pi}{6}} \frac{2\sin^2 x + \sin x - 1}{2\sin^2 x - 3\sin x + 1}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\sin^2 x + (2-1)\sin x - 1}{2\sin^2 x - (2+1)\sin x + 1}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\sin^2 x + 2\sin x - \sin x - 1}{2\sin^2 x - 2\sin x - \sin x + 1}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{2\sin x(\sin x + 1) - 1(\sin x + 1)}{2\sin x(\sin x - 1) - 1(\sin x - 1)}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{(\sin x + 1)(2\sin x - 1)}{(\sin x - 1)(2\sin x - 1)}$$

$$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sin x + 1}{\sin x - 1}$$

$$= \frac{\sin \frac{\pi}{6} + 1}{\sin \frac{\pi}{6} - 1}$$

$$= \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1}$$

$$= \frac{\frac{3}{2}}{\frac{-1}{2}}$$

$$= -3$$

6. Option (a) is correct.*Explanation:*

$$\text{Given: } f(x) = \begin{cases} a + bx, & x < 1 \\ 5, & x = 1 \\ b - ax, & x > 1 \end{cases}$$

As we know if function is continuous at a point then limiting value of a function is equal to the functional value at that point.

So, for $f(x)$ to be continuous at $x = 1$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x \rightarrow 1^-} (a + bx) = \lim_{x \rightarrow 1^+} (b - ax) = 5$$

$$\Rightarrow a + b = b - a = 5$$

$$\therefore a + b = 5$$

Hint:

- If $g(x)$ is continuous at a point then limiting value of $g(x)$ is equal to the functional value of $g(x)$ at that point.

7. Option (b) is correct.*Explanation:*

Consider, $f(x) = \sin\left(\frac{1}{x^2}\right)$

Statement 1: Given $f(0) = 0$ To check continuity to $x = 0$ **(1) Right hand limit ($x > 0$)**

$$\lim_{h \rightarrow 0} \sin\left(\frac{1}{(0+h)^2}\right)$$

$$\lim_{h \rightarrow 0} \sin\left(\frac{1}{h^2}\right) \quad [\text{Put limit, } h = 0]$$

$$= \sin\left(\frac{1}{0}\right)$$

$$= \sin \infty$$

We know, sine function value lies between -1 and $+1$.Hence, $-1 \leq \sin x \leq +1$ So, $-1 \leq \sin \theta \leq 1$ Right hand limit $\neq f(0)$ Hence, $f(x)$ not continuous at $x = 0$.**Statement 2:**To check continuity at $x = \frac{2}{\sqrt{\pi}}$

$$(1) \quad \text{At } x = \frac{2}{\sqrt{\pi}}$$

$$f(x) = \sin\left(\frac{1}{x^2}\right)$$

$$f\left(\frac{2}{\sqrt{\pi}}\right) = \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}$$

(2) Right hand limit

$$\lim_{h \rightarrow 0} \sin\left(\frac{1}{\left(\frac{2}{\sqrt{\pi}} + h\right)^2}\right)$$

$$\lim_{h \rightarrow 0} \sin\left(\frac{1}{\frac{4}{\pi} + h^2 + \frac{4h}{\sqrt{\pi}}}\right)$$

[Put limit, $h = 0$]

$$= \sin\left(\frac{1}{\frac{4}{\pi} + 0}\right)$$

$$= \sin \frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}$$

(3) Left hand limit

$$\lim_{h \rightarrow 0} \sin\left(\frac{1}{\left(\frac{2}{\sqrt{\pi}} - h\right)^2}\right)$$

$$\lim_{h \rightarrow 0} \sin\left(\frac{1}{\frac{4}{\pi} + h^2 - \frac{4h}{\sqrt{\pi}}}\right)$$

[Put limit, $h = 0$]

$$= \sin\left(\frac{\pi}{4}\right)$$

$$= \frac{1}{\sqrt{2}}$$

So,

$$\text{Left hand limit} = \text{Right hand limit} = f\left(\frac{2}{\sqrt{\pi}}\right)$$

Hence, $f(x)$ is continuous at $x = \frac{2}{\sqrt{\pi}}$.

Statement 2 is correct.

Hints:

- Check continuity at $x = 0$.
- Since value of $\sin x$ lies between -1 and $+1$.

8. Option (b) is correct.*Explanation:*

$$f(x) = \frac{\sin x}{x}, x \in \mathbb{R}$$

If $f(x)$ is continuous at $x = 0$, then

$$f(0) = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

9. Option (c) is correct.*Explanation:*At $x = -2$

$$\text{L.H.L} = \lim_{x \rightarrow -2^-} (2x + 1)$$

$$= \lim_{h \rightarrow 0} [2(-2 - h) + 1]$$

$$= 2(-2 - 0) + 1$$

$$= -4 + 1$$

$$= -3$$

$$\text{R.H.L} = \lim_{x \rightarrow -2^+} (x-1)$$

$$= \lim_{h \rightarrow 0} (-2+h-1)$$

$$= -2+0-1$$

$$= -3$$

$$f(-2) = -2-1 = -3$$

Here, L.H.L = R.H.L = $f(-2)$

So, function is continuous at $x = -2$.

At $x = 0$:

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} x-1$$

$$= \lim_{h \rightarrow 0} (0-h-1)$$

$$= -1$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} (x+2)$$

$$= \lim_{h \rightarrow 0} (0+h+2)$$

$$= 2$$

Here, L.H.L $\neq$ R.H.L

So, function is not continuous at $x = 0$

10. Option (d) is correct.

Explanation:

$$f(x) = \begin{cases} -1, & x \leq 0 \\ ax+b, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

at $x = 0$

$$\text{L.H.L} = \lim_{x \rightarrow 0^-} f(x)$$

$$\Rightarrow \text{L.H.L} = \lim_{x \rightarrow 0^-} (-1)$$

$$\Rightarrow \text{L.H.L} = \lim_{h \rightarrow 0} (-1)$$

$$\Rightarrow \text{L.H.L} = -1$$

$$\text{R.H.L} = \lim_{x \rightarrow 0^+} f(x)$$

$$\Rightarrow \text{R.H.L} = \lim_{x \rightarrow 0^+} (ax+b)$$

$$\Rightarrow \text{R.H.L} = \lim_{h \rightarrow 0} a(0+h)+b$$

$$\Rightarrow \text{R.H.L} = \lim_{h \rightarrow 0} ah+b$$

$$\Rightarrow \text{R.H.L} = a \times 0 + b$$

$$\Rightarrow \text{R.H.L} = b$$

Here, function is continuous every where,

So, L.H.L = R.H.L

$$\Rightarrow -1 = b$$

or

$$b = -1$$

Again, at $x = 1$

$$\text{L.H.L} = \lim_{x \rightarrow 1^-} f(x)$$

$$\Rightarrow \text{L.H.L} = \lim_{x \rightarrow 1^-} ax+b$$

$$\Rightarrow \text{L.H.L} = \lim_{h \rightarrow 0} a(1-h)+b$$

$$\Rightarrow \text{L.H.L} = a(1-0)+b$$

$$\Rightarrow \text{L.H.L} = a+b$$

$$\text{R.H.L} = \lim_{x \rightarrow 1^+} f(x)$$

$$\Rightarrow \text{R.H.L} = \lim_{x \rightarrow 1^+} (1)$$

$$\Rightarrow \text{R.H.L} = \lim_{h \rightarrow 0} 1$$

$$\Rightarrow \text{R.H.L} = 1$$

Since, function is continuous every where,

So, L.H.L = R.H.L

$$a+b=1$$

Put $b = -1$

So, $a-1=1$

$$\Rightarrow a=2$$

11. Option (a) is correct.

Explanation:

From the solution of question no 9

$$b = -1$$

12. Option (d) is correct.

Explanation:

Given: $f(x) = e^{|x|}$

Let us generalize the $f(x)$ using the definition of modulus function.

$$\text{So, } f(x) = \begin{cases} e^x; & x \geq 0 \\ e^{-x}; & x < 0 \end{cases}$$

Differentiate the $f(x)$ w.r.t. x , we get

$$f'(x) = \begin{cases} e^x; & x \geq 0 \\ e^{-x} \frac{d}{dx}(-x); & x < 0 \end{cases}$$

$$\left[\because \frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x) \right]$$

$$\Rightarrow f'(x) = \begin{cases} e^x; & x \geq 0 \\ -e^{-x}; & x < 0 \end{cases}$$

Let us check differentiability of $f(x)$ at $x = 0$.

$$\text{Now, L.H.D.} = f'(0^-) = -e^{-0} = -1$$

$$\text{R.H.D.} = f'(0^+) = e^0 = 1$$

$\therefore$ L.H.D. $\neq$ R.H.D.

$\therefore f'(0)$ does not exist.

Hint:

- Use $|x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$
- Function $f(x)$ is differentiable at $x = a$ if L.H.D. = R.H.D. = Finite at $x = a$.
- $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

13. Option (a) is correct.**Explanation:**

$$f(x) = e^{-|x|}$$

We know that $|x|$ is continuous at $x = 0$ but not differentiable at $x = 0$.

Hence, only statement 1 is correct.

14. Option (c) is correct.**Explanation:**

$$f(x) = \begin{cases} -\frac{x}{x}, & x > 0 \\ -\frac{x}{-x}, & x < 0 \\ 0, & x = 0 \end{cases}$$

$$f(x) = \begin{cases} -1, & x > 0 \\ 1, & x < 0 \\ 0, & x = 0 \end{cases}$$

Here, L.H.L. = +1

and R.H.L. = -1

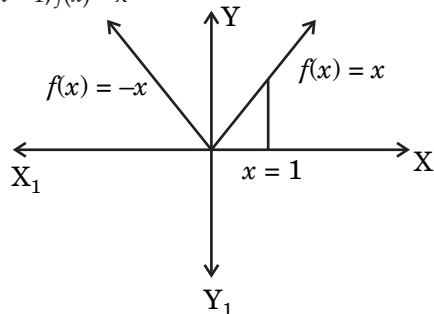
and $f(0) = 0$

Now, L.H.L. $\neq$ R.H.L. $\neq f(0)$
So, function is discontinuous at $x = 0$.

15. Option (b) is correct.**Explanation:**

$$(1) \quad f(x) = |x| = \begin{cases} +x, & x > 0 \\ -x, & x < 0 \end{cases}$$

at $x = 1, f(x) = x$



and we can draw the tangent at $x = 1$ on the $f(x)$.

$f(x)$ is differentiable at $x = 1$.

Hence, statement (1) is false.

$$(2) \quad f(x) = e^x \\ f'(x) = e^x$$

$$f'(0) = e^0$$

$$f'(0) = 1$$

$f'(0)$ is the slope of the line, which means we can draw a tangent at $x = 0$ on e^x .

So, $f(x) = e^x$ is differentiable function.

Therefore, only statement (2) is correct.

16. Option (d) is correct.**Explanation:**

Given that, $f'(2) = 0$

$$f(x) = \frac{m}{x} + 2nx + 1$$

$$f'(x) = \frac{m}{x^2} + 2n$$

$$f'(2) = \frac{m}{4} + 2n = 0$$

$$8n - m = 0$$

Cannot be determined value of $m + 8n$ due to insufficient data.

Hints:

- $f(x)$ is vanish at $x = 2$ i.e., $f'(2) = 0$

17. Option (d) is correct.**Explanation:**

$$\text{Given: } \frac{d}{dx} \left(\frac{1+x^4+x^8}{1-x^2+x^4} \right) = ax + bx^3$$

$$\Rightarrow \frac{d}{dx} \left(\frac{(x^4+x^2+1)(x^4+1-x^2)}{1-x^2+x^4} \right) = ax + bx^3$$

$$\Rightarrow \frac{d}{dx} (x^4 + x^2 + 1) = ax + bx^3$$

$$\Rightarrow 4x^3 + 2x = bx^3 + ax$$

$$\Rightarrow b = 4 \text{ and } a = 2$$

$$\Rightarrow 2a = b$$

Hint: Use $1 + x^4 + x^8 = (x^4 + x^2 + 1)(x^4 + 1 - x^2)$ and solve further.

18. Option (a) is correct.**Explanation:**

$$y = \sec^{-1} \left(\frac{x+1}{x-1} \right) + \sin^{-1} \left(\frac{x-1}{x+1} \right)$$

$$\Rightarrow y = \cos^{-1} \left(\frac{x-1}{x+1} \right) + \sin^{-1} \left(\frac{x-1}{x+1} \right) \left[\because \sec^{-1} \left(\frac{x}{y} \right) = \cos^{-1} \left(\frac{y}{x} \right) \right]$$

$$\Rightarrow y = \frac{\pi}{2} \left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \right]$$

Differentiate both sides w.r.t x

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left(\frac{\pi}{2} \right) = 0$$

19. Option (a) is correct.*Explanation:*

$$y = \cos^{-1} \left(\frac{2x}{1+x^2} \right)$$

Put $x = \tan \theta$

$$\text{So, } y = \cos^{-1} \left[\frac{2 \tan \theta}{1 + \tan^2 \theta} \right]$$

$$\Rightarrow y = \cos^{-1} [\sin 2\theta]$$

$$\Rightarrow y = \cos^{-1} \left[\cos \left(\frac{\pi}{2} - 2\theta \right) \right]$$

$$\Rightarrow y = \frac{\pi}{2} - 2\theta$$

$$\Rightarrow y = \frac{\pi}{2} - 2 \times \tan^{-1} x \quad [\because \tan \theta = x]$$

$$\Rightarrow y = \frac{\pi}{2} - 2 \tan^{-1} x$$

Now, differentiate both sides w.r.t. x

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\pi}{2} \right) - 2 \cdot \frac{d}{dx} (\tan^{-1} x)$$

$$= 0 - 2 \times \frac{1}{1+x^2}$$

$$= \frac{-2}{1+x^2} \quad \forall |x| < 1$$

20. Option (c) is correct*Explanation:*

$$\text{Let } f(x) = x + \frac{1}{x}$$

$$f'(x) = 1 - \frac{1}{x^2}$$

[on differentiating]

$$f'(x) = \frac{x^2 - 1}{x^2}$$

For increasing or decreasing function

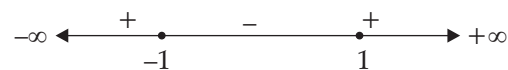
$$\text{Put } f'(x) = 0$$

$$\Rightarrow \frac{x^2 - 1}{x^2} = 0$$

$$\Rightarrow x^2 - 1 = 0$$

$$\Rightarrow (x+1)(x-1) = 0$$

$$\Rightarrow x = 1 \text{ and } x = -1$$

Here, function will decrease from $(-1, 1)$ But it is given that $x \in (0, 1)$ So, function will decrease from $(0, 1)$.**21. Option (b) is correct.***Explanation:*

$$\text{Given : } f(x) = |x-1| + x^2, x \in \mathbb{R}$$

$$|x-1| = \begin{cases} x-1; & x \geq 1 \\ 1-x; & x < 1 \end{cases}$$

$$|x-1| + x^2 = \begin{cases} x^2 + x - 1; & x \geq 1 \\ x^2 - x + 1; & x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x+1; & x \geq 1 \\ 2x-1; & x < 1 \end{cases}$$

$$\left(\because \frac{d}{dx}(x^n) = nx^{n-1} \right)$$

$$\Rightarrow \frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(x) = 1$$

 $f(x)$ is increasing if $f'(x) > 0$ and $f(x)$ is decreasing if $f'(x) < 0$.For $x \geq 1$,

$$f'(x) > 0$$

$$\Rightarrow 2x+1 > 0$$

$$\Rightarrow x > -\frac{1}{2}$$

$$\Rightarrow \boxed{x \geq 1}$$

For $x < 1$

$$2x-1 > 0$$

$$\Rightarrow x > \frac{1}{2}$$

$$\Rightarrow x \in \left(\frac{1}{2}, 1 \right)$$

$$\therefore f(x) \text{ is increasing } \forall x \in \left(\frac{1}{2}, \infty \right)$$

For $f(x)$ to be decreasing

$$f'(x) < 0$$

$$\text{For } x \geq 1, 2x+1 < 0 \Rightarrow x < -\frac{1}{2}$$

$$\Rightarrow x \in \phi$$

$$\text{For } x < 1, 2x-1 < 0 \Rightarrow x < +\frac{1}{2}$$

$$\Rightarrow x < +\frac{1}{2}$$

$$\therefore f(x) \text{ is decreasing } \forall x \in \left(-\infty, +\frac{1}{2} \right).$$

So, $f(x)$ is increasing in $\left(\frac{1}{2}, \infty\right)$ and decreasing in $\left(-\infty, \frac{1}{2}\right)$

Hints :

- $f(x)$ is decreasing wherever $f'(x) < 0$.
- $f(x)$ is increasing wherever $f'(x) > 0$.

22. Option (a) is correct.**Explanation:**

Consider,

$$f(x) = x + \frac{1}{x}$$

Differentiate with respect to x

$$f'(x) = 1 - \frac{1}{x^2} \quad \left[\frac{d}{dx}(x^n) = nx^{n-1} \right]$$

Find critical point by $f'(x) = 0$, we get

$$\Rightarrow 1 - \frac{1}{x^2} = 0$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

Now, critical points are +1, -1

$$\text{Consider, } f'(x) = 1 - \frac{1}{x^2}$$

Again differentiate with respect to x

$$f''(x) = \frac{2}{x^3}$$

Check at $x = +1, -1$ Now, At $x = +1, f''(x)$ is (+)ve, $x = -1, f''(x)$ is negative.

By rule of second differentiation, we get

At $x = -1, f(x)$ is maximum andat $x = +1, f(x)$ is minimum

Hence,

at $x = -1, f(x) = -2$ and at $x = +1, f(x) = +2$

So, maximum value is -2 and minimum value is +2.

Statement 1: It is correct.

Statement 2: It is incorrect.

Hints :

- Find critical point.
- By rule of second differentiation.

23. Option (a) is correct.**Explanation:**

$$y = \cos^{-1} \cos(x)$$

$$\Rightarrow \cos y = \cos x$$

$$\Rightarrow \frac{d}{dx}(\cos y) = \frac{d}{dx}(\cos x)$$

$$\Rightarrow -\sin y \frac{dy}{dx} = -\sin x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x}{\sin y}$$

$$\text{Now, at } x = -\frac{\pi}{4}$$

$$y = \cos^{-1} \left[\cos \left(-\frac{\pi}{4} \right) \right]$$

$$= \cos^{-1} \left(\cos \frac{\pi}{4} \right)$$

$$[\cos(-\theta) = \cos \theta]$$

$$= \frac{\pi}{4}$$

$$\therefore \left(\frac{dy}{dx} \right)_{x=-\frac{\pi}{4}} = \frac{\sin \left(-\frac{\pi}{4} \right)}{\sin \left(\frac{\pi}{4} \right)} \quad [\sin(-\theta) = -\sin \theta]$$

$$= \frac{-\sin \frac{\pi}{4}}{\sin \frac{\pi}{4}}$$

$$= -1$$

$$\text{Hence, slope of tangent at } \left(x = -\frac{\pi}{4} \right) = -1$$

Hints :

$$(1) \text{ At } x = -\frac{\pi}{4}, y = -\cos^{-1}(\cos x).$$

$$(2) \text{ Differentiation, } \frac{dy}{dx} = -1.$$

24. Option (c) is correct.**Explanation:**

$$\text{Given, } y = e^{2x}$$

(i)

Differentiating both sides of eq. (i) w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}(e^{2x})$$

$$\Rightarrow = e^{2x} \cdot \frac{d}{dx}(2x)$$

$$\Rightarrow \frac{dy}{dx} = e^{2x} \cdot 2 \quad \text{(ii)}$$

For slope of tangent at (0, 1) put $x = 0$ and $y = 1$ in eq. (ii),

$$\Rightarrow \left(\frac{dy}{dx} \right)_{\text{at } (0, 1)} = e^0 \times 2 = 2$$

25. Option (c) is correct.**Explanation:**

Now equation of tangent,

$$y - y_1 = m(x - x_1)$$

where $m = 2$

$$\Rightarrow (x_1, y_1) \equiv (0, 1)$$

$$\Rightarrow y - 1 = 2(x - 0)$$

$$\Rightarrow y - 1 = 2x$$

$$\Rightarrow y = 2x + 1$$

This is the equation of tangent.

If this line meet the x -axis, then y -coordinate should be zero. So, put $y = 0$.

$$\Rightarrow 2x + 1 = 0$$

$$\Rightarrow x = -\frac{1}{2}$$

So, the line meet the x -axis at $\left(-\frac{1}{2}, 0\right)\sqrt{2}$.

○○○

INDEFINITE AND DEFINITE INTEGRATION

Indefinite integration

Integration is the inverse process of differentiation. Instead of differentiating a function, we are given the derivative of a function and asked to find its primitive, i.e., the original function. Such a process is called integration $\frac{d}{dx}f(x) = F(x)$ then $\int F(x)dx = f(x) + c$.

Basics of indefinite integrals

Let $\frac{d}{dx}f(x) = F(x)$ then, we write $\int F(x)dx = f(x)$

+ c. These integrals are called indefinite integrals or general integrals;

C is called a constant of integration. All these integrals differ by a constant.

If two functions differ by a constant, they have the same derivative.

Geometrically, the statement $\int F(x)dx = f(x) + c$

= y (say) represents a family of curves. The different values of C correspond to different members of this family and these members can be obtained by shifting any one of the curves parallel to itself. Further, the tangents to the curves at the points of intersection of a line $x = a$ with the curves are parallel.

Some properties of indefinite integrals

- (i) The process of differentiation and integration are inverse of each other, i.e.,

$$\frac{d}{dx}f(x) = f'(x) \text{ and } \int f'(x)dx = f(x) + C \text{ where } C \text{ is}$$

any arbitrary constant.

- (ii) Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent. So, if f and g are two functions such

$$\text{that } \frac{d}{dx}f(x) = \frac{d}{dx}g(x) \text{ then } \int f(x)dx \text{ and}$$

$$\int g(x)dx \text{ are equivalent.}$$

- (iii) The integral of the sum of two functions equals the sum of the integrals of the functions i.e.,

$$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

- iv) A constant factor may be written either before or after the integral sign, i.e., $\int af(x)dx = a \int f(x)dx$ Where a as a constant.

Methods of Integration

We discussed integrals of those functions which were readily obtainable from derivatives of some functions. It was based on inspection, i.e., on the search of a function F whose derivative is f which led us to the integral of f .

However, this method, which depends on inspection, is not very suitable for many functions. Hence, we need to develop additional techniques or methods for finding the integrals by reducing them into standard forms. Prominent among them are methods based on:

1. Integration by substitution
2. Integration using partial fractions
3. Integration by parts

1. Integration by substitution

In this section, we consider the method of integration by substitution.

The given integral $\int f(x)dx$ can be transform into another form by changing the independent variable x to t by substituting $x = g(t)$.

$$\text{Consider } I = \int f(x)dx$$

$$\text{Put, } x = g(t) \text{ so that } \frac{dx}{dt} = g'(t) \text{ we write } dx = g'(t)dt$$

$$\text{Thus } I = \int f(g(t)) g'(t) dt$$

This change of variable formula is one of the important tools available to us in the name of integration by substitution. It is often important to guess what will be the useful substitution. Usually, we make a substitution for a function whose derivative also occurs in the integral.

Integrals of Some Particular Functions

In this section, we mention below some important formulae of integrals and apply them for integrating many other related standard integrals:

$$(i) \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$(ii) \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$$(iii) \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + c$$

$$(iv) \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log x + \sqrt{x^2 + a^2} + c$$

$$(v) \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c$$

$$(vi) \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log(x + \sqrt{x^2 + a^2}) + c$$

Integration of Rational Functions

Recall that a rational function is defined as the ratio of two polynomials in the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and

$Q(x)$ are polynomials in x and $Q(x) \neq 0$. If the degree of $P(x)$ is less than the degree of $Q(x)$, then the rational function is called proper, otherwise, it is called improper. The improper rational functions can be reduced to the proper rational functions by long division process. Thus, if is improper, then $\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$, where $T(x)$ is a polynomial in x and $\frac{P_1(x)}{Q(x)}$ is a proper rational function. As we

know how to integrate polynomials, the integration of any rational function is reduced to the integration of a proper rational function. The rational functions which we shall consider here for integration purposes will be those whose denominators can be factorized into linear and quadratic factors.

Assume that we want to evaluate $\int \frac{P(x)}{Q(x)} dx$, where $\frac{P(x)}{Q(x)}$ is proper rational function. It is always

possible to write the integrand as a sum of simpler rational functions by a method called partial fraction decomposition. After this, the integration can be carried out easily using the already known methods.

S. No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$
	Where $x^2 + bx + c$ cannot be factorized further.	

In the above table, A, B and C are real numbers to be determined suitably.

Integration by Parts

In this section, we describe one more method of integration that is found quite useful in integrating products of functions. If u and v are any two differentiable functions of a single variable x (say). Then, by the product rule of differentiation, we have:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

The integral of the product of two functions = (first function) $\times$ (integral of the second function) – Integral of [(differential coefficient of the first function) $\times$ (integral of the second function)]

Integration Using Trigonometric Identities

When the integrand involves some trigonometric functions, we use some known identities.

Trigonometric:

$$\sin^2 x + \cos^2 x = 1 \quad \tan^2 x + 1 = \sec^2 x$$

$$\cos^2 x = \frac{1 + \cos 2x}{2} \quad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - \sin^2 x$$

$\sqrt{a^2 - x^2}$	$x = a \sin \theta$	$dx = a \cos \theta d\theta$	$\sqrt{a^2 - x^2} = a \cos \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$	$dx = a \sec^2 \theta d\theta$	$\sqrt{a^2 + x^2} = a \sec \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$	$dx = a \sec \theta \tan \theta d\theta$	$\sqrt{x^2 - a^2} = a \tan \theta$

$\cos(\sin^{-1} x) = \sqrt{1-x^2}$	$\sin(\cos^{-1} x) = \sqrt{1-x^2}$
$\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}$	$\sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$
$\tan(\cos^{-1} x) = \frac{\sqrt{1-x^2}}{x}$	$\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$

Definite integral

In the previous sections, we have studied about the indefinite integrals and discussed few methods of finding them including integrals of some special functions. In this section, we shall study what is called definite integral of a function. The definite integral has a unique value. A definite integral is

denoted by $\int_a^b f(x) dx$ where a is called the lower limit of the integral and b is called the upper limit of the integral. The definite integral is introduced either as the limit of a sum or if it has an anti derivative F in the interval $[a, b]$, then its value is the difference between the values of F at the end points, i.e., $F(b) - F(a)$.

Properties of definite integrals

$$P_0: \int_a^b f(x) dx = \int_a^b f(t) dt$$

$$P_1: \int_a^b f(x) dx = - \int_b^a f(x) dx. \quad \text{In particular}$$

$$\int_a^a f(x) dx = 0$$

$$P_2: \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$P_3: \int_0^b f(x)dx = \int_0^b f(a+b-x)dx$$

$$P_4: \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

(Note that P_4 is a particular case of P_3)

$$P_5: \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$$

$$P_6: \int_0^{2a} f(x)dx = 2 \int_0^a f(x)dx \text{ if } f(2a-x) = f(x) \text{ and } 0$$

$$\text{if } f(2a-x) = -f(x)$$

$$P_7: \text{(i) } \int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx, \text{ if } f \text{ is an even function, i.e., if } f(-x) = f(x).$$

$$\text{(ii) } \int_{-a}^a f(x)dx = 0, \text{ if } f \text{ is an odd function, i.e., if } f(-x) = -f(x).$$

Area bounded by curves

Area function

We have defined $\int_b^a f(x)dx$ as the area of the region

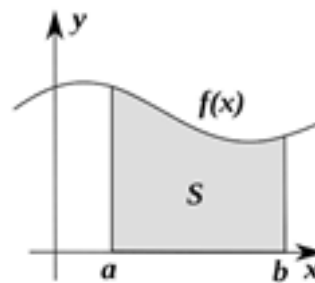
bounded by the curve $y = f(x)$, the ordinates $x = a$ and $x = b$ and x -axis. Let x be a given point in $[a, b]$.

Then $\int_a^x f(x)dx$ represents the area of the shaded region

In Fig. [Here it is assumed that $f(x) > 0$ for $x \in [a, b]$, the assertion made below is equally true for other functions as well]. The area of this shaded region depends upon the value of x .

In other words, the area of this shaded region is a function of x . We denote this function of x by $A(x)$. We call the function $A(x)$ as Area function and is given by

$$A(x) = \int_a^x f(x)dx$$



EXERCISE

- What is $\int e^{(2 \ln x + \ln x^2)} dx$ equal to?
 - $\frac{x^4}{4} + c$
 - $\frac{x^3}{3} + c$
 - $\frac{2x^5}{5} + c$
 - $\frac{x^5}{5} + c$
- What is the integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2 ?
 - $x + \frac{x^3}{3} + \frac{x^5}{5} + C$
 - $\frac{x^3}{3} + \frac{x^5}{5} + C$
 - $x^2 + \frac{x^4}{4} + \frac{x^6}{6} + C$
 - $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$
- If $\int \sqrt{1 - \sin 2x} dx = A \sin x + B \cos x + C$, where $0 < x < \frac{\pi}{4}$, then which one of the following is correct?
 - $A + B = 0$
 - $A + B - 2 = 0$
 - $A + B + 2 = 0$
 - $A + B - 1 = 0$
- Let $f(x)$ and $g(x)$ be twice differentiable functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$ and $g(2) = 9$. Then what is $f(x) - g(x)$ at $x = 4$ equal to?
 - 10
 - 6
 - 4
 - 2
- If $I_1 = \int \frac{e^x dx}{e^x + e^{-x}}$ and $I_2 = \int \frac{dx}{e^{2x} + 1}$, then what is $I_1 + I_2$ equal to?
 - $\frac{x}{2} + c$
 - $x + c$
 - $\ln(e^x + e^{-x}) + c$
 - $\ln(e^x - e^{-x}) + c$
- If $p(x) = (4e)^{2x}$, then what is $\int p(x) dx$ equal to?
 - $\frac{p(x)}{1 + 2 \ln 2} + c$
 - $\frac{p(x)}{2(1 + 2 \ln 2)} + c$
 - $\frac{2p(x)}{1 + \ln 4} + c$
 - $\int e^{(2 \ln x + \ln x^2)} dx$
- What is $\int (e^{\log_e x} + \sin x) \cos x$ equal to?
 - $\sin x + x \cos x + \frac{\sin^2 x}{2} + c$
 - $\sin x - x \cos x + \frac{\sin^2 x}{2} + c$
 - $x \sin x + \cos x + \frac{\sin^2 x}{2} + c$
 - $x \sin x - x \cos x + \frac{\sin^2 x}{2} + c$

8. What is $\int \frac{dx}{2x^2 - 2x + 1}$ equal to?

(a) $\frac{\tan^{-1}(2x-1)}{2} + c$

(b) $2 \tan^{-1}(2x-1) + c$

(c) $\frac{\tan^{-1}(2x+1)}{2} + c$

(d) $\tan^{-1}(2x-1) + c$

9. What is $\int \frac{dx}{x(1+\ln x)^n}$ equal to ($n \neq 1$) ?

(a) $\frac{1}{(n-1)(1+\ln x)^{n-1}} + c$

(b) $\frac{1-n}{(1+\ln x)^{1-n}} + c$

(c) $\frac{n+1}{(1+\ln x)^{n+1}} + c$

(d) $-\frac{1}{(n-1)(1+\ln x)^{n-1}} + c$

10. If

$$l_1 = \frac{d}{dx}(e^{\sin x})$$

$$l_2 = \lim_{h \rightarrow 0} \frac{e^{\sin(x+h)} - e^{\sin x}}{h}$$

$$l_3 = \int e^{\sin x} \cos x dx$$

then, which one of the following is correct?

(a) $l_1 \neq l_2$ (b) $\frac{d}{dx}(l_3) = l_2$

(c) $\int l_3 dx = l_2$ (d) $l_2 = l_3$

11. $\int (\ln x)^{-1} dx - \int (\ln x)^{-2} dx$ is equal to

(a) $x(\ln x)^{-1} + c$ (b) $x(\ln x)^{-2} + c$

(c) $x(\ln x) + c$ (d) $x(\ln x)^2 + c$

12. What is $\int \frac{xe^x dx}{(x+1)^2}$ equal to?

(a) $(x+1)^2 e^x + c$ (b) $(x+1)e^x + c$

(c) $\frac{e^x}{x+1} + c$ (d) $\frac{e^x}{(x+1)^2} + c$

13. What is $\int \frac{dx}{\sec^2(\tan^{-1} x)}$ equal to?

(a) $\sin^{-1} x + c$ (b) $\tan^{-1} x + c$

(c) $\sec^{-1} x + c$ (d) $\cos^{-1} x + c$

Directions: (Q. 26 – 27)

The integral $\int \frac{dx}{a \cos x + b \sin x}$ is of the form

$$\frac{1}{r} \ln \left[\tan \left(\frac{x+\alpha}{2} \right) \right].$$

14. What is r equal to?

(a) $a^2 + b^2$

(b) $\sqrt{a^2 + b^2}$

(c) $a + b$

(d) $\sqrt{a^2 - b^2}$

15. What is α equal to?

(a) $\tan^{-1} \left(\frac{a}{b} \right)$

(b) $\tan^{-1} \left(\frac{b}{a} \right)$

(c) $\tan^{-1} \left(\frac{a+b}{a-b} \right)$

(d) $\tan^{-1} \left(\frac{a-b}{a+b} \right)$

16. What is $\int_0^{\frac{\pi}{4}} \frac{dx}{(\sin x + \cos x)^2}$ equal to?

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{3}{2}$

17. If $f(x) = 2^x$, then what is $\int_2^{10} \frac{f'(x)}{f(x)} dx$ equal to?

(a) $4 \ln 2$

(b) $\ln 4$

(c) $\ln 5$

(d) $8 \ln 2$

18. If $\int_{-2}^5 f(x) dx = 4$ and $\int_0^5 \{1 + f(x)\} dx = 7$ then what

is $\int_{-2}^0 f(x) dx$ equal to?

(a) -3

(b) 2

(c) 3

(d) 5

19. If $0 < a < b$, then $\int_a^b \frac{|x|}{x} dx$ is equal to:

(a) $|b| - |a|$

(b) $|a| - |b|$

(c) $\frac{|b|}{|a|}$

(d) 0

20. $\int_{-1}^1 x|x| dx$ is equal to:

(a) 0

(b) $\frac{2}{3}$

(c) 2

(d) -2

Directions: (Q. 20 – 21)

$$\text{Consider } I = \int_0^{\frac{\pi}{2}} \frac{x dx}{1 + \sin x}$$

21. What is $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \sin x}$ equal to?

(a) 1

(b) 2

(c) 4

(d) -2

22. What is $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ equal to?

(a) $2ab$

(b) $2\pi ab$

(c) $\frac{\pi}{2ab}$

(d) $\frac{\pi}{ab}$

23. What is the area of the region enclosed between the curve $y^2 = 2x$ and the straight line $y = x$?

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) 2

24. What is the area enclosed by the parabola and the line?

(a) 27 square unit (b) 36 square unit
(c) 48 square unit (d) 54 square unit

25. What is the area enclosed by the parabola, the line and the y -axis in the first quadrant?

(a) 7 square unit (b) 14 square unit
(c) 20 square unit (d) 21 square unit

26. What is the area of the parabola $y^2 = 4bx$ bounded by its latus rectum?

(a) $\frac{2b^2}{3}$ square unit (b) $\frac{4b^2}{3}$ square unit

(c) b^2 square unit (d) $\frac{8b^2}{3}$ square unit

Answer Key

1. (d)	2. (d)	3. (b)	4. (a)	5. (b)	6. (b)	7. (c)	8. (d)	9. (d)	10. (b)
11. (a)	12. (c)	13. (b)	14. (b)	15. (a)	16. (b)	17. (d)	18. (b)	19. (a)	20. (a)
21. (c)	22. (c)	23. (c)	24. (a)	25. (c)	26. (d)				

Answers with Explanations

1. Option (d) is correct.

Explanation:

$$\text{Let } I = \int e^{(2\ln x + \ln x^2)} dx$$

$$\Rightarrow I = \int e^{(\ln x^2 + \ln x^2)} dx \quad [\because \log_a b^m = m \log_a b]$$

$$\Rightarrow I = \int e^{(\ln(x^2 \cdot x^2))} dx$$

$$[\because \log_a b + \log_a c = \log_a bc]$$

$$\Rightarrow I = \int e^{\ln(x^4)} dx$$

$$\Rightarrow I = \int x^4 dx \quad [\because {}_a \log_a x = x]$$

$$\Rightarrow I = \frac{x^5}{5} + C$$

Hint:

- Use principle properties of logarithm, $\log_a b^m = m \log_a b$, $\log_a b + \log_a c = \log_a bc$ and $a \log_a x = x$.

2. Option (d) is correct.

Explanation:

$$\text{Given: } f(x) = 1 + x^2 + x^4$$

We have to find w.r.t x^2 , we get:

$$I = \int (1 + x^2 + x^4) d(x^2)$$

Let $y = x^2$, we get

$$I = \int (1 + y + y^2) dy$$

$$I = y + \frac{y^2}{2} + \frac{y^3}{3} + c$$

$$\left[\int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

[put $y = x^2$]

$$I = x^2 + \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} + c$$

$$I = x^2 + \frac{x^4}{2} + \frac{x^6}{3} + c$$

Hints:

- (1) Put $y = x^2$ and Modify function.
- (2) Integrate according to function.

3. Option (b) is correct.

Explanation:

Let

$$I = \int \sqrt{1 - \sin 2x} dx = A \sin x + B \cos x + C \quad \dots(i)$$

Consider,

$$I = \int \sqrt{1 - \sin 2x} dx$$

$$[\text{Putting values, } \sin^2 x + \cos^2 x = 1 \\ \sin 2x = 2 \sin x \cos x]$$

$$I = \int \sqrt{\sin^2 x + \cos^2 x - 2 \sin x \cos x} dx$$

$$[\text{using } (A+B)^2 = A^2 + B^2 - 2AB]$$

$$I = \int \sqrt{(\cos x - \sin x)^2} dx$$

$$I = \int (\cos x - \sin x) dx \quad \left[\because 0 < x < \frac{\pi}{4} \right]$$

$$I = \sin x + \cos x + c$$

Comparing using (i)

$$\cos x + \sin x + c = A \sin x + B \cos x + c$$

We get: $A = 1$

$$B = 1$$

So,

$$\Rightarrow A + B = 1 + 1 = 2$$

$$\Rightarrow A + B = 2$$

$$\Rightarrow A + B - 2 = 0$$

Hint:

(1) Integrate using formula.

(2) Compare with given equation to find value.

4. Option (a) is correct

Explanation:

Given: $f(x)$ and $g(x)$ are twice differentiable function on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$ and $g(2) = 9$.

$$f''(x) = g''(x)$$

Integrate both sides w.r.t. x

$$\int f''(x) dx = \int g''(x) dx$$

$$\Rightarrow f'(x) + c_1 = g'(x) + c_2$$

$$\Rightarrow f'(x) = g'(x) + c \quad \dots(1)$$

Put $x = 1$

$$f'(1) = g'(1) + c$$

$$\Rightarrow 4 = 6 + c \quad (\because f'(1) = 4 \text{ and } g'(1) = 6)$$

$$\Rightarrow \boxed{c = -2}$$

So, equation (1) becomes

$$f'(x) = g'(x) - 2$$

Integrate again w.r.t. x

$$\int f'(x) dx = \int (g'(x) - 2) dx$$

$$\Rightarrow f(x) = g(x) - 2x + c \quad \dots(2)$$

$$(\because \int dx = x + c_1)$$

Put $x = 2$

$$f(2) = g(2) - 4 + c$$

$$\Rightarrow 3 = 9 - 4 + c \quad (\because f(2) = 3, g(2) = 9)$$

$$\Rightarrow \boxed{c = -2}$$

From (2)

$$f(x) = g(x) - 2x - 2$$

$$\Rightarrow f(x) - g(x) = -2x - 2$$

Put $x = 4$

$$\begin{aligned} f(4) - g(4) &= -2 \times 4 - 2 \\ &= -10 \end{aligned}$$

So, $f(x) - g(x) = -10$ at $x = 4$

Hints :

$$(i) \int f''(x) = f'(x) + c_1$$

$$(ii) \int f'(x) dx = f(x) + c_2$$

5. Option (b) is correct.

Explanation:

$$I_1 = \int \frac{e^x}{e^x + e^{-x}} dx$$

$$= \int \frac{e^x}{e^x + \frac{1}{e^x}} dx = \int \frac{e^{2x}}{e^{2x} + 1} dx$$

$$\text{Now, } I_1 + I_2 = \int \left(\frac{e^{2x}}{e^{2x} + 1} + \frac{1}{e^{2x} + 1} \right) dx$$

$$= \int \frac{e^{2x} + 1}{e^{2x} + 1} dx$$

$$= \int 1 dx = x + c$$

Hints:

- Simplify function in I_1 and add $I_1 + I_2$.

6. Option (b) is correct.

Explanation:

$$p(x) = (4e)^{2x}$$

$$\text{Now, } I = \int p(x) dx$$

$$\Rightarrow I = \int (4e)^{2x} dx$$

$$\left[\because \int a^x dx = \frac{a^x}{\log e^a} + c \right]$$

So,

$$\Rightarrow I = \frac{(4e)^{2x}}{\ln(4e)} \times \frac{1}{2} + c$$

$$\Rightarrow I = \frac{p(x)}{2[\ln 4 + \ln e]} + c$$

$$\Rightarrow I = \frac{p(x)}{2[\ln 2^2 + 1]} + c$$

$$[\because \ln_e e = 1]$$

$$\Rightarrow I = \frac{p(x)}{2[2\ln 2 + 1]} + c$$

7. Option (c) is correct.

Explanation:

$$I = \int (e^{\log_e x} + \sin x) \cos x dx$$

$$\Rightarrow I = \int (x + \sin x) \cos x dx \quad \left[\because e^{\log_e x} = x \right]$$

$$\Rightarrow I = \int_1^x \cos x dx + \int \sin x \cdot \cos x dx$$

$$\left[\because \int I \cdot \Pi dx = I \cdot \int \Pi dx - \int \left\{ \frac{d}{dx} I \cdot \int \Pi dx \right\} dx \right]$$

$$\Rightarrow I = x \int \cos x dx - \int \left\{ \frac{d}{dx} x \cdot \int \cos x dx \right\} dx$$

$$+ \int \sin x \cdot \cos x dx$$

$$\Rightarrow I = x \sin x - \int 1 \cdot \sin x dx + \int \sin x \cdot \cos x dx$$

$$\Rightarrow I = x \sin x + \cos x + \int \sin x \cdot \cos x dx$$

Let $\sin x = t$

Differentiate both sides,

$$\cos x dx = dt$$

$$\Rightarrow I = x \sin x + \cos x + \int t \cdot dt$$

$$\Rightarrow I = x \sin x + \cos x + \frac{t^2}{2} + c$$

$$\Rightarrow I = x \sin x + \cos x + \frac{\sin^2 x}{2} + c$$

$$\Rightarrow I = \frac{1}{n} \ln \left| \frac{x^n}{x^n + 1} \right| + c$$

8. Option (d) is correct.

Explanation:

$$\int \frac{1}{2x^2 - 2x + 1} dx$$

Multiply and divide by 2.

$$= \int \frac{2}{4x^2 - 4x + 2} dx$$

$$= \int \frac{2}{1 + (2x - 1)^2} dx$$

Put $u = 2x - 1$, $du = 2dx$.

$$\text{Now, } \int \frac{1}{1 + u^2} du$$

$$= \tan^{-1} u + C$$

$$= \tan^{-1} (2x - 1) + C$$

Shortcut:

$$\text{Given: } \int \frac{1}{2x^2 - 2x + 1} dx$$

$$= \int \frac{2}{4x^2 - 4x + 2} dx$$

$$= \int \frac{2}{1 + (2x - 1)^2} dx$$

Put $u = 2x - 1$, $du = 2dx$

$$\text{Now, } \int \frac{1}{1 + u^2} du$$

$$= \tan^{-1} u + C$$

$$= \tan^{-1} (2x - 1) + C$$

Hint:

- Substitute the result with u and then apply the concept of solving of integration and using standard result of integration solve.

9. Option (d) is correct.

Explanation:

$$\text{Given, } \int \frac{dx}{x(1 + \ln x)^n} \text{ equal to } (n \neq 1)$$

Not substitute $\ln x = t$

$$\text{then } \frac{1}{x} dx = dt$$

$$\text{Now, } \int \frac{dt}{(1+t)^n} = \frac{(1+t)^{-(n-1)}}{-n+1}$$

$$= \frac{1}{(1-n)} \frac{1}{(1+t)^{n-1}} + C$$

$$= \frac{1}{(1-n)(1+\ln x)^{n-1}} + C$$

$$= \frac{-1}{(n-1)(1+\ln x)^{n-1}} + C$$

Shortcut:

Put $\ln x = t$

$$\text{then } \frac{1}{x} dx = dt$$

$$\text{Now, } \int \frac{dt}{(1+t)^n} = \frac{(1+t)^{-(n-1)}}{(-n+1)}$$

$$= \frac{1}{(1-n)} \frac{1}{(1+t)^{n-1}} + C$$

$$= \frac{1}{(1-n)(1+\ln x)^{n-1}} + C$$

$$= \frac{-1}{(n-1)(1+\ln x)^{n-1}} + C$$

Hint:

- Substitute $\ln x = t$ and they apply the operation for integration and solve.

10. Option (b) is correct.

Explanation:

$$l_1 = \frac{d}{dx} e^{\sin x}$$

$$= e^{\sin x} \times \frac{d}{dx} \sin x$$

$$\Rightarrow l_1 = e^{\sin x} \cdot \cos x \quad \dots(i)$$

$$l_2 = \lim_{x \rightarrow 0} \frac{e^{\sin(x+h)} - e^{\sin x}}{h}$$

$$= \frac{d}{dx} (e^{\sin x})$$

[By using first principal formula]

$$= e^{\sin x} \times \frac{d}{dx} \sin x$$

$$\Rightarrow l_2 = e^{\sin x} \cdot \cos x \quad \dots(ii)$$

$$l_3 = \int e^{\sin x} \cdot \cos x \, dx$$

$$= \int e^t \cdot dt \quad \text{Let } \sin x = t$$

$$= e^t + c \quad \text{Differentiate both sides} \\ \cos x \, dx = dt$$

$$\Rightarrow l_3 = e^{\sin x} + c \quad \dots(iii)$$

From equation (i), (ii) and (iii), it is clear that $l_1 = l_2$, which is not given in the options.

Now,

$$\frac{d}{dx} (I_3) = \frac{d}{dx} (e^{\sin x} + c)$$

$$= e^{\sin x} \times \cos x + 0$$

$$= e^{\sin x} \cdot \cos x$$

$$\text{So, } l_1 = l_2 = \frac{d}{dx} (l_3)$$

11. Option (a) is correct.

Explanation:

$$I = \int (\ln x)^{-1} dx - \int (\ln x)^{-2} dx$$

$$= \int \underset{\text{II}}{1} \cdot (\ln x)^{-1} dx - \int \underset{\text{I}}{(\ln x)^{-2}} dx$$

$$= (\ln x)^{-1} \cdot \int 1 \, dx - \int \left\{ \frac{d}{dx} (\ln x)^{-1} \cdot \int 1 \, dx \right\} dx \\ - \int (\ln x)^{-2} dx$$

$$= x(\ln x)^{-1} - \int -1 \cdot (\ln x)^{-2} \times \frac{1}{x} \times x \, dx \\ - \int (\ln x)^{-2} dx + c$$

$$= x(\ln x)^{-1} + \int (\ln x)^{-2} dx - \int (\ln x)^{-2} dx + c$$

$$= x(\ln x)^{-1} + c$$

12. Option (c) is correct

Explanation:

$$\text{Let } I = \int \frac{x \cdot e^x}{(1+x)^2} dx$$

$$\Rightarrow I = \int \frac{(1+x-1)e^x}{(1+x)^2} dx$$

$$\Rightarrow I = \int \left\{ \frac{(1+x)}{(1+x)^2} - \frac{1}{(1+x)^2} \right\} e^x dx$$

$$\Rightarrow I = \int \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} e^x dx$$

$$\Rightarrow I = \frac{1}{(1+x)} \cdot e^x + c$$

$$[\because \int [f(x) + f'(x)] \cdot e^x = e^x \cdot f(x) + c]$$

13. Option (b) is correct.

Explanation:

$$\text{Let } I = \int \frac{dx}{\sec^2(\tan^{-1} x)}$$

$$\text{As we know } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\therefore I = \int \frac{dx}{1 + \tan^2(\tan^{-1} x)}$$

$$\Rightarrow I = \int \frac{dx}{1 + [\tan(\tan^{-1} x)]^2}$$

$$\Rightarrow I = \int \frac{dx}{1 + x^2} \quad [\because \tan(\tan^{-1} x) = x]$$

$$\Rightarrow I = \tan^{-1} x + C$$

Hint:

- Simplify using $1 + \tan^2 \theta = \sec^2 \theta$.
- Use $\tan(\tan^{-1} x) = x$.
- Recall basic formulae of indefinite integration from definition.

14. Option (b) is correct

Explanation:

$$I = \int \frac{dx}{a \cos x + b \sin x} \quad \dots(i)$$

$$\text{Let } a = r \sin \alpha \text{ and } b = r \cos \alpha$$

$$\text{then, } I = \int \frac{dx}{r \sin \alpha \cdot \cos x + r \cos \alpha \cdot \sin x}$$

$$\Rightarrow I = \frac{1}{r} \int \frac{dx}{(\sin \alpha \cdot \cos x + \cos \alpha \cdot \sin x)}$$

$$\Rightarrow I = \frac{1}{r} \int \frac{dx}{\sin(\alpha + x)}$$

$$\Rightarrow I = \frac{1}{r} \int \operatorname{cosec}(\alpha + x) dx$$

$$\Rightarrow I = \frac{1}{r} \log_e \left[\tan \left(\frac{x + \alpha}{2} \right) \right] + c$$

$\Rightarrow$ I = which is equal to the given value.

So, $a = r \sin \alpha$ and $b = r \cos \alpha$ are the right assumptions.

$$\text{Now, } a = r \sin \alpha \quad \dots(ii)$$

$$b = r \cos \alpha \quad \dots(iii)$$

Square and add the equations, we get

$$\begin{aligned} a^2 + b^2 &= r^2 \sin^2 \alpha + r^2 \cos^2 \alpha \\ &= r^2 (\sin^2 \alpha + \cos^2 \alpha) \end{aligned} \quad [\because \sin^2 \alpha + \cos^2 \alpha = 1]$$

$$\begin{aligned} a^2 + b^2 &= r^2 \times 1 \\ \Rightarrow r^2 &= a^2 + b^2 \\ \Rightarrow r &= \sqrt{a^2 + b^2} \end{aligned}$$

15. Option (a) is correct

Explanation:

From the previous solution.

Divide equation (2) by equation (3), we get:

$$\frac{a}{b} = \frac{r \sin \alpha}{r \cos \alpha}$$

$$\Rightarrow \tan \alpha = \frac{a}{b}$$

$$\Rightarrow \alpha = \tan^{-1} \left(\frac{a}{b} \right)$$

16. Option (b) is correct.

Explanation:

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \frac{dx}{(\sin x + \cos x)^2} &= \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{dx}{\left(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x \right)^2} \\ &= \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{dx}{\cos^2 \left(\frac{\pi}{4} - x \right)} \\ &= \frac{1}{2} \int_0^{\frac{\pi}{4}} \sec^2 \left(\frac{\pi}{4} - x \right) dx \\ &= -\frac{1}{2} \left[\tan \left(\frac{\pi}{4} - x \right) \right]_0^{\frac{\pi}{4}} \\ &= -\frac{1}{2} [0 - 1] = \frac{1}{2} \end{aligned}$$

Hints:

- Use

$$a \sin x + b \cos x = \sqrt{a^2 + b^2} \left[\frac{a}{\sqrt{a^2 + b^2}} \sin x + \frac{b}{\sqrt{a^2 + b^2}} \cos x \right]$$

17. Option (d) is correct.

Explanation:

Given: $f(x) = 2^x$

Differentiate with respect to x

$$[\because f(x) = a^x \text{ gives } f'(x) = a^x \log a]$$

$$\Rightarrow f'(x) = 2^x \log 2 \quad \dots(i)$$

Now, we have to find

$$I = \int_2^{10} \frac{f'(x)}{f(x)} dx$$

$$I = \int_2^{10} \frac{2^x \log 2}{2^x} dx \quad [\text{using (i)}]$$

$$I = \int_2^{10} \log 2 dx$$

$$I = \int_2^{10} \log 2 dx$$

[cut 2^x in numerator & denominator]

$$I = \log 2 \int_2^{10} 1 dx$$

[log 2 is constant]

$$I = \log 2 \times [x]_2^{10}$$

[Integration of 1 in x]

$$I = \log 2 \times [10 - 2]$$

$$I = \log 2 \times 8$$

$$I = 8 \log 2$$

Hints:

- (1) Differentiation of a^x is $a^x \log a$.
- (2) Put value and cut common.

18. Option (b) is correct

Explanation:

$$\text{Given: } \int_{-2}^5 f(x) dx = 4 \quad \dots(1)$$

$$\int_0^5 \{1 + f(x)\} dx = 7 \quad \dots(2)$$

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$\int_{-2}^5 f(x) dx = \int_{-2}^0 f(x) dx + \int_0^5 f(x) dx$$

$$4 = \int_{-2}^0 f(x) dx + \int_0^5 f(x) dx \quad \dots(3)$$

From (2)

$$\begin{aligned} \int_0^5 \{1 + f(x)\} dx &= 7 \\ \Rightarrow \int_0^5 dx + \int_0^5 f(x) dx &= 7 \\ \Rightarrow [x]_0^5 + \int_0^5 f(x) dx &= 7 \quad (\because \int dx = x + c) \\ \Rightarrow 5 + \int_0^5 f(x) dx &= 7 \\ \Rightarrow \int_0^5 f(x) dx &= 2 \quad \dots(4) \end{aligned}$$

From (3) and (4)

$$\Rightarrow \int_{-2}^0 f(x) dx = 4 - 2 = 2$$

Hints :

$$(i) \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

$$(ii) \int dx = x + c$$

19. Option (a) is correct

Explanation :

Given, $0 < a < b$

$$\begin{aligned} I &= \int_a^b \frac{|x|}{x} dx \\ &= \int_a^b \frac{x}{x} dx, \quad \text{if } x > 0 \\ &= \int_a^b 1 dx \\ &= [x]_a^b \\ I &= (b - a) \quad \dots(i) \end{aligned}$$

Again,

$$\begin{aligned} I &= \int_a^b \frac{-x}{x} dx, \quad \text{if } x < 0 \\ &= \int_a^b (-1) dx \\ &= -[x]_a^b \\ \Rightarrow I &= -(b - a) \\ &= a - b \end{aligned}$$

which is not possible, as $b > a$

$$\begin{aligned} \text{Hence, } I &= b - a \\ I &= |b| - |a| \end{aligned}$$

20. Option (a) is correct

Explanation :

$$\begin{aligned} I &= \int_{-1}^1 x|x| dx \\ |x| &= 0 \\ \Rightarrow x &= 0 \\ \text{So, function will break at } x &= 0 \\ I &= \int_{-1}^0 x|x| dx + \int_0^1 x|x| dx \\ &= \int_{-1}^0 -x^2 dx + \int_0^1 x^2 dx \\ &= -\left[\frac{x^3}{3}\right]_{-1}^0 + \left[\frac{x^3}{3}\right]_0^1 \\ &= -\left[\frac{0}{3} + \frac{1}{3}\right] + \left[\frac{1}{3} - \frac{0}{3}\right] \\ &= -\frac{1}{3} + \frac{1}{3} \\ &= 0 \end{aligned}$$

21. Option (c) is correct

Explanation :

$$\begin{aligned} \text{Let } I_3 &= \int_0^{\pi} \frac{dx}{1 + \sin x} \\ \Rightarrow I_3 &= \int_0^{\pi} \frac{dx}{1 + \cos\left(\frac{\pi}{2} - x\right)} \quad \left[\because \cos\left(\frac{\pi}{2} - x\right) = \sin x\right] \\ \Rightarrow I_3 &= 2 \int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)} \quad [\because 1 + \cos 2\theta = 2\cos^2 \theta] \\ \Rightarrow I_3 &= 2 \int_0^{\frac{\pi}{2}} \sec^2\left(\frac{\pi}{4} - \frac{x}{2}\right) dx \\ \Rightarrow I_3 &= 2 \cdot (-2) \cdot \left[\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)\right]_0^{\frac{\pi}{2}} \\ &= -4 \left[\tan\left(\frac{\pi}{4} - \frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4} - 0\right)\right] \\ &= -4 \left[\tan 0 - \tan \frac{\pi}{4}\right] \end{aligned}$$

$$= -4[0 - 1]$$

$$= 4$$

22. Option (c) is correct

Explanation:

$$I = \int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

Divide numerator and denominator by $\cos^2 x$.

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\frac{1}{\cos^2 x}}{\frac{a^2 \cos^2 x}{\cos^2 x} + \frac{b^2 \sin^2 x}{\cos^2 x}} dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$$

Let $\tan x = t$

Differentiate both sides

$$\sec^2 x dx = dt$$

for $x = 0, t = 0$

for $x = \frac{\pi}{2}, t \rightarrow \infty$

$$\Rightarrow I = \int_0^{\infty} \frac{dt}{a^2 + b^2 \cdot t^2}$$

$$= \frac{1}{b^2} \int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$$

$$= \frac{1}{b^2} \times \frac{1}{\frac{a}{b}} \cdot \left[\tan^{-1} \frac{t}{\frac{a}{b}} \right]_0^{\infty}$$

$$\left[\because \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \left| \frac{x}{a} \right| + c \right]$$

$$= \frac{b}{ab^2} \cdot \left[\tan^{-1} \infty - \tan^{-1} 0 \right]$$

$$= \frac{1}{ab} \left[\frac{\pi}{2} - 0 \right]$$

$$= \frac{\pi}{2ab}$$

23. Option (c) is correct.

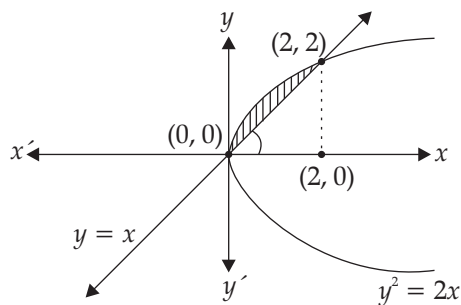
Explanation:

$$y^2 = 2x$$

$$y = x$$

....(i)

....(ii)



Solve equation (i) and (ii),

$$(x)^2 = 2x$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

$$\text{at } x = 0, y = 0$$

$$\Rightarrow \text{at } x = 2, y = 2$$

$$\text{Now, shaded area} = \int_0^2 (\sqrt{2x} - x) dx$$

$$= \left[\sqrt{2} \times \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2} \right]_0^2$$

$$= \left[\sqrt{2} \times \frac{2}{3} \times (2)^{\frac{3}{2}} - \frac{2^2}{2} \right] - \{0\}$$

$$= \sqrt{2} \times \frac{2}{3} \times 2\sqrt{2} - \frac{4}{2}$$

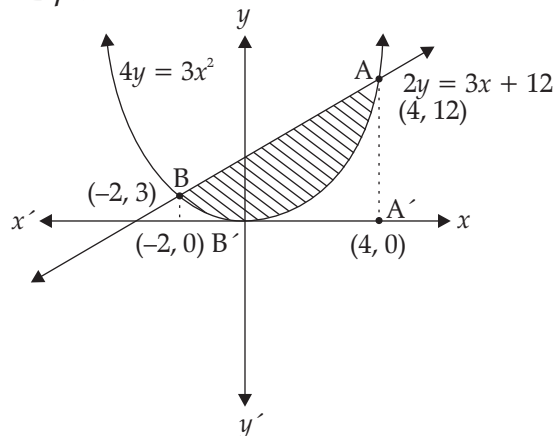
$$= \frac{8}{3} - 2$$

$$= \frac{8-6}{3}$$

$$= \frac{2}{3} \text{ square units}$$

24. Option (a) is correct

Explanation:



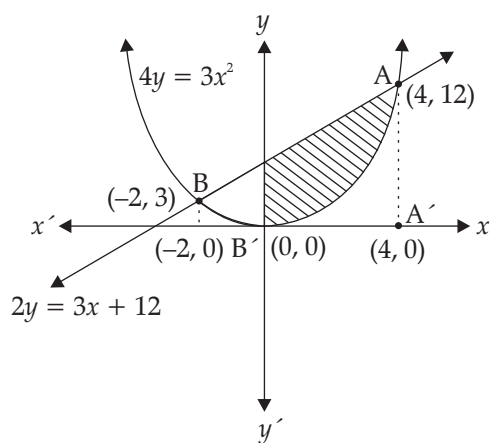
Now,

$$\begin{aligned}
 \text{Shaded area} &= \int_{-2}^4 \left(\frac{3x+12}{2} \right) dx - \int_{-2}^4 \left(\frac{3x^2}{4} \right) dx \\
 &= \frac{1}{2} \left[\frac{3x^2}{2} + 12x \right]_{-2}^4 - \frac{3}{4} \left[\frac{x^3}{3} \right]_{-2}^4 \\
 &= \frac{1}{2} \left[\left(\frac{3 \times 4^2}{2} + 12 \times 4 \right) - \left(\frac{3 \times (-2)^2}{2} + 12 \times -2 \right) \right] \\
 &\quad - \frac{3}{4} \left[\frac{(4)^3}{3} - \frac{(-2)^3}{3} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \left[\left(\frac{48}{2} + 48 \right) - \left(\frac{12}{2} - 24 \right) \right] - \frac{3}{4} \times \left[\frac{64}{3} + \frac{8}{3} \right] \\
 &= \frac{1}{2} [(24 + 48) - (6 - 24)] - \frac{3}{4} \times \frac{(64 + 8)}{3} \\
 &= \frac{1}{2} [72 + 18] - \frac{1}{4} \times 72 \\
 &= \frac{90}{2} - \frac{72}{4} \\
 &= 45 - 18 \\
 &= 27 \text{ square unit}
 \end{aligned}$$

25. Option (c) is correct

Explanation:



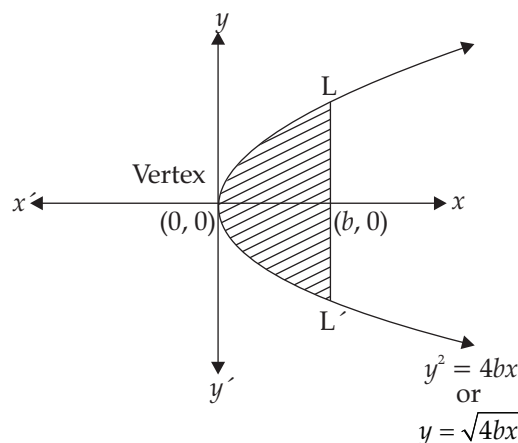
$$\begin{aligned}
 \text{Shaded Area} &= \int_0^4 \left(\frac{3x+12}{2} \right) dx - \int_0^4 \left(\frac{3x^2}{4} \right) dx \\
 &= \frac{1}{2} \times \left[\frac{3x^2}{2} + 12x \right]_0^4 - \frac{3}{4} \times \left[\frac{x^3}{3} \right]_0^4
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} \left[\left(\frac{3 \times 16}{2} + 48 \right) - (0) \right] - \frac{3}{4} \left(\frac{64}{3} - 0 \right) \\
 &= \frac{1}{2} [(24 + 48)] - \frac{3}{4} \times \frac{64}{3} \\
 &= \frac{1}{2} \times 72 - 16 \\
 &= 36 - 16 \\
 &= 20 \text{ square units}
 \end{aligned}$$

26. Option (d) is correct

Explanation:

Let LL' is the latus rectum of the parabola.



Now,

$$\begin{aligned}
 \text{Shaded area} &= 2 \int_0^b \sqrt{4bx} \, dx \\
 &= 2 \times 2\sqrt{b} \int_0^b (x)^{\frac{1}{2}} \, dx \\
 &= 4\sqrt{b} \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^b \\
 &= 4\sqrt{b} \times \frac{2}{3} \left[(b)^{\frac{3}{2}} - 0 \right] \\
 &= 4\sqrt{b} \times \frac{2}{3} \times b\sqrt{b} \\
 &= \frac{8}{3} b^2 \text{ square unit}
 \end{aligned}$$

Determinants, Minors & Co-factors

- (a) **Determinant:** A unique number (real or complex) can be associated to every square matrix $A = [a_{ij}]$ of order m . This number is called the determinant of the square matrix A , where $a_{ij} = (i, j)^{\text{th}}$ element of A .

For instance, if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then, determinant

of matrix A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \det(A)$

and its value is given by $ad - bc$.

- (b) **Minors:** Minors of an element a_{ij} of a determinant (or a determinant corresponding to matrix A) is the determinant obtained by deleting its i^{th} row and j^{th} column in which a_{ij} lies. Minor of a_{ij} is denoted by M_{ij} . Hence, we can get 9 minors corresponding to the 9 elements of a third order (i.e., 3×3) determinant.
- (c) **Co-factors:** Cofactor of an element a_{ij} , denoted by A_{ij} , is defined by $A_{ij} = (-1)^{(i+j)} M_{ij}$, where M_{ij} is minor of a_{ij} . Sometimes C_{ij} is used in place of A_{ij} to denote the co-factor of element a_{ij} .

1. ADJOINT OF A SQUARE MATRIX:

Let $A = [a_{ij}]$ be a square matrix. Also, assume $B = [A_{ij}]$, where A_{ij} is the cofactor of the elements a_{ij} in matrix A . Then the transpose B^T of matrix B is called the **adjoint of matrix A** and it is denoted by "**adj A** ".

To find adjoint of a 2×2 matrix: Follow this,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ or } \text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

For example, consider a square matrix of order 3 as

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 0 & 5 \end{bmatrix}, \text{ then in order to find the adjoint}$$

matrix A , we find a matrix B (formed by the co-factors of elements of matrix A as mentioned above in the definition)

$$\text{i.e., } B = \begin{bmatrix} 15 & -2 & -6 \\ -10 & -1 & 4 \\ -1 & 2 & -1 \end{bmatrix}.$$

$$\text{Hence, } \text{adj } A = B^T = \begin{bmatrix} 15 & -10 & -1 \\ -2 & -1 & 2 \\ -6 & 4 & -1 \end{bmatrix}$$

2. SINGULAR MATRIX AND NON-SINGULAR MATRIX:

- (a) **Singular matrix:** A square matrix A is said to be singular if $|A| = 0$ i.e., its determinant is zero.

$$\text{e.g. } A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 12 \\ 1 & 1 & 3 \end{bmatrix}$$

$$= 1(15 - 12) - 2(12 - 12) + 3(4 - 5) = 3 - 0 - 3 = 0$$

$\therefore A$ is singular matrix.

- (b) **Non-singular matrix:** A square matrix A is said to be non-singular if $|A| \neq 0$.

$$\text{e.g. } A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= 0(0 - 1) - 1(0 - 1) + 1(1 - 0)$$

$$= 0 + 1 + 1 = 2 \neq 0$$

$\therefore A$ is non-singular matrix.

- A square matrix A is **invertible** if and only if A is **non-singular**.

3. ALGORITHM TO FIND A^{-1} BY DETERMINANT METHOD:

STEP 1: Find $|A|$.

STEP 2: If $|A| = 0$, then, write " A is a singular matrix and hence not invertible". Else write " A is a non-singular matrix and hence invertible".

STEP 3: Calculate the co-factors of elements of matrix A .

STEP 4: Write the matrix of co-factors of elements of A and then obtain its transpose to get $\text{adj } A$ (i.e., adjoint A).

STEP 5: Find the inverse of A by using the relation

$$A^{-1} = \frac{1}{|A|} (\text{adj } A).$$

4. AREA OF TRIANGLE:

Area of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by,

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ sq. units}$$

- Since area is a positive quantity, we take absolute value of the determinant.
- If the points (x_1, y_1) , (x_2, y_2) and (x_3, y_3) are collinear, then $\Delta = 0$.
- The equation of a line passing through the points (x_1, y_1) and (x_2, y_2) can be obtained by the expression given here:

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

SOLVING SYSTEM OF EQUATIONS BY MATRIX METHOD [INVERSE MATRIX METHOD]

(a) Homogeneous and Non-homogeneous system:

A system of equations $AX = B$ is said to be a homogeneous system if $B = O$. Otherwise it is called a non-homogeneous system of equations.

$$a_1x + b_1y + c_1z = d_1,$$

$$a_2x + b_2y + c_2z = d_2,$$

$$a_3x + b_3y + c_3z = d_3$$

STEP 1 : Assume

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

STEP 2 : Find $|A|$. Now there may be following situations :

- (i) $|A| \neq 0 \Rightarrow A^{-1}$ exists. It implies that the given system of equations is consistent and

therefore, the system has **unique solution**. In that case, write

$$\begin{aligned} AX &= B \\ \Rightarrow X &= A^{-1}B \end{aligned}$$

$$\left[\text{where } A^{-1} = \frac{1}{|A|}(\text{adj } A) \right]$$

Then by using the definition of equality of matrices, we can get the values of x, y and z .

- (ii) $|A| = 0 \Rightarrow A^{-1}$ does not exist. It implies that the given system of equations may be **consistent** or **inconsistent**.

In order to check proceed as follow:

$\Rightarrow$ Find $(\text{adj } A)B$. Now, we may have either $(\text{adj } A)B \neq O$ or $(\text{adj } A)B = O$.

- If $(\text{adj } A)B = O$, then the given system may be consistent or inconsistent.

To check, put $z = k$ in the given equations and proceed in the same manner in the new two variables system of equations assuming $d_i - c_i k$, $1 \leq i \leq 3$ as constant.

- And if $(\text{adj } A)B \neq O$, then the given system is inconsistent with no solutions.

EXERCISE

- If A is a square matrix of order 3, $|A'| = -3$, then $|AA'| =$
(a) 9 (b) -9 (c) 3 (d) -3
- If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$
(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB
- If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) or 'x' are
(a) 3 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$
- If A is a square matrix of order 3 and $|A| = 5$, then $|\text{adj } A| =$
(a) 5 (b) 25 (c) 125 (d) $\frac{1}{5}$

Q. 5. Let matrix $X = [x_{ij}]$ is given by $X = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$.

Then the matrix $Y = [m_{ij}]$, where $m_{ij} = \text{Minor of } x_{ij}$, is

- (a) $\begin{bmatrix} 7 & -5 & -3 \\ 19 & 1 & -11 \\ -11 & 1 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & -19 & 11 \\ 5 & -1 & -1 \\ 3 & 11 & 7 \end{bmatrix}$
(c) $\begin{bmatrix} 7 & 19 & -11 \\ -3 & 11 & 7 \\ -2 & -1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

- If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of

the other two roots is

- (a) 4 (b) -3 (c) 2 (d) 5
- Three points $P(2x, x+3)$, $Q(0, x)$ and $R(x+3, x+6)$ are collinear, then x is equal to:
(a) 0 (b) 2 (c) 3 (d) 1
 - If C_{ij} denotes the cofactor of element P_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31} \cdot C_{23}$ is:
(a) 5 (b) 24 (c) -24 (d) -5

- If for the matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is:
(a) ± 3 (b) -3 (c) ± 1 (d) 1

- The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is:

- (a) $24 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ (b) $\frac{1}{24} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- (c) $\frac{1}{24} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$
11. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals
(a) 8 (b) 24 (c) 72 (d) 216
12. If A is skew symmetric matrix of order 3, then the value of $|A|$ is:
(a) 3 (b) 0 (c) 9 (d) 27
13. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is:
(a) 3 (b) 0 (c) -1 (d) 1
14. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then $\det(\text{adj } A)$ equals:
(a) a^{27} (b) a^9 (c) a^6 (d) a^2
15. The system of linear equations
 $5x + ky = 5$,
 $3x + 3y = 5$;

will be consistent if:

- (a) $k \neq -3$ (b) $k = -5$ (c) $k = 5$ (d) $k \neq 5$
16. The pair of equations $3x - 5y = 7$ and $6x - 10y = 14$ have:
(a) a unique solution
(b) infinitely many solution
(c) no solution
(d) two solutions
17. For what value of k system of equation $3x + 5y = 0$ and $kx + 10y = 0$ has a non-zero solution?
(a) 2 (b) 2 (c) 6 (d) 8
18. If A a non-singular square matrix of order 3 and $A^2 = 2A$, then find the value of $|A|$.
(a) 8 (b) 7 (c) 6 (d) 5
19. Find $\text{adj } A$, if $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.
(a) $\begin{bmatrix} 3 & -1 \\ 2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$
(c) $\begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & -1 \\ -4 & 2 \end{bmatrix}$
20. Find the value of λ , if the system of equations $2x + 3y = 8$, $7x - 5y + 3 = 0$, $4x - 6y + \lambda = 0$ is solvable.
(a) 5 (b) 9 (c) 8 (d) 10

Answer Key

1. (a)	2. (c)	3. (d)	4. (b)	5. (d)	6. (a)	7. (d)	8. (a)	9. (a)	10. (d)
11. (d)	12. (b)	13. (c)	14. (c)	15. (d)	16. (b)	17. (c)	18. (a)	19. (b)	20. (c)

Answers with Explanations

1. **Option (a) is correct.**
Explanation: $|AA'| = |A| |A'| = (-3)(-3) = 9$
2. **Option (c) is correct.**
Explanation: $(AB^{-1})^{-1} = (B^{-1})^{-1} A^{-1} = BA^{-1}$
3. **Option (d) is correct.**
Explanation: $2 - 20 = 2x^2 - 24$
 $\Rightarrow 2x^2 = 6$
 $\Rightarrow x^2 = 3$
 $\Rightarrow x = \pm \sqrt{3}$
4. **Option (b) is correct.**
Explanation: $|\text{adj } A| = |A|^{n-1}$
 $\Rightarrow |\text{adj } A| = 25$
5. **Option (d) is correct.**
Explanation: $m_{11} = \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 12 - 5 = 7$
 $m_{12} = \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = 9 + 10 = 19$
 $m_{13} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -3 - 8 = -11$

$$m_{21} = \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = -3 + 2 = -1$$

$$m_{22} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1$$

$$m_{23} = \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1 + 2 = 1$$

$$m_{31} = \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = 5 - 8 = -3$$

$$m_{32} = \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = -5 - 6 = -11$$

$$m_{33} = \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 4 + 3 = 7$$

$$\therefore Y = \begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$$

6. Option (a) is correct.

Explanation: Given,
$$\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$$

$$\Rightarrow x(x^2 - 2) - 2(x - 3) + 3(2 - 3x) = 0$$

$$\Rightarrow x^3 - 13x + 13 = 0$$

Since $(x + 4)$ is one root of above cubic equation.

$$\text{Sum roots} = 0$$

$$\therefore \text{Sum of two roots} + (-4) = 0$$

$$\text{Sum of two roots} = 4$$

7. Option (d) is correct.

Explanation: As points are collinear

$\Rightarrow$ Area of triangle formed by 3 points is zero.

$$\Rightarrow \frac{1}{2} \begin{vmatrix} (x_1 - x_2) & (x_2 - x_3) \\ (y_1 - y_2) & (y_2 - y_3) \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2} \begin{vmatrix} (2x - 0) & \{0 - (x + 3)\} \\ (x + 3 - x) & \{x - (x + 6)\} \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 2x & -(x + 3) \\ 3 & -6 \end{vmatrix} = 0$$

$$\Rightarrow -12x + 3(x + 3) = 0$$

$$\Rightarrow -12x + 3x + 9 = 0$$

$$\Rightarrow -9x = -9$$

$$\Rightarrow x = 1$$

8. Option (a) is correct.

Explanation:

Here,
$$C_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3 - 4 = -1$$

and
$$C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = -(2 + 3) = -5$$

Thus,
$$C_{31} \cdot C_{23} = (-1)(-5) = 5$$

9. Option (a) is correct.

Explanation:
$$A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$$

$$\Rightarrow |A| = \alpha^2 - 4 \quad \dots(i)$$

Also, given $|A^3| = 125$

$$\Rightarrow |A|^3 = 125$$

$$\Rightarrow |A| = 5$$

$$\Rightarrow \alpha^2 - 4 = 5$$

[from eq. (i)]

$$\Rightarrow \alpha^2 = 9$$

$$\Rightarrow \alpha = \pm 3$$

10. Option (d) is correct.

Explanation: The inverse of a diagonal matrix is obtained by replacing each element in the diagonal with its reciprocal.

Since,
$$X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Therefore
$$X^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$$

11. Option (d) is correct.

Explanation:

Here $|A| = 8$

Then $|3A| = 3^3 |A| = 27 \times 8 = 216$

12. Option (b) is correct.

Explanation: Determinant value of skew symmetric matrix is always '0'.

13. Option (c) is correct.

Explanation:

$$\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$

On expanding along R_1

$$2(x - 9x) - 3(x - 4x) + 2(9x - 4x) + 3 = 0$$

$$2(-8x) - 3(-3x) + 2(5x) + 3 = 0$$

$$-16x + 9x + 10x + 3 = 0$$

$$3x + 3 = 0$$

$$3x = -3$$

$$x = -\frac{3}{3}$$

$$x = -1$$

14. Option (c) is correct.

Explanation:

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$$

$$\text{Det}(A) = a(a \times a - 0 \times 0) - 0 + 0$$

$$= a^3$$

$$\text{Det}(\text{adj } A) = (a^3)^2$$

$$= a^6$$

15. Option (d) is correct.

Explanation: We have, $5x + ky - 5 = 0$

and $3x + 3y - 5 = 0$

For consistent system

$$\frac{5}{3} \neq \frac{k}{3}$$

$$\Rightarrow k \neq 5$$

16. Option (b) is correct.

Explanation: Given equations are:

$$3x - 5y = 7 \quad \dots(i)$$

$$\text{and } 6x - 10y = 14 \text{ or } 3x - 5y = 7 \quad \dots(ii)$$

Equations (i) and (ii) are same.

Hence it will have infinitely many solutions.

17. Option (c) is correct.

Explanation: For a non-zero solution

$$\begin{bmatrix} 3 & 5 \\ k & 10 \end{bmatrix} = 0$$

$$\Rightarrow 30 - 5k = 0 \Rightarrow k = 6$$

18. **Option (a) is correct.**

Explanation: $A^2 = 2A$

A multiplying with A^{-1}

$$A^{-1} A A = 2A^{-1} A$$

$$A = 2I$$

$$|A| = 8|I|$$

$$|A| = 8$$

19. **Option (b) is correct.**

Explanation: Given, $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

$$C_{11} = 3, \quad C_{12} = -4$$

$$C_{21} = 1, \quad C_{22} = 2$$

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T = \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$$

20. **Option (c) is correct.**

Explanation: Given system of equations :

$$2x + 3y = 8, 7x - 5y + 3 = 0, 4x - 6y + \lambda = 0$$

Solving first two equations, we get $x = 1, y = 2$
substituting this value in third equation, we get

$$4 - 12 + \lambda = 0$$

$$\Rightarrow \lambda = 8$$

○○○

1. MATRIX - BASIC INTRODUCTION:

A matrix is an ordered rectangular **array** of numbers (real or complex) or functions which are known as elements or the entries of the matrix. It is denoted by the uppercase letters i.e. A, B, C etc.

Consider a matrix A given as,

Here in matrix A the horizontal lines of elements are said to constitute **rows** and vertical lines of elements are said to constitute **columns** of the matrix. Thus, matrix A has m **rows** and n **columns**. The array is enclosed by square brackets $[]$, the parentheses $()$ or the double vertical bars $\| \|$.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

- A matrix having m rows and n columns is called a matrix of order $m \times n$ (read as ' m by n ' matrix). A matrix A of order $m \times n$ is depicted as $A = [a_{ij}]_{m \times n}$; $i, j \in N$.
- Also in general, a_{ij} means an element lying in the i^{th} row and j^{th} column.
- Number of elements in the matrix $A = [a_{ij}]_{m \times n}$ is given as mn .

2. TYPES OF MATRICES:

- (i) **Column matrix:** A matrix having only one column is called a **column matrix** or **column vector**.

$$\text{e.g.: } \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}_{3 \times 1}, \begin{bmatrix} 4 \\ 5 \end{bmatrix}_{2 \times 1}$$

General notation : $A = [a_{ij}]_{m \times 1}$

- (ii) **Row matrix:** A matrix having only one row is called a **row matrix** or **row vector**.

General notation : $A = [a_{ij}]_{1 \times n}$

- (iii) **Square matrix:** It is a matrix in which the number of rows is equal to the number of columns i.e., an $n \times n$ matrix is said to constitute a square matrix of order $n \times n$ and is known as a **square matrix of order ' n '**.

General notation : $A = [a_{ij}]_{n \times n}$

- (iv) **Diagonal matrix:** A square matrix $A = [a_{ij}]_{m \times m}$ is said to be a **diagonal matrix** if all the elements, except those in the leading diagonal are zero i.e., $a_{ij} = 0$, for all $i \neq j$.

- (v) **Scalar matrix:** A diagonal matrix $A = [a_{ij}]_{m \times m}$ is said to be a **scalar matrix** if its diagonal elements are equal. i.e.,

$$a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ k, & \text{when } i = j \text{ for some constant } k \end{cases}$$

- (vi) **Unit or Identity matrix:** A square matrix $A = [a_{ij}]_{m \times m}$ is said to be an **identity matrix** if $a_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$

A **unit matrix** can also be defined as the **scalar matrix** in which all diagonal elements are equal to **unity**. We denote the identity matrix of order m by I_m or I .

- (vii) **Zero matrix or Null matrix:** A matrix is said to be a **zero matrix** or **null matrix** if each of its elements is '0'.

- (viii) **Horizontal matrix:** A $m \times n$ matrix is said to be a **horizontal matrix** if $m < n$.

- (ix) **Vertical matrix:** A $m \times n$ matrix is said to be a **vertical matrix** if $m > n$.

3. EQUALITY OF MATRICES:

Two matrices A and B are said to be equal and written as $A = B$, if they are of the **same order** and their **corresponding elements are identical** i.e. $a_{ij} = b_{ij}$ i.e., $a_{11} = b_{11}$, $a_{22} = b_{22}$, $a_{32} = b_{32}$ etc.

4. ADDITION OF MATRICES:

If A and B are two $m \times n$ matrices, then another $m \times n$ matrix obtained by adding the corresponding elements of the matrices A and B is called the sum of the matrices A and B and is denoted by ' $A + B$ '.

Thus if $A = [a_{ij}]$, $B = [b_{ij}]$, or $A + B = [a_{ij} + b_{ij}]$.

5. MULTIPLICATION OF A MATRIX BY A SCALAR:

If a $m \times n$ matrix A is multiplied by a scalar k (say), then the new kA matrix is obtained by multiplying each element of matrix A by scalar k . Thus, if $A = [a_{ij}]$ and it is multiplied by a scalar k , then $kA = [ka_{ij}]$, i.e. $A = [a_{ij}]$ or $kA = [ka_{ij}]$.

6. MULTIPLICATION OF TWO MATRICES:

Let $A = [a_{ij}]$ be a $m \times n$ matrix and $B = [b_{jk}]$ be a $n \times p$ matrix such that the number of columns in A is equal to the number of rows in B , then the $m \times p$ matrix $C = [c_{ik}]$

such that $[c_{ik}] = \sum_{j=1}^n a_{ij} b_{jk}$ is said to be the product of the matrices A and B in that order and it is denoted by AB i.e. " $C = AB$ ".

Properties of matrix multiplication:

- Note that the product AB is defined only when the number of columns in matrix A is equal to the number of rows in matrix B .
- If A and B are $m \times n$ and $n \times p$ matrices, respectively, then the matrix AB will be an $m \times p$ matrix i.e., order of matrix AB will be $m \times p$.
- If A is a $m \times n$ matrix and both AB as well as BA are defined, then B will be a $n \times m$ matrix.
- If A is a $n \times n$ matrix and I_n be the unit matrix of order n , then $A I_n = I_n A = A$.
- Matrix multiplication is not commutative.

7. IDEMPOTENT MATRIX:

A square matrix A is said to be an idempotent matrix if $A^2 = A$.

For example,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

are idempotent matrices.

8. TRANSPOSE OF A MATRIX:

If $A = [a_{ij}]_{m \times n}$ be a $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of matrix A is said to be a **transpose of matrix** A . The transpose of A is denoted by A' or A^T i.e., if $A^T = [a_{ji}]_{n \times m}$.

PROPERTIES OF TRANSPOSE OF MATRICES:

- (i) $(A + B)^T = A^T + B^T$
- (ii) $(A^T)^T = A$
- (iii) $(kA)^T = kA^T$, where k is any constant
- (iv) $(AB)^T = B^T A^T$
- (v) $(ABC)^T = C^T B^T A^T$

Symmetric matrix: A square matrix $A = [a_{ij}]$ is said to be a **symmetric matrix** if $A^T = A$. i.e., if $A = [a_{ij}]$, then $A^T = [a_{ji}] = [a_{ij}]$ or $A^T = A$.

For example:

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}, \begin{bmatrix} 2+i & 1 & 3 \\ 1 & 2 & 3+2i \\ 3 & 3+2i & 4 \end{bmatrix}$$

Skew symmetric matrix: A square matrix $A = [a_{ij}]$ is said to be a **skew symmetric matrix** if

$$A^T = -[A] \text{ i.e., if } A = [a_{ij}], \text{ then } A^T = [a_{ji}] = -[a_{ij}] \text{ or } A^T = -A.$$

$$\text{For example: } \begin{bmatrix} 0 & 1 & -5 \\ -1 & 0 & 5 \\ 5 & -5 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

Orthogonal matrix: A matrix A is said to be **orthogonal** if $A A^T = I$, where A^T is transpose of A .

Invertible Matrix: An invertible matrix is a matrix for which matrix inversion operation exists, given that it satisfies the requisite conditions. Any given square matrix A of order $n \times n$ is called invertible if there exists another $n \times n$ square matrix B such that, $AB = BA = I_n$, where I_n is an identity matrix of order $n \times n$.

Example: Let matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ and matrix $B =$

$$\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\text{Now, } AB = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{and } BA = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, $A^{-1} = B$ and B is called the inverse of A .

So, A can also be the inverse of B or $B^{-1} = A$.

Uniqueness of Inverse of Matrix

If there exists an inverse of a square matrix, it is always unique.

Proof: Let A be a square matrix of order $n \times n$. Let us assume matrices B and C be inverses of matrix A .

Now, $AB = BA = I$, since B is the inverse of matrix A .

Similarly, $AC = CA = I$

But, $B = BI = B(AC) = (BA)C = IC = C$

This proves $B = C$, or B and C are the same matrices.

EXERCISE

1. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n , then

(a) $a_{ij} = \frac{1}{a_{ji}} \forall i, j$ (b) $a_{ij} \neq 0 \forall i, j$

(c) $a_{ij} = 0$, where $i = j$ (d) $a_{ij} \neq 0$ where $i = j$

2. The number of all possible matrices of order 2×3 with each entry 1 or 2 is

(a) 16 (b) 6 (c) 64 (d) 24

3. $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of $ab - cd$ is:

(a) 4 (b) 16 (c) -4 (d) -16

4. For the matrix $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, $(X^2 - X)$ is:

(a) $2I$ (b) $3I$ (c) I (d) $5I$

5. For two matrices $P = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $Q^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$,

$P - Q$ is:

(a) $\begin{bmatrix} 2 & 3 \\ -3 & 0 \\ 0 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 3 \\ -0 & -3 \\ -1 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 3 \\ 0 & -3 \\ 0 & -3 \end{bmatrix}$

6. A matrix $A = [a_{ij}]_{3 \times 3}$ is defined by

$$a_{ij} = \begin{cases} 2i+3j, & i < j \\ 5, & i = j \\ 3i-2j, & i > j \end{cases}$$

The number of elements in A which are more than 5, is:

(a) 3 (b) 4 (c) 5 (d) 6

7. If A is a square matrix such that $A^2 = A$, then $(I - A)^3 + A$ is equal to:

(a) I (b) 0
(c) $I - A$ (d) $I + A$

8. If $[x \ 1] \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = 0$, then x equals:

(a) 0 (b) -2 (c) -1 (d) 2

9. Suppose P and Q are two different matrices of order $3 \times n$ and $n \times p$, then the order of the matrix $P \times Q$ is?

(a) $3 \times p$ (b) $p \times 3$
(c) $n \times n$ (d) 3×3

10. If A and B are symmetric matrices of same order, then $AB - BA$ is a:

(a) Skew-symmetric matrix

(b) Symmetric matrix

(c) Zero matrix

(d) Identity matrix

11. If the matrix A is both symmetric and skew-symmetric, then:

(a) A is a diagonal matrix

(b) A is a zero matrix

(c) A is a square matrix

(d) None of these

12. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then find A^3 .

(a) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ -1 & 3 \end{bmatrix}$

13. Find the value of A^2 , where A is a 2×2 matrix

whose elements are given by $a_{ij} = \begin{cases} 1 & \text{if } i \neq j \\ 0 & \text{if } i = j \end{cases}$

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

14. If $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, write the value of $(x - y)$.

(a) 2 (b) 0 (c) 1 (d) 4

15. Find a matrix A such that $2A - 3B + 5C = O$, where

$$B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}.$$

(a) $\begin{bmatrix} 5 & 1 & -3 \\ 2 & -4 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 5 & 2 \\ -10 & 7 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$

(d) None of these

16. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence

find a matrix X such that $A^2 - 5A + 4I + X = O$.

(a) $\begin{bmatrix} 1 & 3 & 8 \\ -3 & 1 & 10 \\ -5 & 6 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$

- (c) $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & -10 \\ -1 & 5 & 8 \end{bmatrix}$ (d) None of these
17. Matrix $A = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is given to be symmetric, find values of a and b .

- (a) $a = \frac{-2}{3}, b = \frac{3}{2}$ (b) $a = \frac{1}{2}, b = \frac{3}{4}$
- (c) $a = \frac{1}{3}, b = \frac{-2}{3}$ (d) $a = \frac{3}{2}, b = \frac{1}{2}$
18. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, then $A + A' = I$, then the value of α .
- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) π

Answer Key

1. (c)	2. (c)	3. (a)	4. (a)	5. (b)	6. (b)	7. (a)	8. (d)	9. (a)	10. (a)
11. (b)	12. (a)	13. (c)	14. (b)	15. (c)	16. (b)	17. (a)	18. (a)		

Answers with Explanations

1. Option (c) is correct.

Explanation: In a skew-symmetric matrix, the (i, j) th element is negative of the (j, i) th element. Hence, the (i, i) th element = 0

2. Option (c) is correct.

Explanation: The order of the matrix = 2×3

The number of elements = $2 \times 3 = 6$

Each place can have either 1 or 2. So, each place can be filled in 2 ways.

Thus, the number of possible matrices = $2^6 = 64$

3. Option (a) is correct.

Explanation: Given, $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$

$$\therefore \quad 3c + 6 = 12 \quad \dots(i)$$

$$a - d = 2 \quad \dots(ii)$$

$$a + d = -8 \quad \dots(iii)$$

$$2 - 3b = -4 \quad \dots(iv)$$

From eq. (i), we get $c = 2$

On solving eqs. (ii) and (iii), we get $a = -3$ and $d = -5$ from eq. (iv), we get $b = 2$

$$\text{Now,} \quad ab - cd = (-3)2 - 2(-5)$$

$$\Rightarrow \quad ab - cd = -6 + 10 = 4$$

4. Option (a) is correct.

Explanation:

$$\text{Here} \quad X^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad X^2 = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow \quad X^2 - X = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad X^2 - X = 2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 2I$$

5. Option (b) is correct.

Explanation:

$$\text{Here,} \quad Q = (Q^T)^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\text{Now,} \quad P - Q = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

6. Option (b) is correct.

$$\text{Explanation: Here,} \quad A = \begin{bmatrix} 5 & 8 & 11 \\ 4 & 5 & 13 \\ 7 & 5 & 5 \end{bmatrix}$$

Thus, number of elements more than 5, is 4.

7. Option (a) is correct.

Explanation: $(I - A)^3 + A$

$$= I^3 - A^3 + 3A^2I - 3A + A$$

$$= I - A^2A + 3A^2 - 3A + A$$

$$= I - A^2 + 3A - 3A + A$$

$$= I - A + 3A - 3A + A$$

$$= I$$

8. Option (d) is correct.

Explanation:

$$\begin{bmatrix} x & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x-2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$\Rightarrow x-2 = 0 \text{ [By def. of equality]}$$

$$\Rightarrow x = 2$$

9. Option (a) is correct.

10. Option (a) is correct.

Explanation: Given that,

 A and B are symmetric matrices.

$$\Rightarrow A = A' \text{ and } B = B'$$

$$\text{Now, } (AB - BA)' = (AB)' - (BA)' \quad \dots(i)$$

$$\Rightarrow (AB - BA)' = B'A' - A'B' \quad \text{[By reversal law]}$$

$$\Rightarrow (AB - BA)' = BA - AB \quad \text{[From Eq. (i)]}$$

$$\Rightarrow (AB - BA)' = -(AB - BA)$$

$$\Rightarrow (AB - BA) \text{ is a skew-symmetric matrix.}$$

11. Option (b) is correct.

Explanation: If A is both symmetric and skew-symmetric, then we have,

$$A' = A \text{ and } A' = -A$$

$$\Rightarrow A = -A$$

$$\Rightarrow A + A = 0$$

$$\Rightarrow 2A = 0$$

$$\Rightarrow A = 0$$

Therefore, A is a zero matrix.

12. Option (a) is correct.

$$\text{Explanation: } A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

13. Option (c) is correct.

$$\text{Explanation: } A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

14. Option (b) is correct.

$$\text{Explanation: } 2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Equating corresponding entries, we have

$$2+y = 5 \quad 2x+2 = 8$$

$$y = 3 \quad x = 3$$

$$x - y = 3 - 3$$

$$x - y = 0$$

15. Option (c) is correct.

Explanation:

$$2A - \begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} + \begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$

16. Option (b) is correct.

Explanation: Getting $A^2 = A \cdot A$

$$= \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\therefore A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5 \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$+ 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\therefore X = -(A^2 - 5A + 4I)$$

$$\text{or } X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$$

17. Option (a) is correct.

Explanation: As A is a symmetric matrix,

$$\text{or } A' = A$$

$$\therefore \begin{bmatrix} 0 & 3 & 3a \\ 2b & 1 & 3 \\ -2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$$

$$\therefore \text{By equality of matrices, } a = \frac{-2}{3} \text{ and } b = \frac{3}{2}.$$

18. Option (a) is correct.

$$\text{Explanation: Given that, } A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

Also $A + A' = I$

$$\Rightarrow \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} + \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating corresponding entries, we have

 $\Rightarrow$

$$2\cos \alpha = 1$$

 $\Rightarrow$

$$\cos \alpha = \frac{1}{2}$$

 $\Rightarrow$

$$\cos \alpha = \cos \frac{\pi}{3}$$

 $\therefore$

$$\alpha = \frac{\pi}{3}$$

○○○

Chapter 30

VECTORS

1. Vector: Basic Introduction:

- A physical quantity having **magnitude** as well as the direction is called a vector. It is denoted as $\vec{AB}$ or $\vec{a}$. Its magnitude (or modulus) is $|\vec{AB}|$ or $|\vec{a}|$ otherwise, simply AB or a .

- Vectors are denoted by symbols such as $\vec{a}$.


[Pictorial representation of vector]

2. Initial and Terminal Points:

The initial and terminal points means that point from which the vector originates and terminates respectively.

3. Position Vector:

The position vector of a point say $P(x, y, z)$ is $\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and the magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

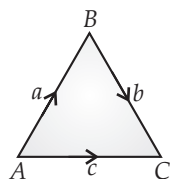
- Also, $\vec{AB} = (\text{Position Vector of } B) - (\text{Position Vector of } A)$. For example, let $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$. Then, $\vec{AB} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$.
- Here, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors along the axes OX , OY and OZ respectively (The discussion about unit vectors is given later under 'types of vectors').

4. Addition of vectors

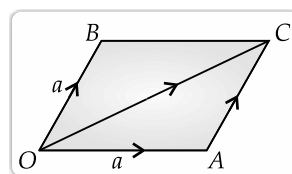
- (a) **Triangular law:** If two adjacent sides (say sides AB and BC) of a triangle ABC are represented by $\vec{a}$ and $\vec{b}$ taken in same order, then the third side of the triangle taken in the reverse order gives the sum of vectors $\vec{a}$ and $\vec{b}$ i.e.,

$$\vec{AC} = \vec{AB} + \vec{BC} \Rightarrow \vec{AC} = \vec{a} + \vec{b}.$$

- Also since $\vec{AC} = -\vec{CA} \Rightarrow \vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$.



- (b) **Parallelogram law:** If two vectors $\vec{a}$ and $\vec{b}$ are represented in magnitude and the direction by the two adjacent sides (say OA and OB) of a parallelogram $OACB$, then their sum is given by that diagonal of parallelogram which is co-initial with $\vec{a}$ and $\vec{b}$ i.e., $\vec{OC} = \vec{OA} + \vec{OB}$.



Note: Multiplication of a vector by a scalar

Let $\vec{a}$ be any vector and k be any non-zero scalar. Then the product $k\vec{a}$ is defined as a vector whose magnitude is $|k|$ times that of $\vec{a}$ and the direction is

- (i) same as that of $\vec{a}$ if k is positive, and
- (ii) opposite as that of $\vec{a}$ if k is negative.

Types of Vectors:

- (a) **Zero or Null vector:** It is that vector whose initial and terminal points are coincident. It is denoted by $\vec{0}$. Of course its magnitude is 0 (zero).
- Any non-zero vector is called a **proper vector**.
- (b) **Co-initial vectors:** Those vectors (two or more) having the same starting point are called the co-initial vectors.
- (c) **Co-terminus vectors:** Those vectors (two or more) having the same terminal point are called the co-terminus vectors.
- (d) **Negative of a vector:** The vector which has the same magnitude as the $\vec{r}$ but opposite direction. It is denoted by $-\vec{r}$. Hence if, $\vec{AB} = \vec{r}$ or $\vec{BA} = -\vec{r}$ i.e., $\vec{AB} = -\vec{BA}$, $\vec{PQ} = -\vec{QP}$ etc.
- (e) **Unit vector:** It is a vector with the unit magnitude.

The unit vector in the direction of vector $\vec{r}$ is given by $\hat{r} = \frac{\vec{r}}{|\vec{r}|}$ such that $|\hat{r}| = 1$, so, if

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then its unit vector is:

$$\hat{r} = \frac{x}{\sqrt{x^2 + y^2 + z^2}}\hat{i} + \frac{y}{\sqrt{x^2 + y^2 + z^2}}\hat{j} + \frac{z}{\sqrt{x^2 + y^2 + z^2}}\hat{k}.$$

- Unit vector perpendicular to the plane $\vec{a}$ and $\vec{b}$ is: $\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.

(f) **Reciprocal of a vector:** It is a vector which has the same direction as the vector $\vec{r}$ but magnitude equal to the reciprocal of the magnitude of $\vec{r}$. It is denoted as $\vec{r}^{-1}$. Hence $|\vec{r}^{-1}| = \frac{1}{|\vec{r}|}$.

(g) **Equal vectors:** Two vectors are said to be equal if they have the same magnitude as well as direction, regardless of the position of their initial points.

$$\text{Thus } \vec{a} = \vec{b} \Leftrightarrow \begin{cases} |\vec{a}| = |\vec{b}| \\ \vec{a} \text{ and } \vec{b} \text{ have same direction} \end{cases}$$

$$\begin{aligned} \text{Also, if } \vec{a} = \vec{b} &\Rightarrow a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} \\ &\Rightarrow a_1 = b_1, a_2 = b_2, a_3 = b_3. \end{aligned}$$

(h) **Collinear or Parallel vector:** Two vectors $\vec{a}$ and $\vec{b}$ are collinear or parallel if there exists a non-zero scalar λ such that $\vec{a} = \lambda\vec{b}$.

- It is important to note that the respective coefficients of $\hat{i}, \hat{j}, \hat{k}$ in $\vec{a}$ and $\vec{b}$ are proportional provided they are parallel or collinear to each other.
- The d.r.'s of parallel vectors are same (or are in proportion).
- The vectors $\vec{a}$ and $\vec{b}$ will have same or opposite direction as λ is positive or negative respectively.
- The vectors $\vec{a}$ and $\vec{b}$ are collinear if $\vec{a} \times \vec{b} = \vec{0}$.

(i) **Free vectors:** The vectors which can undergo parallel displacement without changing its magnitude and direction are called free vectors.

• **Properties/Observations of Dot product**

$$\Rightarrow \hat{i} \cdot \hat{i} = |\hat{i}| |\hat{i}| \cos 0 = 1 \text{ or } \hat{i} \cdot \hat{i} = 1 = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k}$$

$$\Rightarrow \hat{i} \cdot \hat{j} = |\hat{i}| |\hat{j}| \cos \frac{\pi}{2} = 0 \text{ or } \hat{i} \cdot \hat{j} = 0 = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = R, \text{ where } R \text{ is real number i.e., any scalar.}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} \text{ (Commutative property of dot product).}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b} \text{ or } |\vec{a}| = 0 \text{ or } |\vec{b}| = 0.$$

$$\Rightarrow \text{If } \theta = 0, \text{ then } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}|. \text{ Also } \vec{a} \cdot \vec{a} = |\vec{a}|^2 = a^2; \text{ as } \theta \text{ in this case is } 0.$$

$$\text{Moreover if } \theta = \pi, \text{ then } \vec{a} \cdot \vec{b} = -|\vec{a}| |\vec{b}|.$$

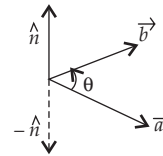
$$\Rightarrow \vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \quad \text{(Distributive property of dot product).}$$

$$\Rightarrow \vec{a} \cdot (-\vec{b}) = -(\vec{a} \cdot \vec{b}) = (-\vec{a}) \cdot \vec{b}.$$

1. The **cross product or vector product of two vectors $\vec{a}$ and $\vec{b}$** is defined by,

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}, \text{ where } \theta \text{ is the angle between the vectors } \vec{a} \text{ and } \vec{b}, 0 \leq \theta \leq \pi \text{ and } \hat{n}$$

is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. For better illustration, see figure.



Consider $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$. then

$$\begin{aligned} \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}. \end{aligned}$$

• **Properties/Observations of Cross Product**

$$\Rightarrow \hat{i} \times \hat{i} = |\hat{i}| |\hat{i}| \sin 0 = 0 \text{ or } \hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}.$$

$$\Rightarrow \hat{i} \times \hat{j} = |\hat{i}| |\hat{j}| \sin \frac{\pi}{2} = \hat{k} \text{ or } \hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}.$$

$\Rightarrow \vec{a} \times \vec{b}$ is a vector $\vec{c}$ (say) then this vector $\vec{c}$ is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$.

$$\Rightarrow \vec{a} \times \vec{b} = 0 \Leftrightarrow \vec{a} \parallel \vec{b} \text{ or } \vec{a} = 0, \vec{b} = 0.$$

$$\Rightarrow \vec{a} \times \vec{a} = 0.$$

- $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ (Commutative property does not hold for cross product).
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$ (Left distributive).
- $(\vec{b} + \vec{c}) \times \vec{a} = \vec{b} \times \vec{a} + \vec{c} \times \vec{a}$ (Right distributive).
(Distributive property of the vector product over cross product)

2. Relationship between Vector product and Scalar product [Lagrange's Identity]

$$\text{or } |\vec{a} \times \vec{b}|^2 + \left(\vec{a} \cdot \vec{b} \right)^2 = |\vec{a}|^2 \cdot |\vec{b}|^2$$

3. Cauchy-Schwarz inequality :

For any two vectors $\vec{a}$ and $\vec{b}$, always have

$$|\vec{a} \cdot \vec{b}| \leq |\vec{a}| |\vec{b}|.$$

EXERCISE

- The area of a triangle with vertices A, B, C is given by
(a) $|\vec{AB} \times \vec{AC}|$ (b) $\frac{1}{2} |\vec{AB} \times \vec{AC}|$
(c) $\frac{1}{4} |\vec{AC} \times \vec{AB}|$ (d) $\frac{1}{8} |\vec{AC} \times \vec{AB}|$
- The scalar projection of the vector $3\hat{i} - \hat{j} - 2\hat{k}$ on the vectors $\hat{i} - 2\hat{j} - 3\hat{k}$ is
(a) $\frac{7}{\sqrt{14}}$ (b) $\frac{7}{14}$ (c) $\frac{6}{13}$ (d) $\frac{7}{2}$
- If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then $|\vec{a} - 2\vec{b}|$ is equal to
(a) $\sqrt{2}$ (b) $2\sqrt{6}$ (c) 24 (d) $2\sqrt{2}$
- If $\vec{a}$ is a non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda \vec{a}$ is unit vector if
(a) $\lambda = 1$ (b) $\lambda = -1$
(c) $a = |\lambda|$ (d) $a = \frac{1}{|\lambda|}$
- What is the vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is
(a) $\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$
(c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$
- The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$
- If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is
(a) 0 (b) 1 (c) $-\frac{2}{3}$ (d) $-\frac{3}{2}$
- Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if
(a) $\theta = \frac{\pi}{4}$ (b) $\theta = \frac{\pi}{3}$
(c) $\theta = \frac{\pi}{2}$ (d) $\theta = \frac{2\pi}{3}$
- The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$

is

- (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

- The value of λ such that the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is
(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $-\frac{5}{2}$
- If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to
(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π
- For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to
(a) a^2 (b) $3a^2$ (c) $4a^2$ (d) $2a^2$
- If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is:
(a) 5 (b) 10 (c) 14 (d) 16
- The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is
(a) one (b) two
(c) three (d) infinite
- Find the area of the parallelogram whose diagonals are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.
(a) 3 sq units (b) 5 sq units
(c) 4 units (d) 8 sq units
- The position vectors of points A, B and C are $\lambda\hat{i} + 3\hat{j}$, $12\hat{i} + \mu\hat{j}$ and $11\hat{i} - 3\hat{j}$ respectively. If C divides the line segment joining A and B in the ratio 3 : 1, find the values of λ and μ .
(a) $d = 1, u = -3$ (b) $d = 5, u = 2$
(c) $d = 8, u = -5$ (d) $d = 4, u = -3$
- If $\hat{a}$ and $\hat{b}$ are unit vectors, then $|\hat{a} + \hat{b}| = \underline{\hspace{2cm}}$, where θ is the angle between them.
(a) $2 \cos \theta$ (b) $2 \cos \frac{\theta}{2}$
(c) $2 \sin \theta$ (d) $\cos \theta$
- Find the angle between two vectors $\vec{a}$ and $\vec{b}$ having the same length $\sqrt{2}$ and their vector product is

$$-\hat{i} - \hat{j} + \hat{k}$$

- (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi}{4}$

19. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $|\vec{b}|$.
- (a) 2 (b) 3 (c) 0 (d) 1

Answer Key

1. (b)	2. (a)	3. (b)	4. (d)	5. (c)	6. (a)	7. (c)	8. (d)	9. (b)	10. (d)
11. (b)	12. (d)	13. (d)	14. (b)	15. (a)	16. (c)	17. (b)	18. (a)	19. (d)	

Answers with Explanations

1. Option (b) is correct.

Explanation: The area of the parallelogram with adjacent sides AB and $AC = |\overrightarrow{AB} \times \overrightarrow{AC}|$. Hence, the area of the triangle with vertices $A, B, C = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$.

2. Option (a) is correct.

Explanation: Scalar Projection of $3\hat{i} - \hat{j} - 2\hat{k}$ on vector $\hat{i} + 2\hat{j} - 3\hat{k} = \frac{(3\hat{i} - \hat{j} - 2\hat{k}) \cdot (\hat{i} + 2\hat{j} - 3\hat{k})}{|\hat{i} + 2\hat{j} - 3\hat{k}|}$

$$= \frac{7}{\sqrt{14}}$$

3. Option (b) is correct.

Explanation: $|\vec{a} - 2\vec{b}|^2 = (\vec{a} - 2\vec{b}) \cdot (\vec{a} - 2\vec{b})$

$$|\vec{a} - 2\vec{b}|^2 = \vec{a} \cdot \vec{a} - 4\vec{a} \cdot \vec{b} + 4\vec{b} \cdot \vec{b}$$

$$= |\vec{a}|^2 - 4\vec{a} \cdot \vec{b} + 4|\vec{b}|^2$$

$$= 4 - 16 + 36 = 24$$

$$|\vec{a} - 2\vec{b}|^2 = 24 \Rightarrow |\vec{a} - 2\vec{b}| = 2\sqrt{6}$$

4. Option (d) is correct.

Explanation:

Vector $\lambda\vec{a}$ is a unit vector if

$$\begin{aligned} |\lambda\vec{a}| &= 1 \\ \Rightarrow |\lambda||\vec{a}| &= 1 \\ \Rightarrow |\vec{a}| &= \frac{1}{|\lambda|} \quad [\lambda \neq 0] \\ \Rightarrow a &= \frac{1}{|\lambda|} \quad [|\vec{a}| = a] \end{aligned}$$

Therefore, vector $\lambda\vec{a}$ is a unit vector if $a = \frac{1}{|\lambda|}$.

5. Option (c) is correct.

Explanation:

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Any vector in the direction of a vector $\vec{a}$ is given by

$$\begin{aligned} \frac{\vec{a}}{|\vec{a}|} &= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} \\ &= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} \end{aligned}$$

$\therefore$ Vector in the direction of $\vec{a}$ with magnitude 9

$$\begin{aligned} &= 9 \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} \\ &= 3(\hat{i} - 2\hat{j} + 2\hat{k}) \end{aligned}$$

6. Option (a) is correct.

Explanation:

Let $\vec{a} = 3\hat{i} - 6\hat{j} + \hat{k}$
and $\vec{b} = 2\hat{i} - 4\hat{j} + \lambda\hat{k}$
Since, $\vec{a} \parallel \vec{b}$

$$\Rightarrow \frac{3}{2} = \frac{-6}{-4} = \frac{1}{\lambda}$$

$$\Rightarrow \lambda = \frac{2}{3}$$

7. Option (c) is correct.

Explanation:

Projection- $\vec{a} \cdot \vec{b} = 0$

$$\begin{aligned} &|\vec{b}| \\ \Rightarrow \vec{a} \cdot \vec{b} &= 0 \\ 2 + 3\lambda &= 0 \\ \Rightarrow \lambda &= \frac{-2}{3} \\ \text{C. } \frac{-2}{3} \end{aligned}$$

8. Option (d) is correct.

Explanation: Let $\vec{a}$ and $\vec{b}$ be two-unit vectors and θ be the angle between them.

$$\begin{aligned}
 |\vec{a} + \vec{b}| &= 1 \\
 \Rightarrow (\vec{a} + \vec{b})^2 &= 1 \\
 \Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) &= 1 \\
 \Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} &= 1 \\
 \Rightarrow |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 &= 1 \\
 \Rightarrow 1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 &= 1 \\
 \Rightarrow \cos\theta &= -\frac{1}{2} \\
 \Rightarrow \theta &= \frac{2\pi}{3}
 \end{aligned}$$

So that, $|\vec{a} + \vec{b}|$ is a unit vector if $\theta = \frac{2\pi}{3}$.

9. Option (b) is correct.

Explanation:

Here, $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$ [Given]

We know that,

$$\begin{aligned}
 \vec{a} \cdot \vec{b} &= |\vec{a}||\vec{b}|\cos\theta \\
 \Rightarrow 2\sqrt{3} &= \sqrt{3} \cdot 4 \cdot \cos\theta \\
 \Rightarrow \cos\theta &= \frac{2\sqrt{3}}{4\sqrt{3}} \\
 &= \frac{1}{2} \\
 \therefore \theta &= \frac{\pi}{3}
 \end{aligned}$$

10. Option (d) is correct.

Explanation:

Since, two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal,

i.e., $\vec{a} \cdot \vec{b} = 0$

$$\therefore (2\hat{i} + \lambda\hat{j} + \hat{k}) \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 0$$

$$\Rightarrow 2 + 2\lambda + 3 = 0$$

$$\therefore \lambda = \frac{-5}{2}$$

11. Option (b) is correct.

Explanation: Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$\begin{aligned}
 |\vec{a} \cdot \vec{b}| &= |\vec{a} \times \vec{b}| \\
 \Rightarrow |\vec{a}||\vec{b}|\cos\theta &= |\vec{a}||\vec{b}|\sin\theta \\
 \Rightarrow \cos\theta &= \sin\theta \\
 &[\because |\vec{a}| \text{ and } |\vec{b}| \text{ are positive.}] \\
 \Rightarrow \tan\theta &= 1 \\
 \Rightarrow \theta &= \frac{\pi}{4}
 \end{aligned}$$

So that, $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ when θ is equal to $\frac{\pi}{4}$.

12. Option (d) is correct.

Explanation:

$$\text{Let } \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\therefore \vec{a}^2 = x^2 + y^2 + z^2$$

$$\begin{aligned}
 \therefore \vec{a} \times \hat{i} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 0 & 0 \end{vmatrix} \\
 &= \hat{i}[0] - \hat{j}[-z] + \hat{k}[-y] \\
 &= z\hat{j} - y\hat{k}
 \end{aligned}$$

$$\begin{aligned}
 \therefore (\vec{a} \times \hat{i})^2 &= (z\hat{j} - y\hat{k}) \cdot (z\hat{j} - y\hat{k}) \\
 &= y^2 + z^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Similarly, } (\vec{a} \times \hat{j})^2 &= x^2 + z^2 \text{ and } (\vec{a} \times \hat{k})^2 = x^2 + y^2 \\
 (\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 &= y^2 + z^2 + x^2 + z^2 + x^2 + y^2 \\
 &= 2(x^2 + y^2 + z^2) \\
 &= 2\vec{a}^2
 \end{aligned}$$

13. Option (d) is correct.

Explanation:

Here, $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$ [Given]

$$\therefore \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$12 = 10 \times 2 \cos\theta$$

$$\begin{aligned}
 \Rightarrow \cos\theta &= \frac{12}{20} \\
 &= \frac{3}{5}
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \sin\theta &= \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{9}{25}} \\
 \sin\theta &= \pm \frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 \therefore |\vec{a} \times \vec{b}| &= |\vec{a}||\vec{b}|\sin\theta \\
 &= 10 \times 2 \times \frac{4}{5} \\
 &= 16
 \end{aligned}$$

14. Option (b) is correct.

Explanation: The number of vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$ is c (say)

$$\text{i.e., } \vec{c} = \pm(\vec{a} \times \vec{b})$$

So, there will be two vectors of unit length perpendicular to the vectors $\vec{a}$ and $\vec{b}$.

15. Option (a) is correct.

$$\begin{aligned}
 \text{Explanation: } \vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 4 \\ 2 & -1 & 2 \end{vmatrix} \\
 &= -2\hat{i} + 4\hat{j} + 4\hat{k} \\
 |\vec{a} \times \vec{b}| &= \sqrt{4 + 16 + 16} = 6
 \end{aligned}$$

$$\text{Area of the parallelogram} = \frac{|\vec{a} \times \vec{b}|}{2} = 3 \text{ sq. units.}$$

16. Option (c) is correct.

$$\begin{aligned} \text{Explanation: } 11\hat{i} - 3\hat{j} &= \frac{3(12\hat{i} + \mu\hat{j}) + 1(\lambda\hat{i} + 3\hat{j})}{4} \\ 44 &= 36 + \lambda, -12 = 3\mu + 3 \\ \lambda &= 8, \mu = -5 \end{aligned}$$

17. Option (b) is correct.

$$\begin{aligned} \text{Explanation: } (\hat{a} + \hat{b}) \cdot (\hat{a} + \hat{b}) &= |\hat{a}|^2 + |\hat{b}|^2 + 2(\hat{a} \cdot \hat{b}) \\ |\hat{a} + \hat{b}|^2 &= 1 + 1 + 2 \cos \theta \cdot 1 \\ &= 2(1 + \cos \theta) \\ &= 4 \cos^2 \frac{\theta}{2} \\ \therefore |\hat{a} + \hat{b}| &= 2 \cos \frac{\theta}{2} \end{aligned}$$

18. Option (a) is correct.

$$\begin{aligned} \text{Explanation: } \sin \theta &= \frac{|\hat{i} - \hat{j} + \hat{k}|}{\sqrt{2} \cdot \sqrt{2}} \\ \sin \theta &= \frac{\sqrt{3}}{2} \\ \text{or } \theta &= 60^\circ \\ \text{or } \theta &= \frac{\pi}{3} \end{aligned}$$

19. Option (d) is correct.

$$\begin{aligned} \text{Explanation: } \text{Given, } \vec{a} &= \hat{i} + \hat{j} + \hat{k} \\ \vec{a} \cdot \vec{b} &= 1 \end{aligned}$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k}$$

Let

$$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$$

Now,

$$\vec{a} \cdot \vec{b} = 1$$

$$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) \cdot (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) = 1$$

$$\Rightarrow b_1 + b_2 + b_3 = 1 \quad \dots(i)$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k}$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{j} - \hat{k}$$

$$\Rightarrow \hat{i}(b_3 - b_2) - \hat{j}(b_3 - b_1) + \hat{k}(b_2 - b_1) = \hat{j} - \hat{k}$$

On comparing both sides, we get

$$-(b_3 - b_1) = 1 \text{ and } b_2 - b_1 = -1$$

$$\Rightarrow b_3 - b_1 = -1 \text{ and } b_2 - b_1 = -1$$

$$\Rightarrow b_3 = -1 + b_1 \text{ and } b_2 = -1 + b_1 \dots(ii)$$

Now from eq. (i), we get

$$b_1 + (-1 + b_1) + (-1 + b_1) = 1$$

$$\Rightarrow 3b_1 = 3$$

$$\Rightarrow b_1 = 1$$

From eq. (ii), we get

$$b_2 = 0 \text{ and } b_3 = 0$$

$$\therefore \vec{b} = \hat{i}$$

$$\text{Therefore, } |\vec{b}| = 1$$

PHYSICAL WORLD
AND MEASUREMENTS**Units**

- Unit refers to the quantity of a constant magnitude, used to measure the magnitudes of other quantities of the same nature.

SI unit system

- It is based on the following seven basic units and two supplementary units.

SI Units

Name of Quantity	Unit
Length	Metre (m)
Mass	Kilogram (kg)
Time	Second (s)
Electric Current	Ampere (A)
Temperature	Kelvin (K)
Luminous intensity	Candela (cd)
Amount of substance	Mole (mol)

Supplementary Units➤ **Important Results and Formulae :**1. **Fundamental Quantities :**

Fundamental quantity		Unit	Symbol
(a)	Length	Metre	m
(b)	Mass	Kilogram	kg
(c)	Time	Second	s
(d)	Electric current	Ampere	A
(e)	Thermodynamic temperature	Kelvin	K
(f)	Luminous intensity	Candela	cd
(g)	Amount of Substance	Mole	mol

2. **Dimensional Formulae**

Quantity	Symbol	Formula	S.I. Unit	D.F.
Displacement	s	—	Metre or m	$[M^0 L^1 T^0]$
Area	A	$l \times b$	$(\text{Metre})^2$ or m^2	$[M^0 L^2 T^0]$
Volume	V	$l \times b \times h$	$(\text{Metre})^3$ or m^3	$[M^0 L^3 T^0]$
Velocity	v	$v = \frac{\Delta s}{\Delta t}$	m/s	$[M^0 L T^{-1}]$
Momentum	p	$p = mv$	kgm/s	$[MLT^{-1}]$
Acceleration	a	$a = \frac{\Delta v}{\Delta t}$	m/s^2	$[M^0 L T^{-2}]$
Force	F	$F = ma$	Newton or N	$[MLT^{-2}]$
Impulse	I	$F \times t$	N.s	$[MLT^{-1}]$
Work	W	$F.d$	N.m	$[ML^2 T^{-2}]$
Energy	KE	$K.E. = \frac{1}{2}mv^2$	Joule or J	$[ML^2 T^{-2}]$
Potential Energy	U	$U = mgh$		

Name of Quantity	Name of unit
Plane angle	Radian (rad)
Solid Angle	Steradian (sr)

Derived Units

- These units expressed in terms of two or more fundamental units. e.g., Velocity - (m/s), Acceleration - m/s^2 , Pressure - Pa, Force - N or $kgms^{-2}$, etc.

Bigger Units

$$1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$$

$$1 \text{ Parsec} = 3.086 \times 10^{16} \text{ m} = 3.26 \text{ ly}$$

$$1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

$$1 \text{ metric ton} = 10^3 \text{ kg}$$

$$1 \text{ quintal} = 10^2 \text{ kg}$$

- Dimensions** – All physical quantities can be expressed in terms of powers of M, L, K, A are called dimensions.

$$\text{Dimension of } \frac{\text{energy}}{\text{work}} = [M^0 L^0 T^0]$$

$$\text{Dimension of force} = [M^1 L^1 T^{-2}]$$

$$\text{Dimension of voltage} = [M^1 L^2 T^{-3} A^{-1}]$$

Power	P	$P = \frac{W}{t}$	watt or W	$[ML^2T^{-3}]$
Density	d	$d = \text{mass/volume}$	kg/m^3	$[ML^{-3}T^0]$
Pressure	P	$P = F/A$	Pascal or Pa	$[ML^{-1}T^{-2}]$
Torque	τ	$\tau = r \times F$	Nm	$[ML^2T^{-2}]$
Angular displacement	θ	$\theta = \frac{\text{arc}}{\text{radius}}$	radian or rad	$[M^0L^0T^0]$
Angular velocity	ω	$\omega = \frac{\theta}{t}$	rad/s	$[M^0L^0T^{-1}]$
Angular acceleration	α	$\alpha = \frac{\Delta\omega}{\Delta t}$	rad/s^2	$[M^0L^0T^{-2}]$
Moment of Inertia	I	$I = mr^2$	$kg\cdot m^2$	$[ML^2T^0]$
Angular momentum	J or L	$J = mvr$	$\frac{\ddot{u}}{s}^2$	$[ML^2T^{-1}]$
Frequency	ν or f	$f = \frac{1}{T}$	hertz or Hz	$[M^0L^0T^{-1}]$
Stress	—	F/A	N/m^2	$[ML^{-1}T^{-2}]$
Strain	—	$\frac{\Delta l}{l}; \frac{\Delta A}{A}; \frac{\Delta V}{V}$	—	$[M^0L^0T^0]$
Young's modulus (Bulk modulus)	Y	$Y = \frac{F/A}{\Delta l/l}$	N/m^2	$[ML^{-1}T^{-2}]$
Surface tension	T	$\frac{F}{l}$ or $\frac{W}{A}$	$\frac{N}{m}; \frac{J}{m^2}$	$[ML^0T^{-2}]$
Force constant (spring)	k	$k = \frac{F}{m}$	N/m	$[ML^0T^{-2}]$
Coefficient of viscosity	η	$F = \eta \left(\frac{dv}{dx} \right) A$	kg/ms (poise in C.G.S)	$[ML^{-1}T^{-1}]$
Gravitational constant	G	$F = \frac{Gm_1m_2}{r^2}$	$\frac{N\cdot m^2}{kg^2}$	$[M^{-1}L^3T^{-2}]$
Gravitational potential	V_g	$V_g = \frac{PE}{m}$	$\frac{J}{kg}$	$[M^0L^2T^{-2}]$
Temperature	θ	—	Kelvin or K	$[M^0L^0T^0K^1]$
Heat	Q	$Q = m \times S \times \Delta t$	Joule or Calorie	$[ML^2T^{-2}]$
Specific heat	S	$S = \frac{Q}{m\Delta t}$	$\frac{\text{Joule}}{kg \cdot \text{Kelvin}}$	$[M^0L^2T^{-2}K^{-1}]$
Latent heat	L	$L = \frac{Q}{m}$	$\frac{\text{Joule}}{kg}$	$[M^0L^2T^{-2}]$
Coefficient of thermal conductivity	K	$Q = \frac{KA(\theta_1 - \theta_2)t}{d}$	$\frac{\text{Joule}}{msK}$	$[MLT^{-3}K^{-1}]$
Universal gas constant	R	$PV = nRT$	$\frac{\text{Joule}}{mol.K}$	$[ML^2T^{-2}K^{-1}mol^{-1}]$

EXERCISE

- What is the dimension of gravitational constant?
(a) $[M L^3 T^{-2}]$ (b) $[M^{-1} L^3 T^{-2}]$
(c) $[M^2 L^{-2} T^{-2}]$ (d) $[M^2 L^{-1} T^{-2}]$
- Consider the following statements about Light year:
(1) Light year is a unit for measurement of very large distances.
(2) Light year is a unit for measurement of very large time intervals.
(3) Light year is a unit for measurement of intensity of light.
Which of the statements given above is/are correct?
(a) 1, 2 and 3 (b) 2 and 3 only
(c) 1 and 2 only (d) 1 only
- Which one of the following cannot be the unit of frequency of a sound wave?
(a) dB (b) s^{-1} (c) Hz (d) min^{-1}
- The unit of the ratio between thrust and impulse is same as that of
(a) frequency (b) speed
(c) wavelength (d) acceleration
- Which one of the following is NOT the unit of energy?
(a) Joule (b) Watt-hr
(c) Newton-metre (d) $kg\text{-metre/s}^2$
- Light year is a unit for measurement of :
(a) age of universe
(b) very small-time intervals
(c) very high temperature
(d) very large distance
- Which one of the following physical quantities has the same unit as that of pressure?
(a) Angular momentum (b) Stress
(c) Strain (d) Work
- Match List-I with List-II and select the correct answer using the code given below the Lists:

List-I (Physical quantity)		List-II (Unit)	
A.	Distance	(1)	Mole
B.	Amount of material	(2)	Coulomb
C.	Amount of electric charge	(3)	Light year
D.	Energy	(4)	Watt hour

Code:

A	B	C	D
(a) 3	1	2	4
(b) 3	2	1	4
(c) 4	2	1	3
(d) 4	1	2	3
- Which one of the following is the SI unit of the thermal conductivity of a material?
(a) $Wm^{-1}K^{-1}$ (b) $\frac{Wm}{K}$
(c) $\frac{Wm^{-1}}{K^{-1}}$ (d) $Js^{-1}m^{-1}K$
 - In SI unit of force 'Newton' (N) is given by (where m stands for metre and s stands for second):
(a) $1 N = 1 kg/ms^2$ (b) $1 N = 1 kgm/s^2$
(c) $1 N = 1 kg s^2/m$ (d) $1 N = 1 kg m s^2$

- The dimension of 'impulse' is the same as that of :
(a) pressure (b) angular momentum
(c) work (d) linear momentum
- An object is made of two equal parts by volume; one part has density ρ_0 and the other part has density $2\rho_0$. What is the average density of the object?
(a) $3\rho_0$ (b) $\frac{3}{2}\rho_0$ (c) ρ_0 (d) $-\rho_0$
- Which one of the following regarding density of water at atmospheric pressure is correct?
(a) Density of water at $4^\circ C$ is $1000 kg/m^3$.
(b) Density of water at $0^\circ C$ is $1000 kg/m^3$.
(c) Density of water at $0^\circ C$ is $100 kg/m^3$.
(d) Density of water at $4^\circ C$ is $10 kg/m^3$.
- Fundamental laws of physics require
(a) conservation of energy and non-conservation of charge.
(b) conservation of charge and non-conservation of linear momentum.
(c) conservation of charge and non-conservation of energy.
(d) conservation of energy, momentum and charge.
- Density of water is:
(a) maximum at $0^\circ C$ (b) minimum at $0^\circ C$
(c) maximum at $4^\circ C$ (d) minimum at $-4^\circ C$
- Which of the followings is the proper representation of a 10 cm long scale?
(a) The scale of the length is 10 cm.
(b) This is a 10 cm. long scale.
(c) This a 10 c.m. long scale.
(d) The scale of the length is 10 c.m.
- Which one of the following representation of temperature is not correct?
(a) $50^\circ K$ (b) $100^\circ K$ (c) $225^\circ C$ (d) $125^\circ F$
- Which one of the following is correct representation of current?
(a) 5 a (b) 5 Ampere
(c) 5 ampere (d) 5 amperes
- Number of significant figures in 0.0024500 is
(a) 3 (b) 5 (c) 8 (d) 7
- The number of significant figures in 0.06900 is
(a) 5 (b) 4 (c) 2 (d) 3
- The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is
(a) 663.821 (b) 664 (c) 663.8 (d) 663.82
- The mass and volume of a body are 4.237 g and $2.5 cm^3$, respectively. The density of the material of the body in correct significant figures is
(a) $1.6048 g cm^{-3}$ (b) $1.69 g cm^{-3}$
(c) $1.7 g cm^{-3}$ (d) $1.695 g cm^{-3}$
- The numbers 2.745 and 2.735 on rounding off to 3 significant figures will give
(a) 2.75 and 2.74 (b) 2.74 and 2.73
(c) 2.75 and 2.73 (d) 2.74 and 2.74
- The length and breadth of a rectangular sheet are 16.2 cm and 10.1 cm, respectively. The area of the sheet in appropriate significant figures and error is
(a) $164 \pm 3 cm^2$ (b) $163.62 \pm 2.6 cm^2$
(c) $163.6 \pm 2.6 cm^2$ (d) $163.62 \pm 3 cm^2$

25. Which of the following pairs of physical quantities does not have same dimensional formula?
 (a) Work and torque.

- (b) Angular momentum and Planck's constant.
 (c) Tension and surface tension.
 (d) Impulse and linear momentum.

Answer Key

1. (b)	2. (d)	3. (a)	4. (a)	5. (d)	6. (d)	7. (b)	8. (a)	9. (a)	10. (b)
11. (d)	12. (b)	13. (a)	14. (d)	15. (c)	16. (a)	17. (b)	18. (c)	19. (b)	20. (b)
21. (c)	22. (c)	23. (d)	24. (d)	25. (c)					

Answers with Explanations

1. **Option (b) is correct.**

Explanation:

$$\text{Since, } F_g = G \frac{m_1 m_2}{r^2}$$

$$\text{Or } G = \frac{F_g r^2}{m_1 m_2}$$

$$= \frac{[M^1 L^2 T^{-2}][L^2]}{[M^2]}$$

$$= [M^{-1} L^3 T^{-2}]$$

2. **Option (d) is correct.**

Explanation:

- The light-year express astronomical distances and is a unit of length which is equivalent about 9.46 trillion kilometers or 9.461×10^{15} meters.

3. **Option (a) is correct.**

Explanation:

- The frequency of a sound wave refers to the rate of the vibration of the sound travelling through the air.
- So, in sound, the frequency is also known as Pitch and the frequency of the vibrating source of sound is calculated in cycles per second.
- The SI Unit of frequency being hertz which is defined as $\frac{1}{T}$.
- T refers to the time period of the wave.

4. **Option (a) is correct.**

Explanation:

$$\frac{\text{Thrust}}{\text{Impulse}} = \frac{F}{F \times t} = \frac{1}{t}, \text{ this is equivalent to frequency.}$$

5. **Option (d) is correct.**

Explanation: The unit kg-meter/sec² is equivalent to newton, which is unit of force and not energy.

6. **Option (d) is correct.**

Explanation: A light year is a measurement of distance and not time. So, a light year is the distance a beam of light travels in a single Earth year. 1 ly = 9.46 trillion kilometers = 9.46×10^{12} km or 5.88 trillion miles = 5.88×10^{12} miles.

7. **Option (b) is correct.**

Explanation: Pressure and stress both are the ratio of force and surface area, so their dimensions are also same.

Strain is a dimension less and unit less quantity.

8. **Option (a) is correct.**

Explanation: Option (a) correctly matches the physical quantities to their respective units.

9. **Option (a) is correct.**

Explanation: Thermal conductivity is the measure of the ability of a material to transfer heat. Its SI unit is watts per meter-kelvin ($\text{Wm}^{-1}\text{K}^{-1}$).

10. **Option (b) is correct.**

Explanation: SI unit of force kg m/s²

One newton is the force needed to accelerate one kilogram of mass at the rate of one meter per second squared in the direction of the applied force. Hence, 1 N = 1 kg m/s²

11. **Option (d) is correct.**

Explanation: Impulse $I = F \times t$, $= mv$ hence impulse and linear momentum have same dimensions.

12. **Option (b) is correct.**

Explanation:

Given, $V_2 = V_1 = V$

$$\rho_1 = \rho_0, \rho_2 = 2\rho_0$$

$$\text{Average density} = \frac{\text{Total mass}}{\text{Total volume}}$$

$$\frac{(m_1 \quad m_2)}{(V_1 \quad V_2)} = \frac{(\rho_1 V_1 + \rho_2 V_2)}{(V_1 + V_2)}$$

$$= \frac{(\rho_0 V + 2\rho_0 V)}{(V + V)} = \frac{3}{2\rho_0}$$

13. **Option (a) is correct.**

Explanation: We know density of water on 4°C is maximum. i.e., 1gm/cm³ or 1000 kg/m³

14. **Option (d) is correct.**

Explanation: Fundamental laws of physics require conservation of energy, linear and angular momentum and charge.

15. **Option (c) is correct.**

Explanation: Ice is less dense than water, as ice molecules arrange themselves in a rigid tetrahedral structure because to which cage like empty spaces remain in their bonding. On increasing the temperature, the ice melts and the water molecules come closer to each other due to which its volume decreases and hence density increases the density is maximum at 4°C.

16. **Option (a) is correct.**

Explanation: While units are symbolically represented, there should not be any dots. But if it appears at the ends of a sentence, then a dot may appear as a full-stop.

17. Option (a,b) is correct.

Explanation: While representing temperature in Kelvin scale, °K should not be written.

18. Option (c) is correct.

Explanation: When symbolically written, the first letter of the unit in the name of scientist should be a capital letter. But when the unit is written in complete word form then it should be written in small letters.

19. Option (b) is correct.

Explanation: All the zeros on the right immediate after the decimal are not significant.

All the zeros those are on right of the last non-zero digit, after the decimal point, are significant.

20. Option (b) is correct.**21. Option (c) is correct.****22. Option (c) is correct.**

Explanation:

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{4.237\text{g}}{2.5\text{cm}^3} = 1.6948\text{gcm}^{-3}$$

23. Option (d) is correct.**24. Option (d) is correct.**

Explanation: $l = 16.2 \pm 0.1\text{cm} = 16.2\text{cm} \pm .6\%$

$$b = 10.1 \pm 0.1\text{cm} = 10.1\text{cm} \pm 1\%$$

$$A = l \times b = 163.62 \pm 1.6\%$$

$$= 163.62 \pm 2.6$$

$$= 163.62 \pm 3\text{cm}^2$$

25. Option (c) is correct.

Explanation:

$$\text{Tension} = \text{Force} = [\text{MLT}^{-2}]$$

$$\text{Surface Tension} = \frac{\text{Force}}{\text{length}} = \frac{[\text{MLT}^{-2}]}{[\text{L}]}$$

$$= [\text{ML}^0\text{T}^{-2}]$$

Chapter 32

KINEMATICS & LAWS OF MOTION

In physics, we use various physical quantities, which can be classified as scalars and vectors.

- **Scalar quantities** – These are the physical quantities that have magnitude only, e.g. – speed, mass, volume, work, time, distance, power, energy, etc.
- **Vector quantities** – These are physical quantities that have magnitude and direction both, e.g. displacement, velocity, acceleration, momentum, force, etc.

Distance

- The length of the path covered by an object, or body is called distance. It is always positive.
- SI unit of distance is metre.
- Average speed = $\frac{\text{Total distance travelled}}{\text{Time taken}}$

Displacement

- The shortest distance between the final and initial position of an object is called displacement.
- It may be positive, negative or zero.
- SI unit of displacement is metre.
- Displacement $\leq$ Distance

Speed

- When a moving body returns to its starting point, then its effective displacement is zero.
- The distance covered by a moving body in a unit time interval is the speed.
- It is always equal to or greater than the magnitude of the velocity.

Velocity

- The rate of change of displacement is called velocity.
- It is a vector quantity.
- Its SI unit is m/s
- Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}}$
- Relative velocity is the rate of change of displacement of one body relative to that of another and vice-versa.

Acceleration

- It is the rate of change of velocity. Its SI unit is m/s^2 . It is a vector quantity.
- It can be positive or negative. Negative acceleration is called retardation
- Equation of Motion – for a straight line motion with initial velocity (u), final velocity (v) and constant acceleration (a), equations of motion are:
 - (i) $v = u + at$
 - (ii) $s = ut + \frac{1}{2}at^2$
 - (iii) $v^2 = u^2 + 2as$
- Slope of displacement – time graph give velocity.
- Slope of distance – time graph give speed.
- Slope of $u - t$ graph gives acceleration.
- Area under $u - t$ graph gives distance.
- Area under $v - t$ graph gives displacement.
- Area under $a - t$ graph gives velocity.

- In case of straight line motion displacement and distance are equal.
- **Forces** are an external effort in the form of push or pull which try to produce motion in a body at rest or stops or try to stop a moving body or can change or try to change the direction of motion of the body.
- **Impulse:** When a large force acts on a body for a short time, then the measure of the total effect of force is called impulse of force.

$$\text{Impulse} = \text{Force} \times \text{Time} = F \times \Delta t$$

Newton's Laws of Motion

- **First law** – Every object maintains the state of rest or uniform motion unless non-zero external force acts on it. It is called Galileo's law of inertia. For example – While jumping from a moving bus or train, one must follow the direction of motion for a while.
- **Second law** – The force acting upon an object is directly proportional to the product of masses of the object and the acceleration product on it.
i.e. $F = \frac{dP}{dt} = \frac{mdv}{dt} = ma$
- **Third law** – Every action has an equal and opposite reaction.

For example: Rocket moves up due to reaction of downward ejection of gases.

Circular Motion

- When an object moves in a circular path, its motion is called circular motion.
- Tangent at a circle gives the direction of motion at that point in a circular motion.
- Centripetal force is the external force required to act radially inward over the circular motion of the body.
- Centrifugal force is such a pseudo force that is equal and opposite to centripetal force.
- Uniform circular motion is the motion when a point object is moving on a circular path at a constant speed.

$$\text{Speed} = \frac{2\pi r}{T}$$

Friction

Friction is a force that opposes motion. It is set-up between the surface of contact of two bodies when one body slides on rolls or tends to do so on the surface of another body.

- We are able to move on the surface of Earth due to friction.
- The brakes stop, the automobile because of friction.

➤ Important Results and Formulae :

1. Newton's Laws :

(i) **First Law** : It gives the concept of inertia. i.e., mass is the measure of the inertia of the body.(ii) **Second Law** :

$$\vec{F} \propto \frac{d\vec{p}}{dt}$$

$$\vec{F} = \frac{m d\vec{v}}{dt}$$

$$\vec{F} = m\vec{a}$$

(ii) **Third Law** :

$$\vec{F}_{21} = -\vec{F}_{12}$$

2. Linear Momentum :

$$\vec{p} = m\vec{v}$$

Conservation of momentum :

$$\Delta \vec{p} = \text{Constant}$$

$$\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$$

3. Impulse :

$$I = F_{\text{avg}} \times t$$

$$I = p_2 - p_1$$

4. Types of Frictional Forces :

1. Static frictional force
2. Kinetic frictional force
3. Rolling frictional force

5. Angle of friction (θ) and angle of repose (ϕ) :

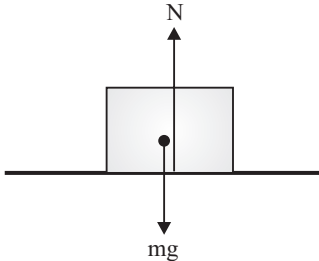
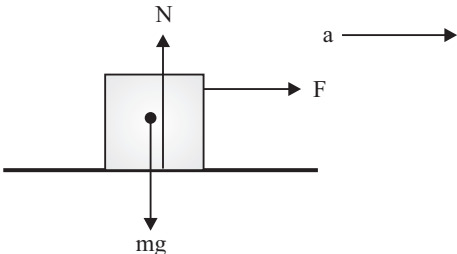
$$\tan \theta = \mu_s$$

$$\tan \phi = \mu_s$$

$$\text{i.e., } \theta = \phi$$

6. Circular Motion :

Different Cases for Application of Newton's Second and Third Law (Without Friction) :

Case	Equation
	$N = mg$
	$N = mg$ $F = ma$

$$\text{Angular Displacement} = \frac{\text{arc}}{\text{radius}}$$

$$\theta = \frac{l}{r}$$

$$\text{Angular Velocity, } \omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation Between Angular Velocity and Linear Velocity,

$$\omega = \frac{v}{r}$$

$$v = r\omega$$

Time Period of Uniform Circular motion,

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Centripetal Acceleration,

$$a = \frac{v^2}{r}$$

$$a = r\omega^2$$

Centripetal Force

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

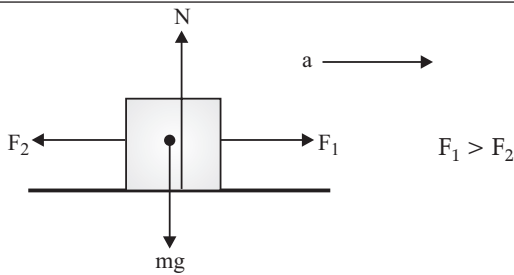
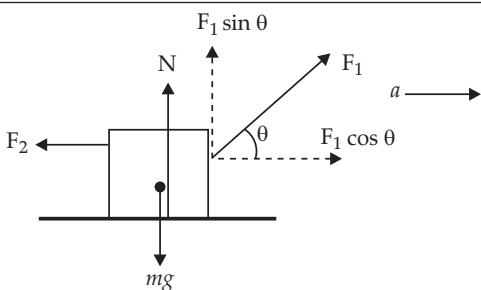
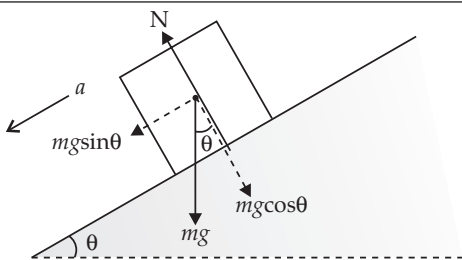
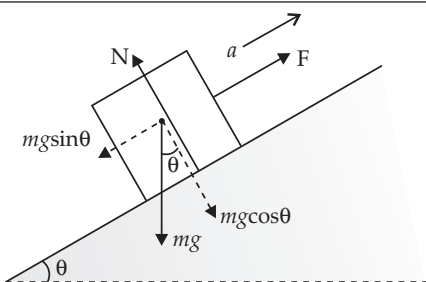
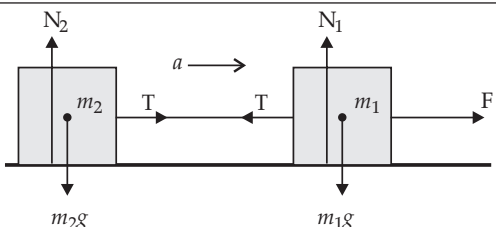
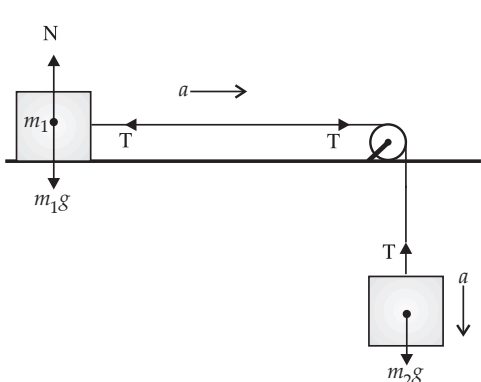
8. Condition for Banking of Road :

Without friction

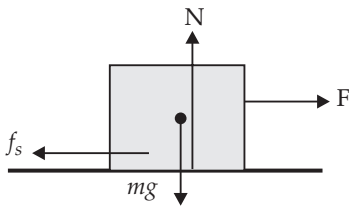
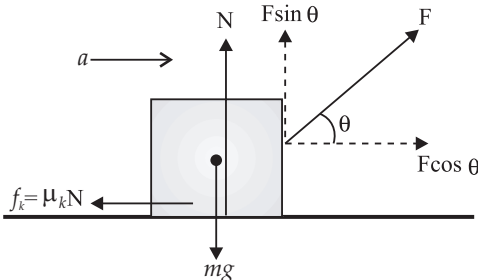
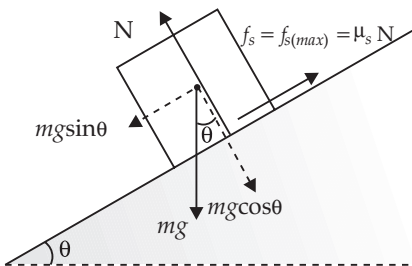
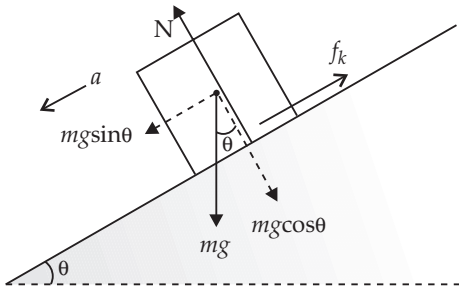
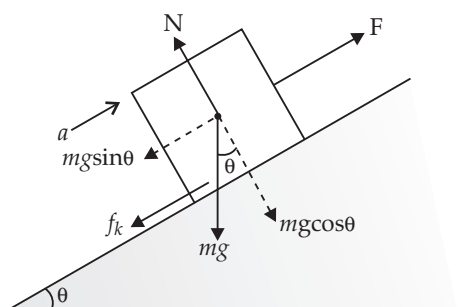
$$\tan \theta = \frac{v^2}{rg}$$

With Friction

$$v_{\text{max}} = \left(r_s \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta} \right)^{\frac{1}{2}}$$

	$N = mg$ $F_1 - F_2 = ma$
	$N + F_1 \sin \theta = mg$ $F_1 \cos \theta - F_2 = ma$
	$N = mg \cos \theta$ $mg \sin \theta = ma$
	$N = mg \cos \theta$ $F - mg \sin \theta = ma$
	$N_1 = m_1 g$ $N_2 = m_2 g$ $F - T = m_1 a$ $T = m_2 a$
	$N = m_1 g$ $T = m_1 a$ $m_2 g - T = m_2 a$

Different Cases for Application of Newton's Second and Third Law (With Friction) :

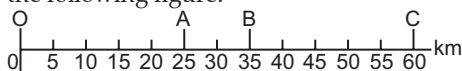
Case	Equations
	$N = mg$ $F = f_s = \mu_s N$
	$N + F \sin \theta = mg$ $F \cos \theta - f_k = ma$ $F \cos \theta - \mu_k N = ma$
	$N = mg \cos \theta$ $mg \sin \theta = f_s = \mu_s N$
	$N = mg \cos \theta$ $mg \sin \theta - f_k = ma$ $mg \sin \theta - \mu_k N = ma$
	$N = mg \cos \theta$ $F - f_k - mg \sin \theta = ma$ $F - \mu_k N - mg \sin \theta = ma$

EXERCISE

1. Which one of the following statements about speed and velocity is correct?
- (a) Speed and velocity both are vector quantities.
- (b) Speed and velocity both are scalar quantities.

- (c) Speed is vector quantity and velocity is scalar quantity.
- (d) Speed is scalar quantity and velocity is vector quantity.

2. The motion of a car along a straight path is shown by the following figure:



The car starts from O and reaches at A, B and C at different instants of time. During its motion from O to C and back to B, the distance covered and the magnitude of the displacement are, respectively.

- (a) 25 km and 60 km (b) 95 km and 35 km
(c) 60 km and 25 km (d) 85 km and 35 km
3. Pressure is a scalar quantity because:
(a) it is the ratio of force to area and both force and area are vectors.
(b) it is the ratio of magnitude of force to area.
(c) it is the ratio of component of force (normal to area) to area.
(d) none of the above
4. If an object moves at a non-zero constant acceleration for a certain interval of time, then the distance it covers in that time
(a) depends on its initial velocity.
(b) is independent of its initial velocity.
(c) increases linearly with time.
(d) depends on its initial displacement.
5. A car starts from Bengaluru, goes 50 km in a straight line towards south, immediately turns around and returns to Bengaluru. The time taken for this round trip is 2 hours. The magnitude of the average velocity of the car for this round trip
(a) is 0
(b) is 50 km/hr
(c) is 25 km/hr
(d) cannot be calculated without knowing acceleration
6. The speed of a car travelling on a straight road is listed below at successive intervals of 1 s:

Time (s)	0	1	2	3	4
Speed (m/s)	0	2	4	6	8

Which of the following is/are correct? The car travels

- (1) with a uniform acceleration of 2 m/s^2 .
(2) 16 m in 4 s.
(3) with an average speed of 4 m/s.
Select the correct answer using the code given below:
(a) 1, 2 and 3 (b) 2 and 3 only
(c) 1 and 2 only (d) 1 only
7. A particle is moving with uniform acceleration along a straight line ABC, where $AB = BC$. The average velocity of the particle from A to B is 10 m/s and from B to C is 15 m/s. The average velocity for the whole journey from A to C in m/s is :
(a) 12 (b) 12.5 (c) 13 (d) 13.5
8. A ball is thrown vertically upward with a speed of 40 m/s. The time taken by the ball to reach the maximum height would be approximately :
(a) 2 s (b) 3 s (c) 4 s (d) 5 s
9. A racing car accelerates on a straight road from rest to a speed of 50 m/s in 25 s. Assuming uniform acceleration of the car throughout, the distance covered in this time will be :
(a) 625 m (b) 1250 m (c) 2500 m (d) 50 m

10. A brass ball is tied to a thin wire and swung so as to move uniformly in a horizontal circle. Which of the following statements in this regard is/are true?

- (1) The ball moves with constant velocity.
(2) The ball moves with constant speed.
(3) The ball moves with constant acceleration.
(4) The magnitude of the acceleration of the ball is constant.

Select the correct answer using the code given below:

- (a) 1 only (b) 1 and 3 (c) 1, 2 and 4 (d) 2 and 4 only

11. **Statement I:** A body moving in a circular path is acted upon by the centripetal force.

Statement II: Centripetal force acting on the body is doing work to keep it rotating in the circular path.

12. A bullet is fired vertically up from a 400 m tall tower with a speed 80 m/s. If g is taken as 10 m/s^2 , the time taken by the bullet to reach the ground will be :
(a) 8 s (b) 16 s (c) 20 s (d) 24 s

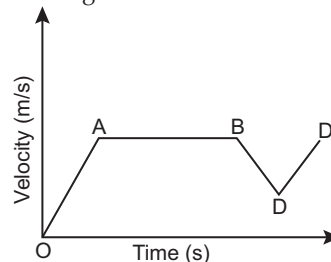
13. What is the nature of velocity-time graph for a car moving with uniform acceleration?

- (a) Parabola (b) Logarithmic
(c) Straight line (d) Exponential

14. An object is moving with uniform acceleration a . Its initial velocity is u and after time t its velocity is v . The equation of its motion is $v = u + at$. The velocity (along y -axis) time (along x -axis) graph shall be a straight line.

- (a) passing through origin (b) with x -intercept u
(c) with y -intercept u (d) with slope u

15. The following figure represents the velocity-time graph of a moving car on a road:



Which segment of the graph represents the retardation?

- (a) AB (b) BC (c) CD (d) None

16. A 5 N force is defined when a mass of 10 kg is accelerated with :

- (a) 5.0 cm/s^2 (b) 0.5 m/s^2
(c) 0.5 cm/s^2 (d) 5.0 m/s^2

17. The statement "friction force is a contact force while magnetic force is a non-contact force" is :

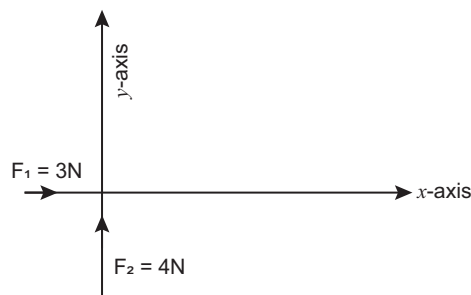
- (a) always true.
(b) true only at 0°C .
(c) a false statement.
(d) either true or false depending upon the temperature of the surroundings.

18. Weight and mass of an object are defined with Newton's laws of motion. Which among the following is true?

- (a) Weight is a constant of proportionality.
(b) Mass is a constant of proportionality.
(c) Mass is not a constant of proportionality.
(d) Weight is a universal constant.

19. Which one of the following is not a conservative force?
 (a) Frictional force (b) Electric force
 (c) Gravitational force (d) Spring force
20. Which one of the following has maximum inertia?
 (a) An atom (b) A molecule
 (c) A one-rupee coin (d) A cricket ball
21. A ball balanced on a vertical rod is an example of :
 (a) stable equilibrium
 (b) unstable equilibrium
 (c) neutral equilibrium
 (d) perfect equilibrium
22. An object moves in a circular path with a constant speed. Which one of the following statements is correct?
 (a) The centripetal acceleration of the object is smaller for a gentle curve (i.e., curve of larger radius) than that for a sharp curve (i.e., curve of smaller radius).
 (b) The centripetal acceleration is greater for a gentle curve than that for a sharp curve.
 (c) The centripetal acceleration is the same for both, the gentle and sharp curves.
 (d) The centripetal acceleration causes the object to slow down.
23. When a force of 1 newton acts on a mass of 1 kg which is able to move freely, the object moves in the direction of force with a/an.
 (a) speed of 1 km/s
 (b) acceleration of 1 m/s²
 (c) speed of 1 m/s
 (d) acceleration of 1 km/s²
24. The displacement-time graph of a particle acted upon by a constant force is :
 (a) a straight line
 (b) a circle
 (c) a parabola
 (d) any curve depending upon initial condition
25. A brass ball is tied to a thin wire and swung so as to move uniformly in a horizontal circle. Which of the following statements in this regard is/ are true?
 (1) The ball moves with constant velocity.
 (2) The ball moves with constant speed.
 (3) The ball moves with constant acceleration.
 (4) The magnitude of the acceleration of the ball is constant.
- Select the correct answer using the code given below:
 (a) 1 only (b) 1 and 3

- (c) 1, 2 and 4 (d) 2 and 4 only
26. Two forces, one of 3 newton and another of 4 newton are applied on a standard 1 kg body, placed on a horizontal and frictionless surface, simultaneously along the x-axis and the y-axis, respectively, as shown below:



The magnitude of the resultant acceleration is:

- (a) 7 m/s² (b) 1 m/s²
 (c) 5 m/s² (d) $\sqrt{7}$ m/s²
27. A passenger in a moving train tosses a coin upward which falls behind him. It implies that the motion of the train is :
 (a) accelerated
 (b) uniform
 (c) retarded
 (d) along the circular tracks
28. **Statement I:** Body moving in a circular path is acted upon by the centripetal force.
Statement II: Centripetal force acting on the body is doing work to keep it rotating in the circular path.
Codes:
 (a) Both the statements are individually true and Statement II is the correct explanation of Statement I.
 (b) Both the statements are individually true but Statement II is not the correct explanation of Statement I.
 (c) Statement I is true but Statement II is false.
 (d) Statement I is false but Statement II is true.
29. A force applied on a body is represented as

$$\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$$
 and accelerates it at 1 m/s². The mass of the body is :
 (a) 10 kg (b) $10\sqrt{2}$ kg (c) $2\sqrt{10}$ kg (d) 8 kg
30. Application of lubricants cannot reduce
 (a) Static friction (b) Sliding friction
 (c) Rolling friction (d) Inertia

Answer Key

1. (d)	2. (d)	3. (c)	4. (a)	5. (a)	6. (a)	7. (a)	8. (c)	9. (a)	10. (d)
11. (c)	12. (c)	13. (c)	14. (c)	15. (b)	16. (b)	17. (a)	18. (b)	19. (a)	20. (d)
21. (b)	22. (a)	23. (b)	24. (c)	25. (d)	26. (c)	27. (a)	28. (c)	29. (b)	30. (d)

Answers with Explanations

1. Option (d) is correct.

Explanation: Speed is a scalar quantity. Velocity of an object is its speed in a particular direction, so it is a vector quantity.

2. Option (d) is correct.

Explanation: Displacement = shortest distance between initial point O and final point B = 35 km
 Distance = OC + BC = 60 + (60 - 35)

$$= 60 + 25 = 85 \text{ km}$$

3. **Option (c) is correct.**

Explanation:

Pressure (P) = component of force normal to the surface/contact area

Here, both the denominator and the numerator are scalar quantities, so their ratio, pressure is also a scalar quantity.

4. **Option (a) is correct.**

Explanation:

$$As, = ut + \frac{1}{2}at^2$$

so, distance depends upon initial velocity and time.

5. **Option (a) is correct.**

Explanation:

Average velocity = Total displacement / Total Time

$$\text{Average Velocity} = \frac{(50 - 50)}{2} = 0$$

6. **Option (a) is correct.**

Explanation: Speed of vehicle is increasing equally in equal interval of time, hence acceleration is uniform.

Area under speed time graph given distance, distance = $\frac{1}{2} \times 8 \times 4 = 16\text{m}$

$$\text{average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{16}{4} = 4\text{ms}^{-1}$$

All three statements are correct, hence option (a) is the correct answer.

7. **Option (a) is correct.**

Explanation: Let AB = BC = x

$$\text{Time to reach B from A} = t_1 = \frac{x}{10} = 0.1x$$

$$\text{Time to reach C from B} = t_2 = \frac{x}{15} = 0.067x$$

Average velocity between A and C

$$= (x + x)(0.1x + 0.067x) = 12 \text{ m/s}$$

8. **Option (c) is correct.**

Explanation:

Here, $u = 40 \text{ m/s}$, At maximum height velocity = $v = 0 \text{ m/s}$, $a = -10 \text{ m/s}^2$

Applying $v = u + at$

$$\Rightarrow 0 = 40 + (-10)t$$

$$\Rightarrow t = 4\text{s}$$

In 4 s ball will reach at the maximum height.

9. **Option (a) is correct.**

Explanation:

$u = 0$, $v = 50 \text{ m/s}$, $t = 25 \text{ s}$

$$a = \frac{v - u}{t} = \frac{50 - 0}{25} = 2 \text{ m/s}^2$$

Using second equation of motion, $s = ut + \frac{1}{2}at^2$

$$s = 0 \times 25 + \frac{1}{2} \times 2 \times 25 \times 25$$

$$s = 625 \text{ m}$$

10. **Option (d) is correct.**

Explanation: In uniform horizontal circular motion, speed of an object remains same but the direction of motion changes at every point. So, velocity is changing continuously.

Since, velocity is different at every point, this means due to change in direction acceleration is also changing at every point but its magnitude remains same.

11. **Option (c) is correct.**

Explanation: An object moving in a circular path, is acted upon by a force directed towards the center of the path. This force is known as centripetal force and is responsible for holding the object in curved path. So, statement I is correct.

Centripetal force acts towards the center of a circular path, while velocity or displacement at any point is along the tangential direction. So, both force and displacement are perpendicular to each other. Hence, work done by the centripetal force is zero. $W = Fs = Fs \cos \theta = Fs \cos 90^\circ = 0$. So, statement II is incorrect.

12. **Option (c) is correct.**

Explanation:

$$u = 80 \text{ m/s}, v = 0, a = -10 \text{ m/s}^2$$

$$\text{Net displacement } s = -400 \text{ m}$$

$$\text{Applying 2}^{\text{nd}} \text{ equation of motion } S = ut + \frac{1}{2}at^2$$

$$-400 = 80 \times t + \frac{1}{2}(-10) \times t^2$$

$$t^2 - 16t - 80 = 0$$

$$t = 20 \text{ s}$$

Time taken to reach ground: 20 seconds

13. **Option (c) is correct.**

Explanation: Uniform or constant acceleration means rate of change of velocity is constant. This is possible only when graph of velocity-time graph is a straight line.

14. **Option (c) is correct.**

Explanation: Since, object does have initial velocity so it will start from a point above origin on y axis i.e., intercepts y axis.

15. **Option (b) is correct.**

Explanation: In segment BC velocity is decreasing with time which means acceleration is negative. In segment AB velocity is constant so acceleration is zero. While in segments OA and CD acceleration is positive.

16. **Option (b) is correct.**

Explanation:

Here, $F = 5 \text{ N}$, $m = 10 \text{ kg}$, $a = ?$

$$a = \frac{F}{m} = \frac{5}{10} = 0.5 \text{ ms}^{-2}$$

17. **Option (a) is correct.**

Explanation: Friction is a contact surface phenomenon. It works only when surfaces are in contact with each other. Magnetic force does not depend upon surface in contact, it works till an object is in the influence of a magnetic field.

18. Option (b) is correct.

Explanation: When an object undergoes circular motion, speed remains same but its direction changes continuously. Due to the change in direction of speed its velocity changes at every point. Since momentum depends upon velocity, it also changes.

19. Option (a) is correct.

Explanation: Work done under the action of a conservative force depends only upon the initial and final position of an object and is independent of the path. Friction is a surface phenomenon, so it is not a conservative force.

20. Option (d) is correct.

Explanation: Inertia $\propto$ Mass of object

21. Option (b) is correct.

Explanation: If we touch the ball, it will fall from its place, hence it is unstable equilibrium, in case of stable equilibrium ball will try to reach at its original position again.

22. Option (a) is correct.

Explanation: Centripetal force is given by, $a = \frac{v^2}{r}$, we can see if radius is more, acceleration would be less.

23. Option (b) is correct.

Explanation:

$$a = \frac{F}{m} = \frac{1}{1} = 1 \text{ m/s}^2$$

24. Option (b) is correct.

Explanation: As constant force is acting on an object, it will keep on increasing its displacement. Hence, we will get parabola.

25. Option (d) is correct.

Explanation: Here, ball moves with constant speed but due to the change in direction the velocity will be different. In uniform circular motion, acceleration is always constant and is directed towards the center.

26. Option (c) is correct.

Explanation: net force =

$$\sqrt{3^2 + 4^2} + 2.3.4 \cdot \cos 90 = \sqrt{9 + 16} = 5 \text{ N} / 5^2$$

27. Option (a) is correct.

Explanation: When coin is in hand, both move with same velocity. When coin is in the air, the passenger moves forward, it means his speed has increased but the coin falls behind it means train is accelerating.

28. Option (c) is correct.

Explanation: Since direction of centripetal force and displacement is always perpendicular, work done by this force is always zero.

29. Option (b) is correct.

Explanation:

$$m = \frac{F}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = 10\sqrt{2} \text{ kg m/s}^2$$

30. Option (d) is correct.

Work

- Work is said to be done, if force acting on a body can move it through some distance in the direction of the force.
- SI unit of work is a joule
- $W = Fs \cos \theta$, Where, F = force, s = displacement and θ is the angle between the direction of force and displacement.
- If $\theta > 90^\circ$, then work will be negative.
- If $\theta < 90^\circ$, then work will be positive.
- If $\theta = 90^\circ$, then work will be zero.
- If a coolie carrying a load on his head is moving on a horizontal platform, then theoretically is not doing any work because $\theta = 90^\circ$, $W = Fs \cos 90^\circ = 0$

Energy

- Energy is the capacity of doing work by an object.
- It is a scalar quantity.
- SI unit of energy is Joule (J)
- Mechanical energy is of two type
 - **Kinetic Energy (K):** It is the energy possessed by a body due to its motion.
 - $K = \frac{1}{2} mv^2$ Where, m is mass and v is the velocity.
 - **Potential Energy (U):** Energy possessed by the body due to virtue of its position. Potential energy does not depend upon the path of the object. It is given by, $U = mgh$
- **The law of conservation of energy** states that the sum of all kinds of energy in an isolated system remains constant.
- **The law of conservation of linear momentum:** In the absence of any external force, total linear momentum of a system remains conserved.

Power

- Power is the rate of doing work.
- SI unit of power is watt.
- Power = Work done/Time Taken
 - 1 watt hour = 3600 joule
 - 1 kilowatt-hour = 3.6×10^6 joule
 - 1 HP = 746 watt

➤ Important Results and Formulae :

1. Different Formulae for Work Done :

$$W = (F \cos \theta) \times s$$

$$W = F s \cos \theta$$

$$W = \vec{F} \cdot \vec{s}$$

$$dW = \vec{F} \cdot d\vec{s}$$

$$W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$$

$$W = \int_{t_1}^{t_2} \vec{F} \cdot \vec{v} dt$$

2. Different cases of Work Done :

Positive Work Done :

$$\text{if, } \theta = 0^\circ$$

$$W = F s \cos \theta$$

$$W = F s \cos 0^\circ$$

$$W = F s$$

Zero Work Done :

$$\text{if, } \theta = 90^\circ$$

$$W = F s \cos \theta$$

$$W = F s \cos 90^\circ$$

$$W = 0$$

Negative Work Done :

$$\text{if, } \theta = 180^\circ$$

$$W = F s \cos \theta$$

$$W = F s \cos 180^\circ$$

$$W = -F s$$

3. Kinetic Energy :

$$\text{K.E.} = \frac{1}{2} m v^2$$

$$K = \frac{p^2}{2m} \quad [\because p = mv]$$

4. Potential Energy :

$$\text{P.E.} = mgh$$

5. Restoring Force and Elastic Potential energy :

$$F_R = kx$$

$$\text{E.P.E.} = \frac{1}{2} kx^2$$

6. Work Energy theorem :

$$W = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$W = K_2 - K_1$$

$$W = \Delta K$$

7. Power :

$$P_{\text{avg}} = \frac{\text{Total work done}}{\text{Total time taken}} = \frac{\Delta W}{\Delta t}$$

$$P_{\text{inst}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$P_{\text{inst}} = \vec{F} \cdot \vec{v}$$

8. Coefficient of Restitution :

$$e = 1 \text{ for perfectly elastic collision}$$

$$e = 0 \text{ for perfectly inelastic collision}$$

$$e < 1 \text{ for perfectly elastic and inelastic collision}$$

9. Elastic Collision in 1-D

Velocities of both bodies after collision

$$v_1 = \frac{(m_1 - m_2)u_1}{(m_1 + m_2)} + \frac{2m_2u_2}{(m_1 + m_2)}$$

$$v_2 = \frac{(m_2 - m_1)u_2}{(m_1 + m_2)} + \frac{2m_1u_1}{(m_1 + m_2)}$$

10. Elastic collision in 2-D:

Resultant of momentum along x -axis = Resultant of momentum along x -axis and

Resultant of momentum along y -axis = Resultant of momentum along y -axis

$$\text{So, } m_1u_1\cos\theta_1 + m_2u_2\cos\theta_2 = m_1v_1\cos\phi_1 + m_2v_2\cos\phi_2$$

$$m_1u_1\sin\theta_1 + m_2u_2\sin\theta_2 = m_1v_1\sin\phi_1 + m_2v_2\sin\phi_2$$

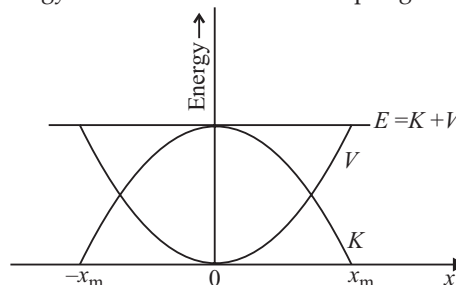
11. Perfectly Inelastic Collision on 1-D

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

$$\text{Loss of KE} = \frac{m_1m_2(u_1 - u_2)^2}{2(m_1 + m_2)}$$

➤ Important Graphs and Figures :

Parabolic plots of the potential energy V and kinetic energy K of a block attached to a spring.

**EXERCISE**

- A mass M is dragged by a pulley on a horizontal plane by a force anti-parallel to its displacement. The work done in pulling the mass M is :
(a) zero (b) positive
(c) infinite (d) negative
- Work is said to be one Joule when a force of :
(a) 4 N moves an object by 25 cm.
(b) 2 N moves an object by 1 m.
(c) 1 N moves an object by 1 cm.
(d) 1 N moves an object by 50 cm.
- An object of mass 2000 g possesses 100 J kinetic energy. The object must be moving with a speed of :
(a) 10.0 m/s (b) 11.1 m/s
(c) 11.2 m/s (d) 12.1 m/s
- When a ball bounces off the ground, which of the following changes suddenly?
(Assume no loss of energy to the floor)
(a) Its speed (b) Its momentum
(c) Its kinetic energy (d) Its potential energy
- How is the kinetic energy of a moving object affected if the net work done on it is positive?
(a) Decreases (b) Increases
(c) Remains constant (d) Becomes zero
- Conservation of momentum in a collision between particles can be understood on the basis of:
(a) Newton's first law of motion.
(b) Newton's second law of motion only.
(c) Both Newton's second law of motion and Newton's third law of motion.
(d) Conservation of energy
- A boy of mass 52 kg jumps with a horizontal velocity of 2 m/s onto a stationary cart of mass 3 kg. The cart is fixed with frictionless wheels. Which one of the following would be the speed of the cart?
(a) 2.15 m/s (b) 1.89 m/s
(c) 1.51 m/s (d) 2.51 m/s
- The energy possessed by a body due to its change in position or shape is called :
(a) thermal energy (b) potential energy
(c) kinetic energy (d) electric energy
- A bullet of mass 10 g is horizontally fired with velocity 300 m s^{-1} from a pistol of mass 1 kg. What is the recoil velocity of the pistol?
(a) 0.3 ms^{-1} (b) 3 ms^{-1}
(c) -3 ms^{-1} (d) -0.3 ms^{-1}
- Which one among the following happens when a swing rises to a certain height from its rest position?
(a) Its potential energy decreases while kinetic energy increases.
(b) Its kinetic energy decreases while potential energy increases.
(c) Both potential and kinetic energy decrease.
(d) Both potential and kinetic energy increase.
- The correct sequence of energy transfer that occurs when an apple falls to the ground is
(a) Gravitational potential energy → heat energy to air → kinetic energy → heat energy to ground and apple → sound energy
(b) Gravitational potential energy → sound energy → kinetic energy → heat energy to air → heat energy to ground and apple
(c) Gravitational potential energy → kinetic energy → heat energy to air → heat energy to ground and apple → sound energy
(d) Gravitational potential energy → kinetic energy → sound energy → heat energy to air → heat energy to ground and apple
- Which one of the following is not a form of stored energy?
(a) Nuclear energy (b) Potential energy
(c) Electrical energy (d) Chemical energy
- A body has a free fall from a height of 20 m. After falling through a distance of 5 m, the body would :
(a) lose one-fourth of its total energy.
(b) lose one-fourth of its potential energy.
(c) gain one-fourth of its potential energy.
(d) gain three-fourth of its total energy.
- Two bodies A and B are moving with equal velocities. The mass of B is double that of A. In this context, which one of the following statements is correct?
(a) Momentum of B will be double that of A.

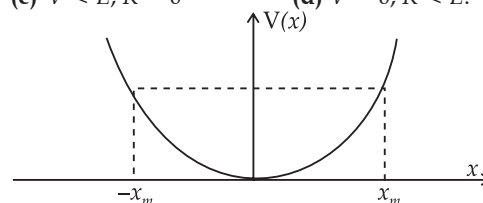
- (b) Momentum of A will be double that of B.
 (c) Momentum of B will be four times that of A.
 (d) Momenta of both A and B will be equal.
15. A cyclist comes to a skidding stop in 20 m. During this process, the force on the cycle due to the road is 100 N and is directly opposed to the motion. Work done by the road on the cycle is
 (a) -2000 J (b) 2000 J (c) 1000 J (d) 100 J
16. A ball is dropped from a height of 1 m. If the coefficient of restitution between the surface and ball is 0.6, the ball rebounds to a height of
 (a) 0.6 m (b) 0.36 m (c) 3.6 m (d) 1 m
17. A body of mass 100 g falls from a height of 10 m. Its increase in kinetic energy is
 (a) 9800 J (b) 9.8 J (c) 980 J (d) 100 J
18. 1 joule = 1 _____
 (a) Nm (b) Nm² (c) N²m² (d) N²m
19. If momentum rises by 20%, kinetic energy rises by
 (a) 44% (b) 40% (c) 88% (d) 20%
20. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is because,
 (a) the two magnetic forces are equal and opposite, so they produce no net effect.
 (b) the magnetic forces do not work on each particle.
 (c) the magnetic forces do equal and opposite (but non-zero) work on each particle.
 (d) the magnetic forces are necessarily negligible.
21. A proton is kept at rest. A positively charged particle is released from rest at a distance d in its field. Consider two experiments: one in which the charged particle is also a proton and in another, a positron. In the same time t, the work done on the two moving charged particles is
 (a) same as the same force law is involved in the two experiments.
 (b) less for the case of a positron, as the positron moves away more rapidly and the force on it weakens.
 (c) more for the case of a positron, as the positron moves away a larger distance.
 (d) same as the work done by charged particle on the stationary proton.
22. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is
 (a) constant and equal to mg in magnitude.
 (b) constant and greater than mg in magnitude.
 (c) variable but always greater than mg.
 (d) at first greater than mg, and later becomes equal to mg.
23. A bicyclist comes to a skidding stop in 10m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is
 (a) + 2000 J (b) -200 J
 (c) zero (d) -20,000 J
24. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities

remain constant during the fall?

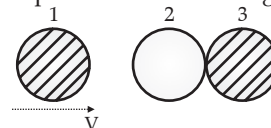
- (a) Kinetic energy.
 (b) Potential energy.
 (c) Total mechanical energy.
 (d) Total linear momentum.
25. During inelastic collision between two bodies, which of the following quantities always remain conserved?
 (a) Total kinetic energy.
 (b) Total mechanical energy.
 (c) Total linear momentum.
 (d) Speed of each body.
26. The potential energy function for a particle executing linear SHM is given by $V(x) = \frac{1}{2}kx^2$ where k is the

force constant of the oscillator. For $k = 0.5\text{N/m}$. The graph of $V(x)$ versus x is shown in the figure. A particle of total energy E turns back when it reaches $x = \pm x_m$. If V and K indicate the PE. and K.E., respectively of the particle at $x = +x_m$, then which of the following is correct?

- (a) $V = 0, K = E$ (b) $V = E, K = 0$
 (c) $V < E, K = 0$ (d) $V = 0, K < E$.



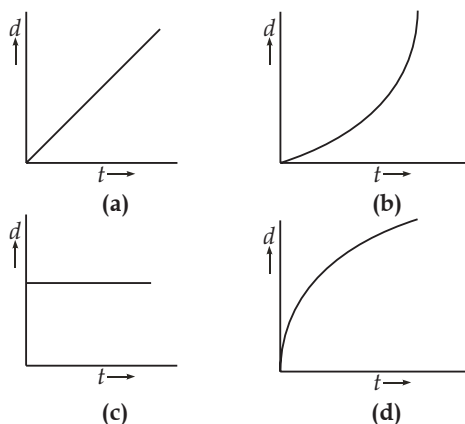
27. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v as shown in Fig.



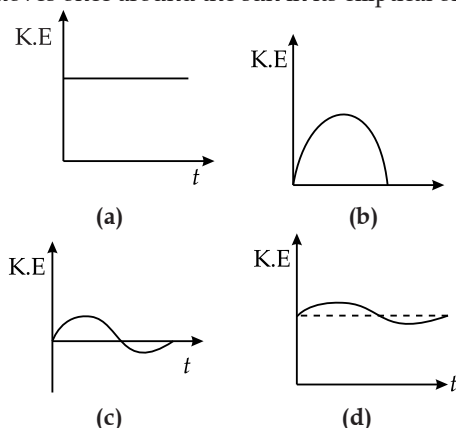
If the collision is elastic, which of the following is a possible result after collision?

- (a)
- (b)
- (c)
- (d)

28. A body is moving uni-directionally under the influence of a source of constant power supplying energy. Which of the diagrams shown below correctly shows the displacement - time curve for its motion?



29. Which of the diagrams shown below most closely shows the variation in kinetic energy of the Earth as it moves once around the sun in its elliptical orbit?

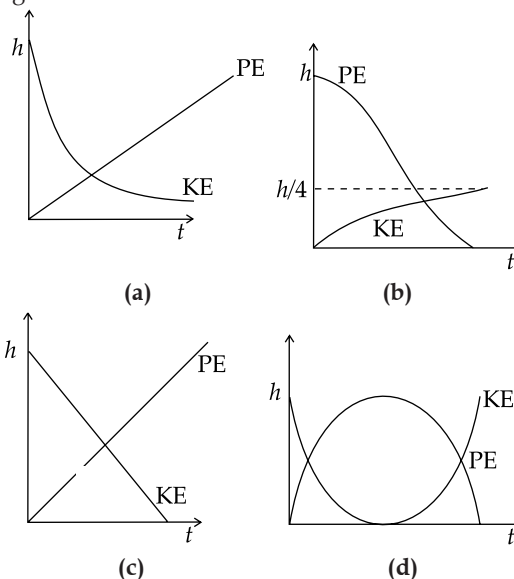


30. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be

- (a) $250\pi^2$ (b) $100\pi^2$ (c) $5\pi^2$ (d) 0

31. A raindrop falling from a height h above ground, attains a near terminal velocity when it has fallen through a height $(3/4)h$. Which of the diagrams shown in figure correctly the change in kinetic and

potential energy of the drop during its fall up to the ground?



32. In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1 m/s, at 45° from a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s^2 , the kinetic energy of the shotput when it just reaches the ground will be

- (a) 2.5 J (b) 5.0 J (c) 52.5 J (d) 155.0 J

33. A cricket ball of mass 150 g moving with a speed of 126 km/h hits at the middle of the bat, held firmly at its position by the batsman. The ball moves straight back to the bowler after hitting the bat. Assuming that collision between ball and bat is completely elastic and the two remain in contact for 0.001 s, the force that the batsman had to apply to hold the bat firmly at its place would be

- (a) 10.5 N (b) 21.0 N
(c) $1.05 \times 10^4 \text{ N}$ (d) $2.1 \times 10^4 \text{ N}$

Answer Key

1. (d)	2. (a)	3. (a)	4. (b)	5. (b)	6. (c)	7. (b)	8. (b)	9. (c)	10. (b)
11. (c)	12. (c)	13. (b)	14. (a)	15. (a)	16. (b)	17. (b)	18. (a)	19. (a)	20. (b)
21. (c)	22. (d)	23. (c)	24. (c)	25. (c)	26. (b)	27. (b)	28. (b)	29. (d)	30. (a)
31. (b)	32. (d)	33. (c)							

Answers with Explanations

1. Option (d) is correct.

Explanation: Force and displacement are in opposite direction.

So, $\theta = 180^\circ$

Work done is given by,

$$\begin{aligned} W &= Fs \cos \theta \\ W &= Fs \cos 180^\circ \\ &= Fs \times -1 = -Fs \end{aligned}$$

2. Option (a) is correct.

Explanation: Work done, $W = Fs$

$$\begin{aligned} \text{(a) } F &= 4 \text{ N, } s = 0.25 \text{ m; } W = 4 \times 0.25 = 1 \text{ N} \\ \text{(b) } F &= 2 \text{ N, } s = 1 \text{ m; } W = 2 \times 1 = 2 \text{ N} \\ \text{(c) } F &= 1 \text{ N, } s = 0.01 \text{ m; } W = 1 \times 0.01 = 0.01 \text{ N} \\ \text{(d) } F &= 1 \text{ N, } s = 0.50 \text{ m; } W = 1 \times 0.5 = 0.5 \text{ N} \end{aligned}$$

3. Option (a) is correct.

Explanation:

Here, $m = 2 \text{ kg}$, $\text{KE} = 100 \text{ J}$, $v = ?$

$$\text{KE} = \frac{1}{2}mv^2$$

$$\Rightarrow 100 = \frac{1}{2} \times 2 \times v^2$$

$$\Rightarrow v^2 = 100$$

$$\Rightarrow v = 10 \text{ m/s}$$

4. Option (b) is correct.

Explanation: There will be various changes but the sudden change will be treated as the momentum of the ball. Here it states the sudden change of velocity of ball.

5. Option (b) is correct.

Explanation: Positive work means work is done on the system. External energy is supplied to the system which increases its overall energy.

6. Option (c) is correct.

Explanation:

We know for a system :

- $F_{\text{ext}} = dp/dt$ (from Newton's second law of motion)
If $F_{\text{ext}} = 0$, $dp = 0 \Rightarrow p = \text{constant}$
- Hence, based on this the momentum of a system will remain conserved if external force on the system is zero. In case of collision between particles equal and opposite forces will act on individual particles in accordance with the Newton's third law.

Hence, net force on the system will be zero.

7. Option (b) is correct.

Explanation: As boy jumps on cart, so both cart and boy will move together.

Applying law of conservation of linear momentum,

$$\Rightarrow 52 \times 2 + 3 \times 0 = (52+3) \times v$$

$$\Rightarrow v = \frac{52 \times 2}{55} = 1.89 \text{ m/s}$$

8. Option (b) is correct.

Explanation: Energy held by an object due to change in its shape or position is called potential energy.

9. Option (c) is correct.

Explanation: Applying law of conservation on momentum, $P_i = P_f$

$$0 = 1 \times v + \frac{10}{1000} \times 300$$

10. Option (b) is correct.

Explanation: When an external force is applied to a swing, it moves upwards, as the force imparts certain kinetic energy to the swing. During its upward journey the swing continuously losses this kinetic energy and gains potential energy. Such that the total energy of the swing remains constant. When the swing reaches at its highest point its entire kinetic energy is converted into potential energy.

11. Option (c) is correct.

Explanation:

- The energy of any object can never be created nor can it be destroyed. It can only be converted from one form to another form.
- When an apple starts falling due to gravitational force it will be accelerated toward the ground and will have some kinetic energy. As apple falls because of air resistance a friction is provided to it

and hence produce heat energy.

- When an apple hits the ground, then both heat and kinetic energy is transferred to ground and rest is been transmitted in the form of sound energy.

- Thus, potential energy is converted to kinetic energy heat and sound energy.

12. Option (c) is correct.

Explanation: Electrical energy, is due to the flow of current, when there is no current in a circuit, electrical energy is zero. We cannot store it in its original form. In batteries energy is present in the form of chemical which gets converted into electrical energy, when batteries start their operation.

13. Option (b) is correct.

Explanation: As height of object decreases, its potential energy decreases and kinetic energy increases. As the body falls one fourth of its height, it will lose one fourth of potential energy which get transferred into kinetic energy.

14. Option (a) is correct.

Explanation: We know that $P = mv$,

At constant velocity, $P \propto m$

Mass of object B is double that of a, hence its momentum will also be double.

15. Option (a) is correct.

Explanation: $W = Fs \cos 180^\circ = 100 \times 20 \times (-1) = -2000 \text{ J}$

16. Option (b) is correct.

Explanation: The velocity of the ball when it reaches the ground $= v_0 = \sqrt{2gH}$

$$\text{or, } v_0 = \sqrt{2g \times 1}$$

$$\therefore v_0 = \sqrt{2g}$$

The ball rebounds with a velocity v'

$$v' = ev_0 = e\sqrt{2g}$$

Maximum height travelled by the ball starting with

$$\text{a velocity } v' \text{ is } h = \frac{v'^2}{2g}$$

$$\text{or, } h = \frac{e^2 \times 2g}{2g}$$

$$\therefore h = e^2 = 0.36 \text{ m}$$

17. Option (b) is correct.

Explanation:

Loss of PE = Gain in KE

$$\text{Loss in PE} = mgh = \left(\frac{100}{1000} \right) \times 9.8 \times 10 = 9.8 \text{ J}$$

So, increase in KE = 9.8 J

18. Option (a) is correct.

Explanation: 1 Joule = 1 Newton $\times$ 1 meter = 1 Nm

19. Option (a) is correct.

Explanation: Initially, $KE = \frac{p^2}{2m}$

$$\text{Finally, } (KE)' = \frac{p'^2}{2m}$$

$$\text{Putting, } p' = 1.20p$$

$$(KE)' = \frac{(1.20p)^2}{2m}$$

$$\text{or, } (KE)' = \frac{1.44p^2}{2m}$$

$$\text{or, } (KE)' = 1.44 (KE)$$

So, KE increases by 44%

20. Option (b) is correct.

Explanation: As the magnetic field due to motion of electron and proton act in a direction perpendicular to the direction of motion, no work is done by the forces. This is why one ignores the magnetic force of one particle on another.

21. Option (c) is correct.

Explanation: Force between two protons = force between a proton and a positron. Because of having much lighter weight than proton, positron moves away a larger distance compared to proton.

As work done = force $\times$ displacement, therefore in the same time t , work done in case of positron is more than that of proton.

22. Option (d) is correct.

Explanation: In the whole process, the man exerts a variable force (F) on the ground to set his body in motion. This force is in addition to the force required to support his weight (mg). Once the man is in standing position, F become zero.

23. Option (c) is correct.

Explanation: Just because road does not move at all. So, the work done by the cycle on the road must be zero.

24. Option (c) is correct.

25. Option (c) is correct.

26. Option (b) is correct.

Explanation: At any instant of time, the total energy of an oscillator is the sum of kinetic energy and potential energy.

$$\text{Total energy } E = U + K$$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

At $x = \pm x_m$, the particle turns back

$\therefore$ its velocity at this point is zero, i.e., $v = 0$

$$\therefore K = 0 \quad \therefore E = V$$

27. Option (b) is correct.

Explanation: Let m be the mass of each ball bearing. Total kinetic energy of the system before collision,

$$= \frac{1}{2}mv^2 + 0$$

$$= \frac{1}{2}mv^2$$

In (A), K of the system after collision,

$$K_1 = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$$

In (B), K of the system after collision,

$$K_2 = \frac{1}{2}(m)(v)^2 = \frac{1}{2}mv^2$$

In (C), K of the system after collision,

$$K_3 = \frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$$

In (D), K of the system after collision,

$$K_4 = \frac{1}{2}mv^2 + \frac{1}{2}m\left(\frac{v}{2}\right)^2 + \frac{1}{2}m\left(\frac{v}{3}\right)^2$$

$$= \frac{49}{72}mv^2$$

K is only conserved in (B)

$\therefore$ (B) is the only possibility

28. Option (b) is correct.

Explanation: For constant power, displacement $d \propto t^{3/2}$

29. Option (d) is correct.

Explanation: As Earth moves once around the sun in its elliptical orbit, its kinetic energy is maximum when it is farthest from the sun. As kinetic energy is never zero during its motion, $\therefore$ Option (d) is right.

30. Option (a) is correct.

Explanation: Given mass, $m = 5\text{kg}$,

Radius, $R = 1\text{m}$

$$v = 300\text{rpm} = \frac{300}{60}\text{rps} = 5\text{ rps}$$

The angular speed,

$$\omega = 2\pi v = 2\pi \times 5 = 10\pi \text{ rad/s}$$

The linear speed is

$$v = \omega R = (10\pi)(\text{rad/s})(1\text{m})$$

$$= 10\pi \text{ m/s}$$

$$K.E. = K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2} \times (5\text{kg})(10\pi \text{ m/s})^2$$

$$= 250\pi^2 \text{ J}$$

31. Option (b) is correct.

Explanation: At h , PE of raindrop is maximum and $KE=0$. As raindrop falls its PE goes on decreasing and KE goes on increasing up to point $\frac{h}{4}$ above

the ground. At this stage, rain drop has acquired near terminal velocity (=constant). $\square$ At this stage, KE tends to be constant. PE becomes zero when raindrop falls on the ground. Hence, (B) is most appropriate.

32. Option (d) is correct.

Explanation: Initial K.E. = $\frac{1}{2}mv^2$

$$= \frac{1}{2}(10\text{kg}) \times (1\text{m/s})^2$$

$$= 5 \text{ J}$$

Initial PE at height 1.5m

$$= mgh$$

$$= (10\text{kg})(10\text{m/s}^2)(1.5\text{m})$$

$$= 150 \text{ J}$$

Total initial energy

$$= 155 \text{ J}$$

33. Option (c) is correct.

Explanation:

$$F = \frac{\Delta p}{t} = \frac{mv - (-mv)}{t}$$

$$= \frac{2mv}{t}$$

$$= \frac{2 \times 150 \times 10^{-3} \text{ kg} \times 35 \text{ m/s}}{0.001 \text{ s}}$$

$$= 10500 \text{ N}$$

$$= 1.05 \times 10^4 \text{ N}$$

○○○

Chapter 34

MOTION OF SYSTEM OF PARTICLES AND RIGID BODY

Moment of inertia of a body about a given axis is the property by virtue of which, the body opposes any change in its state of rest or state of uniform rotation about that axis. For a single particle, moment of inertia (I) is equal to product of mass (m) of the particle and square of perpendicular distance (r) of the particle from the axis of rotation. i.e., $I = mr^2$.

- Moment of inertia is a scalar quantity, its unit is kgm^2
- It plays the same role in rotational motion as is played by the mass in linear motion.

Moment of inertia depends upon:

- Mass of the body
- Shape and size of the body
- Distribution of mass about axis of rotation
- Position and orientation of the axis of rotation w.r.t the body

Rotational Kinetic Energy: It is the energy possessed by the body on account of its rotation about a given axis. It is given by, $\text{KE} = \frac{1}{2} I\omega^2$

➤ Important Results and Formulae :

1. Centre of Mass : of n -particle system

$$R_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + \dots + m_n \vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$\therefore R_{\text{cm}} = \frac{\sum_{i=1}^{i=n} m_i \vec{r}_i}{M}$$

$$\text{where, } M = \sum_{i=1}^n m_i$$

2. Torque :

$$\vec{\tau} = \vec{r} \times \vec{F}$$

3. Angular Momentum :

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L_z = xp_y - yp_x$$

$$L_x = yp_z - zp_y, L_y = zp_x - xp_z$$

$$L = rp \sin \phi$$

4. Relation between Torque and Angular Momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

5. Law of Conservation of Angular Momentum :

$$\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \dots + \vec{L}_n = \text{constant}$$

6. Moment of Inertia :

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots + m_n r_n^2$$

$$= \sum_{i=1}^{i=n} m_i r_i^2$$

7. Radius of gyration :

$$K = \sqrt{\frac{r_1^2 + r_2^2 + r_3^2 + \dots + r_n^2}{n}}$$

8. Theorem of perpendicular axes :

$$I_z = I_x + I_y$$

9. Theorem of parallel axes :

$$I = I_c + Mh^2$$

10. K.E. of rotation : $K_R = \frac{1}{2} I\omega^2$

11. Moment of inertia and torque :

$$\tau = I\alpha$$

where α = angular acceleration

12. Moment of Inertia of some bodies :

- (i) M.I. of a rod about an axis through its c.m. and perpendicular to rod,

$$I = \frac{1}{12} ML^2$$

- (ii) M.I. of a circular ring about an axis through its centre and perpendicular to its plane,

$$I = MR^2$$

- (iii) M.I. of a circular disc about an axis through its centre and perpendicular to its plane,

$$I = \frac{1}{2} MR^2$$

- (iv) M.I. of a right circular solid cylinder about its symmetry axis,

$$I = \frac{1}{2} MR^2$$

- (v) M.I. of a right circular hollow cylinder about its axis $I = MR^2$

- (vi) M.I. of a solid sphere about its diameter,

$$I = \frac{2}{5} MR^2$$

- (vii) M.I. of spherical shell about its diameter,

$$I = \frac{2}{3} MR^2$$

Comparison between Linear and Rotational Motion:

S. No.	Linear Motion	Rotational Motion
1.	Distance/ displacement (s)	Angular displacement (θ)
2.	Linear velocity, $v = \frac{ds}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$
3.	Linear acceleration, $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$	Angular acceleration, $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

4.	Mass (m)	Moment of inertia (I)
5.	Linear momentum, $p = mv$	Angular momentum, $L = I\omega$
6.	Force, $F = ma$	Torque, $\tau = Ia$
7.	Also, force, $F = \frac{dp}{dt}$	Also, torque, $\tau = \frac{dL}{dt}$
8.	Translational KE, $K_T = \frac{1}{2}mv^2$	Rotational KE, $K_R = \frac{1}{2}I\omega^2$

9.	Work done, $W = Fs$	Work done, $W = \tau\theta$
10.	Power, $P = Fv$	Power, $P = \tau\omega$
11.	Equations of translatory motion: (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$	Equations of rotational motion: (i) $\omega_2 = \omega_1 + \alpha t$ (ii) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ (iii) $\omega_2^2 - \omega_1^2 = 2\alpha\theta$

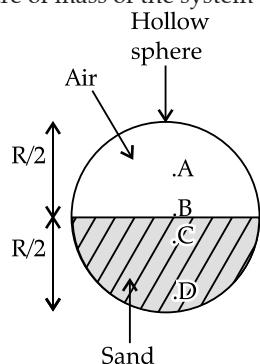
EXERCISE

- A thin disc and a thin ring, both have mass M and radius R . Both rotate about axes through their center of mass and are perpendicular to their surfaces at the same angular velocity. Which of the following is true?
 - The ring has higher kinetic energy
 - The disc has higher kinetic energy
 - The ring and the disc have the same kinetic energy
 - Kinetic energies of both the bodies are zero since they are not in linear motion
- Let there be an object having some chemicals in it. It starts moving with a uniform velocity v and a chemical reaction starts happening. In this case, which of the following statement's is/are correct?
 - Chemical reactions happening in the system cannot change the velocity v and the center of mass of the object.
 - Chemical reactions happening in the system cannot change kinetic energy of the particles inside with respect to the center of mass of object.

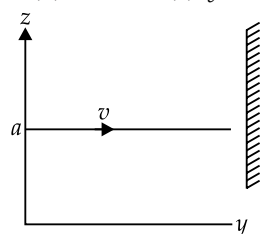
Select the correct answer using the code given below:

 - 1 only
 - 2 only
 - Both 1 and 2
 - Neither 1 nor 2
- A solid disc and a solid sphere have the same mass and same radius. Which one has the higher moment of inertia about its centre of mass?
 - The disc
 - The sphere
 - Both have the same moment of inertia
 - The information provided is not sufficient to answer the question
- If the length of the Equator is about 40000 km and the velocity of rotation is about 1700 km/hr, what would be the velocity of rotation the Pole?
 - Zero
 - 850 km/hr
 - 1700 km/hr
 - 3400 km/hr
- On which of the following factors moment of inertia of an object does not depend?
 - Axis of rotation
 - Angular velocity
 - Distribution of mass
 - Mass of an object
- When consideration of moment of inertia of a body becomes important?
 - When the motion is linear
 - When the motion is rotational
 - When the motion is along a curved path
 - In all of the above cases
- Which of the following have the same moment of inertia?
 - Moment of inertia of a thin circular ring of radius R about an axis perpendicular to the plane, through the centre.
 - Moment of inertia of a hollow cylinder of radius R about the axis of the cylinder.
 - Moment of inertia of a thin circular ring of radius R about an axis perpendicular to the plane, through the centre.
 - Moment of inertia of a solid cylinder of radius R about the axis of the cylinder.
 - Moment of inertia of a thin circular ring of radius R about an axis perpendicular to the plane, through the centre.
 - Moment of inertia of a solid sphere of radius R about the diameter of the sphere.
 - Moment of inertia of a solid cylinder of radius R about the axis of the cylinder.
 - Moment of inertia of a hollow cylinder of radius R about the axis of the cylinder.
- For a vector product of two vectors $\vec{a}$ and $\vec{b}$ which of the following statements is correct?
 - magnitude of resultant vector $= ab \sin \theta$ where, a and b are magnitudes of $\vec{a}$ and $\vec{b}$ and θ is the angle between the two vectors.
 - Resultant vector is perpendicular to the Plane containing $\vec{a}$ and $\vec{b}$.
 - if we take a right handed screw with its head lying in the plane of $\vec{a}$ and $\vec{b}$ and the screw perpendicular to this plane, and if we turn the head in the direction from $\vec{a}$ to $\vec{b}$, then the tip of the screw advances in the direction of resultant vector.
 - All of the above
- Moment of inertia of a solid sphere about its diameter is $\frac{2}{5}Mr^2$. Its radius of gyration is

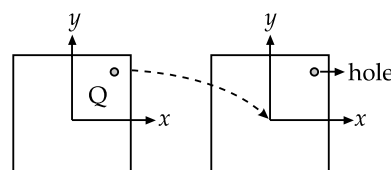
- (a) $\frac{2}{5}r$ (b) $\frac{5}{2}r$ (c) $\sqrt{\frac{2}{5}}r$ (d) $\sqrt{\frac{5}{2}}r$
10. For which of the following does the centre of mass lie outside the body?
 (a) A pencil (b) A shotput
 (c) A dice (d) A bangle
11. Which of the following point is the likely position of the centre of mass of the system shown in Fig. ?



- (a) A (b) B (c) C (d) D
12. A particle of mass m is moving in yz -plane with a uniform velocity v with its trajectory running parallel to $+ve$ y -axis and intersecting z -axis at $z=a$. The change in its angular momentum about the origin as it bounces elastically from a wall at $y=\text{constant}$ is
- (a) $mva\hat{e}_x$ (b) $2mva\hat{e}_x$ (c) $ymv\hat{e}_x$ (d) $2ymv\hat{e}_x$



13. When a disc rotates with uniform angular velocity, which of the following is not true?
 (a) The sense of rotation remains same.
 (b) The orientation of the axis of rotation remains same.
 (c) The speed of rotation is non-zero and remains same.
 (d) The angular acceleration is non-zero and remains same.
14. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. The moment of inertia about the z -axis is then
 (a) increased
 (b) decreased
 (c) the same
 (d) changed in unpredicted manner.



15. In problem 10, the CM of the plate is now in the following quadrant of x - y plane,
 (a) I (b) II (c) III (d) IV
16. A merry-go-round, made of a ring-like platform of radius R and mass M , is revolving with angular speed ω . A person of mass M is standing on it. At one instant, the person jumps off the round, radially away from the centre of the round (as seen from the round). The speed of the round afterwards is
 (a) 2ω (b) ω (c) $\frac{\omega}{2}$ (d) 0

Answer Key

1. (a)	2. (a)	3. (a)	4. (a)	5. (b)	6. (b)	7. (a)	8. (d)	9. (c)	10. (d)
11. (c)	12. (b)	13. (d)	14. (b)	15. (c)	16. (a)				

Answers with Explanations

1. Option (a) is correct.

Explanation:

- The moment of inertia of the ring about an axis passing through centre and perpendicular to its plane is given by:

$$I_{\text{ring}} = MR^2$$

- Moment of inertia of the disc about an axis passing through centre and perpendicular to its plane is given by-

$$I_{\text{disc}} = \frac{1}{2}MR^2$$

- So, mathematically rotational kinetic energy can be written as-

$$KE = \frac{1}{2}I\omega^2$$

- As per asked question, that angular velocity of thin disc and a thin ring is same. Therefore, Kinetic energy is depending on to moment of inertia.
- In that case, a body having more moment of inertia will have more kinetic energy and vice versa.
- So, as per the mentioned equation $I_{\text{ring}} > I_{\text{disc}}$
 Hence, $K_{\text{ring}} > K_{\text{disc}}$
 The ring has higher kinetic energy

2. Option (a) is correct.

Explanation:

- The chemical reactions generally happening in a system influence the rate of reaction.
- The factors such as temperature, pressure and concentration. In this case the reaction cannot change the velocity and centre of mass of the object.

- However, chemical reaction can change the Kinetic energy of the reaction.

3. **Option (a) is correct.**

Explanation:

- **Moment of inertia** is the property of a body due to which it opposes any change in its state of rest or of uniform rotation. It plays the same role in rotational motion as mass plays in linear motion.
- Moment of inertia of the disc is

$$I_{disc} = \frac{1}{2} MR^2 = 0.5 MR^2$$

- Moment of inertia of the solid sphere

$$I_{sphere} = \frac{2}{5} MR^2 = 0.4 MR^2$$

So, it is very clear that the moment of inertia of the disc is greater than the moment of inertia of the solid sphere.

4. **Option (a) is correct.**

Explanation: At poles distance from axis of rotation is zero, hence zero velocity of rotation.

5. **Option (b) is correct.**

6. **Option (b) is correct.**

7. **Option (a) is correct.**

Explanation: In both the cases moment of inertia is MR^2

8. **Option (d) is correct.**

9. **Option (c) is correct.**

Explanation: Moment of inertia $= I = Mk^2$
 k is the radius of gyration.

In this case,

$$\frac{2}{5} Mr^2 = Mk^2$$

$$\text{or, } k^2 = \frac{2}{5} r^2$$

$$\therefore k = \sqrt{\frac{2}{5}} r$$

10. **Option (d) is correct.**

11. **Option (c) is correct.**

12. **Option (b) is correct.**

Explanation: Initial velocity, $\vec{V}_i = v\hat{e}_y$

After reflection, final velocity $\vec{V}_f = -v\hat{e}_y$

The trajectory is at constant distance a on z -axis and as particle moves along y -axis, its y component changes

Now, position vector, $\vec{r} = y\hat{e}_y + a\hat{e}_z$

Hence, change in angular momentum-

$$\vec{r} \times m(\vec{V}_f - \vec{V}_i) = 2mva\hat{e}_x$$

13. **Option (d) is correct.**

14. **Option (b) is correct.**

Explanation: The small cut piece comes closer to Z axis.

15. **Option (c) is correct.**

16. **Option (a) is correct.**

Explanation: As no external torque acts on the system, angular momentum should be considered i.e., $I\omega = \text{constant}$

from equation (1), $I_1\omega_1 = I_2\omega_2$

[ω_1 & ω_2 are angular velocities before and after jumping]

$$\therefore I = mr^2, m_1 = 2M, m_2 = M$$

$$r_1 = r_2 = R, \therefore m_1 r_1^2 \omega_1 = m_2 r_2^2 \omega_2$$

$$\text{or } 2MR^2 \omega_1 = MR^2 \omega_2$$

$$\text{or } \omega_2 = 2\omega_1$$

Gravitational Force

- It is the force of attraction between two objects having some mass
- It is a long-range force but weakest among all fundamental forces.
- It is represented as $F = G \frac{m_1 m_2}{r^2}$; Here, $G = 6.67 \times 10^{-11} \text{ N-m}^2/\text{kg}^2$

Gravity

- The gravitational force of earth is called gravity
- The acceleration in a body due to the force of gravity is called acceleration due to gravity (g).
- It is given by, $g = \frac{GM}{R^2}$; g on earth is 9.8 m/s^2
- Value of g decreases with height or depth from the earth surface.
 - g is maximum at poles and minimum at equator.
 - g decreases if the angular speed of the earth increases and increases if the angular speed of the earth decreases.
- The acceleration due to gravity does not depend upon (a) mass, (b) shape or size of the body
- The g at the moon is one-sixth that of the Earth.

Mass of a body is the quantity of matter possessed by the body.

- Mass of an object always remains constant. It is a scalar quantity. Its SI unit is kg.

Weight is the gravitational force with which a body is attracted towards any celestial body.

- Weight of an object is given by, $W = mg$
- Its value differs according to the place due to change in the value of ' g '.
- Weight is a vector quantity. Its SI unit is N (Newton)

Escape velocity is the minimum velocity at which an object just crosses the earth's gravitational field and never returns.

- Escape velocity at earth's surface is 11.2 km/s and at moon's surface is 2.4 km/s .

Satellites

- A satellite is an object (either natural or artificial) in space that circles around a bigger object.
- The period of revolution of satellite revolving near the surface of Earth is 1 hour 24 minutes.
- Geo-stationary satellite revolves around the Earth at a height of 36000 km (approx.)
- Time period of rotation of geo-stationary satellite is 24 h.
- Polar satellites revolve around the Earth at a height of 800 km (approx.)
- Time period of polar satellites is 84 minutes.

► Important Results and Formulae :

1. Kepler's Laws :

- Law of orbits** : All planets move in elliptical orbits with the Sun situated at one of the foci of the ellipse.
- Law of areas** : The line that joins any planet to the Sun sweeps equal areas in equal intervals of time.
- Law of periods** : The square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

$$T^2 \propto r^3$$

$$T^2 = Kr^3; K = \text{constant}$$

2. Newton's Law of Gravitational Attraction :

$$F = G \frac{m_1 m_2}{r^2}$$

Where, $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$

3. Acceleration due to gravity (g) :

$$g = \frac{GM}{R^2}$$

4. Variation of acceleration due to gravity :

- Effect of altitude, $g' = g(1 - 2h/R)$
- Effect of depth, $g' = g(1 - d/R)$
- Effect of rotation of earth :

$$g' = g - R\omega^2 \cos^2 \lambda$$

5. Gravitational potential

$$V = \frac{W}{m_o} = -\frac{GM}{r}$$

6. Gravitational potential energy :

$$U = -\frac{GM}{r} \times m$$

7. Escape Velocity :

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

8. Satellite :

Orbital Velocity :

$$v_o = R \sqrt{\frac{g}{R+h}}$$

9. Time Period :

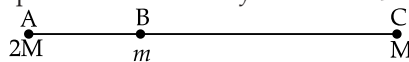
$$T = 2\pi \sqrt{\frac{(R+h)^3}{GM}}$$

$$T = \frac{2\pi}{R} \sqrt{\frac{(R+h)^3}{g}}$$

EXERCISE

- Suppose there are two planets, 1 and 2, having the same density but their radii are R_1 and R_2 respectively, where $R_1 > R_2$. The accelerations due to gravity on the surface of these planets are related as
 (a) $g_1 > g_2$ (b) $g_1 < g_2$
 (c) $g_1 = g_2$ (d) Can't say anything
- A planet has a mass M_1 and radius R_1 . The value of acceleration due to gravity on its surface is g_1 . There is another planet 2, whose mass and radius both are two times that of the first planet. Which one of the following is the acceleration due to gravity on the surface of planet 2?
 (a) g_1 (b) $2g_1$ (c) $\frac{g_1}{2}$ (d) $\frac{g_1}{4}$
- The acceleration due to gravity ' g ' for objects on or near the surface of earth is related to the universal gravitational constant ' G ' as (' M ' is the mass of the earth and ' R ' is its radius):
 (a) $G = g \frac{M}{R^2}$ (b) $g = G \frac{M}{R^2}$
 (c) $M = \frac{gG}{R^2}$ (d) $R = \frac{gG}{M^2}$
- Two planets orbit the Sun in circular orbits, with their radius of orbits as $R_1 = R$ and $R_2 = 4R$. Ratio of their periods $\left(\frac{T_1}{T_2}\right)$ around the Sun will be :
 (a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$
- Two bodies of mass M each are placed R distance apart. In another system, two bodies of mass $2M$ each are placed $\frac{R}{2}$ distance apart. If F be the gravitational force between the bodies in the first system, then the gravitational force between the bodies in the second system will be :
 (a) $16F$ (b) $1F$
 (c) $4F$ (d) None of these
- Which one of the following statements about a satellite orbiting around the Earth is correct?
 (a) Satellite is kept in orbit by remote control from ground station.
 (b) Satellite is kept in orbit by retro-rocket and solar energy keeps it moving around the Earth.
 (c) Satellite requires energy from solar panels and solid fuels for orbiting.
 (d) Satellite does not require any energy for orbiting.
- Which one of the following is not a contact force?
 (a) Push force (b) Frictional force
 (c) Gravitational force (d) Strain force
- LIGO stands for
 (a) Laser Interferometer Gravitational wave Observatory
 (b) Light Interferometer Gravitational wave Observatory.
 (c) Light Induced Gravity Observatory
 (d) Laser Induced Gaseous Optics
- Which one of the following statements about gravitational force is NOT correct?
 (a) It is experienced by all bodies in the universe.
 (b) It is a dominant force between celestial bodies.
 (c) It is a negligible force for atoms.
 (d) It is same for all pairs of bodies in our universe.
- Which one of the following statements is true for the relation $F = \frac{Gm_1m_2}{r^2}$?
 (All symbols have their usual meanings)
 (a) The quantity G depends on the local value of g , acceleration due to gravity.
 (b) The quantity G is greatest at the surface of the Earth.
 (c) The quantity G is used only when earth is one of the two masses.
 (d) The quantity G is a universal constant.
- Suppose the force of gravitation between two bodies of equal masses is F . If each mass is doubled keeping the distance of separation between them unchanged, the force would become :
 (a) F (b) $2F$ (c) $4F$ (d) $\frac{1}{4}F$
- A man weighing 70 kg is coming down in a lift. If the cable of the lift breaks suddenly, the weight of the man would become :
 (a) 70 kg (b) 35 kg (c) 140 kg (d) Zero
- The free fall acceleration g increases as one proceeds, at sea level from the equator toward either pole. The reason is :
 (a) Earth is a sphere with same density everywhere.
 (b) Earth is a sphere with different density at the polar regions than in the equatorial regions.
 (c) Earth is approximately an ellipsoid having its equatorial radius greater than its polar radius by 21 km
 (d) Earth is approximately an ellipsoid having its equatorial radius smaller than its polar radius by 21 km.
- If radius of the earth were to shrink by 1%, its mass remaining the same, g would decrease by nearly
 (a) 1% (b) 2% (c) 3% (d) 4%
- The radius of the Moon is about one-fourth that of the Earth and acceleration due to gravity on the Moon is about one-sixth that on the Earth. From this, we can conclude that the ratio of the mass of Earth to the mass of the Moon is about:
 (a) 10 (b) 100 (c) 1,000 (d) 10,000
- Escape velocity of an object of mass m is proportional to
 (a) m^2 (b) m (c) m^{-1} (d) m^0
- An object of mass m is lifted at a height equal to the radius (R) of the earth. If the mass of the earth is M and acceleration due to gravity is g , then the change in gravitational potential energy of the object is
 (a) mgR (b) $2MgR$
 (c) $\frac{MgR}{2}$ (d) $\frac{mgR}{2}$

18. Two particles of equal mass m are moving along a circle of radius r under the action of their mutual gravitational attraction. The speed of each particle is
- (a) $\sqrt{\frac{Gm}{4r}}$ (b) $\sqrt{\frac{Gm}{2r}}$
 (c) $\sqrt{\frac{Gm}{r}}$ (d) $\sqrt{Gmr}$
19. If R is the radius of earth, then the acceleration due to gravity will be $g/4$ at a height _____ from the earth surface.
 (a) R (b) $2R$ (c) $3R$ (d) $\frac{R}{2}$
20. The relation between gravitational potential (V) and gravitational intensity (E) is
- (a) $E = -\frac{d^2V}{dt^2}$ (b) $E = -\frac{dV}{dr}$
 (c) $E = \frac{d^2V}{dt^2}$ (d) $E = \frac{dV}{dr}$
21. The Earth is an approximate sphere. If the interior contained matter which is not of the same density everywhere, then on the surface of the Earth, the acceleration due to gravity
- (a) will be directed towards the centre but not the same everywhere.
 (b) will have the same value everywhere but not directed towards the centre.
 (c) will be same everywhere in magnitude directed towards the centre.
 (d) cannot be zero at any point.
22. As observed from Earth, the Sun appears to move in an approximate circular orbit. For the motion of another planet like Mercury as observed from Earth, this would
- (a) be similarly true.
 (b) not be true because the force between Earth and Mercury is not inverse square law.
 (c) not be true because the major gravitational force on Mercury is due to Sun.
 (d) not to be true because Mercury is influenced by forces other than gravitational forces.
23. Different points in Earth are at slightly different distances from the Sun and hence experience different forces due to gravitation. For a rigid body, we know that if various forces act at various points in it, the resultant motion is as if a net force acts on the c.m. (centre of mass) causing translation and a net torque at the c.m. causing rotation around an axis through the c.m. For the Earth-Sun system (approximating the Earth as a uniform density sphere)
- (a) the torque is zero.
 (b) the torque causes the Earth to spin.
 (c) the rigid body result is not applicable since the Earth is not even approximately a rigid body.
 (d) the torque causes the Earth to move around the Sun.
24. Satellites orbiting the Earth have finite life and sometimes debris of satellites fall to the Earth. This is because,
- (a) the solar cells and batteries in satellites run out.
 (b) the laws of gravitation predict a trajectory spiralling inwards.
 (c) of viscous forces causing the speed of satellite and hence height to gradually decrease.
 (d) of collisions with other satellites.
25. Both Earth and Moon are subject to the gravitational force of the Sun. As observed from the Sun, the orbit of the Moon
- (a) will be elliptical
 (b) will not be strictly elliptical because the total gravitational force on it is not central.
 (c) is not elliptical but will necessarily be a closed curve.
 (d) deviates considerably from being elliptical due to influence of planets other than Earth.
26. In our solar system, the inter-planetary region has chunks of matter (much smaller in size compared to planets) called asteroids. They
- (a) will not move around the Sun since they have very small masses compared to the Sun.
 (b) will move in an irregular way because of their small masses and will drift away into outer space.
 (c) will move around the Sun in closed orbits but not obey Kepler's laws.
 (d) will move in orbits like planets and obey Kepler's laws.
27. Choose the wrong option.
- (a) Inertial mass is a measure of difficulty of accelerating body by an external force whereas the gravitational mass is relevant in determining the gravitational force on it by an external mass.
 (b) That the gravitational mass and inertial mass are equal is an experimental result.
 (c) That the acceleration due to gravity on Earth is the same for all bodies is due to the equality of gravitational mass and inertial mass.
 (d) Gravitational mass of a particle like proton can depend on the presence of neighbouring heavy objects but the inertial mass cannot.
28. Particles of masses $2M$, m and M are respectively at points A, B and C with $AB = \frac{1}{2}(BC)$. m is much smaller than M and at time $t = 0$. They are all at rest.
- At subsequent times before any collision takes place:
- (a) m will remain at rest.
 (b) m will move towards M .
 (c) m will move towards $2M$.
 (d) m will have oscillatory motion.



Answer Key

1. (a)	2. (c)	3. (b)	4. (b)	5. (a)	6. (d)	7. (c)	8. (a)	9. (d)	10. (d)
11. (c)	12. (d)	13. (c)	14. (b)	15. (b)	16. (d)	17. (d)	18. (a)	19. (a)	20. (b)
21. (d)	22. (c)	23. (a)	24. (c)	25. (b)	26. (d)	27. (d)	28. (c)		

Answers with Explanations

1. Option (a) is correct.

Explanation:

Acceleration due to gravity, $g = \frac{GM}{R^2}$

$$g = G \cdot \frac{4}{3} \frac{\pi R^3 \rho}{R^2}$$

$$g = \frac{4\pi G \rho R}{3}$$

$$\Rightarrow g \propto R$$

$$\text{As } R_1 > R_2$$

$$\Rightarrow g_1 > g_2$$

2. Option (c) is correct.

Explanation:

$$g_1 = G \frac{M_1}{R_1^2}$$

$$g_2 = G \frac{M_2}{R_2^2}$$

On substitution $M_2 = 2M_1$ and $R_2 = 2R_1$ in equation (2)

$$g_2 = G \frac{2M_1}{4R_1^2} \quad \dots(3)$$

$$\text{On } \frac{(3)}{(1)}; \frac{g_2}{g_1} = \frac{1}{2}$$

$$g_2 = \frac{g_1}{2}$$

3. Option (b) is correct.

Explanation:

Let gravitational force be acting on a mass "m"

$$\text{Then, } mg = \frac{GMm}{R^2}$$

$$\text{or } g = \frac{GM}{R^2}$$

4. Option (b) is correct.

Explanation:

According to Kepler's law $T_2 \propto r^3$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2} \right)^{\frac{3}{2}}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{R}{4R} \right)^{\frac{3}{2}}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{1}{4} \right)^{\frac{3}{2}} = \left(\frac{1}{2} \right)^3 = \frac{1}{8}$$

5. Option (a) is correct.

Explanation:

$$\text{Here, } F = G \frac{MM}{r^2}$$

$$\begin{aligned} \text{now } F' &= G \frac{(2M)(2M)}{\left(\frac{R}{2}\right)^2} \\ &= G \frac{4MM}{\frac{R^2}{4}} \\ &= 16F \end{aligned}$$

6. Option (d) is correct.

Explanation: A force that makes a body follow a curved path is called as a centripetal force. The gravitational force between satellite and earth provides the necessary centripetal force for the satellite to orbit the earth. Also, this is why artificial satellites orbiting the Earth stay in their orbits and do not fall from the sky.

7. Option (c) is correct.

Explanation: Gravitational force acts between two objects irrespective of the distance between them. Whereas, for push, strain and friction force the two objects must be in contact.

8. Option (a) is correct.

Explanation: LIGO is a large-scale experiment and observatory to detect cosmic gravitational waves.

9. Option (d) is correct.

Explanation: Gravitational force between two objects depends upon the product of their mass.

10. Option (d) is correct.

Explanation: G is gravitational constant; its value always remains constant.

11. Option (c) is correct.

Explanation: If distance of separation remains unchanged, $F_g \propto m_1 m_2$. On doubling both mass, force will also get four times.

12. Option (d) is correct.

Explanation: In case of free fall, weight becomes zero.

13. Option (c) is correct.

Explanation: Since, $F_g \propto \frac{1}{r^2}$

Since $R_e > R_p$, Force of gravitation at pole is more than that at equator.

14. Option (b) is correct.

Explanation: We know $g = \frac{GM}{R^2}$

$$\text{Hence, } \frac{\Delta g}{g} = \frac{-2\Delta R}{R} = -2 \times 1\% = -2\%$$

- sign indicates decrease in the value of R

15. Option (b) is correct.

Explanation:

$$g_e = \frac{GM_e}{R_e^2} \quad \dots\dots(i)$$

$$g_m = G \frac{M_m}{R_m^2}$$

$$\frac{g_e}{6} = G \frac{Mm}{4R_e^2} \quad \dots\dots(ii)$$

$$\text{On } \frac{(i)}{(ii)}, \text{ we get } \frac{M_e}{M_m} = \sim 100$$

16. Option (d) is correct.

Explanation: Escape velocity = $\sqrt{2gR}$,

It is independent of mass.

17. Option (d) is correct.

Explanation: Change in gravitational potential

$$\text{energy} = \Delta U = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right) = \frac{GMm}{2R}$$

$$\text{Putting } g = \frac{GM}{R^2}$$

$$\Delta U = \frac{mgR}{2}$$

18. Option (a) is correct.

Explanation: Gravitational attraction between the

$$\text{two particles} = F = \frac{Gm^2}{(2r)^2}$$

$$\text{Required centripetal force} = \frac{mv^2}{r}$$

$$\therefore \frac{Gm^2}{4r^2} = \frac{mv^2}{r}$$

$$\therefore v = \sqrt{\frac{Gm}{4r}}$$

19. Option (a) is correct.

$$\text{Explanation: } \frac{g'}{g} = \frac{R^2}{(R+h)^2}$$

$$\text{or } \frac{1}{4} = \frac{R^2}{(R+h)^2}$$

$$\text{or } \frac{1}{2} = \frac{R}{(R+h)}$$

$$\therefore h = R$$

20. Option (b) is correct.

21. Option (d) is correct.

22. Option (c) is correct.

23. Option (a) is correct.

24. Option (c) is correct.

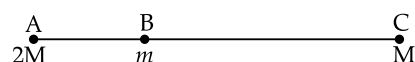
25. Option (b) is correct.

26. Option (d) is correct.

27. Option (d) is correct.

28. Option (c) is correct.

Explanation:



$$\text{Gravitational force, } F = \frac{GMm}{r^2}$$

Let $AB = r$

$\therefore$ Force on B due to A,

$$\therefore F_{BA} = \frac{G(2Mm)}{r^2} = 2F$$

Now, Force on B due to C, -

$$\therefore F_{BC} = \frac{GMm}{(BC)^2} = \frac{GMm}{4r^2} \quad [\because BC = 2AB]$$

$$F_{BC} = \frac{F}{4} \text{ or } F_{BA} > F_{BC}$$

Hence, m will move towards A i.e., $2M$.

Chapter 36

MECHANICS OF SOLIDS AND FLUIDS

Pressure

- The force, acting normally on a unit area of the surface is pressure.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

- SI unit of pressure is N/m^2 . It is a scalar quantity.
- If gravitational attraction is negligible in equilibrium condition (approx.) pressure is same at all points in a liquid.
- Hydraulic lift, hydraulic press and hydraulic brakes are based on Pascal's law of pressure

Archimedes' Principle

- When a body is immersed partially or wholly in a liquid, there is an apparent loss in the weight of the body, which is equal to the weight of the liquid displaced by the body.
- The weight of water displaced by an iron ball is less than its own weight whereas water displaced by the immersed portion of a ship is equal to its weight. So, a small ball of iron sinks in water, but a large ship float.

Laws of Floatation

- A body floats in a liquid if the density of the material of the body is less than or equal to the density of the liquid.
- When the density of the material of the body is equal to the density of liquid, the body floats fully submerged in liquid in natural equilibrium.
- The weight of the body is equal to the weight of displaced liquid when body floats in neutral equilibrium. The center of gravity of the body and center of gravity of the displaced liquid should be in one vertical line for this condition.

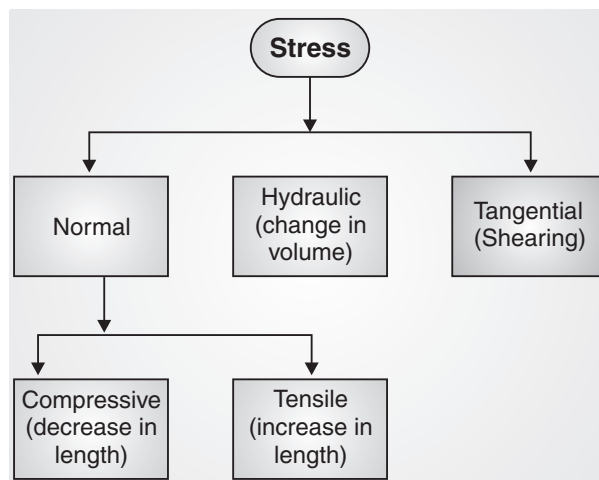
Surface Tension: It is the property of the liquid by virtue of which the free surface of the liquid at rest tends to have the minimum surface area and as such it behaves as if covered with a stretched membrane.

- Surface tension is a molecular phenomenon, and it arises due to electromagnetic forces.
- S.I. unit of surface tension is Nm^{-1} . It is a scalar quantity.
- Surface tension of liquids decreases with increase in temperature. It does not depend upon the area of the free surface at liquid.

➤ Important Results and Formulae :

1. Stress and Its Types :

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$



2. Types of Strain

$$\text{Longitudinal Strain} = \frac{\text{Change in Length}}{\text{Original Length}}$$

$$\text{Volumetric strain} = \frac{\text{Change in Volume}}{\text{Original Volume}}$$

$$\text{Shearing Strain} = \phi = \frac{\Delta L}{L}$$

Where ΔL = displacement of the face on which force is applied and L is the height of the face.

3. Hooke's Law

Stress $\propto$ strain

$$\text{or } \frac{\text{Stress}}{\text{Strain}} = \text{constant} = E$$

E is called modulus of elasticity

4. Young's Modulus (Y) :

$$Y = \frac{\text{Normal Stress}}{\text{Longitudinal Strain}} = \frac{F/A}{l/L} = \frac{FL}{Al} = \frac{MgL}{\pi r^2 l}$$

5.

Total energy stored in wire (U)	Energy stored in per unit volume of wire (U_v)
$\frac{1}{2}Fl$	$\frac{1}{2} \frac{Fl}{\text{volume}}$
$\frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$	$\frac{1}{2} \times \text{Stress} \times \text{Strain}$
$\frac{1}{2} \times Y \times (\text{Strain})^2 \times \text{Volume}$	$\frac{1}{2} \times Y \times (\text{Strain})^2$
$\frac{1}{2} \times (\text{Stress})^2 \times \text{Volume}$	$\frac{1}{2} \times (\text{Stress})^2$

6. Bulk Modulus :

$$K = \frac{\text{Normal Stress}}{\text{Volumetric Strain}}$$

$$K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

(-ve sign means decrease in volume, if volume increases, then it will be +ve)

$$C = \text{compressibility} = \frac{1}{k} = \frac{\Delta V}{pV}$$

7. Modulus of Rigidity :

$$\eta = \frac{\text{Shearing stress}}{\text{Shearing strain}}$$

8. Poisson's Ratio :

$$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\sigma = \frac{-dr/r}{dL/L}$$

9. Pressure exerted by liquid column :

$$P = \rho gh$$

10. Variation of pressure with depth :

$$P = P_a + \rho gh$$

11. Atmospheric pressure :

At sea level, atmospheric pressure = Pressure of 0.76 m of Hg column. Mathematically 1 atm = $1.013 \times 10^5 \text{ Nm}^{-2}$

12. Pascal's Law :

A change in pressure at any point in an enclosed incompressible fluid at rest is transmitted equally and undiminished to all points in all directions throughout the fluid, and acts at right angle to the enclosure.

13. Newton's formula for Viscous force :

$$F = \eta A(dv/dx)$$

14. Poiseuille's equation :

$$V = \pi Pr^4/8\eta l$$

15. Stoke's law :

$$F = 6\pi\eta rv$$

16. Terminal velocity :

$$v = \frac{2r^2(d-\rho)g}{9\eta}$$

17. Critical velocity :

$$v_c = \frac{k\eta}{\rho r}$$

18. Theorem of continuity :

$$a_1v_1 = a_2v_2$$

19. Bernoulli's theorem :

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{Constant}$$

$$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{Constant}$$

20. Surface tension (T) :

$$T = \frac{F}{l}$$

21. Surface energy :

$$E_s = \frac{\text{Work done in increasing the surface area}}{\text{Increase in area}}$$

22. Excess of pressure inside a liquid drop = $2T/R$.

Excess of pressure inside a soap bubble = $4T/R$

Excess of pressure inside an air bubble = $2T/R$

23. Rise of liquid through capillary tube :

$$h = \frac{2T \cos \theta}{r \rho g}$$

24. Temperature Conversion :

$$\frac{F \ddot{u}}{\ddot{u}} = \frac{C-0}{100}$$

$$= \frac{K-273}{100} = \frac{R-0}{80}$$

$$T_K = t_C + 273.15$$

25. Heat Transfer Processes :

(a) Conduction is a procedure where heat is transferred between objects through physical contact.

(b) Convection is the type of heat transmission where there is an energy transform within the fluid.

(c) Radiation refers to the process by which heat is transferred between things without direct physical touch.

26. Thermal Expansion :**Linear Expansion :**

$$\alpha = \frac{\Delta L}{L \Delta T}$$

Area Expansion :

$$\beta = \frac{\Delta A}{A \Delta T}$$

Volume Expansion :

$$\gamma = \frac{\Delta V}{V \Delta T}$$

Relation between α , β , and γ :

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3}$$

27. Specific Heat :

$$C = \frac{\Delta Q}{m \Delta T}$$

28. Newton's Law of Cooling :

$$-\frac{dQ}{dt} = K(T - T_0)$$

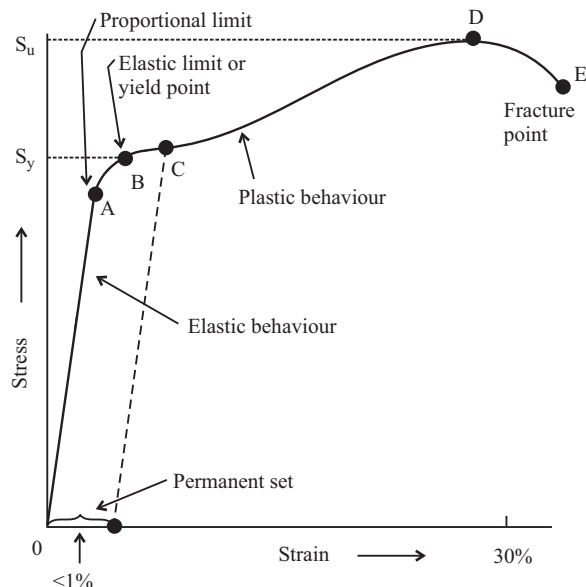
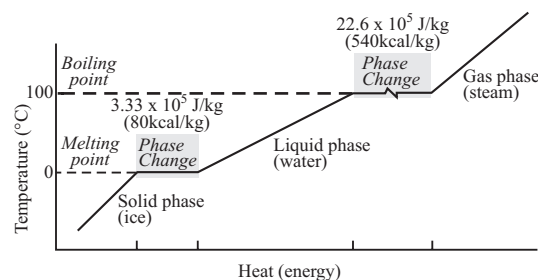
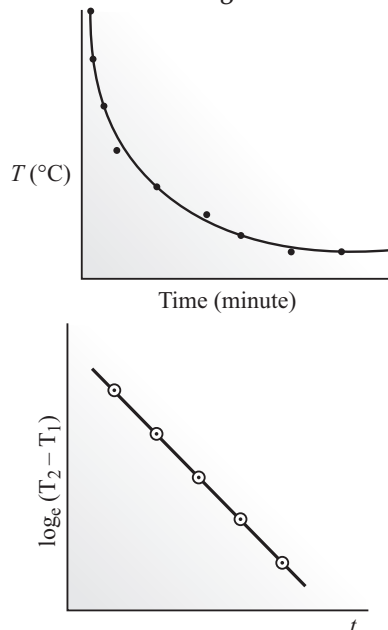
29. Stefan's law: For an ideal absorber/emitter / black body, total energy radiated per unit surface area per unit time is proportional to the fourth power of the black body's temperature, T.

$$E = \sigma(T^4 - T_0^4)$$

30. Wien's displacement law:

As the temperature of a black body rises, the radiation of highest intensity shift towards the lower wavelength.

$$\lambda_m T = b$$

Important Graphs :**A Typical Stress Strain Curve for metal :****Change of State (Water at 1 atm pressure)****Newton's Law of Cooling :****EXERCISE**

- All objects experience a buoyancy when they are immersed in a fluid. Buoyancy is :
 - a downward force
 - a downward pressure
 - an upward force
 - an upward pressure
- The volume of a sealed packet is 1 litre and its mass is 800 g. The packet is first put inside water with density 1 g cm^{-3} and then in another liquid B with density 1.5 g cm^{-3} . Then which one of the following statements holds true?
 - The packet will float in both water and liquid B.
 - The packet will sink in both water and liquid B.
 - The packet will sink in water but will float in liquid B.
 - The packet will float in water and sink in liquid B.
- A wooden box of mass 2 kg and dimensions (30 cm $\times$ 5 cm $\times$ 10 cm) is placed on a table with sides 30 cm and 10 cm touching the tabletop. Which one of the following is the approximate pressure exerted on the table?

(a) 111.1 N/m ²	(b) 222.2 N/m ²
(c) 333.3 N/m ²	(d) 666.6 N/m ²
- A liquid is kept in a glass beaker. Which one of the following statements is correct regarding the pressure exerted by the liquid column at the base of the beaker?
 - The pressure depends on the area of the base of the beaker.
 - The pressure depends on the height of liquid column.
 - The pressure does not depend on the density of the liquid.
 - The pressure neither depends on the area of the base of the beaker nor on the height of liquid column.
- If some object is weighed when submerged in water, what will happen to its weight compared to its weight in air?
 - Increase
 - Decrease
 - Remain exactly the same
 - Increase or decrease cannot be predicted
- A container is first filled with water and then the entire water is replaced by mercury. Mercury has a density of $13.6 \times 10^3 \text{ kg/m}^3$. If X is the weight of the water and Y is the weight of the mercury, then

(a) $X = Y$	(b) $X = 13.6Y$
(c) $Y = 13.6X$	(d) None of the above

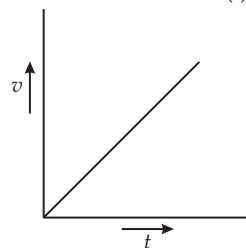
7. Whether an object will float or sink in a liquid, depends on
 (a) mass of the object only.
 (b) mass of the object and density of liquid only.
 (c) difference in the densities of the object and liquid.
 (d) mass and shape of the object only.
8. Which of the following statements about a fluid at rest in a cup is/are correct?
 (1) Pressure is same at all the points in the fluid.
 (2) Pressure is exerted on the walls.
 (3) Pressure exists everywhere in the fluid.
 Select the correct answer using the code given below:
 (a) 1 and 2 only (b) 2 and 3 only
 (c) 1 only (d) 1, 2 and 3
9. Which one of the following devices is used to measure atmospheric pressure?
 (a) Ammeter (b) Barometer
 (c) Potentiometer (d) Lactometer
10. Along a streamline flow of fluid
 (a) the velocity of all fluid particles at a given instant is constant
 (b) the speed of a fluid particle remains constant
 (c) the velocity of all fluid particles crossing a given position is constant
 (d) the velocity of a fluid particle remains constant
11. Which one of the following is not a result of surface tension?
 (a) Nearly spherical drop of rain
 (b) Capillary rise
 (c) Removal of dirt by soap or detergent
 (d) Flow of a liquid
12. A person stands on his two feet over a surface and experiences a pressure P . Now the person stands on only one foot. He would experience a pressure of magnitude :
 (a) $4P$ (b) P (c) $\frac{1}{2}P$ (d) $2P$
13. Which one among the following does not wet the walls of the glass vessel in which it is kept?
 (a) Water (b) Alcohol
 (c) Mercury (d) Phenol
14. Bernoulli's principle is based on which one among the following laws?
 (a) Conservation of mass
 (b) Conservation of momentum
 (c) Conservation of angular momentum
 (d) Conservation of energy
15. A particle falling through a viscous liquid reaches its terminal velocity. The acceleration then is
 (a) g (b) $> g$ (c) $< g$ (d) 0
16. Bernoulli's principle is based on principle of conservation of _____.
 (a) Energy (b) Mass
 (c) Linear momentum (d) Angular momentum
17. Water in a capillary tube rises to a height 6 cm. The area of cross-section of the capillary is made one fourth. Water will rise upto
 (a) 6 cm (b) 12 cm (c) 3 cm (d) 24 cm

18. The excess pressure in a soap bubble is
 (a) $\frac{4T}{R}$ (b) $\frac{T}{4R}$ (c) $\frac{2T}{R}$ (d) $\frac{R}{2T}$

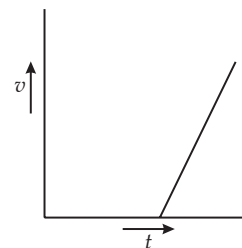
19. 1 torr of pressure is the pressure exerted by

- (a) 1 m high column of water
 (b) 1 cm high column of water
 (c) 1 mm high column of water
 (d) None of the above

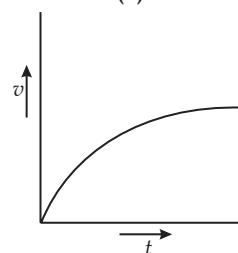
20. A tall cylinder is filled with viscous oil. A round pebble is dropped from the top with zero initial velocity. From the plot shown in Fig. indicate the one that represents the velocity (v) of the pebble as a function of time (t).



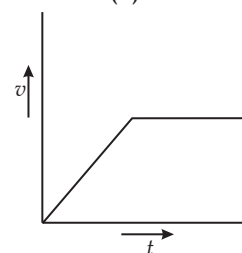
(a)



(b)

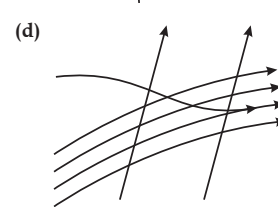
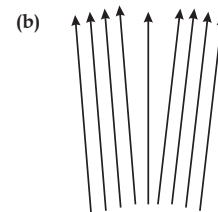


(c)



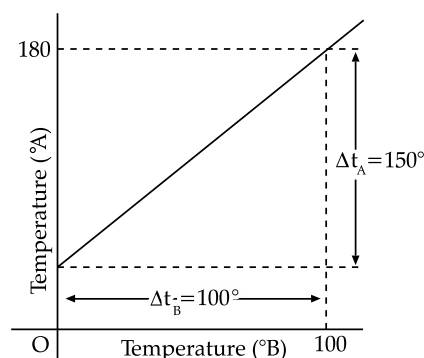
(d)

21. Which of the following diagrams does not represent a streamline flow?



22. Along a streamline
 (a) the velocity of a fluid particle remains constant.
 (b) the velocity of all fluid particles crossing a given position is constant.
 (c) the velocity of all fluid particles at a given instant is constant.
 (d) the speed of a fluid particle remains constant.
23. An ideal fluid flows through a pipe of circular cross-section made of two sections with diameters 2.5 cm and 3.75 cm. The ratio of the velocities in two pipes is
 (a) $9:4$ (b) $3:2$ (c) $\sqrt{3}:\sqrt{2}$ (d) $\sqrt{2}:\sqrt{3}$

24. 100g ice at 0°C is mixed with 100g water at 20°C . The final temperature will be
 (a) 0°C (b) 10°C
 (c) 20°C (d) -10°C
25. When temperature of water decreases from 4°C to 0°C its density _____, volume _____.
 (a) Increases, increases. (b) Decreases, decreases.
 (c) Increases, decreases. (d) Decreases, increases.
26. Which of following process of transmission of heat will not work in space station?
 (a) Conduction
 (b) Convection
 (c) Radiation
 (d) Both conduction and convection
27. Two ends of a metal rod are maintained at temperatures 200°C and 210°C . The rate of heat flow was found to be 10 J/s. If the ends of the rod are maintained at 400°C and 410°C , the rate of heat flow will be
 (a) 10J/s (b) 20J/s (c) 80J/s (d) 30J/s
28. When a metal sphere is heated, the largest percentage increase will occur in its
 (a) Volume (b) Diameter
 (c) Area (d) Mass
29. A bimetallic strip is made of aluminium and steel ($\alpha_{\text{AL}} > \alpha_{\text{steel}}$).
 On heating, the strip will
 (a) remain straight
 (b) get twisted
 (c) will bend with aluminium on concave side.
 (d) will bend with steel on concave side.
30. A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly
 (a) its speed of rotation increases.
 (b) its speed of rotation decreases.
 (c) its speed of rotation remains same.
 (d) its speed increases because its moment of inertia increases.
31. The graph between two temperature scales A and B is shown in Fig. Between upper fixed point and lower fixed point there are 150 equal division on scale A and 100 on scale B. The relationship for conversion between the two scales is given by
 (a) $\frac{t_A - 180}{100} = \frac{t_B}{150}$ (b) $\frac{t_A - 30}{150} = \frac{t_B}{100}$
 (c) $\frac{t_B - 180}{150} = \frac{t_A}{100}$ (d) $\frac{t_B - 40}{100} = \frac{t_A}{180}$



32. An aluminium sphere is dipped into water. Which of the following is true?
 (a) Buoyancy will be less in water at 0°C than that in water at 4°C .
 (b) Buoyancy will be more in water at 0°C than that in water at 4°C .
 (c) Buoyancy in water at 0°C will be same as that in water at 4°C .
 (d) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere.
33. As the temperature is increased, the time period of a pendulum
 (a) increases as its effective length increases even though its centre of mass still remains at the centre of the bob.
 (b) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob.
 (c) increases as its effective length increases due to shifting of centre of mass below the centre of the bob.
 (d) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob.
34. Heat is associated with
 (a) kinetic energy of random motion of molecules.
 (b) kinetic energy of orderly motion of molecules.
 (c) total kinetic energy of random and orderly motion of molecules.
 (d) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other.
35. Rigidity modulus and Young's modulus of are respectively η and Y . A copper wire of length L and area of cross-section A is so pulled that its length becomes $5L$ and area of cross-section becomes $\frac{A}{5}$. So
 (a) Its Y increases, η decreases.
 (b) Its η increases, Y decreases.
 (c) Its Y and η both increases.
 (d) Its Y and η both remains unchanged

Answer Key

1. (c)	2. (a)	3. (d)	4. (b)	5. (b)	6. (c)	7. (c)	8. (b)	9. (b)	10. (c)
11. (d)	12. (d)	13. (c)	14. (d)	15. (d)	16. (a)	17. (b)	18. (a)	19. (d)	20. (c)
21. (d)	22. (b)	23. (a)	24. (a)	25. (d)	26. (b)	27. (a)	28. (a)	29. (d)	30. (b)
31. (b)	32. (a)	33. (a)	34. (a)	35. (d)					

Answers with Explanations

1. Option (c) is correct.

Explanation: Buoyance is an upward force, responsible for floating of all the objects Immersed in fluids.

2. Option (a) is correct.

Explanation:

$$\text{Density of object, } \rho = \frac{\text{mass}}{\text{volume}} = \frac{800}{1000} = 0.8 \text{ gcm}^{-3}$$

Since density of object is less than both water and liquid B. So it will float on them.

3. Option (d) is correct.

Explanation:

$$\text{Here, } A = 30 \times 10 \times 10^{-4} = 3 \times 10^{-2} \text{ m}^2$$

$$F = mg = 2 \times 10 = 20 \text{ N}$$

$$\text{Pressure, } P = \frac{F}{A} = \frac{20}{3 \times 10^{-2}} = 6.66 \times 10^2 \frac{\text{N}}{\text{m}^2}$$

4. Option (b) is correct.

Explanation:

- The pressure depends on the height of liquid column
- The pressure increases as the depth inside the liquid increases.

5. Option (b) is correct.

Explanation: When an object is weighted in fluid, buoyant force exerted by fluid in upwards direction reduces the effect of force exerted by gravity. Since water is more dense compare to air, more buoyant force acts on object and its weigh is less in water as compare to its weight in air.

6. Option (c) is correct.

Explanation: Since, mercury is 13.6 times denser than water for the same volume its weight will be 13.6 times more than the weight of water.

$$Vw = Vm$$

$$\frac{W_w}{D_w} = \frac{W_m}{D_m}$$

$$\frac{X}{1} = \frac{Y}{13.6}$$

$$Y = 13.6X$$

7. Option (c) is correct.

Explanation: Law of flotation depends upon density of object and liquid. If the density of an object is more than that of liquid it will sink, if it is less it will float.

8. Option (b) is correct.

Explanation: Fluid exerts pressure everywhere, at wall but fluid pressure is same at same height/depth.

9. Option (b) is correct.

Explanation:

Ammeter – Electric Current

Barometer – Atmospheric Pressure

Potentiometer - EMF of two cell

Lactometer - To check milk

10. Option (c) is correct.

Explanation: In streamline flow there is uniformity, hence velocity of all fluid particles passing through a point remains same.

11. Option (d) is correct.

Explanation: flow of liquid depends upon fluid pressure or level of surface.

12. Option (d) is correct.

$$\text{Explanation: Since, } P \propto \frac{1}{A}$$

Since we are lifting one leg, it means only one leg is on surface. As area becomes half pressure will become double.

13. Option (c) is correct.

Explanation: Surface tension in the mercury is very high, it means force of attraction between molecule of mercury is much higher than between mercury and glass. That's why mercury won't stick on glass wall.

14. Option (d) is correct.

Explanation: Bernoulli's principle is based on conservation of energy.

15. Option (d) is correct.

Explanation: After achieving terminal velocity the particle moves with a constant velocity. Hence the acceleration is 0.

16. Option (a) is correct.

17. Option (b) is correct.

$$\text{Explanation: } h = \frac{2T}{r\rho g}$$

$$\text{Initial area} = A_1 = \pi r_1^2$$

$$\text{Final area} = A_2 = \pi r_2^2$$

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

$$\text{Or, } 4 = \frac{r_1^2}{r_2^2}$$

$$\therefore \frac{r_1}{r_2} = 2$$

$$h = \frac{2T}{r\rho g}$$

$$\therefore h \propto \frac{1}{r}$$

$$\text{or } \frac{h_1}{h_2} = \frac{r_2}{r_1}$$

$$\text{or } \frac{6}{h_2} = \frac{1}{2}$$

$$\therefore h_2 = 12 \text{ cm}$$

18. Option (a) is correct.

19. Option (d) is correct.

Explanation: 1 torr = pressure exerted by 1 mm high column of mercury.

20. Option (c) is correct.

Explanation: As the pebble acquires terminal velocity after some time.

21. Option (d) is correct.

Explanation: In streamline flow of flow do not intersect each other.

22. **Option (b) is correct.**

23. **Option (a) is correct.**

Explanation: According to equation of continuity

$$a_1 v_1 = a_2 v_2$$

$$\text{or } \frac{v_1}{v_2} = \frac{a_2}{a_1} = \frac{\pi \frac{d_2^2}{4}}{\pi \frac{d_1^2}{4}} = \left(\frac{d_2}{d_1} \right)^2 = \left(\frac{3.75}{2.50} \right)^2 = \frac{9}{4}$$

24. **Option (a) is correct.**

Explanation: Heat required by ice to melt completely = $80 \times 100 = 8000$ calorie.

If the temperature of mixture becomes 0°C , then the heat released by water is 20°C

$$\begin{aligned} &= ms\Delta\theta \\ &= 100 \times 1 \times 20 \\ &= 2000 \text{ calorie} \end{aligned}$$

This heat will not be capable to melt the ice completely.

So, the temperature of the mixture will remain 0°C .

25. **Option (d) is correct.**

Explanation: Due to anomalous expansion of water if the temperature of water decreases from 4°C to 0°C its density decreases, volume increases.

26. **Option (b) is correct.**

Explanation: Convection process depends on gravity. So, where there is no gravity the convection process does not work.

27. **Option (a) is correct.**

Explanation: Since, the temperature difference remains same, the rate of heat flow will also remain same.

28. **Option (a) is correct.**

Explanation: Since, volume = $\frac{4}{3}\pi r^3$ and area =

$4\pi r^2$, the percentage volume increase will be maximum. Density decreases with temperature. Mass is a constant quantity.

29. **Option (d) is correct.**

Explanation: $\because \alpha_{\text{AL}} > \alpha_{\text{steel}}$

$\therefore$ Aluminium will expand more than that of steel strip. Due to it, this steel strip will bend on concave side.

30. **Option (b) is correct.**

Explanation: Its M.I increases .

Angular momentum, $L = I\omega$,

ω is angular speed which decreases to conserve L.

31. **Option (b) is correct.**

Explanation: For the scale A,

Lower fixed point = 30°A

Upper fixed point = 180°A

For the scale B,

Lower fixed point = 0°B

Upper fixed point = 100°B

$\therefore$ The relationship between the two scales A & B is given by

$$\frac{T_A - 30}{150} = \frac{T_B - 0}{100}$$

32. **Option (a) is correct.**

Explanation: Density of water at 4°C is more than that at 0°C .

33. **Option (a) is correct.**

Explanation: Effective length increases with temperature. As length increases, time period increases.

34. **Option (a) is correct.**

35. **Option (d) is correct.**

Explanation: Modulus of elasticity does not change with dimension. It depends on material only.

Chapter 37

HEAT AND THERMODYNAMICS

Heat

- It is a form of energy,
- Its unit are calorie, kilocalorie or joule.
- 1 calorie = 4.18 joule

Temperature

- Temperature is the measurement of hotness or coldness of a body.
- When two bodies are placed in contact, heat always flows from a body at a higher temperature to the body at a lower temperature.
 - $K = C + 273.15$
 - $F = \frac{9}{5} C + 32$

Thermal Expansion

- Thermal Expansion is the increase in the size of objects on heating
- A solid undergo three types of expansion:
 - (i) Linear expansion
 - (ii) Surface expansion
 - (iii) Cubical expansion
- When the temperature of the water is increased from 0°C then its volume decreases up to 4°C , becomes minimum at 4°C , and then increases. This behavior of water around 4°C is called the anomalous expansion of water.

Specific Heat

- It is defined as the amount of energy required to raise the temperature of 1 kg of a substance per unit mass by 1°C . In SI system it is expressed in $\text{J kg}^{-1} \text{K}^{-1}$. It is given by
(S), $Q = ms\Delta T$

Latent heat

- Latent heat is heat energy absorbed or released at constant temperature per unit mass for change of state. It is given by, $Q = mL$
- Latent heat of fusion of ice is 80 cal/g
- Latent heat of vaporization of steam is 536 cal/g .

Note: Latent heat is for phase change, specific heat is for temperature change.

Evaporation

- Evaporation is the process of conversion of a liquid into vapor below its boiling point.
- Relative humidity is measured by a hygrometer.

➤ Important Results and Formulae :

1. Boyle's Law :

$$V \propto \frac{1}{P} \text{ or } PV = \text{constant}$$

2. Charle's Law :

$$V \propto T \text{ or } \frac{V}{T} = \text{constant}$$

3. Gay Lussac's Law :

$$P \propto T \text{ or } \frac{P}{T} = \text{constant}$$

4. Ideal Gas Equation :

$$PV = nRT$$

$$\text{for 1 mole, } PV = RT$$

5. Universal Gas Constant :

$$R = 8.31 \text{ Jmol}^{-1}\text{K}^{-1}$$

6. Pressure Exerted by Gas :

$$P = \frac{1}{3} \cdot \frac{M}{V} \bar{v}^2 = \frac{1}{3} \rho \bar{v}^2$$

$$= \frac{1}{3} mn \bar{v}^2$$

where, n = no. of molecules per unit volume.

m = mass of each molecule.

$\bar{v}^2$ = mean of square speeds.

V = volume

M = mass of gas

7. Mean K.E. per mole of a gas :

$$\bar{E} = \frac{1}{2} M \bar{v}^2 = \frac{3}{2} PV$$

$$= \frac{3}{2} RT = \frac{3}{2} k_B N_B T$$

Mean K.E. per molecule of gas,

$$\bar{E} = \frac{1}{2} M \bar{v}^2 = \frac{1}{2} k_B T$$

8. Graham's Law of diffusion :

$$\frac{r_1}{r_2} = \sqrt{\frac{\rho_1}{\rho_2}}$$

9. RMS Speed :

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + \dots + v_n^2}{n}}$$

$$v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{m}} = \sqrt{\frac{3PV}{m}}$$

10. Most Probable Speed :

$$v_{m.p.} = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2RT}{m}} = \sqrt{\frac{2PV}{m}}$$

11. Average Speed :

$$v_{av} = \sqrt{\frac{8K_B T}{\pi m}} = \sqrt{\frac{8RT}{\pi M}}$$

Note : $v_{rms} > v_{av} > v_{m.p.}$

[In short you can remember RAM]

12. Degree of freedom :

$$f = 3N - k$$

where, f = no. of degrees of freedom.

N = no. of atoms in a molecule.

k = no. of independent relation between the atoms.

For Monoatomic gas:

3 Degree of freedom.

For Diatomic gas:

5 Degree of freedom.

13. Law of equipartition of energy : At thermal equilibrium, the average kinetic energy of each

molecule of a substance is equally divided among its degrees of freedom. Average kinetic energy

associated with each degree of freedom is $\frac{1}{2} k_B T$.

14. Mean free path : It is the average distance covered by a molecule between two successive collisions. It is given by,

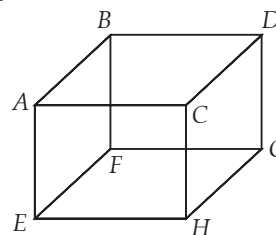
$$\bar{\lambda} = \frac{1}{\sqrt{2}(n\pi d^2)} = \frac{kT}{\sqrt{2} \pi d^2 P}$$

where, n is no. density and ' d ' is diameter of the

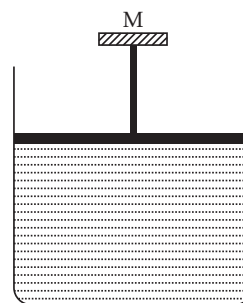
EXERCISE

- Which one of the following is the lowest possible temperature?
(a) 0° Celsius (b) -073° Celsius
(c) -173° Celsius (d) -273° Celsius
- Numerically two thermometers, one in Fahrenheit scale and another in Celsius scale shall read same at :
(a) -40° (b) 0° (c) -273° (d) 100°
- The temperature of a place on one sunny day is 113° in Fahrenheit scale. The Kelvin scale reading of this temperature will be :
(a) 318 K (b) 45 K
(c) 62.8 K (d) 335.8 K
- The brightness of a star depends on its :
(a) Size and temperature only
(b) Size and distance from the earth
(c) Size, temperature and mass
(d) Size, temperature and distance from the earth
- What is the mass of a material, whose specific heat capacity is $400 \text{ J/(kg}^\circ\text{C)}$ for a rise in temperature from 15°C to 25°C , when heat received is 20 kJ?
(a) 0.1 kg (b) 1 kg (c) 10 kg (d) 5 kg
- The specific latent heat of vaporization of a substance is the quantity of heat needed to change unit mass from :
(a) liquid to vapour with a change of temperature.
(b) liquid to vapour without a change of temperature.
(c) vapour to liquid without a change of temperature.
(d) vapour to liquid with a change of temperature.
- 10 g of ice at -10°C is mixed with 10 g of water at 0°C . The amount of heat required to raise the temperature of mixture to 10°C is :
(a) 400 cal (b) 550 cal
(c) 1050 cal (d) 1200 cal
- Heat given to a body which raises its temperature by 1°C is known as :
(a) water equivalent
(b) thermal capacity
(c) specific heat
(d) temperature gradient
- Evaporation from the surface of a given liquid takes place more rapidly when
(a) the temperature is high and the surface area of the liquid is large.
(b) the temperature is low and the surface area of the liquid is large.
(c) the temperature is low and the surface area of the liquid is small.
(d) the temperature is high and the surface area of the liquid is small.
- In which of the following phenomena do heat waves travel along a straight line with the speed of light?
(a) Thermal conduction
(b) Thermal convection
(c) Thermal radiation
(d) Both, thermal conduction and radiation
- If the work done on the system or by the system is zero, which one of the following statements for a gas kept at a certain temperature is correct?
(a) Change in internal energy of the system is equal to flow of heat in or out of the system.
(b) Change in internal energy of the system is less than heat transferred.
(c) Change in internal energy of the system is more than the heat flow.
(d) Cannot be determined.
- The statement that 'heat cannot flow by itself from a body at a lower temperature to a body at a higher temperature', is known as
(a) Zeroth law of thermodynamics
(b) First law of thermodynamics
(c) Second law of thermodynamics
(d) Third law of thermodynamics
- A stainless steel chamber contains Ar gas at a temperature T and pressure P . The total number of Ar atoms in the chamber is n . Now Ar gas in the chamber is replaced by CO_2 atoms in the number of CO_2 atoms in the chamber is $\frac{n}{2}$ at the same temperature T . The pressure in the chamber now is P' . Which one of the following relations holds true? (Both the gases behave as ideal gases)
(a) $P' = P$ (b) $P' = 2P$
(c) $P' = \frac{P}{2}$ (d) $P' = \frac{P}{4}$
- If we plot a graph between volume V and inverse of pressure P (Le, $1/P$) for an ideal gas at constant temperature T , the curve so obtained is
(a) straight line (b) circle
(c) parabola (d) hyperbola
- The pressure of an ideal gas undergoing isothermal change is increased by 10%. The volume of the gas must decrease by about
(a) 0.1% (b) 9% (c) 10% (d) 0.9%

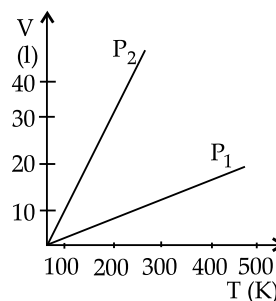
- (d) The pressure on EFGH would be half that of ABCD.



23. Boyle's law is applicable for an
 - (a) adiabatic process.
 - (b) isothermal process.
 - (c) isobaric process.
 - (d) isochoric process.
24. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction. If the temperature is increased,



- (a) both P and V of the gas will change.
(b) only P will increase according to Charles's law.
(c) V will change but not P.
(d) P will change but not V.
25. Volume versus temperature graphs for a given mass of an ideal gas are shown in Fig. at two different values of constant pressure. What can be inferred about relation between P_1 & P_2 ?
- (a) $P_1 > P_2$ (b) $P_1 = P_2$
(c) $P_1 < P_2$ (d) data is insufficient



1. (d)	2. (a)	3. (a)	4. (d)	5. (d)	6. (b)	7. (c)	8. (b)	9. (a)	10. (c)
11. (a)	12. (c)	13. (c)	14. (a)	15. (b)	16. (c)	17. (c)	18. (a)	19. (d)	20. (a)
21. (b)	22. (d)	23. (b)	24. (c)	25. (a)					

Answers with Explanations

1. Option (d) is correct.

Explanation: According to Charles's law, the lowest possible temperature is -273° and the volume of any gas at -273° is zero.

2. Option (a) is correct.

Explanation:

- Celsius and Fahrenheit are two temperature scales.
- Both the Fahrenheit and Celsius scales have one point at which they intersect and that point they are equal at -40°C and -40°F .

3. Option (a) is correct.

Explanation: Temperature of a place is 113 in Fahrenheit scale. So,

$$(^{\circ}\text{F} - 32) \times \frac{5}{9} + 273.15 = \text{K}$$

$$(113^{\circ}\text{F} - 32) \times \frac{5}{9} + 273.15 = 318.15\text{K}$$

$$\Rightarrow \text{K} = 318.15$$

4. Option (d) is correct.

Explanation: The luminosity of a star is defined as the amount of light and other forms of radiant energy it radiates per unit of time. The luminosity of a star is determined by its radius (size), surface temperature and how far it is from the observer. Astronomers define star brightness in terms of apparent magnitude- how bright the star appears from Earth and absolute magnitude-how bright the star appears at a standard distance of 32.6 light- years

5. Option (d) is correct.

Explanation: Formula used $q = ms\Delta T$

$$S = \frac{400\text{J}}{\text{kg } ^{\circ}\text{C}}$$

$$q = 20\text{kJ} = 20 \times 10^3\text{J}$$

$$= 10 \times 0.5 \times 10 + 10 \times 80 + 20 \times 1 \times 10$$

By putting all the values :

$$20 \times 10^3\text{J} = m \times \frac{400\text{J}}{\text{kg } ^{\circ}\text{C}} \times 10^{\circ}\text{C}$$

$$m = \frac{20 \times 10^3\text{J}}{400 \frac{20 \times 10^3\text{J}}{\text{kg } ^{\circ}\text{C}}} = 5\text{kg}$$

6. Option (b) is correct.

Explanation: The specific latent heat of vaporization of a substance is the quantity of heat needed to change unit mass from liquid to vapour without change of temperature.

7. Option (c) is correct.

Explanation: Heat required to raise the temperature = Heat required to (raise temperature of ice to $^{\circ}\text{C}$ + convert ice to water + raise total solution to 10°C)

$$ms\Delta T_{\text{ice}} + mL + ms\Delta T_{\text{water}}$$

$$= 10 \times 0.5 \times 10 + 10 \times 80 + 20 \times 1 \times 10$$

$$= 50 + 800 + 200$$

$$= 1050\text{ cal}$$

8. Option (b) is correct.

Explanation: Heat capacity or thermal capacity of a body is the amount of heat required to raise its temperature by 1°C .

Specific heat is the amount of heat required to raise the temperature of a unit mass of substance by 1°C .

9. Option (a) is correct.

Explanation: Evaporation is the process in which the conversion of liquid into vapour takes place at all temperatures.

It increases with increase in temperature, surface area and wind speed.

10. Option (c) is correct.

Explanation: Thermal radiation refers the energy emitted by a body in the form of radiation on account of its temperature. During thermal radiations, the heat waves travel along straight lines with speed of light. All bodies having a temperature above 0 K emit thermal radiation continuously. For example, the radiation emitted by red-hot iron or light from a filament lamp is thermal radiation.

11. Option (a) is correct.

Explanation: According to 1st law

$$dQ = dU + dW$$

$$\text{when } dW = 0 \text{ then } dQ = dU$$

12. Option (c) is correct.

Explanation: Energy flows from higher temperature to lower temperature, to make it reverse external supply is needed.

13. Option (c) is correct.

Explanation: Temperature is same for both the gases. Container is same which means volume occupied by both the gas is also same.

$$\text{also } n' = \frac{n}{2} \text{ and } V' = V$$

$$\text{Using ideal gas equation } \frac{P'V'}{n'} = \frac{PV}{n}$$

$$\frac{2P'V}{n} = \frac{PV}{n}$$

$$P' = \frac{P}{2}$$

14. Option (a) is correct.

Explanation: For an ideal gas,

$PV = nRT$ since temperature is constant it means $PV = \text{constant} = k$

$$V = \frac{k}{P} \propto \left(\frac{1}{P}\right)$$

since, volume is directly proportion to $\left(\frac{1}{P}\right)$, we will get a straight line graph.

15. Option (b) is correct.

Explanation: Given $P_1 = 100\text{P}$ $P_2 = 110\text{P}$

$$V_1 = V \quad V_2 = ?$$

From Boyle's law

$$P_1 V_1 = P_2 V_2$$

$$100P \times V = 110P \times V_2$$

$$V_2 = \frac{100P \times V}{110P} = 0.909V$$

$$\% \text{ change} \Rightarrow \frac{V - 0.909V}{V} \times 100 = 9.1\% \cong 9\%$$

16. Option (c) is correct.

Explanation: The molecule of a monatomic gas has only three translational degrees of freedom.

$$\text{So, } C_V = \frac{5}{2} R$$

$$\text{Since, } C_P - C_V = R$$

$$\therefore C_P = \frac{5}{2} R$$

$$\frac{C_P}{C_V} = \frac{5}{3}$$

A diatomic molecule treated having a rigid dumb-bell shape, has 3 translational and 2 rotational degrees of freedom.

$$\text{So, } C_V = \frac{5}{2} R$$

$$\text{Since, } C_P - C_V = R$$

$$\therefore C_P = \frac{7}{2} R$$

$$\frac{C_P}{C_V} = \frac{7}{5}$$

17. Option (c) is correct.

18. Option (a) is correct.

Explanation: The average kinetic energy of a molecule is proportional to the absolute temperature of the gas; it is independent of pressure, volume or the nature of the ideal gas. This is a fundamental result relating temperature.

19. Option (d) is correct.

$$\text{Explanation: } \lambda = \frac{1}{\sqrt{2n\pi d^2}}$$

20. Option (a) is correct.

21. Option (b) is correct.

$$\text{Explanation: } P = \frac{nRT}{V}$$

P remains unaffected as n , R , T and V are constant.

22. Option (d) is correct.

23. Option (b) is correct.

Explanation: Since, $\propto \frac{1}{T}$ at constant temperature.

24. Option (c) is correct.

$$\text{Explanation: } P = \frac{F}{A} = \frac{mg}{A} = \text{constant}$$

$$\therefore V \propto T \text{ (at constant pressure)}$$

25. Option (a) is correct.

Explanation: $V \propto T$

$$\frac{V}{T} = \text{constant} = \text{slope of the graph}$$

$$\text{Again, } V = \frac{nRT}{P}$$

So, as slope increases, P decreases.

In graph, slope at P_2 is more than slope at P_1 ,

$$\therefore P_1 > P_2$$

○○○

Simple Pendulum

- A heavy point mass suspended from rigid support using an elastic and inextensible string, is called a simple pendulum. Time period is given by, $T = \frac{2\pi}{\omega}$

$$= 2\pi \sqrt{\frac{l}{g}}$$
- If a lift falls freely under gravity when the time of the pendulum is infinite
- Time period of the pendulum will increase if a simple pendulum is suspended in a lift with acceleration.
- If lift is ascending, then the time period of pendulum will decrease.

Waves

- Waves are disturbance, which propagates energy from one place to the other without transportation of matter
- The waves are classified into two types:
 - Mechanical wave (longitudinal wave and transverse wave)
 - Electromagnetic force.

Wave velocity: Distance covered by a wave in unit time in its direction of propagation. Speed with which wave propagates in a medium. $v = f\lambda = \frac{\lambda}{T}$

Longitudinal wave

- In this wave, particles of the medium vibrate in the direction of propagation of the wave.
- Sound waves in air is an example of longitudinal waves.

Transverse wave

- In this wave, particles of the medium vibrate perpendicular to the direction of propagation of a wave.
- Waves on strings under tension, waves on the surface of water are examples of transverse waves.

Electromagnetic wave

- Electromagnetic waves are those waves, which do not require a medium for their propagation i.e., which can propagate through the vacuum.
- Light, radio waves, microwaves, UV rays, Infrared, X-rays, UV rays are examples of electromagnetic waves

Sound waves

- Sound waves are longitudinal mechanical waves; sound waves are grouped as follows:
 - Audible waves : 20 Hz to 20,000 Hz
 - Infrasonic or subsonic waves : frequency < 20 Hz
 - Ultrasonic waves : frequency > 20,000 Hz
- Speed of sound wave is higher in denser medium. Hence, it is maximum in solid, least in gases.
- Speed of sound decreases with increase in temperature.
- Speed of sound increases with humidity.
- Speed of sound is independent of frequency of wave and pressure.

► Important Results and Formulae :

1. Harmonic Functions :

$$f(t) = \sin \frac{2\pi t}{T}$$

and

$$g(t) = \cos \frac{2\pi t}{T}$$

- All Harmonic functions are periodic but all periodic functions are not harmonic.

3. Characteristics of S.H.M. :

- $x = A \cos(\omega t + \theta_0)$
- $v = \frac{dx}{dt} = -\omega \sqrt{A^2 - x^2}$, $v_{\max} = \omega A$
- $a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi_0)$
 $= -\omega^2 x$
 $a_{\max} = \omega^2 A$

4. Restoring force :

$$F = -kx = -m\omega^2 x$$

where, k = force constant & $\omega^2 = \frac{k}{m}$

5. Angular frequency : $\omega = 2\pi v = \frac{2\pi}{T}$

6. Time Period :

$$T = 2\pi \sqrt{\frac{\text{Inertia Factor}}{\text{Spring Factor}}} = 2\pi \sqrt{\frac{m}{k}}$$

7. Energy in S.H.M. :

P.E. at displacement 'y' from mean position :

$$\begin{aligned} E_P &= \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2 y^2 \\ &= \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t \end{aligned}$$

K.E. at displacement 'y' from the mean position :

$$\begin{aligned} E_K &= \frac{1}{2}k(A^2 - y^2) \\ &= \frac{1}{2}m\omega^2 (A^2 - y^2) \\ &= \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t \end{aligned}$$

Total energy at any point :

$$\begin{aligned} E_T &= \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 \\ &= 2\pi^2 mA^2 v^2 \end{aligned}$$

8. Oscillation in a Spring :

Spring Factor : $k = F/y$

Period of oscillation of a mass ' m ' suspended from a massless spring of force constant ' k ' :

$$T = 2\pi\sqrt{\frac{m}{k}}$$

For two spring factors k_1 and k_2 connected in parallel effective spring factor :

$$k = k_1 + k_2$$

$\therefore$

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

For two spring connected in series, effective spring factor ' k ' :

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

or

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$

9. Oscillation of simple pendulum :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$v = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

10. Wave equation :

If wave is travelling along + x -axis

$$(i) \quad Y = A \sin(\omega t - kx),$$

$$\text{where, } k = \frac{2\pi}{\lambda}$$

$$(ii) \quad Y = A \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$(iii) \quad Y = A \sin \frac{2\pi}{\lambda}(vt - x)$$

If wave is travelling along -ve x -axis :

$$(i) \quad Y = A \sin(\omega t + kx),$$

$$\text{where, } k = \frac{2\pi}{\lambda}$$

$$(ii) \quad Y = A \sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)$$

$$(iii) \quad Y = A \sin \frac{2\pi}{\lambda}(vt + x)$$

11. Phase and phase difference :

Phase is the argument of the sine or cosine function representing the wave.

$$\phi = 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

12. Angular wave number : It is phase change per unit distance.

$$\text{i.e., } k = \frac{2\pi}{\lambda},$$

S.I. unit of k is radian per metre.

13. Velocity of Transverse wave :

(i) In solid molecules having modulus of rigidity ' η ' and density ' ρ ' :

$$v = \sqrt{\frac{\eta}{\rho}}$$

(ii) In string for mass per unit length ' m ' and

$$\text{tension 'T' : } v = \sqrt{\frac{T}{m}}$$

Velocity of longitudinal wave :

(a) In solid

$$v = \sqrt{\frac{Y}{\rho}}, \quad (Y = \text{Young's modulus})$$

(b) In liquid

$$v = \sqrt{\frac{K}{\rho}}, \quad (K = \text{bulk modulus})$$

$$(c) \text{ In gases } v = \sqrt{\frac{K}{\rho}}, \quad (K = \text{bulk modulus})$$

According to Newton's formula,

$$v = \sqrt{\frac{P}{\rho}}$$

According to Laplace's law

$$v = \sqrt{\frac{P\gamma}{\rho}}$$

$$\text{where, } \gamma = \frac{C_p}{C_v}$$

14. Stationary Waves in a Stretched String of length L :

$$y_1(x, t) = a \sin(kx - \omega t)$$

(wave travelling in the positive direction of x -axis)

$$\text{and } y_2(x, t) = a \sin(kx + \omega t)$$

(wave travelling in the negative direction of x -axis)

The principle of superposition gives, for the combined wave,

$$\begin{aligned} y(x, t) &= y_1(x, t) + y_2(x, t) \\ &= a \sin(kx - \omega t) + a \sin(kx + \omega t) \\ &= (2a \sin kx) \cos \omega t \end{aligned}$$

$$\text{nodes are at } x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \dots$$

$$\text{antinodes are at } x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$$

$$\text{Length of the string : } L = n \frac{\lambda}{2}$$

$$\text{Wavelength : } \lambda = \frac{2L}{n}$$

$$\text{Frequency : } v = n \frac{v}{2L}$$

15. Beats :

Consider the two sounds,

$$S_1 = a \cos \omega_1 t \text{ and } S_2 = a \cos \omega_2 t$$

$$\text{Frequency } \omega = 2\pi v$$

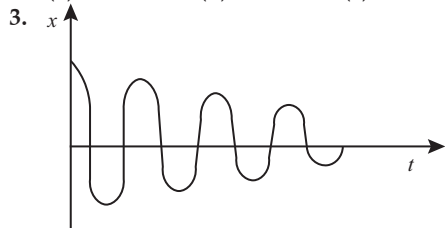
Then Beats Frequency :

$$v_{\text{beat}} = v_1 - v_2$$

EXERCISE

- The time period of a 1m long pendulum approximates to :
(a) 6 s (b) 4 s (c) 2 s (d) 1 s
- The time period of oscillation of a simple pendulum having length L and mass of the bob m is given as T . If the length of the pendulum is increased to $4L$ and the mass of the bob is increased to $2m$, then which one of the following is the new time period of oscillation?

- (a) T (b) $2T$ (c) $4T$ (d) $\frac{T}{2}$



The displacement (x)-time (t) graph given above approximately represents the motion of a :

- (a) simple pendulum placed in vacuum.
(b) simple pendulum immersed in water.
(c) simple pendulum placed in outer space.
(d) point mass moving in air.
- A sound wave has a frequency of 1 kHz and wavelength 50 cm. How long will it take to travel 1 km?
(a) 5 s (b) 4 s (c) 3 s (d) 2 s
- When the pitch of sound increases, which one of the following increases?
(a) Intensity (b) Loudness
(c) Wavelength (d) Frequency
- SONAR is a device that is used to measure the distance of underwater objects by a ship. Which of the following types of waves does it use for this purpose?
(a) Infrasonic waves
(b) Sound waves in audible range for human beings
(c) Ultrasonic waves
(d) All of the above
- Which one of the following statements about the speed of sound waves is not correct?
(a) The speed of sound waves in steel is higher than that in water.
(b) The speed of sound waves in air decreases with increase in temperature.
(c) The speed of sound waves in air increases with increase in temperature.
(d) The speed of sound waves in water is higher than that in air.
- Which one of the following statements about sound is NOT correct?
(a) Sound travels at a speed slower than the speed of light.
(b) Sound waves are transverse waves.
(c) Sound waves are longitudinal waves.
(d) Sound travels faster in moist air than in dry air.
- The speed of a body that has Mach number more than 1 is :
(a) supersonic (b) subsonic
(c) 300 m/s (d) about 10 m/s

- Ultrasonic waves of frequency 3×10^5 Hz are passed through a medium where speed of sound is 10 times that in air (Speed of sound in air is 300 m/s). The wavelength of this wave in that medium will be of the order of
(a) 1 cm (b) 10 cm (c) 100 cm (d) 0.1 cm

- Which one among the following waves bats use to detect the obstacles in their flying path?

- (a) Infrared waves
(b) Electromagnetic waves
(c) Ultrasonic waves
(d) Radio waves

- The part of the human ear that converts the pressure variations associated with audible sound waves to electrical signals is :

- (a) auditory nerve (b) cochlea
(c) eardrum (d) eustachian tube

- The sound created in a big hall persists because of the repeated reflections. The phenomenon is called

- (a) Reverberation. (b) Dispersion.
(c) Refraction. (d) Diffraction.

- 'Beats' is a phenomenon that occurs when frequencies of two harmonic waves are

- (a) equal. (b) far apart.
(c) multiples of each other. (d) nearly same.

- Reverberation is a phenomenon associated with a

- (a) multiple refraction of sound
(b) multiple reflection of sound
(c) single refraction of sound.
(d) single reflection of sound.

- Which one of the following frequency ranges is sensitive to human ears?

- (a) 0-200 Hz (b) 20-20,000 Hz
(c) 200-20,000 Hz only (d) 2,000-20,000 Hz only

- A particle is executing simple harmonic motion. When its displacement is x , its total energy is

- (a) Proportional to x (b) Proportional to $\frac{1}{x}$
(c) Proportional to x^2 (d) Independent of x

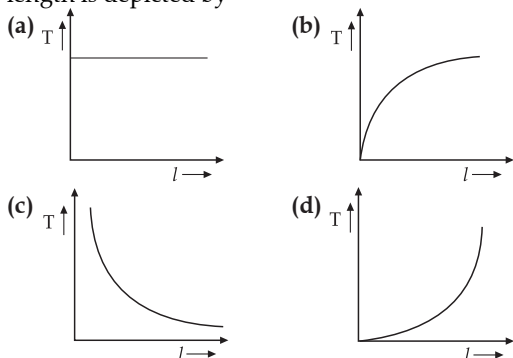
- Which of the following statement is incorrect?

- (a) Every periodic motion is oscillatory, but every oscillatory motion need not be periodic.
(b) Simple harmonic motion is the simplest form of oscillatory motion.
(c) There is no significant difference between oscillations and vibrations. It seems that when the frequency is small, we call it oscillation, while when the frequency is high, we call it vibration.
(d) Oscillating bodies eventually come to rest at their equilibrium positions.

- A particle moves such that its acceleration a is given by $a = -bx$, where x is the displacement from equilibrium position and b is a constant. The period of oscillation is

- (a) $\frac{2\pi}{\sqrt{b}}$ (b) $\frac{\sqrt{b}}{2\pi}$ (c) $\frac{2\pi}{\sqrt{b}}$ (d) $2\pi b$

20. In case of a simple pendulum, time period versus length is depicted by



21. A particle executes S.H.M. with a period of 8 second and amplitude of 4 cm. Its maximum speed in cm/s is

- (a) π (b) 2π (c) 3π (d) $\frac{\pi}{2}$

22. The displacement of a particle is represented by the equation $y = 3 \cos\left(\frac{\pi}{4} - 2\omega t\right)$. The motion of the particle is

- (a) simple harmonic with period $2\pi/\omega$.
(b) simple harmonic with period π/ω .
(c) periodic but not simple harmonic.
(d) non-periodic.

23. The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is

- (a) non-periodic.
(b) periodic but not simple harmonic.
(c) simple harmonic with period $2\pi/\omega$.
(d) simple harmonic with period π/ω .

24. The relation between acceleration and displacement of four particles are given below. Which one of the particles is executing simple harmonic motion?

- (a) $a_x = +2x$. (b) $a_x = +2x^2$.
(c) $a_x = -2x^2$. (d) $a_x = -2x$.

25. Motion of an oscillating liquid column in a U-tube is

- (a) periodic but not simple harmonic.
(b) non-periodic.
(c) simple harmonic and time period is independent of the density of the liquid.
(d) simple harmonic and time-period is directly proportional to the density of the liquid.

26. A particle is acted simultaneously by mutually perpendicular simple harmonic motions $x = a \cos \omega t$ and $y = a \sin \omega t$. The trajectory of motion of the particle will be

- (a) an ellipse. (b) a parabola.
(c) a circle (d) a straight line.

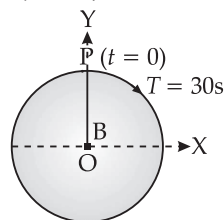
27. The displacement of a particle varies with according to the relation $y = a \sin \omega t + b \cos \omega t$.

- (a) The motion is oscillatory but not S.H.M.
(b) The motion is S.H.M. with amplitude $a + b$.
(c) The motion is S.H.M. with amplitude $a^2 + b^2$.
(d) The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$

28. Figure shows the circular motion of a particle. The radius of the circle, the period, sense of revolution

and the initial position are indicated on the figure. The simple harmonic motion of the x -projection of the radius vector of the rotating particle P is

- (a) $x(t) = B \sin\left(\frac{2\pi t}{30}\right)$. (b) $x(t) = B \sin\left(\frac{\pi t}{15}\right)$.
(c) $x(t) = B \sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$. (d) $x(t) = B \sin\left(\frac{\pi t}{15} + \frac{\pi}{2}\right)$.



29. The equation of motion of a particle is $x = a \cos(\alpha t)^2$. The motion is

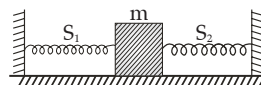
- (a) Periodic but not oscillatory.
(b) Periodic and oscillatory.
(c) Oscillatory but not periodic.
(d) Neither periodic nor oscillatory.

30. A particle executing S.H.M. has a maximum speed of 30 cm/s and a maximum acceleration of 60 cm/s². The period of oscillation

- (a) π s. (b) $\frac{\pi}{2}$ s. (c) 2π s (d) $\frac{\pi}{t}$ s.

31. When a mass m is connected individually to two springs, S_1 and S_2 , the oscillation frequencies are v_1 and v_2 . If the same mass is attached to the two springs as shown in Fig., the oscillation frequency would be

- (a) $v_1 + v_2$ (b) $\sqrt{v_1^2 + v_2^2}$
(c) $\left(\frac{1}{v_1} + \frac{1}{v_2}\right)^{-1}$ (d) $\sqrt{v_1^2 - v_2^2}$



32. Following are the equations of transverse waves:

- (i) $Z = A \cos(kx - \omega t)$
(ii) $Z = A \cos(kx + \omega t)$
(iii) $Z = A \cos(ky - \omega t)$

Which may superimpose to form stationary wave?

- (a) (i) and (ii) (b) (i) and (iii)
(c) (ii) and (iii) (d) (i) and (iii)

33. In a stationary wave, the distance between two consecutive nodes is

- (a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) λ (d) 2λ

34. Fundamental tone of same frequency is produced by an open organ pipe and a closed organ pipe. The ratio of their lengths is

- (a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4

35. Waves produced in a cylinder containing a liquid by moving its piston back and forth are

- (a) Transverse and longitudinal
(b) Longitudinal
(c) Transverse and longitudinal
(d) Longitudinal

36. Identify the incorrect statement.
 (a) Echo is an example of reflection by a rigid boundary.
 (b) Modification of Newton's formula for speed of light is referred to as the Laplace correction.
 (c) In a longitudinal wave, the constituents of the medium oscillate forward and backward in the direction of propagation of the wave.
 (d) Matter waves are associated with constituents of matter : electrons, protons, neutrons, atoms and molecules
37. Water waves produced by a motor boat sailing in water are
 (a) neither longitudinal nor transverse.
 (b) both longitudinal and transverse.
 (c) only longitudinal.
 (d) only transverse.
38. Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where its speed is $2v$ m/s. Wavelength of sound waves in the second medium is
 (a) λ (b) $\frac{\lambda}{2}$ (c) 2λ (d) 4λ
39. Speed of sound wave in air
 (a) is independent of temperature.
 (b) increases with pressure.
 (c) increases with increase in humidity.
 (d) decreases with increase in humidity.
40. Change in temperature of the medium changes
 (a) frequency of sound waves.
 (b) amplitude of sound waves.
 (c) wavelength of sound waves.
 (d) loudness of sound waves.
41. With propagation of longitudinal waves through a medium, the quantity transmitted is
 (a) matter. (b) energy.
 (c) energy and matter.
 (d) energy, matter and momentum.
42. Equation of a plane progressive wave is given by $y = 0.6 \sin 2\pi \left(t - \frac{x}{2} \right)$. On reflection from a denser medium its amplitude becomes $\frac{2}{3}$ of the amplitude of the incident wave. The equation of the reflected wave is
 (a) $y = 0.6 \sin 2\pi \left(t + \frac{x}{2} \right)$ (b) $y = -0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$
 (c) $y = 0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$ (d) $y = -0.4 \sin 2\pi \left(t - \frac{x}{2} \right)$
43. A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, the disturbance will reach the other end in
 (a) one second
 (b) 0.5 second
 (c) 2 seconds
 (d) data given is insufficient.

Answer Key

1. (c)	2. (b)	3. (b)	4. (d)	5. (d)	6. (c)	7. (c)	8. (b)	9. (a)	10. (a)
11. (c)	12. (b)	13. (a)	14. (d)	15. (b)	16. (b)	17. (d)	18. (a)	19. (a)	20. (b)
21. (a)	22. (b)	23. (b)	24. (d)	25. (c)	26. (c)	27. (d)	28. (a)	29. (c)	30. (a)
31. (b)	32. (a)	33. (b)	34. (c)	35. (b)	36. (b)	37. (b)	38. (c)	39. (c)	40. (c)
41. (b)	42. (b)	43. (b)							

Answers with Explanations

1. Option (c) is correct.

Explanation:

Time period of pendulum

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$\Rightarrow$

$$T = 2\pi \sqrt{\frac{1}{10}}$$

$\Rightarrow$

$$T \approx 2 \text{ seconds}$$

2. Option (b) is correct.

Explanation:

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Time period of oscillation of pendulum is independent on mass of bob; hence it won't have any impact.

New length $L' = 4L$

$$\text{New Time period } T' = 2\pi \sqrt{\frac{L'}{g}} = 2\pi \sqrt{\frac{4L}{g}}$$

$$T' = 2T$$

3. Option (b) is correct.

Explanation: When a simple pendulum is kept in water, due to friction some of its energy gets dissipated in water. Hence, amplitude of oscillations decreases continuously.

4. Option (d) is correct.

Explanation:

Given, $f = 1 \text{ kHz}$ or 1000 Hz

$d = 1 \text{ km}$ or 1000 m

$\lambda = 50 \text{ cm}$ or 0.5 m

speed of wave, $v = f \times \lambda$

$$= 1000 \times 0.5 = 500 \text{ ms}^{-1}$$

$$\text{Time taken} = \frac{\text{Distance}}{\text{speed}} = \frac{1000}{500} \text{ s}$$

5. Option (d) is correct.

Explanation: Pitch of a sound is directly proportional to its frequency. If pitch is increasing, this means frequency of sound, wave velocity will increase while wavelength will decrease.

6. Option (c) is correct.

Explanation: SONAR technology uses ultrasonic waves, whose frequency is greater than $20,000 \text{ Hz}$. Many animals use this techniques for navigation purpose. These waves don't lie in audible range.

7. Option (c) is correct.

Explanation: Speed of sound is solid > liquid > Gas. This means speed of sound will be higher in steel, than in water, than in air. So, option (a) and (d) are correct.

Speed of sound decrease with increase in temperature. Hence, option (c) is incorrect.

8. Option (b) is correct.

Explanation: Sound is a vibration that propagates as an acoustic wave, through a transmission medium such as a gas or solid.

9. Option (a) is correct.

Explanation: If the speed of an object is more than the speed of sound or Mach number is more than 1, then it is travelling at supersonic speed. Approximately this speed is greater than 343 m/s.

10. Option (a) is correct.

Explanation:

$$\text{Wavelength} = \frac{\text{wave velocity}}{\text{frequency}} = \frac{v}{f}$$

$$\lambda = \frac{10 \times 300}{3 \times 10^5} = 0.01 \text{ m or } 1 \text{ cm}$$

11. Option (c) is correct.

Explanation: Bats use ultrasonic waves for navigational purpose. These are the sound waves with frequency greater than 20,000 Hz. These waves are also used in sonography and sonar system.

12. Option (b) is correct.

Explanation: Cochlea changes the sound vibration in electrical signal form and send to brain for further processing.

13. Option (a) is correct.

Explanation: Persistence of sound wave for a long time because of overlapping of sounds caused by repeated (multiple) reflections are called reverberation.

14. Option (d) is correct.

Explanation:

- The beat frequency is the difference in frequency of two waves.
- So due to constructive and destructive interference, the beat frequency is the difference in frequency of two waves

15. Option (b) is correct.

Explanation: Reverberation is the persistence of sound even after original sound has lapsed. It occurs due to multiple reflection of sound wave and as a result the brain treats the reflected waves as a continuous wave.

16. Option (b) is correct.

Explanation: Human ears can only sense the sound of frequency bandwidth between 20-20,000 Hz

17. Option (d) is correct.

$$\text{Explanation: Total energy} = \frac{1}{2} m \omega^2 A^2$$

So, it is independent of x .

18. Option (a) is correct.

Explanation: Every oscillatory motion is periodic, but every periodic motion need not be oscillatory. Example: Circular motion is a periodic motion, but it is not oscillatory.

19. Option (a) is correct.

$$\text{Explanation: Time period} = 2\pi \sqrt{\left(\frac{\text{Displacement}}{\text{Acceleration}} \right)}$$

$$= 2\pi \sqrt{\left(\frac{x}{bx} \right)}$$

$$= 2\pi \sqrt{\left(\frac{1}{b} \right)}$$

$$= \frac{2\pi}{\sqrt{b}}$$

20. Option (b) is correct.

Explanation: For SHM, $T^2 \propto l$

It is similar to equation of parabola, $y^2 = 4ax$

21. Option (a) is correct.

Explanation:

$$v_{\max} = A\omega$$

$$\text{or } v_{\max} = A \frac{2\pi}{T}$$

$$\text{or } v_{\max} = 4 \times \frac{2\pi}{8}$$

$$\therefore v_{\max} = \pi$$

22. Option (b) is correct.

Explanation:

$$y = 3 \cos \left(\frac{\pi}{4} - 2\omega t \right) = 3 \cos \left[- \left(2\omega t - \frac{\pi}{4} \right) \right]$$

$$= 3 \cos \left(2\omega t - \frac{\pi}{4} \right) \quad \{ \because \cos(-\theta) = \cos \theta \}$$

This shows simple harmonic motion with time

$$\text{period } T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

23. Option (b) is correct.

Explanation: as $\frac{d^2y}{dt^2}$ is not $\propto (-y)$

$\therefore$ It is not SHM but it is a periodic motion with period $\frac{2\pi}{\omega}$

24. Option (d) is correct.

Explanation: because for SHM, $a_x \propto (-x)$

25. Option (c) is correct.

26. Option (c) is correct.

Explanation:

$$x = a \cos \omega t \quad (i)$$

$$y = a \sin \omega t \quad (ii)$$

squaring and adding (i) & (ii),

we get

$$x^2 + y^2 = a^2 \cos^2 \omega t + a^2 \sin^2 \omega t$$

$$= a^2$$

The trajectory of motion will be a circle.

27. Option (d) is correct.

Explanation: Let

$$a = A \cos \phi \quad (i)$$

$$b = A \sin \phi \quad (ii)$$

Squaring and adding (i) & (ii)

$$a^2 + b^2 = A^2$$

$$y = a \sin \omega t + b \cos \omega t \quad (iii)$$

$$\begin{aligned} \text{or } y &= A \cos \phi \sin \omega t + A \sin \phi \cos \omega t \\ &= A \sin(\omega t + \phi) \end{aligned}$$

This is an equation of SHM with amplitude $A = \sqrt{a^2 + b^2}$

28. Option (a) is correct.

Explanation: Projection of OP on x -axis at time t is

$$x(t) = B \cos\left(\frac{\pi}{2} - \frac{2\pi}{T}t\right) = B \sin\left(\frac{2\pi}{T}t\right)$$

$$x(t) = B \sin\left(\frac{2\pi}{30}t\right) \quad \{\because T = 30\text{s}\}$$

29. Option (c) is correct.

30. Option (a) is correct.

Explanation:

$$v_{\max} = \omega A \quad \dots(i)$$

$$a_{\max} = \omega^2 A \quad \dots(ii)$$

$$(ii) \div (i), \frac{a_{\max}}{v_{\max}} = \frac{\omega^2 A}{\omega A} = \omega$$

$$\therefore \frac{a_{\max}}{v_{\max}} = \frac{2\pi}{T}$$

$$T = 2\pi \left(\frac{v_{\max}}{a_{\max}} \right) = 2\pi \left(\frac{30}{60} \right) = \frac{2\pi}{2} = \pi \text{ s}$$

31. Option (b) is correct.

Explanation: If k_1 and k_2 be the spring constant of spring S_1 and S_2 respectively, then

$$v_1 = \frac{1}{2\pi} \sqrt{\frac{k_1}{m}} \quad (i)$$

$$v_2 = \frac{1}{2\pi} \sqrt{\frac{k_2}{m}} \quad (ii)$$

If k be the effective spring constant,

$$k = k_1 + k_2 \quad (\because \text{springs are parallel})$$

$$\begin{aligned} v &= \frac{1}{2\pi} \sqrt{\frac{k}{m}} \\ &= \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} \\ &= \frac{1}{2\pi} \sqrt{\frac{k_1}{m} + \frac{k_2}{m}} \\ &= \frac{1}{2\pi} \sqrt{v_1^2 + v_2^2} \quad \{\text{by (i) \& (ii)}\} \\ &= \sqrt{v_1^2 + v_2^2} \end{aligned}$$

32. Option (a) is correct.

33. Option (b) is correct.

34. Option (c) is correct.

Explanation: Frequency of fundamental tone pro-

$$\text{duced by open organ pipe} = v_0 = \frac{v}{2L_{\text{open}}}$$

Frequency of fundamental tone produced by closed

$$\text{gan pipe} = v_0 = \frac{v}{4L_{\text{closed}}}$$

or

$$\frac{v}{2L_{\text{open}}} = \frac{v}{4L_{\text{closed}}}$$

or,

$$\therefore L_{\text{open}} : L_{\text{closed}} = 2 : 1$$

35. Option (b) is correct.

36. Option (b) is correct.

Explanation: Modification of Newton's formula for speed of sound is referred to as the Laplace correction.

$$v = \sqrt{\left(\frac{\gamma P}{\rho}\right)}$$

This relation was first given by Newton and is known as Newton's formula.

$$v = \sqrt{\left(\frac{\gamma P}{\rho}\right)}$$

This modification of Newton's formula is referred to as the Laplace correction.

37. Option (b) is correct.

38. Option (c) is correct.

Explanation: Wavelength in I medium,

$$\lambda = \frac{v}{\nu} \quad (i)$$

Wavelength in II medium,

$$\lambda' = \frac{v'}{\nu} \quad (ii)$$

$$\because \nu = \nu' \quad \{\therefore \text{frequency remains unchanged}\}$$

(ii) $\div$ (i), we get

$$\frac{\lambda'}{\lambda} = 2$$

$$\therefore \lambda' = 2\lambda$$

39. Option (c) is correct.

40. Option (c) is correct.

41. Option (b) is correct.

42. Option (b) is correct.

Explanation: Amplitude of reflected wave

$$\begin{aligned} &= \frac{2}{3} \times 0.6 \\ &= 0.4 \end{aligned}$$

There will be phase change of ϕ because of reflection from denser to rarer medium

$$y = 0.4 \sin 2\pi \left(t + \frac{x}{2} + \pi \right) = -0.4 \sin 2\pi \left(t + \frac{x}{2} \right)$$

43. Option (b) is correct.

$$\text{Explanation: } \mu = \frac{M}{L} = \frac{2.5}{20 \text{ m}} \text{ kg} = 0.125 \text{ kg/m}$$

$$\begin{aligned} v &= \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200 \text{ N}}{0.125 \text{ kg/m}}} \\ &= 40 \text{ m/s} \end{aligned}$$

$$t = \frac{L}{v} = \frac{20 \text{ m}}{40 \text{ m/s}} = 0.5 \text{ s}$$

Chapter 39

ELECTROSTATICS

Electric charge is the property of a matter due to which, it experiences a force when placed in an electromagnetic field. Two electrons or two protons will tend to repel each other as they carry like charges. The electron and proton will get attracted towards each other due to their unlike charges.

Charge on a proton = - charge on an electron

$$= 1.6 \times 10^{-19} \text{C}$$

Net charge on a body is given by:

Charging by friction: Charging insulators. On rubbing a glass rod and silk cloth piece together, glass rod gets positively charged whereas silk cloth gets negatively charged. If a plastic rod is rubbed with wool, it becomes negatively charged.

- **Charging by conduction:** Charging metals and other conductors.
- **Charging by induction:** It means wireless charging. For example, if a negatively charged rod is brought near neutral metal with insulator mounting, it repels free electrons and attracts positive charges on metal.

Electric field: The space around a charge up to which its electric force can be experienced is called electric field. Electric field inside a metal/conductor is zero as all the

charges lie at their surface.

- Electric field lines are used to represent electric field.
- Electric field lines are imaginary lines that originate from the positive charge and terminate at negative charge.
- Electric field has the same direction as force on the positive test charge.
- The electric field lines never intersect each other. Strength of electric field is encoded in density of field lines.

Gauss's theorem:

The net outward normal electric flux through any closed surface of any shape is equal to $1/\epsilon$ times to net charge enclosed by the surface. i.e. $\Phi = E \cdot ds = q/\epsilon^0$

Electric potential is the amount of work done by an external force in moving a unit positive charge from infinity to a point in an electrostatic field without producing an acceleration.

It is written as $V = W/Q$ where, W is work done in moving charge Q through the electric field.

The SI units of electric potential are J/C, Volt, Nm/C.

Units and Dimensions

Physical quantity	Symbol	Dimension	Unit
Electrical potential	V	$[ML^2T^{-3}A^{-1}]$	V
Capacitance	C	$[M^{-1}L^{-2}T^4A^2]$	F
Polarisation density	P	$[L^{-2}AT]$	Cm^{-2}
Dielectric constant	K	[dimensionless]	

Important Results and Formulae

1. Electric potential due to a point source charge q at a distance r from it is given by:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

Unit: $1 \text{ V} = 1 \text{ JC}^{-1}$

2. The electric field at a point is related to the negative potential gradient as follows:

$$\vec{E} = -\left(\frac{dV}{dr}\right)$$

Unit: Vm^{-1}

- (i) Electric potential due to a dipole at a point at distance r and making angle θ with the dipole moment p_e is given by:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{p_e \cos \theta}{r^2}$$

- (ii) On the axial line, the electric potential is given by:

$$V_a = \frac{1}{4\pi\epsilon_0} \cdot \frac{p_e}{r^2}$$

- (iii) On the equatorial line, the electric potential is: $V_e = 0$

3. Electric potential due to an isolated conducting charged sphere of radius R carrying charge Q :

- (a) The electric potential at the surface of the sphere and at every point inside the sphere is:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R}$$

- (b) The electric potential at a point outside the sphere is:

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r}$$

where $r > R$ is the distance of the point from the centre of the sphere.

4. Potential energy of system of two point charges: U

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

5. The ratio of the charge q and potential V of a conductor is called capacitance (C).

$$C = \frac{q}{V}$$

Unit: 1 Farad = 1 coulomb/volt

6. (a) Electrostatic potential energy of a conductor carrying charge q , capacitance C and potential V , is given by:

$$U_p = \int_0^q V dq = \int_0^q \frac{q}{C} dq = \frac{1}{2} \frac{q^2}{C}$$

- (b) Since, $q = VC$,

$$\therefore U_p = \frac{1}{2} V^2 C \Rightarrow U_p = \frac{1}{2} qV.$$

7. If a number of capacitors of capacitances C_1, C_2, C_3 are connected in series, their equivalent capacitance is given by:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

8. If a number of capacitors of capacitances C_1, C_2, C_3 C_n are connected in parallel, then their equivalent capacitance is given by:

$$C_p = C_1 + C_2 + C_3 + \dots + C_n$$

9. Let a parallel plate capacitor consists of two plates of area ' A ', separated by distance ' d ', having slab of same thickness and area with dielectric constant ' K ' between the plates. Then the capacitance of the capacitor is given by:

$$C = \frac{K\epsilon_0 A}{d}$$

10. (i) If we have a number of slabs of same area as the plates of the capacitor with thickness $t_1, t_2, t_3, \dots$ and dielectric constants $K_1, K_2, K_3, \dots$ inserted between the plates, the capacitance of the capacitor is given by:

$$C = \frac{\epsilon_0 A}{\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + \dots}$$

- (ii) If a single slab of thickness ' t ' and dielectric constant ' K ' is introduced between the plates having separation ' d ' then the capacitance of the capacitor is given by:

$$C = \frac{\epsilon_0 A}{\frac{t}{K} + \frac{(d-t)}{1}} = \frac{K\epsilon_0 A}{K(d-t) + t}$$

- (iii) If a single slab of conductor of thickness ' t ' is introduced between the plates, having separation ' d ' then the capacitance of the capacitors is given by:

$$C = \frac{\epsilon_0 A}{\frac{t}{\infty} + \frac{(d-t)}{1}} = \frac{\epsilon_0 A}{d-t}$$

11. (i) Dielectrics are non-conducting substances having no or negligible number of charge carriers.

(ii) **Types of dielectrics:**

- (a) **Non-polar:** When centre of positive charge coincides with the centre of negative charge in a molecule.

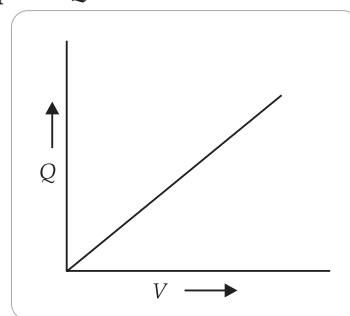
Example: O_2, N_2, H_2 , etc.

- (b) **Polar:** When centre of positive charge does not coincide with the centre of negative charge in a molecule. Example: H_2O, CO_2, N_2O , etc.

(iii) When an external electric field is applied to a non-polar dielectric, the centre of positive charge gets separated from the centre of negative charge. An electric dipole moment is induced and the dielectric is said to be polarised.

Important Graphs and Figures

Graph for Q vs V :



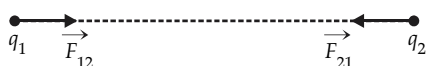
EXERCISE

- Which of the following statements correctly explains/explain the existence of a positive force between two electric charges?
 - Both the charges are positive.
 - Both the charges are negative.
 - Both the charges are oppositely charged.
 Select the correct answer using the code given below.

(a) 1 only	(b) 2 only
(c) 1 and 2 only	(d) 1, 2 and 3
- The electric field lines forms an isolated positively charged conducting sphere are :
 - tangential to the conducting surface.
 - at right angles to the conducting surface and towards the centre of the sphere.
 - at any angle to the conducting surface.
 - at right angles to the conducting surface and outwards from the centre of the sphere.
- Which one of the following can charge an insulator?

(a) Current electricity	(b) Static electricity
(c) Magnetic field	(d) Gravitational field
- Which one of the following statements is not correct?
 - The SI unit of charge is ampere-second.
 - Debye is the unit of dipole moment.
 - Resistivity of a wire of length l and area of cross-section a depends upon both l and a .
 - The kinetic energy of an electron of mass m kg and charge e coulomb, when accelerated through a potential difference of V volt, is eV joule
- When you walk on a woollen carpet and bring your finger near the metallic handle of a door an electric shock is produced. This is because :
 - charge is transferred from your body to the handle.
 - a chemical reaction occurs when you touch the handle.

- (c) the temperature of the human body is higher than that of the handle.
 (d) the human body and the handle arrive at thermal equilibrium by the process.
6. Why are the tyres of aircrafts made of conducting rubber?
1. So that the charge accumulated on the aircraft in flight, by rubbing the air, can easily be transferred to ground on landing.
 2. So that the charge accumulated due to the operation of various electronic equipments in the aircraft in flight can easily be transferred to ground on landing.
- Select the correct answer using the code given below.
- (a) 1 only (b) 2 only
 (c) Both 1 and 2 (d) Neither 1 nor 2
7. A positive charge $+q$ is placed at the centre of a hollow metallic sphere of inner radius a and outer radius b . The electric field at a distance r from the centre is denoted by E . In this regard, which one of the following statements is correct?
- (a) $E = 0$ for $a < r < b$
 (b) $E = 0$ for $r < a$
 (c) $E = q/4\pi\epsilon_0 r$ for $a < r < b$
 (d) $E = q/4\pi\epsilon_0 a$ for $r < a$
8. Lightning conductors are used to protect building from lightning strikes. Which of the following statements is/are true about lightning conductors?
- (1) Lightning conductors create an electric field at its top so that lightning strikes it preferentially.
 - (2) Lightning conductors reduce the effect of the strike by uniformly distributing the charge (current) over the surface of the building.
 - (3) Lightning conductors take all charge (current) to deep down in the earth.
 - (4) Lightning conductors must be installed at a place taller than the building.
- Select the correct answer using the code given below:
- (a) 1 and 2 (b) 3 and 4 only
 (c) 1, 3 and 4 (d) 4 only
9. The electric field inside a perfectly conducting hollow object is :
- (a) 4π
 (b) infinite
 (c) zero
 (d) dependent upon the shape of the object
10. If a free electron moves through a potential difference of 1 kV, then the energy gained by the electron is given by :
- (a) 1.6×10^{-19} J (b) 1.6×10^{-16} J
 (c) 1×10^{-19} J (d) 1×10^{-16} J
11. Van de Graaff generator is used for :
- (a) accelerating charged particles
 (b) generating large currents
 (c) generating electric field
 (d) generating high frequency voltage
12. According to Coulomb's law, which is the correct relation for the following figure?



- (a) $q_1 q_2 > 0$ (b) $q_1 q_2 < 0$
 (c) $q_1 q_2 = 0$ (d) $1 > \frac{q_1}{q_2} > 0$
13. A negatively charged object X is repelled by another charged object Y. However an object Z is attracted to object Y. Which of the following is the most possibility for the charge of object Z?
- (a) positively charged only
 (b) negatively charged only
 (c) neutral or positively charged
 (d) neutral or negatively charged
14. In an experiment three microscopic latex spheres are spread into a chamber and became charged with charges $+3e$, $+5e$ and $-3e$ respectively. All the three spheres came in contact simultaneously for a moment and got separated. Which one of the following are possible values for the final charge on the spheres?
- (a) $+5e, -4e, +5e$ (b) $+6e, +6e, -7e$
 (c) $-4e, +3.5e, +5.5e$ (d) $+5e, -8e, +7e$
15. An object has charge of 1 C and gains 5.0×10^{18} electrons. The net charge on the object becomes:
- (a) -0.80 C (b) $+0.80$ C (c) $+1.80$ C (d) $+0.20$ C
16. The magnitude of electric field due to a point charge $2q$, at distance r is E . Then the magnitude of electric field due to a uniformly charged thin spherical shell of radius R with total charge q at a distance $\frac{r}{2}$ ($r \gg R$) will be
- (a) $\frac{E}{4}$ (b) 0 (c) $2E$ (d) $4E$
17. The magnitude of electric force, F is:
- (a) directly proportional to the product of both charges.
 (b) directly proportional to the distance between both charges.
 (c) directly proportional to the square of the distance between both charges.
 (d) constant.
18. Gauss' law is effective for:
- (a) any enclosed area
 (b) only near uniform surfaces
 (c) any unobstructed surface
 (d) only open irregular surfaces
19. The electric flux depends on:
- (a) The intensity of the electric field
 (b) Area of the surface
 (c) Angle between the surface and the electric field
 (d) Each one of these
20. The electric potential on the axis of an electric dipole at a distance r from its centre is V . Then the potential at a point at the same distance on its equatorial line will be
- (a) $2V$ (b) $-V$ (c) $\frac{V}{2}$ (d) Zero
21. A variable capacitor is connected to a 200 V battery. If its capacitance is changed from $2 \mu\text{F}$ to $X \mu\text{F}$, the decrease in energy of the capacitor is
- (a) $1 \mu\text{F}$ (b) $2 \mu\text{F}$ (c) $3 \mu\text{F}$ (d) $4 \mu\text{F}$

22. Equipotentials at a large distance from a collection of charges whose total sum is not zero are
 (a) spheres (b) planes
 (c) ellipsoids (d) paraboloids
23. Equipotential surface at a great distance from a collection of charges whose total sum is not zero are approximately
 (a) spheres. (b) planes.
 (c) paraboloids. (d) ellipsoids.
24. The work done to move a charge along an equipotential surface from A to B
 (a) cannot be defined.
 (b) is a negative quantity.
 (c) is zero.
 (d) is a positive quantity.
25. Electric potential inside a conducting sphere
 (a) is zero.
 (b) remains constant.
 (c) decreases from centre to surface.
 (d) increases from centre to surface.
26. Capacitance of a parallel plate capacitor can be increased by
 (a) increasing the distance between the plates.
 (b) decreasing the distance between the plates.
 (c) decreasing the area of plates.
 (d) increasing the thickness of the plates.
27. A parallel plate capacitor is charged by connecting it to a battery. Which of the following will remain constant if the distance between the plates of the capacitor is increased in this situation?
 (a) Energy stored
 (b) Electric field
 (c) Potential difference
 (d) Capacitance
28. The capacitance of a parallel plate capacitor is $10 \mu\text{F}$. When a dielectric plate is introduced in between the plates, its potential becomes $1/4$ th of its original value. What is the value of the dielectric constant of the plate introduced?
 (a) 4
 (b) 40
 (c) 2.5
 (d) none of the above

Answer Key

1. (c)	2. (d)	3. (b)	4. (c)	5. (a)	6. (a)	7. (a)	8. (b)	9. (c)	10. (b)
11. (a)	12. (b)	13. (c)	14. (b)	15. (d)	16. (c)	17. (a)	18. (a)	19. (d)	20. (d)
21. (a)	22. (a)	23. (a)	24. (c)	25. (b)	26. (b)	27. (c)	28. (a)		

Answers with Explanations

1. **Option (c) is correct.**

Explanation: $F_e = \frac{kq_1 q_2}{r^2}$

When both charges are of same sign then only, force will be positive. When both charges are of opposite sign (one is positive and another one is negative), magnitude of force will be negative.

2. **Option (d) is correct.**

Explanation: Electric field lines always start from positive charge and end at negative charge. In electrostatics, field lines are always at right angle to the conducting surface because if they are at any other angle then horizontal component of field will make charges move. If charges are not static, condition of electrostatic breaches and it becomes electrodynamics.

3. **Option (b) is correct.**

Explanation: Insulators are made up of materials that lack conduction of electricity. It can be charged by methods of friction and induction. Charge stored in this process will be static.

4. **Option (c) is correct.**

Explanation: SI unit of charge is coulomb. Ampere-second is equivalent to coulomb

Resistivity of wire is independent of the length and area of cross section of wire, it only depends on the nature of material and temperature. So, option (c) is an incorrect statement.

5. **Option (a) is correct.**

Explanation: When we move on a woolen carpet, electric charges get accumulated on our body which

remain in static condition. When we touch any metallic object, these static charges flow from our body to that object which gives shock.

6. **Option (a) is correct.**

Explanation: Tires of aircrafts are made conducting, so that excess charge can be passed to the ground and safeguarding the aircraft and passengers.

7. **Option (a) is correct.**

Explanation: $\int E \cdot ds = \frac{q_{in}}{\epsilon_0}$ since all the charges are present in outer surface so $q_{in} = 0$

$$\int E \cdot ds = \frac{q_{in}}{\epsilon_0} = \frac{0}{\epsilon_0} = 0 \text{ which means } E = 0$$

8. **Option (b) is correct.**

Explanation: Lightning conductors must be installed at a place higher than the building. So, when the lightning strikes, it hits the conductor instead of the building. These conductors take down all the charges (or current) directly to the ground and prevent damages.

9. **Option (c) is correct.**

Explanation: Electric Field inside a conducting body is always zero because all the charges reside at its surface of conducting body.

$$E = \frac{q}{\epsilon_0} = \frac{0}{\epsilon_0} = 0$$

10. **Option (b) is correct.**

Explanation:

$$W = q\Delta V = 1.6 \times 10^{-19} \times 10^3 = 1.6 \times 10^{-16} \text{ J}$$

11. Option (a) is correct.

Explanation: A Van de Graaff Generator uses a moving belt to accumulate very high electrostatic voltage on a hollow metal sphere. This creates a large electric field which is used to accelerate charged particles.

12. Option (b) is correct.

Explanation: Since, the force is attractive, the charges must be unlike.

$$\text{So, } q_1 q_2 < 0$$

13. Option (c) is correct.

Explanation: X is negatively charged. Since it is repelled by Y, this means Y is also negatively charged. Now, if Z is neutral or positively charged then it will be attracted by Y.

14. Option (b) is correct.

Explanation: When three spheres are brought in contact the net charge will be $-3e + 3e + 5e = 5e$. When separated this $5e$ charge will be distributed. In option (a), Total charge is $6e$. So, this option is not correct.

In option (b), Total charge is $5e$. So, this option is correct. In option (c), Total charge is $5e$. But option have charges in fraction which violates quantisation of charge principle. Hence, this option is not correct. In option (d), Total charge is $4e$. Hence, this option is not correct.

So, (B) is the correct option.

15. Option (d) is correct.

Explanation: Charge on 1 electron $= -1.6 \times 10^{-19} \text{ C}$
So, Charge on 5×10^{18} electrons
 $= -5 \times 10^{18} \times 1.6 \times 10^{-19} \text{ C} = -0.8 \text{ C}$ [$\because q = ne$]
Already existing charge $= 1 \text{ C}$
So, net charge after electron gain $= 1 + (-0.8) = 0.2$

16. Option (c) is correct.

Explanation: Electric field due to the point charge

$$= E = \frac{K \times 2q}{r^2}$$

Electric field due to the spherical shell

$$= E' = \frac{K \times q}{(r/2)^2} = 2E$$

17. Option (a) is correct.

Explanation: The magnitude of the electric force F is directly proportional to the amount of an electric charge, q_1 , multiplied by the other, q_2 , and inversely proportional to the square of the distance ' r ' between their centres.

18. Option (a) is correct.

Explanation: The Gauss' law is applicable for any closed surface. Hence, **Option (a) is correct.**

19. Option (d) is correct.

Explanation: If the electric field is E and the area is A , the electric flux associated with the area is $= EA \cos \theta$. Where θ represents the angle formed between the surface and the electric field.

The electric flux is clearly dependent on the electric field strength, area, and angle between the surface and the electric field.

20. Option (d) is correct.

Explanation: Potential at point (r, θ) is

$$V = \frac{Kp \cos \theta}{r^2} \quad [p = \text{dipole moment}]$$

On equatorial line $\theta = 90^\circ$

$$\therefore V = \frac{Kp \cos 90^\circ}{r^2} = 0$$

21. Option (a) is correct.

Explanation: Energy $= \frac{1}{2} CV^2$

$$\text{In 1st case: } E_1 = \frac{1}{2} \times 2 \times 10^{-6} \times 200^2$$

$$= 4 \times 10^{-2} \text{ J}$$

$$\text{In 2nd case: } E_2 = \frac{1}{2} \times (X) \times 10^{-6} \times 200^2 \text{ J}$$

Decrease in energy $= E_1 - E_2$

$$\text{or, } 2 \times 10^{-2} = 4 \times 10^{-2} - 2 \times (X) \times 10^{-2}$$

$$\text{or, } 2 = 4 - 2X$$

$$\therefore X = 1 \mu\text{F}$$

22. Option (a) is correct.

Explanation: The collection of charges, whose total sum is not zero, at great distance may be considered as a point charge.

So, potential is inversely proportional to the distance from the charge.

So, electric potentials due to point charge are the same for all equidistant points. The locus of these equidistant points, which are at same potential, is a sphere.

23. Option (a) is correct.

Explanation: For equipotential surface, these surfaces are perpendicular to the field lines. So, there must be electric field, which cannot be without charge. So, the algebraic sum of all charges must not be zero. Equipotential surface at a great distance means that space of charge is negligible as compared to distance. So, the collection of charges is considered as a point charge.

Electric potential due to point charge is,

$$V = k_e \frac{q}{r}$$

which explains that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, forms spherical surface.

24. Option (c) is correct.

Explanation: For an equipotential surface, $V_A = V_B$
So, work done $= 0$

25. Option (b) is correct.

Explanation: As the electric field inside a conductor is zero. So, the potential at any point is constant.

26. Option (b) is correct.

Explanation: $C = \epsilon_0 A/d$

So, by decreasing the distance between the plates, capacitance increases.

27. Option (c) is correct.

Explanation: As the battery remains connected with the capacitor, the potential difference remains constant.

28. Option (a) is correct.

Explanation: $C' = KC$ (where K is the dielectric constant).

$$V = Q/C$$

$$V' = Q/C'$$

$$V' = V/4 = Q/C' = Q/KC = V/K$$

$$\therefore K = 4$$

Chapter 40

CURRENT & ELECTRICITY

Electric Current: It is defined as the rate of flow of charge, across the cross section of conductor i.e., $I = q/t = ne/t$

Ohm's law states that "The flow of current through conductor is directly proportional to the potential difference established across the conductor, provided other physical conditions such as temperature, pressure remains same."

i.e. $I = V/R$ where R is resistance, V = voltage, I = current

Resistance depends upon

1. Area of conductor, $R \propto 1$

2. Length of conductor, $R \propto \frac{1}{A}$

3. Nature of material and

4. Temperature

- Devices such as carbon resistors and metals obey the Ohm's law and have a constant resistance, so they are termed as **ohmic devices**. This means that the V-I curve is a straight line passing through the origin.
- A circuit component has a non-linear V-I characteristic if the resistance is not constant throughout, diode, triode for example. Devices which do not follow ohm's law are known as **non-ohmic devices**.

Equivalent resistance

In series: $R = R_1 + R_1 + R_1 + \dots R_n$

In parallel: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \frac{1}{R_n}$

- **Resistivity (ρ):** It is defined as the resistance of a material of 1 m² length and 1 m² area of cross section.

$$R = \rho \frac{1}{A}$$

Ohm-meter or Ω -m

- The reciprocal resistivity of a conductor is called its conductivity. Its unit is mho or ohm^{-1} .

Heating effect of electric current: Both ac and dc current in a circuit gives rise to heat, which is given by :

$$H = I^2 R t$$

Heat generated in a circuit is:

- directly proportional to the square of current.
- directly proportional to the resistance of the conductor.
- directly proportional to the amount of time of current flow.
- **Electric energy:** Amount of work done in a unit time.

$$\Rightarrow E = VIT = I^2 R t = \frac{V^2}{R} t$$

$\Rightarrow$ SI unit is watt-sec or J.

$\Rightarrow$ Commercial unit is kWh or kJ. (1kWh = 3.6×10^6 J)

- **Electric Power** is the work done per unit time which is the rate of energy consumed in a circuit.

$$\Rightarrow P = \frac{W}{t} \text{ or } V \frac{q}{t}$$

$$\Rightarrow P = VI = I^2 R = \frac{V^2}{R}$$

$\Rightarrow$ SI unit of power is J/s or W (watt).

$\Rightarrow$ Commercial unit is kW.

UNITS AND DIMENSIONS

Physical quantity	Symbol	Dimension	Unit
Electric current	I	[A]	A
Charge	Q, q	[TA]	C
Electric potential	V	[ML ² T ⁻³ A ⁻¹]	V
Potential difference	V	[ML ² T ⁻³ A ⁻¹]	V
Electromotive force	E	[ML ² T ⁻³ A ⁻¹]	V
Resistance	R	[ML ² T ⁻³ A ⁻²]	Ω
Resistivity	ρ	[ML ³ T ⁻³ A ⁻²]	Ωm
Electrical conductivity	σ	[M ⁻¹ L ⁻³ T ³ A ²]	($\text{Ohm}^{-1}\text{m}^{-1}$) or, Sm^{-1}
Electric field	E	[MLT ⁻³ A ⁻¹]	Vm^{-1}
Drift velocity	v_d	[LT ⁻¹]	ms^{-1}
Relaxation time	τ	[T]	s
Current density	$\vec{J}$	[L ⁻² A]	Am^{-2}
Mobility	μ	[M ⁻¹ T ² A]	$\text{m}^2\text{V}^{-1}\text{s}^{-1}$

IMPORTANT RESULTS AND FORMULAE

1. **Electric current:** Electric current is defined as the rate of flow of charge, *i.e.*,

$$I = \frac{dq}{dt}$$

When charge flows at a constant rate, the corresponding electric current can be written as:

$$I = \frac{q}{t}$$

Unit: 1 Ampere = 1 Coulomb second⁻¹

2. When electric current is set up in a conductor, the electrons drift through the conductor with velocity v_d , which is drift velocity and is given by:

$$v_d = \frac{I}{neA}$$

3. **Ohm's law:** Current I in a conductor is directly proportional to the potential difference V applied across the ends of the conductor provided the physical conditions such as the temperature, mechanical strain, etc. remain unchanged.

$$I \propto V$$

$$\text{Or, } V = RI$$

where, R is a constant called resistance of the conductor.

4. **Resistance:**

$$R = \rho \frac{l}{A}$$

5. **Resistivity:** $\rho = \frac{m}{ne^2\tau}$

6. **Conductivity:** Reciprocal of resistivity

$$\sigma = \frac{1}{\rho}$$

Unit = Ohm⁻¹m⁻¹, or Siemens m⁻¹

7. The temperature coefficient for conductors is positive *i.e.*, resistance increases as the temperature rises.

The temperature coefficient for the insulators and semiconductors is negative *i.e.*, their resistance decreases as the temperature increases.

8. The internal resistance of a cell is

$$r = \left(\frac{E - V}{V} \right) R$$

9. **Electrical power:**

$$P = \frac{V^2}{R} = I^2 R$$

Unit: Watt

10. **The emf and terminal potential difference of a cell:** Let the emf of a cell is ' E ' and its internal resistance is r . If an external resistance R is connected across the cell through a key, then $IR = V =$ potential difference across the external resistance R . This is equal to the terminal potential difference across the cell.

$$E = V + Ir$$

$$\therefore I = \frac{E - V}{r}$$

$$\text{So, } V = E - Ir$$

$$\therefore V < E.$$

When current is drawn from a cell, its terminal potential difference is less than the emf.

11. **Series combination of cells:**

In n cells having internal resistance r and emf E for each.

$$I_s = \frac{nE}{R + nr}$$

12. **Parallel combination of cells:**

If n cells having internal resistances $r_1, r_2, r_3, \dots, r_n$ and of emf $E_1, E_2, E_3, \dots, E_n$ respectively.

$$I_p = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} + \dots + \frac{E_n}{r_n}}{1 + R \left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \dots + \frac{1}{r_n} \right)}$$

14. **Mixed combination of cells:**

Here nm cells are combined in m rows, each row having n cells such that:

$$I_m = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr}$$

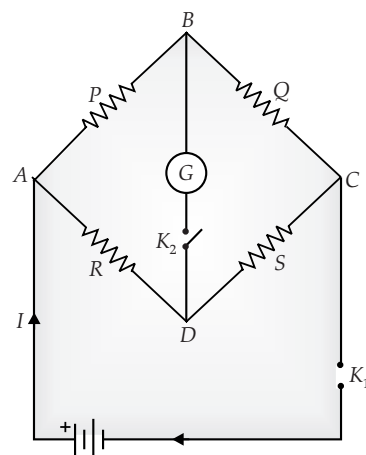
15. **Kirchhoff's rules:**

First law: The algebraic sum of current at a junction is zero *i.e.*, $\Sigma I = 0$. This implies that the total current entering a junction is equal to the total current leaving the junction.

Second law: In a closed loop, the algebraic sum of the emfs is equal to the algebraic sum of the product of resistance and the respective current flowing through them *i.e.*,

$$\Sigma E = \Sigma IR.$$

16. **Wheatstone bridge:** This is a circuit consisting of four resistances P, Q, R and S , a galvanometer and a battery connected as follows:



The Wheatstone bridge is balanced when there is no current flow through the galvanometer. It means potential at B is equal to that at D . In such a case

$$\frac{P}{Q} = \frac{R}{S}$$

EXERCISE

- A current of 0.6 A is drawn by an electric bulb for 10 minutes. Which one of the following is the amount of electric charge that flows through the circuit?
(a) 6 C (b) 0.6 C (c) 360 C (d) 36 C
 - The instrument used for detecting the presence of electric current in a circuit is :
(a) Refractometer (b) Galvanometer
(c) Viscometer (d) Diffractometer
 - A simple circuit contains a 12 V battery and a bulb having 24 ohm resistance. When you turn on the switch, the ammeter connected in the circuit would read :
(a) 0.5 A (b) 2 A (c) 4 A (d) 5 A
 - When the short circuit condition occurs, the current in the circuit :
(a) becomes zero
(b) remains constant
(c) increases substantially
(d) keeps on changing randomly
 - Which one of the following devices is non-ohmic?
(a) Conducting copper coil (b) Electric heating coil
(c) Semi conductor diode (d) Rheostat
 - An electric wire of resistance 50 ohm is cut into five equal wires. These wires are then connected in parallel. What is the equivalent resistance of this combination?
(a) 2 ohm (b) 10 ohm. (c) 0.5 ohm (d) 5 ohm
 - Which one of the following correctly represents the SI unit of resistivity?
(a) Ω (b) Ω/m (c) $\Omega \text{ cm}$ (d) $\Omega \text{ m}$
 - In an incandescent electric bulb, the filament of the bulb is made up of which metal?
(a) Aluminium (b) Copper
(c) Tungsten (d) Silver
 - When three resistors, each having resistance r , are connected in parallel, their resultant resistance is x . If these three resistances are connected in series, the total resistance will be:
(a) $3x$ (b) $3rx$ (c) $9x$ (d) $\frac{3}{x}$
 - Two conducting wires A and B are made of same material. If the length of B is twice that of A and the radius of circular cross-section of A is twice that of B, then their resistances R_A and R_B are in the ratio :
(a) 2:1 (b) 1:2 (c) 1:8 (d) 1:4
 - What is the current required to light a 60 W incandescent bulb in a domestic supply of 240 V?
(a) 0.5 A (b) 0.25 A (c) 1.0 A (d) 5.0 A
 - The commercial unit of electrical energy is kilowatt-hour (kWh), which is equal to
(a) $3.6 \times 10^6 \text{ J}$ (b) $3.6 \times 10^3 \text{ J}$
(c) 10^3 J (d) 1 J
 - Water is heated with a coil of resistance R connected to domestic supply. The rise of temperature of water will depend on :
(1) Supply voltage.
(2) Current passing through the coil.
(3) Time for which voltage is supplied.
- Select the correct answer from among the following:
- (a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 only (d) 2 and 3 only
 - A given conductor carrying a current of 1A produces an amount of heat equal to 2000 J. If the current through the conductor is doubled, the amount of heat produced will be :
(a) 2000 J (b) 4000 J (c) 8000 J (d) 1000 J
 - A fuse is used in an electric circuit to :
(a) break the circuit when excessive current flows through the circuit.
(b) break the circuit when power gets off.
(c) indicate if the current is flowing uninterrupted.
(d) complete the circuit for flow of current.
 - Kirchhoff's first rule $\Sigma I = 0$ and second rule $\Sigma IR = \Sigma E$ (where the symbols have their usual meanings) are respectively based on
(a) conservation of momentum and conservation of charge
(b) conservation of energy, conservation of charge
(c) conservation of charge, conservation of momentum
(d) conservation of charge, conservation of energy
 - The electric power consumed by a 220 V = 100 W bulb when operated at 110 V is
(a) 25 W (b) 30 W (c) 35 W (d) 45 W
 - A potential difference of 200 V is maintained across a conductor of resistance 100 Ω . The number of electrons passing through it in 1 s is
(a) 1.25×10^{19} (b) 2.5×10^{18}
(c) 1.25×10^{18} (d) 2.5×10^{16}
 - In a dc circuit the direction of current inside the battery and outside the battery respectively are
(a) positive to negative terminal and negative to positive terminal
(b) positive to negative terminal and positive to negative terminal
(c) negative to positive terminal and positive to negative terminal
(d) negative to positive terminal and negative to positive terminal
 - A car battery is charged by a 12 V supply, and energy stored in it is $7.20 \times 10^5 \text{ J}$. The charge passed through the battery is
(a) $6.0 \times 10^4 \text{ C}$ (b) $5.8 \times 10^3 \text{ J}$
(c) $8.64 \times 10^6 \text{ J}$ (d) $1.6 \times 10^5 \text{ C}$
 - If n , e , τ and m have their usual meanings, then the resistance of a wire of length l and cross-sectional area A is given by
(a) $\frac{ne^2A}{2m\tau l}$ (b) $\frac{ml}{ne^2\tau A}$
(c) $\frac{m\tau A}{ne^2l}$ (d) $\frac{ne^2\tau A}{2ml}$
 - Two sources of equal emf are connected in series. This combination is connected to an external resistance R . The internal resistance of two sources are r_1 and r_2 ($r_2 > r_1$). If the potential difference across the source of internal resistance r_2 is zero, then R equals

to

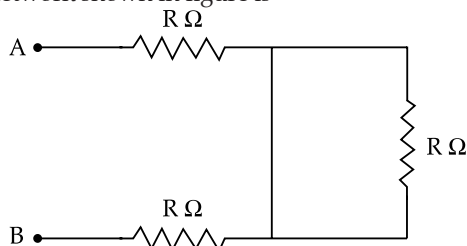
(a) $\frac{r_1 + r_2}{r_2 - r_1}$

(b) $r_2 - r_1$

(c) $\frac{r_1 r_2}{r_2 - r_1}$

(d) $\frac{r_1 + r_2}{r_1 r_2}$

23. The equivalent resistance between A and B of the network shown in figure is



(a) $3R \Omega$ (b) $\left(\frac{3}{2}\right)R \Omega$

(c) $2R \Omega$ (d) $\left(\frac{2}{3}\right)R \Omega$

24. Which of the following characteristics of electrons

determines the current in a conductor?

- (a) Drift velocity alone
(b) Thermal velocity alone
(c) Both drift velocity and thermal velocity
(d) Neither drift nor thermal velocity.

25. When cell of e.m.f. E is connected with an external resistance R , the p.d. across the cell becomes V . The expression for the internal resistance of the cell is

(a) $\frac{E - V}{V} R$ (b) $\frac{E + V}{E} R$

(c) $\frac{V - E}{V} R$ (d) $\frac{V - E}{E} R$

26. Kirchhoff's current rule is based on the law of conservation of

- (a) charge. (b) energy.
(c) mass. (d) (B) and (C)

27. Wheatstone Bridge is not suitable for measurement of

- (a) very high value resistances.
(b) very low value resistances.
(c) both (A) and (B).
(d) medium value resistances.

Answer Key

1. (c)	2. (b)	3. (a)	4. (c)	5. (c)	6. (a)	7. (d)	8. (c)	9. (c)	10. (c)
11. (b)	12. (a)	13. (a)	14. (c)	15. (a)	16. (d)	17. (a)	18. (a)	19. (c)	20. (a)
21. (b)	22. (b)	23. (c)	24. (a)	25. (a)	26. (a)	27. (c)			

Answers with Explanations

1. Option (c) is correct.

Explanation:

$$q = It = 0.6 \times 10 \times 60 = 360 \text{ C}$$

2. Option (b) is correct.

Explanation: Galvanometer is used to detect less ampere current. It also tell us about the direction of current.

3. Option (a) is correct.

Explanation:

$$R_{eq} = \frac{R'}{5} = \frac{10}{5} = 2 \Omega$$

4. Option (c) is correct.

Explanation: When the short circuit condition occurs, the current in the circuit increase abruptly as short circuit is mainly due to removal or decrease in resistance.

5. Option (c) is correct.

Explanation: All the semiconductor devices such as diode, triode, LED are non-ohmic in nature since these devices doesn't follow ohm's law.

6. Option (a) is correct.

Explanation:

Resistance of wire, $R = 50 \Omega$

Since, $R \propto l$, on cutting resistance into 5 equal parts, resistance of each part will be 10Ω . Now all parts are connected in parallel.

$$\text{So, } R_{eq} = \frac{R'}{5} = \frac{10}{5} = 2 \Omega$$

7. Option (d) is correct.

Explanation:

$$\rho = R \frac{A}{l} = \Omega \cdot \frac{m^2}{m} = \Omega m$$

8. Option (c) is correct.

Explanation: Tungsten have resistivity and high melting point, so it is used in bulb filaments.

9. Option (c) is correct.

Explanation: In parallel connection;

$$\frac{r}{3} = x$$

$$r = 3x$$

When these 3 resistors are connected in series then equivalent resistance is:

$$R = 3x + 3x + 3x = 9x$$

10. Option (c) is correct.

Explanation:

$$L_A = \frac{L_B}{2} \text{ and } r_A = 2r_B$$

$$\text{Resistance of wire B: } R_B = \rho \frac{L_B}{\pi r_B^2}$$

Resistance of wire A:

$$R_A = \rho \frac{L_A}{\pi r_A^2} = \rho \frac{\frac{L_B}{2}}{4 \pi r_B^2} = \rho \frac{L_B}{8 \pi r_B^2}$$

or $\frac{R_A}{R_B} = \frac{1}{8}$

11. Option (b) is correct.

Explanation:

Electric power, $P = VI$

$$\Rightarrow I = \frac{60}{240} = 0.25 \text{ A}$$

12. Option (a) is correct.

Explanation:

$$1 \text{ kWh} = 1000 \text{ W} \times 60 \times 60 \text{ sec} = 3.6 \times 10^3 \text{ J}$$

13. Option (a) is correct.

Explanation:

According to Joule's law of heating $H = VI t$ Based on this, the rise of temperature of water depends on :

- Supply voltage
- Current passing through the coil.
- Time for which voltage is supplied

14. Option (c) is correct.

Explanation:

According to Joules law, $H \propto I^2$

$$\frac{H_2}{H_1} = \frac{I_2^2}{I_1^2}$$

$$\frac{H_2}{2000} = \frac{2^2}{1}$$

$$H^2 = 4 \times 2000 = 8000 \text{ J}$$

15. Option (a) is correct.

Explanation: A fuse is an electric device designed to prevent the flow of excess current caused by abnormalities in the circuit. It breaks the circuit and protects the equipment from the heat generated by the excess current.

16. Option (d) is correct.

17. Option (a) is correct.

Explanation: Resistance of the bulb = R

$$R = \frac{V^2}{P}$$

or, $R = \frac{220^2}{100}$

$$\therefore R = 484 \Omega$$

Power consumed by the bulb when connected to 110

$$V \text{ is } \frac{V^2}{R} = \frac{110^2}{484} = 25 \text{ W}$$

18. Option (a) is correct.

Explanation: $I = \frac{V}{R} = \frac{200}{100} = 2 \text{ A}$

Again, $I = \frac{ne}{t}$

or, $2 = ne(\text{since, } t = 1 \text{ s})$
 or, $n = 2/e = 2/(1.6 \times 10^{-19})$
 $\therefore n = 1.25 \times 10^{19}$

19. Option (c) is correct.

Explanation: The direction of an electric current is by convention the direction in which a positive charge would move. Thus, the current in the external circuit is directed from positive terminal and to negative terminal of the battery.

When the battery is giving power it discharges, then the current inside the cell flows from negative terminal to positive terminal.

20. Option (a) is correct.

Explanation: $Q = W/V$
 $= 7.2 \times 10^5 / 12$
 $= 6 \times 10^4 \text{ C}$

21. Option (b) is correct.

Explanation: Drift velocity $= v_d = \frac{eV\tau}{ml}$... (i)

$$\text{Current} = I = enAv_d \quad \dots \text{(ii)}$$

Using equations (i) and (ii)

$$\frac{V}{I} = \frac{ml}{ne^2\tau A}$$

When m, l, n, e, τ are constant then

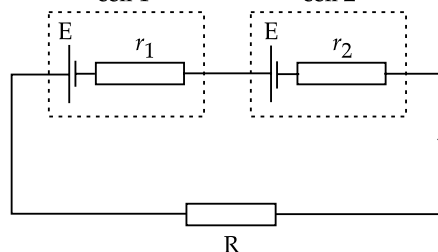
$$\frac{V}{I} = R = \frac{ml}{ne^2\tau A}$$

22. Option (b) is correct.

Explanation:

$$\text{Net emf} = E_{\text{net}} = E + E = 2E$$

$$\text{Net resistance} = R_{\text{net}} = R + r_1 + r_2$$



$$\begin{aligned} \text{Current in circuit} &= I = \frac{E_{\text{net}}}{R_{\text{net}}} \\ &= \frac{2E}{(R + r_1 + r_2)} \end{aligned}$$

Since, the potential difference of the cell of internal resistance r_2 is 0,

$$E - Ir_2 = 0$$

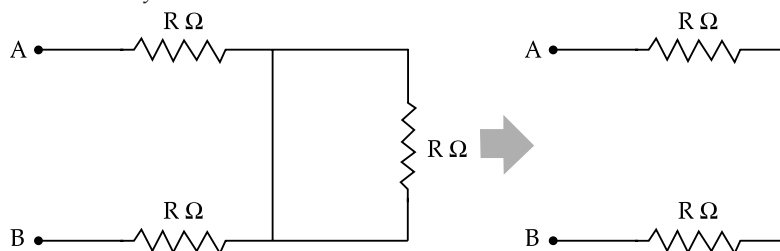
or, $E = \frac{2Er_2}{(R + r_1 + r_2)}$

or, $1 = \frac{2r_2}{(R + r_1 + r_2)}$

$$\therefore R = r_2 - r_1$$

23. **Option (c) is correct.**

Explanation: The given circuit may be further reduced as:



So, the equivalent resistance is $R + R = 2R \Omega$

24. **Option (a) is correct.**

Explanation: As we know that,

$$I = AneV_d$$

So, current,

$$I \propto V_d$$

25. **Option (a) is correct.**

Explanation: Current in the circuit $= I = V/R$

If r = internal resistance of the cell, then $V = E - Ir$

$$\therefore r = \frac{E - V}{I} = \frac{E - V}{V/R} = \frac{E - V}{V} R$$

26. **Option (a) is correct.**

Explanation: Kirchhoff's current rule is based on the law of conservation of charge.

27. **Option (c) is correct.**

Explanation: Wheatstone bridge is suitable for measurement of medium value resistances because to ensure sensitivity, other resistors must be of comparable values.

MAGNETIC EFFECT OF CURRENT AND MAGNETISM

Magnets is a material or object that produces a magnetic field.

- A magnet always has two poles.
- Like poles of a magnet repel each other while unlike poles attract each other.
- Electromagnet is a current-carrying coil containing a soft iron core, which is utilized in the electric bell, telegraph receiver, telephone, transformer, dynamo, bullet train, etc.

A **magnetic field** is a region around a magnet or a moving charge where the force of magnetism acts which affects other magnets and magnetic materials. Magnetic field is represented by imaginary magnetic field lines.

- Magnetic field lines always form continuous closed loops.
- The tangent to the magnetic field line at a given point represents the direction of the net magnetic

field at that point.

- The larger the number of field lines crossing per unit area, the stronger is the magnetic field.
- In a 'uniform' magnetic field, the field lines are parallel and equidistant.
- Magnetic field lines do not intersect.

Solenoid is a tightly wound helical coil of wire whose diameter is small compared to its length. The magnetic field generated in the center, or core of a current carrying solenoid is uniform and is directed along the axis of solenoid. Magnetic field outside an ideal solenoid is zero.

Lorentz Force: When a charge q moving with velocity enters a region where both magnetic fields and electric fields exist, both fields exert a force on it. Lorentz Force,

$$\vec{F} = q(\vec{v} \times \vec{B}) + q\vec{E}$$

UNITS AND DIMENSIONS

Physical quantity	Symbol	Nature	Dimension	Unit
Permeability of free space	μ_0	Scalar	$[MLT^{-2}A^{-2}]$	TmA^{-1}
Magnetic field	$\vec{B}$	Vector	$[MT^{-2}A^{-1}]$	T(tesla)
Magnetic moment	$\vec{m}$	Vector	$[L^2A]$	Am^2 or J/T
Torsional constant	k	Scalar	$[ML^2T^{-2}]$	$Nmrad^{-1}$

IMPORTANT RESULTS AND FORMULAE

- Force:** When a test charge q_0 enters a magnetic field $\vec{B}$ directed along the negative Z-axis with a velocity $\vec{v}$ making an angle ϕ with the Z-axis, then:

$$\vec{F}_m = q_0(\vec{v} \times \vec{B})$$

- Force on a current-carrying conductor in a

magnetic field $\vec{B}$ is $\vec{F}_m = I(\vec{l} \times \vec{B})$, where I is the current, l is the length of conductor and $\vec{l}$ is along the direction of current.

- Force per unit length between infinitely long straight current carrying conductor is:

$$F_m/l = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r}$$

where, F_m/l is the force per unit length.

- Lorentz force:** $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
- Torque on a magnetic dipole moment $\vec{p}_m$ due to a magnetic field $\vec{B}$ is $\vec{\tau} = \vec{p}_m \times \vec{B}$.
- Magnetic field due to a straight conductor

carrying current I and at a distance ' a ' from the conductor is:

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{a} [\sin \phi_2 + \sin \phi_1]$$

- Magnetic field due to infinite straight conductor at a distance a from it,

$$B = \frac{\mu_0}{4\pi} \times \frac{2I}{a}$$

- Magnetic field on the axis of a current (I) carrying coil of turns n , radius R and at a distance x from the centre of the coil is:

$$B = \frac{\mu_0}{4\pi} \cdot \frac{In \cdot 2\pi R^2}{(R^2 + x^2)^{3/2}}$$

and $B_{(\text{centre})} = \frac{\mu_0}{4\pi} \cdot \frac{n \cdot 2\pi I}{R}$

$$B_{(\text{far away})} = \frac{\mu_0}{4\pi} \cdot \frac{n \cdot 2\pi IR^2}{x^3}$$

- Ampere's circuital law:** The line integral of the magnetic field around a closed path is μ_0 times the total current enclosed by the path and is given as:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

10. The magnetic field due to a straight solenoid well within it and on the axis is given by:

$$B = \mu_0 n I,$$

where, n = number of turns per unit length and I is the current in the coil.

11. **Biot Savart's law:** The magnetic field due to a current element is given by the following relation:

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\vec{l} \times \hat{r}}{r^2}$$

- (a) If θ be the angle between $I d\vec{l}$ and $\hat{r}$, then:

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I dl}{r^2} \sin \theta$$

- (b) The unit of magnetic field B is called tesla (T).

- (c) $\mu_0 = 4\pi \times 10^{-7} \text{ Tm A}^{-1}$

- (d) B is perpendicular to both $I d\vec{l}$ and $\hat{r}$.

12. (i) **Voltage sensitivity:**

$$\frac{\theta}{V} = \frac{nBA}{kR} \text{ (rad V}^{-1}\text{)}$$

- (ii) **Current sensitivity:**

$$\frac{\theta}{I} = \frac{nBA}{k} \text{ (rad A}^{-1}\text{)}$$

- (iii) Conversion of galvanometer into ammeter (by connecting a shunt parallel to galvanometer).

$$S = \frac{I_s}{(I - I_s)} \times G$$

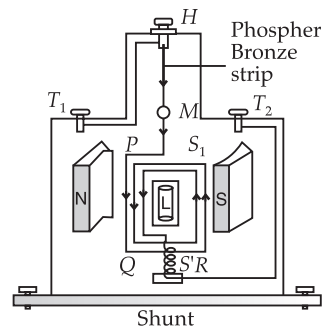
- (iv) Conversion of galvanometer into voltmeter (by connecting a high value

resistance in series with galvanometer).

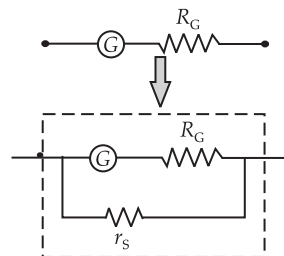
$$R = \frac{V}{I_s} - G$$

IMPORTANT

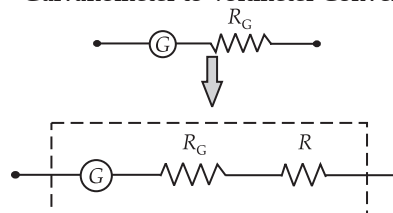
Moving Coil Galvanometer:



Galvanometer to Ammeter Conversion:

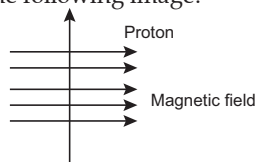


Galvanometer to Voltmeter Conversion:



EXERCISE

- Which one of the following statements regarding magnetic field is NOT correct?
 - Magnetic field is a quantity that has direction and magnitude.
 - Magnetic field lines are closed curves.
 - Magnetic field lines are open curves.
 - No two magnetic field lines are found to cross each other.
- Consider the following image:



A proton enters a magnetic field at right angles to it, as shown above. The direction of force acting on the proton will be :

- to the right
 - to the left
 - out of the page
 - into the page
3. What is the net force experienced by a bar magnet placed in a uniform magnetic field?

- Zero
 - Depends upon length of the magnet
 - Never zero
 - Depends upon temperature
4. Which one of the following statements about magnetic field lines is NOT correct?
- They can emanate from a point.
 - They do not cross each other.
 - Field lines between two poles cannot be precisely straight lines at the ends.
 - There are no field lines within a bar magnet.
5. If a charged particle ($+q$) is projected with certain velocity parallel to the magnetic field, then it will :
- trace helical path
 - trace circular path
 - continue its motion without any change
 - come to rest instantly
6. The magnetic field produced by a current-carrying straight wire at a point outside the wire depends :
- inversely on the distance from it.
 - directly on the distance from it.
 - inversely at short distances and directly at large distance from it.

- (d) directly on the distance (at short distances) and inversely on the distance (at long distances) from it.
7. Imagine a current-carrying straight conductor with magnetic field of lines in anti-clockwise direction. Then the direction of current is determined by :
- the Right Hand Thumb rule and it would be in the downward direction.
 - the Left-Hand Thumb rule and it would be in the downward direction.
 - the Right-Hand Thumb rule and it would be in the upward direction.
 - the Left-Hand Thumb rule and it would be in the upward direction.
8. The magnetic field strength of a current-carrying wire at a particular distance from the axis of the wire :
- Depends upon the current in the wire.
 - Depends upon the radius of the wire.
 - Depends upon the temperature of the surroundings.
 - None of the above
9. At which place Earth's magnetic field becomes horizontal?
- Magnetic meridian
 - Magnetic equator
 - Geographical pole
 - Tropic of Cancer
10. Two long wires each carrying a d.c. current in the same direction are placed close to each other. Which one of the following statements is correct?
- The wires will attract each other.
 - The wires will repel each other.
 - There will be no force between the wires.
 - There will be a force between the wires only at the moment when the current is switched ON or OFF.
11. According to Fleming's right-hand rule, if the forefinger indicated the direction of magnetic field and thumb shows the direction of motion of conductor, then the stretched middle finger will predict the direction of :
- force acting on the conductor
 - electric field
 - induced current
 - current
12. Which one of the following statements regarding a current-carrying solenoid is *not* correct?
- The magnetic field inside the solenoid is uniform.
 - The current-carrying solenoid behaves like a bar magnet.
 - The magnetic field inside the solenoid increases with increase in current.
 - If a soft iron bar is inserted inside the solenoid, the magnetic field remains the same.
13. The presence of magnetic field can be determined using which one of the following instruments?
- Ammeter
 - Voltmeter
 - Magnetic needle
 - Motor
14. Consider the following statements about a solenoid:
- (1) The magnetic field strength in a solenoid depends upon the number of turns per unit length in the solenoid.
 - (2) The magnetic field strength in a solenoid depends upon the current flowing in the wire of the solenoid.
 - (3) The magnetic field strength in a solenoid depends upon the diameter of the solenoid
- Which of the statements given above are correct?
- 1, 2 and 3
 - 1 and 3 only
 - 2 and 3 only
 - 1 and 2 only
15. In a solenoid, the current flowing through the wire is I and number of turns per unit length is n . This gives a magnetic field B inside the solenoid. If number of turns per unit length is increased to $2n$, what will be the value of magnetic field in the solenoid?
- B
 - $2B$
 - $\frac{B}{2}$
 - $\frac{B}{4}$
16. Two wires carrying currents I_1 and I_2 lie, one slightly above the other, in a horizontal plane as shown in figure. The region of vertically upward strongest magnetic field is
-
- I
 - II
 - III
 - IV
17. Two parallel conductors carrying current of 4.0 A and 10.0 A are placed 2.5 cm apart in vacuum. The force per unit length between them is
- $6.4 \times 10^{-5} \text{ N/m}$
 - $6.4 \times 10^{-2} \text{ N/m}$
 - $4.6 \times 10^{-4} \text{ N/m}$
 - $3.2 \times 10^{-4} \text{ N/m}$
18. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a
- low resistance in parallel
 - low resistance in series
 - high resistance in parallel
 - high resistance in series
19. The magnetic field at the centre of a current carrying circular loop of radius R , is B_1 . The magnetic field at a point on its axis at a distance R from the center of the loop is B_2 . Then the ratio (B_1/B_2) is
- $2\sqrt{2}$
 - $\frac{1}{\sqrt{2}}$
 - $\sqrt{2}$
 - 2
20. A current carrying wire kept in a uniform magnetic field, will experience a maximum force when it is
- perpendicular to the magnetic field
 - parallel to the magnetic field
 - at an angle of 45° to the magnetic field
 - at an angle of 60° to the magnetic field
21. A straight conducting rod of length l and mass m is suspended in a horizontal plane by a pair of flexible strings in a magnetic field of magnitude B . To remove the tension in the supporting strings, the magnitude of the current in the wire is
- $\frac{mgB}{l}$
 - $\frac{mgl}{B}$
 - $\frac{mg}{lB}$
 - $\frac{lB}{mg}$
22. A constant current is flowing through a solenoid. An iron rod is inserted in the solenoid along its axis. Which of the following quantities will not increase?
- The magnetic field at the centre

- (b) The magnetic flux linked with the solenoid
(c) The rate of heating
(d) The self-inductance of the solenoid
23. Two concentric and coplanar circular loops P and Q have their radii in the ratio 2:3. Loop Q carries a current 9 A in the anti-clockwise direction. For the magnetic field to be zero at the common centre, loop P must carry
(a) 3 A in clockwise direction
(b) 9 A in clockwise direction
(c) 6 A in anti-clockwise direction
(d) 6 A in the clockwise direction.
24. A long straight wire of circular cross section of radius a carries a steady current I . The current is uniformly distributed across its cross section. The ratio of the magnitudes of magnetic field at a point at a distance $\frac{a}{2}$ above the surface of wire to that at a point at a distance $\frac{a}{2}$ below its surface is
(a) 4 : 1 (b) 1 : 1 (c) 4 : 3 (d) 3 : 4
25. Magnetic field due to a straight solenoid at any point inside it is $B = \mu_0 ni$. Magnetic field at the end of the solenoid is
(a) B (b) B/2 (c) 2B (d) B/4
26. The strength of the magnetic field at distance r from a long straight current carrying wire is B. The field at a distance $r/2$ will be
(a) B (b) 2B (c) B/2 (d) B/4
27. A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field B . The work done to rotate the loop by 30° about an axis perpendicular to its plane is
(a) MB (b) $\frac{\sqrt{3}MB}{2}$ (c) $\frac{MB}{2}$ (d) zero.
28. The deflecting torque acting on the coil of a galvanometer is
(a) inversely proportional to number of turns.
(b) inversely proportional to current flowing.
(c) inversely proportional to area of the coil.
(d) directly proportional to the magnetic field strength.
29. A galvanometer can be converted into a voltmeter by connecting a
(a) high resistance in series.
(b) high resistance in parallel.
(c) low resistance in parallel.
(d) low resistance in series.
30. If the magnetising field on a ferromagnetic material is increased, its permeability
(a) decreases
(b) increases
(c) remains unchanged
(d) first decreases and then increases
31. A bar magnet has magnetic dipole moment $\vec{M}$. Its initial position is parallel to the direction of uniform magnetic field $\vec{B}$. In this position, the magnitudes of torque and force acting on it respectively are
(a) 0 and MB (b) MB and MB
(c) 0 and 0 (d) $|\vec{M} \times \vec{B}|$ and 0
32. Time period of oscillation of a magnetic needle is
(a) $T = \sqrt{\frac{I}{MB}}$ (b) $T = 2\pi \sqrt{\frac{I}{MB}}$
(c) $T = 2\pi \sqrt{\frac{MB}{I}}$ (d) $T = \pi \sqrt{\frac{MB}{I}}$
33. A magnetic needle is kept in a non-uniform magnetic field experiences
(a) a force as well as a torque
(b) a torque but not a force
(c) a force and a torque
(d) a force but not a torque
34. A ferromagnetic substance is heated above its curie temperature. Which of the following statements is correct?
(a) Ferromagnetic domains get perfectly arranged.
(b) Ferromagnetic domains get randomly arranged.
(c) Ferromagnetic domains are not at all influenced.
(d) Ferromagnetic material transforms into diamagnetic substance.

Answer Key

1. (c)	2. (d)	3. (a)	4. (d)	5. (c)	6. (a)	7. (b)	8. (a)	9. (a)	10. (a)
11. (c)	12. (d)	13. (c)	14. (d)	15. (b)	16. (b)	17. (d)	18. (a)	19. (a)	20. (a)
21. (c)	22. (c)	23. (d)	24. (c)	25. (b)	26. (b)	27. (d)	28. (d)	29. (a)	30. (a)
31. (c)	32. (b)	33. (a)	34. (b)						

Answers with Explanations

1. **Option (c) is correct.**
Explanation: Magnetic field always form a close curve, its electric field lines which forms open curves.
2. **Option (d) is correct.**
Explanation: As per Fleming's left hand rule, direction of force will be into the page.
3. **Option (a) is correct.**
Explanation: Two poles of magnet experiences equal and opposite forces when bar magnet is placed in magnetic field. These two forces cancel out each other hence net force experience by bar magnet is zero.
4. **Option (d) is correct.**
Explanation: Magnetic field lines always form close loop. They are present inside as well as outside a magnet. Inside their direction is from south to north Pole inside a magnet while it is north to south outside a magnet.
5. **Option (c) is correct.**

Explanation: Force acting on an object moving in a magnetic field is given by $\vec{F} = q(\vec{v} \times \vec{B})$

Since velocity and magnetic field are parallel to each other, $\vec{v} \times \vec{B} = 0$, so $\vec{F} = 0$.

6. Option (a) is correct.

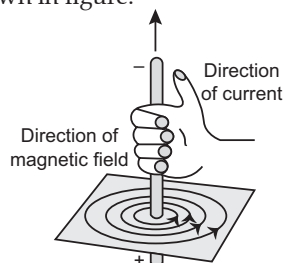
Explanation: Magnetic field at a point outside of a current carrying wire is given, $B = \frac{\mu_0 I}{2\pi r}$

Since $B \propto \frac{1}{r}$,

magnetic field will decrease with increase in distance, so Option (a) is correct.

7. Option (b) is correct.

Explanation: To find the direction of magnetic field or current, we use right hand thumb rule. So options (b) and (d) are incorrect. If we curl our fingers in the direction of magnetic field lines, direction of thumb will represent the direction of current, which is upward as shown in figure.



8. Option (a) is correct.

Explanation:

$$dB \propto \frac{Idl}{a^2} \sin \theta$$

At a particular distance, angle and a will be fixed so, $dB \propto I$

9. Option (a) is correct.

Explanation: At magnetic meridian, earth's magnetic field becomes horizontal.

10. Option (a) is correct.

Explanation: Using right hand rule we can find the direction of magnetic force on each wire due to the current flowing in other wire. Conductors will attract each other if the current flowing in them is in same direction while they will repel each other if current flowing in them is in opposite direction.

11. Option (c) is correct.

Explanation: Fleming's right hand rule is used to determine the direction of induced current.

According to this rule, if thumb represent force, middle finger represent magnetic field, then the direction of induced current is given by middle finger, provided all are mutually perpendicular.

12. Option (d) is correct.

Explanation: Magnetic field inside a current carrying solenoid is uniform, hence it will act like a bar magnet. So, statement (a) and (b) both are correct.

Inside a solenoid magnetic field $B = \mu_0 nI$,

This means, $B \propto I$ and $B \propto n$.

If a soft iron core is placed inside a magnetic field, it will change the magnetization of solenoid. Hence its magnetization will decrease. So, statement (d) is incorrect.

13. Option (c) is correct.

Explanation: Magnetic needle has a small magnet. Its magnetic field in the presence of other magnetic field reacts and needle shows deflection. So magnetic needle or compass can be used to measure presence or direction of magnetic field. Ammeter and voltmeter are used to measure current and voltages respectively.

14. Option (d) is correct.

Explanation:

- The magnetic field inside a solenoid is proportional to both the applied current and the number of turns per unit length.
- There is no dependence on the diameter of the solenoid, and the field strength doesn't depend on the position inside the solenoid i.e., the field inside is constant.

15. Option (b) is correct.

Explanation: In case of solenoid $B \propto n$, at constant current. If number of turns are doubled then magnetic field will also get doubled.

16. Option (b) is correct.

Explanation: Applying right hand thumb rule, the strongest magnetic field will be in region II.

17. Option (d) is correct.

Explanation:
$$F = \frac{\mu_0 i_1 i_2}{2\pi r}$$
$$= \frac{4\pi \times 10^{-7} \times 4 \times 10}{2\pi \times 2.5 \times 10^{-2}}$$
$$= 3.2 \times 10^{-4} \text{ N/m}$$

18. Option (a) is correct.

Explanation: Ammeter and voltmeter are basically galvanometer.

When a high resistance is connected in series with the galvanometer it works as a voltmeter and when a low resistance is connected in parallel with the galvanometer it works as an ammeter.

19. Option (a) is correct.

Explanation: Magnetic field at the centre

$$B_1 = \frac{\mu_0 i}{2R}$$

Magnetic field on the axis

$$B_2 = \frac{\mu_0 i R^2}{2(x^2 + R^2)^{3/2}}$$
$$= \frac{\mu_0 i R^2}{2(R^2 + R^2)^{3/2}}$$
$$= \frac{\mu_0 i R^2}{2 \times 2^{3/2} R^3} = \frac{\mu_0 i}{2^{5/2} R}$$
$$= \frac{\mu_0 i}{2^{5/2} R}$$

$$\therefore \frac{B_1}{B_2} = 2^{3/2} = 2\sqrt{2}$$

20. Option (a) is correct.

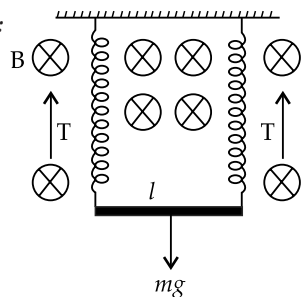
Explanation: $F = B i \sin \theta$

θ is the angle between the direction of current and the direction of magnetic field.

So, when $\theta = 90^\circ$, the force is maximum.

21. Option (c) is correct.

Explanation:



$$F = mg = T$$

$$\text{Magnetic force} = F_{\text{mag}} = ilB$$

To remove the tension,

$$T = F_{\text{mag}}$$

$$\text{or, } mg = ilB$$

$$\therefore i = \frac{mg}{lB}$$

To remove the tension,

$$T = F_{\text{mag}}$$

$$\text{or, } mg = ilB$$

$$\therefore i = \frac{mg}{lB}$$

22. Option (c) is correct.

Explanation: Since, the current remains constant, the rate of heating will not increase.

23. Option (d) is correct.

Explanation: Magnetic field at the centre will be of same magnitude and opposite direction.

$$\text{So, } \frac{\propto i_P}{2r_P} = \frac{\propto i_Q}{2r_Q}$$

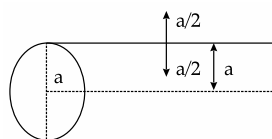
$$\text{Or, } \frac{i_P}{i_Q} = \frac{R_P}{R_Q}$$

$$\text{Or, } \frac{i_P}{9} = \frac{2}{3}$$

$$\therefore i_P = 6A \quad 1$$

24. Option (c) is correct.

Explanation:



$$\text{Magnetic field, } B = \frac{\mu_0}{2\pi} \times \frac{I}{r} \quad (\text{for } r > a)$$

$$\text{And } B = \frac{\mu_0}{2\pi} \times \frac{Ir}{a^2} \quad (\text{for } r < a)$$

$$\text{For point P, } r > a$$

$$\text{So, } B_P = \frac{\mu_0}{2\pi} \times \frac{I}{r_1} \quad \left[r_1 = \frac{3a}{2} \right]$$

$$\text{For point Q, } r < a$$

$$\text{So, } B_Q = \frac{\mu_0}{2\pi} \times \frac{Ir_2}{a^2} \quad \left[r_2 = \frac{a}{2} \right]$$

$$\text{So, the ratio } \frac{B_P}{B_Q} = \frac{r_1}{Ir_2} \times \frac{a^2}{a^2}$$

$$= \frac{I}{r_1} \times \frac{a^2}{Ir_2}$$

$$= \frac{a^2}{\frac{3a}{2} \times \frac{a}{2}}$$

$$= \frac{a^2}{\frac{3a^2}{4}}$$

$$= 4 : 3$$

25. Option (b) is correct.

Explanation: Magnetic field at the end of a current carrying solenoid is half of the magnetic field inside it.

26. Option (b) is correct.

Explanation: $B \propto 1/r$

$$B_1/B_2 = r_2/r_1 = \frac{r/2}{r}$$

$$\therefore B_2 = 2B_1 = 2B$$

27. Option (d) is correct.

Explanation: The work done to rotate the loop in magnetic field, $W = MB (\cos \theta_1 - \cos \theta_2)$.

When current carrying coil is rotated then there will be no change in angle between magnetic moment and magnetic field.

$$\text{Here, } \theta_1 = \theta_2 = \alpha$$

$$\Rightarrow W = MB (\cos \alpha - \cos \alpha) = 0.$$

28. Option (d) is correct.

Explanation: $\tau = nBIA$.

So, torque is directly proportional to the magnetic field strength, area of the coil, number of turns and current flowing.

29. Option (a) is correct.

Explanation: To convert a galvanometer into a voltmeter, a high value resistance is to be connected in series with it.

30. Option (a) is correct.

Explanation: $\mu = 1/H$

So, as magnetising field (H) increases, permeability (μ) decreases.

31. Option (c) is correct.

Explanation: Torque $= \tau = MB \sin \theta$

Since, M and B are parallel, then $\theta = 0$ and hence, $\tau = 0$.

Torque is 0. So, in this case force is also zero since the distance is not equal to zero.

32. Option (b) is correct.

Explanation: Time period of oscillation of a magnetic

$$\text{needle is } T = 2\pi \sqrt{\frac{I}{MB}}$$

33. Option (a) is correct.

Explanation: Field being non-uniform, the poles of the needle will experience non-uniform forces. Hence, the needle experiences a force as well as a torque.

34. Option (b) is correct.

Explanation: On heating above Curie temperature, Ferromagnetic domains get randomly arranged and it transforms into paramagnetic substance.

ELECTROMAGNETIC INDUCTION AND ALTERNATING CURRENT

Faraday's First Law of Electromagnetic induction states that whenever a conductor is placed in varying magnetic field, an emf is induced which is known as induced emf and if the conductor circuit is closed, current is also induced which is called induced current.

Faraday's Second Law of Electromagnetic induction states that the induced emf is equal to the rate of change of flux linkage where flux linkage is the product of number of turns in the coil and flux associated with the coil.

$$e = -N \frac{d\phi}{dt}$$

Here, $\phi = NBA \cos \theta$, θ is the angle between B and A and sign indicate that induced emf opposes its cause.

- **AC generator** is an electrical machine which converts mechanical energy into alternating electrical energy. It works on the principle of electromagnetic induction where a coil gets rotated in uniform magnetic field, sets an induced emf which is given by,

$$e = NBA\omega \sin \omega t$$

Here, $\omega = 2\pi f$

In India frequency (f) of ac is 50 Hz and standard voltage is 220 V.

- **Transformer** is an electrical device used for changing the alternating voltages. It is based on the phenomenon of mutual induction. The main use of transformer is in transmission of ac over long distances at extremely high voltages which reduce the energy losses in transmission.

It comprises of two sets of coils which are insulated from each other and are wound on **soft-iron** core. In this, one of the coil is called as primary (input coil) having N_p turns while other coil is secondary (output coil) having N_s turns, so we have $\frac{E_s}{E_p} = \frac{N_s}{N_p}$

- **Step-up transformer:** If secondary coil has more number of turns than primary ($N_s > N_p$), voltage gets stepped up ($V_s > V_p$) and there is less current in secondary ($I_s < I_p$). The value of transformation ratio $k > 1$.
- **Step-down transformer:** In this, the secondary coil has less number of turns than primary ($N_s < N_p$). In this, $V_s < V_p$ and $I_s > I_p$ as voltage gets stepped down with increase in current. In this, value of transformation ratio $k < 1$.

Quantity	Symbol	Dimension	Unit
Magnetic flux	ϕ	$[ML^2T^{-2}A^{-1}]$	Wb (weber)
EMF	E	$[ML^2T^{-3}A^{-1}]$	V (volt)
Mutual inductance	M	$[ML^2T^{-2}A^{-2}]$	H (henry)
Self inductance	L	$[ML^2T^{-2}A^{-2}]$	H (henry)

IMPORTANT RESULTS AND FORMULAE

1. The inductance of a coil depends on the following factors:
 - (a) area of cross-section
 - (b) number of turns
 - (c) permeability of the core.
2. Self-induction

$$\phi \propto I \text{ or } \phi = LI$$

$$E = -\frac{d\phi}{dt} = -L \frac{dI}{dt}$$

$$L = \frac{-E}{dI/dt}$$

The inductance of a circular coil is given by:

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \frac{\mu}{4\pi} \times \frac{(2\pi NI)}{rI} \times AN$$

$$\left[\because B = \frac{\mu}{4\pi} \cdot \frac{2\pi NI}{r} \right]$$

$$= \frac{\mu N^2}{2r} A = \frac{\mu N^2}{2r} \times \pi r^2$$

$$\text{Or, } L = \frac{\mu N^2 \pi r}{2}$$

here, ϕ = magnetic flux from the coil, I = current through the coil, A = area of coil, r = radius of coil, N = total number of turns of the coil, μ = permeability of the medium.

3. The inductance of a solenoid of length l , is given by:

$$L = \frac{\phi}{I} = \frac{BAN}{I} = \left(\frac{\mu NI}{l} \right) \frac{AN}{I} \quad \left[\because B = \frac{\mu NI}{l} \right]$$

$$\text{Or, } L = \frac{\mu N^2 A}{l} = \mu n^2 Al = \mu n^2 V$$

$$\left[\because n = \frac{N}{l} \right]$$

here, $n = N/l$ = number of turns per unit length and $V = Al$ = volume of the solenoid.

4. The inductance in series combination is given by:

$$L_s = L_1 + L_2 + L_3 + \dots$$

The inductance in parallel combination is given by:

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots$$

5. If two coils of inductance L_1 and L_2 are coupled together, then their mutual inductance is given by:

$$M = k\sqrt{L_1 L_2}$$

6. Mutual inductance:

$$\phi \propto I \text{ or } \phi = MI$$

$$E = -\frac{d\phi}{dt} = -M \frac{dI}{dt}$$

$$M = -E/(dI/dt)$$

$$M = \frac{-E}{dI/dt}$$

The mutual inductance of two coils is given by:

$$M = \frac{\mu_0 N_p N_s A}{l}$$

7. Magnetic flux is number of magnetic field lines crossing a surface normally.

The magnetic flux is denoted by ϕ . If $\vec{B}$ be the magnetic field through the area element $d\vec{A}$. Then, the flux through the area is given by:

$$d\phi = \vec{B} \cdot d\vec{A}$$

8. Faraday's laws of electromagnetic induction:

- Whenever, the magnetic flux linked with a circuit changes, an emf is induced.
- The induced emf lasts as long as the change in magnetic flux continues.
- The magnitude of induced emf (ϵ) is directly proportional to the rate of change in magnetic flux.

Thus, if $d\phi$ be the change in magnetic flux during the time dt , then:

$$\epsilon \propto \frac{d\phi}{dt} \text{ or } \epsilon = k \frac{d\phi}{dt}$$

where, k is the constant of proportionality. In

$$\text{SI, } k = -1, \text{ hence, } \epsilon = -\frac{d\phi}{dt}$$

9. Lenz's law:

It states that induced current produced in a circuit always flows in such a direction that it opposes the change or the cause that produces it.

The Lenz's law explains the negative sign in

$$\text{the Faraday's flux rule, } \epsilon = -\frac{d\phi}{dt}$$

10. If a rod of length l moves perpendicular to a magnetic field B with a velocity v , then the

induced emf produced across it is given by:

$$\epsilon = vBl$$

In general, we have $\epsilon = \vec{B} \cdot (\vec{v} \times \vec{l})$

11. If a current I is set up in a coil of inductance L , then the magnetic field energy stored in it is given by:

$$U_p = \frac{1}{2} LI^2$$

12. Instantaneous value of emf produced by AC generator, $e = NBA\omega \sin \omega t$

UNITS AND DIMENSIONS

Physical quantity	Symbol	Dimension	Unit
rms voltage	V_{rms}	$[ML^2T^{-3}A^{-1}]$	V
rms current	I_{rms}	$[A]$	A
Inductive reactance	X_L	$[ML^2T^{-3}A^{-2}]$	Ω
Capacitive reactance	X_C	$[ML^2T^{-3}A^{-2}]$	Ω
Impedance	Z	$[ML^2T^{-3}A^{-2}]$	Ω
Resonant frequency	f or f_0	$[T^{-1}]$	Hz
Quality factor	Q	Dimensionless	
Power factor	$\cos \phi$	Dimensionless	

IMPORTANT RESULTS AND FORMULAE

1. (a) Instantaneous value of alternating current

$$I = I_0 \sin \omega t \text{ or } I = I_0 \cos \omega t$$

- (b) Peak value of a.c. = I_0

- (c) Alternating emf, $E = E_0 \sin \omega t$
or $E = E_0 \cos \omega t$

- (d) Mean or average value of a.c.

$$I_m \text{ or } I_a = \frac{2I_0}{\pi} = 0.636 I_0$$

- (e) R.M.S. value of a.c. $I_{rms} = I_0 / \sqrt{2} = 0.707 I_0$

2. The part of the impedance in which the phase difference between the current and emf is $\pi/2$, is called reactance.

If the emf leads the current by $\pi/2$, the reactance is called purely inductive.

If the emf lags behind the current by $\pi/2$, the reactance is called purely capacitive.

If the emf is in phase with the current, the circuit is called purely resistive.

3. Sign for phase difference (ϕ) between I and E for series LCR circuit:

ϕ is positive, when $X_L > X_C$.

ϕ is negative, when $X_L < X_C$.

ϕ is zero, when $X_L = X_C$.

4. The LCR circuit is said to be in resonance when:

$$X_L = X_C \text{ i.e., when } \omega L = \frac{1}{\omega C} \text{ and}$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} \text{ is called resonant frequency.}$$

5. At series resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}}$, we have:

(i) $Z = R$ = minimum value of impedance.

(ii) $I_0 = E_0/R$ = maximum value of peak current.

- (iii) $\phi = 0$, i.e., I and E are in phase with each other.
- (iv) V_L is equal and opposite to V_C .
- (v) Potential drop across C and L together is zero.
- (vi) $E = V_R$.

6. Energy stored in an inductor:

$$U = \frac{1}{2}LI_0^2$$

7. Energy stored in a capacitor:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}q_0V$$

8. Power in a.c. circuit: The power in LCR circuit is given by:

$$P = EI = E_0I_0 \sin \omega t \sin (\omega t - \phi).$$

Power in LCR circuit consists of two components:

(i) Virtual power component

$$= \frac{1}{2}E_0I_0 \cos (2\omega t - \phi).$$

It has frequency twice as that of a.c. Its value over the complete cycle is zero.

(ii) Real power component = $\frac{1}{2}E_0I_0 \cos \phi$. It dissipates power.

$\cos \phi$ is called power factor.

9. Inductive reactance: $X_L = \omega L$

$$\text{Inductive susceptance: } B_L = \frac{1}{\omega L}$$

$$\text{Capacitive reactance: } X_C = \frac{1}{\omega C}$$

$$\text{Capacitive susceptance: } B_C = \omega C$$

$$\text{Reactance: } X = X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$\text{Susceptance: } B = B_C - B_L = \omega C - \frac{1}{\omega L}$$

10. Impedance of LCR circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan \phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\text{Power, } P = E_{rms} \times i_{rms} \times \frac{R}{Z}$$

11. Bandwidth = $\omega_2 - \omega_1 = 2\Delta\omega$

12. Sharpness of resonance = $\frac{\omega_0}{2\Delta\omega}$

$$\text{Or, } \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

13. Q factor: $Q = \frac{\text{voltage across } L \text{ or } C}{\text{applied voltage}}$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

14. Transformer:

(i) $\frac{E_s}{E_p} = \frac{N_s}{N_p} = k$ (say) (transformer ratio)

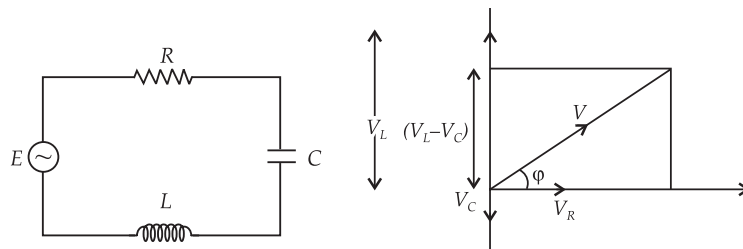
(ii) For step up transformer, $k > 1$ and for step down transformer, $k < 1$.

(iii) For step up transformer, $N_s > N_p$, therefore, $E_s > E_p$. And for the step down transformer, $N_s < N_p$, therefore $E_s < E_p$.

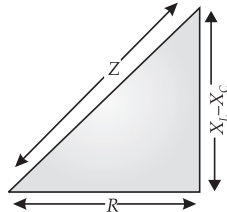
(iv) The efficiency of the transformer is given by:

$$\eta = \frac{\text{output power}}{\text{input power}}$$

LCR circuit:



Impedance triangle:



Transformer:

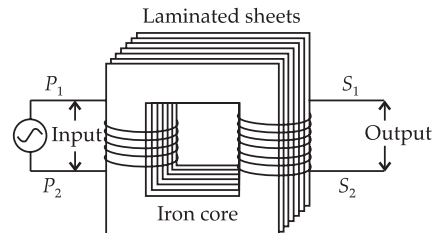
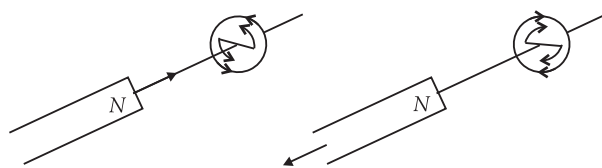
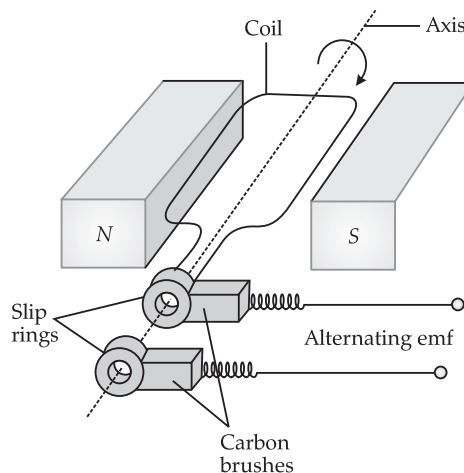


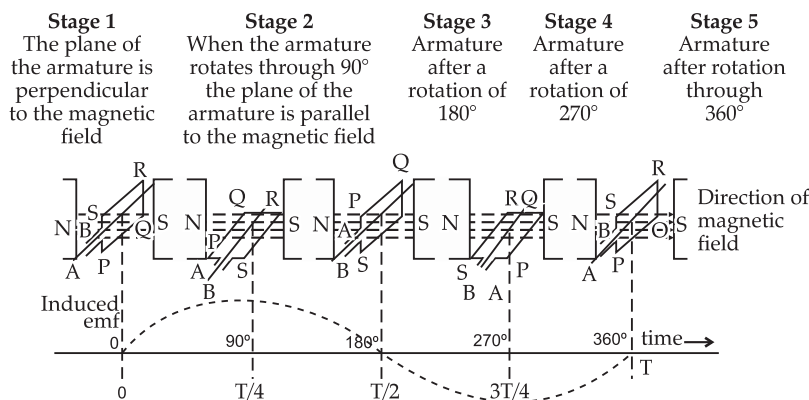
Illustration of Lenz's law:



AC generator:



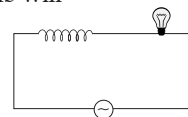
Working of AC generator:



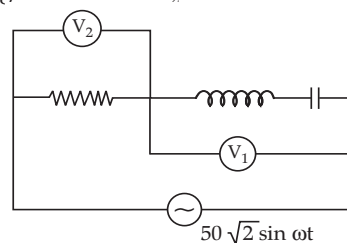
EXERCISE

- A DC generator works on the principle of :
 - Ohm's law
 - Joule's law of heating
 - Faraday's laws of electromagnetic induction
 - None of the above
- The frequency of an alternating current is 3 Hz. It implies that :
 - there are 6 cycles/s
 - there are 3 cycles/s
 - there are 2 cycles/s
 - there is only 1 cycle/s
- Which one of the following devices changes low voltage alternating current to high voltage alternating current and vice versa ?
 - Generator
 - Motor
 - Transformer
 - Vibrator
- Which one of the following is the correct relation between frequency f and angular frequency ω ?
 - $f = \pi\omega$
 - $\omega = 2\pi f$
 - $f = 2\omega/\pi$
 - $f = 2\pi\omega$
- Step-up transformers are used for :
 - increasing electrical power
 - decreasing electrical power
 - decreasing voltage
 - increasing voltage
- The phenomenon of electromagnetic induction implies a production of induced :
 - resistance in a coil when the magnetic field changes with time.
 - current in a coil when an electric field changes with time.
 - current in a coil when a magnetic field changes with time.
 - voltage in a coil when an electric field changes with time.

7. Copper wires are generally used for electrical power transmission instead of iron wire because:
 (a) Copper is cheaper than iron
 (b) Copper is lighter than iron
 (c) Copper is a better conductor of electricity than iron
 (d) Copper can take higher power than iron
8. The self-inductance of a solenoid of 600 turns is 108 mH. The self-inductance of a coil having 500 turns with the same length, the same radius and the same medium will be
 (a) 95 mH (b) 90 mH
 (c) 85 mH (d) 75 mH
9. The current in the primary coil of a pair of coils changes from 7 A to 3 A in 0.4 s. The mutual inductance between the two coils is 0.5 H. The induced emf in the secondary coil is
 (a) 50 V (b) 75 V
 (c) 100 V (d) 220 V
10. A coil of area 100 cm^2 is kept at an angle of 30° with a magnetic field of 10^{-1} T . The magnetic field is reduced to zero in 10^{-4} s . The induced emf in the coil is
 (a) $5\sqrt{3} \text{ V}$ (b) $50\sqrt{3} \text{ V}$
 (c) 5.0 V (d) 50.0 V
11. A rectangular, a square, a circular and an elliptical loop, all in the $(x-y)$ plane, are moving out of a uniform magnetic field with a constant velocity $\vec{v} = v\hat{i}$. The magnetic field is directed along the negative z -axis direction. The induced emf, during the passage of these loops, out of the field region, will not remain constant for
 (a) any of the four loops
 (b) the circular and elliptical loops
 (c) the rectangular, circular and elliptical loops
 (d) only the elliptical loops
12. A square of side L metres lies in the $x-y$ plane in a region where the magnetic field is given by $B = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$ Tesla, where, B_0 is constant. The magnitude of flux passing through the square is
 (a) $2B_0L^2 \text{ Wb}$ (b) $3B_0L^2 \text{ Wb}$
 (c) $4B_0L^2 \text{ Wb}$ (d) $\sqrt{29}B_0L^2 \text{ Wb}$
13. The magnetic flux linked with a coil is given by an equation $\phi = 5t^2 + 2t + 3$. The induced e.m.f. in the coil at the third second will be
 (a) 32 units (b) 54 units
 (c) 40 units (d) 65 units
14. Lenz's law is a consequence of the law of conservation of
 (a) charge (b) mass
 (c) energy (d) momentum
15. The polarity of induced emf is defined by
 (a) Ampere's circuital law.
 (b) Biot-Savart's law.
 (c) Lenz's law.
 (d) Fleming's right hand rule.
16. The self-inductance L of a solenoid of length l and area of cross-section A , with a fixed number of turns N increases as
 (a) l and A increase.
 (b) l decreases and A increases.
 (c) l increases and A decreases.
 (d) both l and A decrease.
17. Magnetic field energy stored in a coil is
 (a) Li^2 (b) $\frac{1}{2} Li$
 (c) Li (d) $\frac{1}{2} Li^2$
18. The rms current in a circuit connected to 50 Hz ac source is 15 A. The value of the current in the circuit $\left(\frac{1}{600}\right)$ s after the instant the current is zero, is
 (a) $\frac{15}{\sqrt{2}} \text{ A}$ (b) $15\sqrt{2} \text{ A}$
 (c) $\frac{\sqrt{2}}{15} \text{ A}$ (d) 8 A
19. When an alternating voltage $E = E_0 \sin \omega t$ is applied to a circuit, a current $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ flows through it. The average power dissipated in the circuit is
 (a) $E_{\text{rms}} \cdot I_{\text{rms}}$ (b) $E_0 I_0$
 (c) $\frac{E_0 I_0}{\sqrt{2}}$ (d) Zero
20. The voltage across a resistor, an inductor, and a capacitor connected in series to an ac source are 20 V, 15 V and 30 V respectively. The resultant voltage in the circuit is
 (a) 5 V (b) 20 V (c) 25 V (d) 65 V
21. A circuit is connected to an ac source of variable frequency. As the frequency of the source is increased, the current first increases and then decreases. Which of the following combinations of elements is likely to comprise the circuit?
 (a) L, C and R (b) L and C
 (c) L and R (d) R and C
22. A 15 W resistor, an 80 mH inductor and a capacitor of capacitance C are connected in series with a 50 Hz ac source. If the source voltage and current in the circuit are in phase, then the value of capacitance is
 (a) 100 mF (b) 127 mF
 (c) 142 mF (d) 160 mF
23. The core of a transformer is laminated to reduce the effect of
 (a) flux leakage (b) copper loss
 (c) hysteresis loss (d) eddy current
24. An iron cored coil is connected in series with an electric bulb with an AC source as shown in figure. When iron piece is taken out of the coil, the brightness of the bulb will



- (a) decrease (b) increase
 (c) remain unaffected (d) fluctuate
25. If the reading of the voltmeter V_1 is 40 V, then the reading of voltmeter V_2 is



- (a) 30 V (b) 58 V
(c) 29 V (d) 15 V
26. In a circuit the phase difference between the alternating current and the source voltage is $\frac{\pi}{2}$. Which of the following cannot be the element(s) of the circuit?
(a) only C (b) only L
(c) L and R (d) L or C
27. Relation between r.m.s. voltage and instantaneous voltage of an AC
(a) $V_0 = V_{\text{rms}} / \sqrt{2}$ (b) $V_{\text{rms}} = V_0 / \sqrt{2}$
(c) $V_{\text{rms}} = 0.707V_0$ (d) Both (B) and (C)
28. The impedance of a series LCR circuit is
(a) $R + X_L + X_C$ (b) $\sqrt{\frac{1}{X_C^2} + \frac{1}{X_L^2} + R^2}$
(c) $\sqrt{X_L^2 - X_C^2 + R^2}$ (d) $\sqrt{R^2 + (X_L - X_C)^2}$
29. In which of the following circuit power dissipation is maximum?
- (a) Pure capacitive circuit
(b) Pure inductive circuit
(c) Pure resistive circuit
(d) LR or CR circuit
30. With increase in frequency of an A.C. supply, the impedance of a series L-C-R circuit
(a) remains constant.
(b) increases.
(c) decreases.
(d) decreases at first, becomes minimum and then increases.
31. Parameter that remains unchanged in a transformer is
(a) voltage. (b) current.
(c) frequency. (d) none of these.
32. The efficiency of transformer is very high because
(a) There is no moving part
(b) It uses AC only
(c) It uses the copper wire for the coils
(d) None of the above

Answer Key

1. (c)	2. (b)	3. (c)	4. (b)	5. (d)	6. (c)	7. (c)	8. (d)	9. (a)	10. (b)
11. (a)	12. (d)	13. (c)	14. (d)	15. (b)	16. (a)	17. (a)	18. (d)	19. (b)	20. (a)
21. (a)	22. (d)	23. (b)	24. (b)	25. (a)	26. (a)	27. (a)	28. (d)	29. (b)	30. (a)
31. (a)	32. (d)								

Answers with Explanations

1. Option (c) is correct.

Explanation: DC generator converts mechanical energy into electrical energy. It applies concept of electromagnetic induction, according to which, "when a conductor is placed in changing magnetic field, current gets induced in it"

2. Option (b) is correct.

Explanation: 3 Hz means in one second there are 3 cycles of a.c. (Hz = 1 cycle/s)

3. Option (c) is correct.

Explanation: Transformer changes the value of electric voltage and current at various levels in from power generation to distribution. According to the need they either increase or decrease their value.

4. Option (b) is correct.

5. Option (d) is correct.

Explanation: Step up transformers are used in leveling up electrical voltage, while step-down transformers are used to decrease the level of electrical voltage.

6. Option (c) is correct.

Explanation: According to the law of electromagnetic induction, the change in flux (or magnetic field) linked with a coil induces current in it. Flux can change due to change in, either direction/ magnitude of current, relative motion of magnet and coil or intensity of magnetic field.

7. Option (c) is correct.

Explanation: Copper has less resistance than iron, thus transmission loss reduces.

8. Option (d) is correct.

Explanation: $L = \frac{\mu_0 N^2 A}{l}$

So, $L \propto N^2$

$\therefore \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$

Or, $\frac{108}{L_2} = \frac{600^2}{500^2}$

$\therefore L_2 = 108 \times \frac{25}{36} = 75 \text{ mH}$

9. Option (a) is correct.

Explanation: Induced emf = $|e| = M \frac{di}{dt}$
 $= 0.5 \times (4/0.04)$
 $= 50\text{V}$

10. Option (a) is correct.

Explanation: Flux through the coil $\phi = BA \cos \theta$
 or, $\phi = 10^{-1} \times (100 \times 10^{-4}) \times \cos 30^\circ$

$\therefore \phi = 10^{-3} \times \frac{\sqrt{3}}{2} \text{ Wb}$

Induced emf = $e = \frac{-d\phi}{dt} = \frac{-(\phi_f - \phi_i)}{\Delta t}$

$= \frac{-\left(0 - 10^{-3} \times \frac{\sqrt{3}}{2}\right)}{10^{-4}} = 5\sqrt{3} \text{ V}$

11. Option (b) is correct.

Explanation: When the circular and the elliptical loops come out of the field, equal areas will not be traced in equal intervals of time.

So, for these two loops induced emfs will not remain constant.

12. Option (c) is correct.

Explanation: Magnetic flux is defined as the total number of magnetic lines of force passing normally through an area placed in a magnetic field and is equal to the magnetic flux linked with that area.

Square lies in x - y plane in $\vec{B}$. So, $\vec{A} = L^2 \hat{k}$

$$\phi = B \cdot A$$

$$= B_0 (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (L^2 \hat{k})$$

$$= B_0 [2 \times \hat{i} \cdot \hat{k} + 3 \times \hat{j} \cdot \hat{k} + 4 \times \hat{k} \cdot \hat{k}]$$

$$= B_0 L^2 [0 + 0 + 4]$$

$$= 4B_0 L^2 \text{ Wb.}$$

13. Option (a) is correct.

Explanation:

$$\text{Induced e.m.f} = \frac{d\phi}{dt}$$

$$= -(5t^2 + 2t + 3)$$

$$= -(10t + 2) = -32$$

14. Option (c) is correct.

Explanation: Lenz's law is a consequence of the law of conservation of energy.

Lenz's law says that induced current always tends to oppose the cause which produces it. So work is done against opposing force. This work is transformed into electrical energy. So, it is a consequence of law of conservation of energy.

15. Option (c) is correct.

Explanation: According to Lenz's law, the direction of an induced e.m.f. always opposes the change in magnetic flux that causes the e.m.f.

16. Option (b) is correct.

Explanation: As we know that,

$$L = \mu_r \mu_0 \frac{N^2 A}{l}$$

As L is constant for a coil,

$$L \propto A \text{ and } L \propto \frac{1}{l}$$

As μ_r and N are constant here so, to increase L for a coil, area A must be increased and l must be decreased. So, answer (b) is correct.

17. Option (d) is correct.

Explanation: If current I flows through a coil of self-inductance L , then magnetic field energy stored in it is $\frac{1}{2} Li^2$

18. Option (a) is correct.

Explanation: $I = I_0 \sin \omega t$

$$\text{or, } I = I_{\text{rms}} \times \sqrt{2} \times \sin(2\pi ft)$$

$$\text{or, } I = 15 \times \sqrt{2} \times \sin\left(\frac{2 \times \pi \times 50}{600}\right)$$

$$\text{or, } I = 15 \times \sqrt{2} \times \sin\left(\frac{\pi}{6}\right)$$

$$\therefore I = \frac{15}{\sqrt{2}} \text{ A}$$

19. Option (d) is correct.

Explanation: Power = $V_{\text{rms}} \times I_{\text{rms}} \times \cos \phi$

$$\text{Here } \phi = \frac{\pi}{2}$$

$$\text{So, Power} = 0$$

20. Option (c) is correct.

Explanation: The resultant voltage in the LCR series circuit is calculated as,

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

Here, all alphabets are in their usual meanings.

$$V_R = 20 \text{ V}, V_C = 30 \text{ V and } V_L = 15 \text{ V}$$

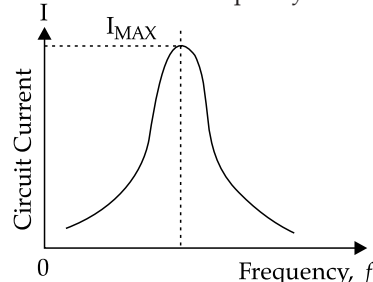
$$\text{So, } V = \sqrt{(20)^2 + (30 - 15)^2}$$

$$V = \sqrt{400 + 225} = \sqrt{625}$$

$$V = 25 \text{ V}$$

21. Option (a) is correct.

Explanation: The frequency vs. circuit current graph in a series LCR circuit is as follows, where current initially increases, reaches a maximum and then decreases with increase in frequency.



Hence, the circuit contains a combination of L , R and C .

22. Option (b) is correct.

Explanation: Voltage and current will be in phase when $X_C = X_L$

$$\text{Or, } \frac{1}{\omega_c} = \omega_L$$

$$\text{Or, } \frac{1}{2\pi f_c} = 2\pi fL$$

$$\text{Or, } C = \frac{1}{4\pi^2 f^2 L}$$

$$\text{Or, } C = \frac{1}{4 \times (3.14)^2 \times (50)^2 \times 80 \times 10^{-3}}$$

$$\therefore C = 127 \text{ mF}$$

23. Option (d) is correct.

Explanation: Lamination increases the resistance and hence reduced the eddy current.

24. Option (d) is correct.

Explanation: As the iron piece is taken out the self inductance of the coil decreases.

So, ω_L , i.e., X_L decreases.

So, the current in the circuit increases.
Hence, the brightness of the bulb increases.

25. Option (a) is correct.

Explanation: In the given circuit,

$$V = 50\sqrt{2} \sin \omega t$$

$$\therefore V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{50\sqrt{2}}{\sqrt{2}} = 50V$$

$$\text{Now, } V_{rms} = \sqrt{V_2^2 + V_1^2}$$

$$\text{Or, } 50 = \sqrt{V_2^2 + 40^2}$$

$$\text{Or, } V_2^2 = 2500 - 1600$$

$$\therefore V_2 = 30V$$

26. Option (c) is correct.

Explanation: Only pure C and L can provide the $\frac{\pi}{2}$ phase difference. But if R is also present there then

the phase difference deviates from $\frac{\pi}{2}$.

27. Option (d) is correct.

$$\text{Explanation: } V_{rms} = V_0/\sqrt{2} = 0.707V_0$$

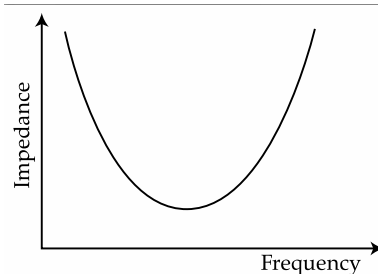
28. Option (d) is correct.

29. Option (c) is correct.

Explanation: Since in pure resistive circuit the current and voltage are in phase, the power dissipation is maximum.

30. Option (d) is correct.

Explanation: The frequency vs. Impedance graph of a series LCR circuit is as follows:



With increase in frequency, the impedance decreases at first, becomes minimum and then increases.

31. Option (c) is correct.

Explanation: Transformer does not change the frequency of the applied AC.

32. Option (a) is correct.

Explanation: Transformer is a static device which transforms power from one circuit to other through electromagnetic induction. In electrical transformer as there are no moving parts, no friction. Losses in the transformer are very less compared to any other rotating machine, hence efficiency of transformers is very high which is about 95% to 98%.

○○○

Chapter 43

ELECTROMAGNETIC WAVES

Electromagnetic waves

- EM waves are produced by accelerated charged particles.
- EM waves are propagated as electric and magnetic fields oscillating in mutually perpendicular directions.
- EMW are transverse in nature.
- These waves can travel through vacuum and do not need the presence of material medium for propagation.
- Their speed in vacuum is 3×10^8 m/s.
- EM waves are not affected by electric and magnetic fields.

Type of radiation	Wavelength range	Production	Application
Radio	$>1.0 \times 10^{-1}$ m	Rapid acceleration and deaccelerations of electrons in aerials.	Radio, TV, Cellular communication,
Microwave	0.1 m- 1.0×10^{-3} m	Klystron valve or magnetron valve.	Radar system, Microwave oven, To detect speed of objects.
Infra-red	1.0×10^{-3} m- 700×10^{-9} m	Vibration of atoms and molecules.	Wireless LANs, Fiber optics remote control, Thermal imaging, Food processing
Light	700×10^{-9} m- 400×10^{-9} m	Electrons in atoms emit when they move from one energy level to a lower level.	The eyes, Photocells photographic film kill bacteria and germs.
Ultraviolet	400×10^{-9} m- 1.0×10^{-12} m	Inner shell electrons in atoms moving from one energy level to a lower level.	Study crystal structure, In X ray imaging, Detect explosive, drugs, to detect cracks
X-rays	1.0×10^{-9} m- 1.0×10^{-12} m	X-ray tubes or inner shell electrons	Cancer treatment, Gieger tubes, Ionization chamber
Gamma rays	$<1.0 \times 10^{-12}$ m	Radioactive decay of the nucleus	Photographic film, Gieger tubes, Ionization chamber

IMPORTANT RESULTS AND FORMULAE

1. EM wave spectrum:

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and de acceleration of electrons in aerials	Receiver's aerials
Microwave	0.1 m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1 mm to 700 nm	Vibration of atoms and molecules	Thermopiles, bolometer, infra photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from a higher energy level to a lower energy level	The eye, photocells, photographic film
Ultraviolet	400 nm to 1 nm	Inner shell electrons in atoms moving from a higher energy level to a lower energy level	Photocells, photographic films
X-rays	1 nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic films, Geiger tubes, ionisation chamber
Gamma rays	$< 10^{-3}$ nm	Radioactive decay of the nucleus	Observing the effects they have on matter

2. Displacement current: $I_D = \epsilon_0 \frac{d\phi_E}{dt}$

(i) $\oint \vec{E} \cdot d\vec{A} = Q / \epsilon_0$ (Gauss's law for electricity)

3. Maxwell's equations:

(ii) $\oint \vec{B} \cdot d\vec{A} = 0$ (Gauss's law for magnetism)

(iii) $\oint \vec{E} \cdot d\vec{l} = \frac{-d\phi_B}{dt}$ (Faraday's law)

(iv) $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_c + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$ (Ampere - Maxwell law)

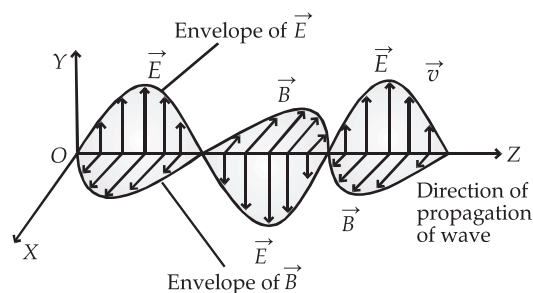
4. **Ampere-Maxwell's circuital law:**

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_c + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

5. **Poynting vector:** $\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$

IMPORTANT GRAPH

Electric and magnetic field for EM waves



EXERCISE

- Which one of the following wavelengths corresponds to the wavelength of X-rays?
(a) 500 nm (b) 5000 nm
(c) 100 nm (d) 1 nm
- Light of uniform intensity impinges perpendicularly on a totally reflecting surface. If the area of the surface is halved, the radiation force on it will become :
(a) double (b) half
(c) four times (d) one fourth
- Which one of the following types of radiations has the smallest wavelength?
(a) Microwaves (b) Infra-red
(c) Visible light (d) X-rays
- Which of the following statements about electromagnetic waves, sound waves and water waves is/are correct?
(1) They exhibit reflection
(2) They carry energy
(3) They exert pressure
(4) They can travel in vacuum
Select the correct answer using the code given below:
(a) 1, 2 and 3 (b) 2 and 4
(c) 1 and 3 only (d) 1 only
- Which one of the following statements is not correct?
(a) The longest wavelength of light visible to human eye is about 700 nm.
(b) The shortest wavelength of light visible to human eye is about 400 nm.
(c) The wavelength of gamma rays is longer than that of X-rays.
(d) The ability of a telescope to form separable images of close objects is called its resolving power.
- The working of a microwave oven involves
(a) absorption of microwaves by matter.
(b) reception of microwaves by optical fibre.
(c) microwave amplification by stimulated emission of radiation.
(d) transmission of microwaves through a metal.
- Radiations coming from the Sun are mostly in the form of
(a) light only.

- light and long wavelength infrared.
(c) light and short wavelength infrared.
(d) both short and long wavelength infrared.
- Which of the following are the characteristics of electromagnetic waves?
(1) They are elastic waves.
(2) They can also move in vacuum.
(3) They have electric and magnetic components which are mutually perpendicular.
(4) They move with a speed equal to 3 lakh metres per second.
Select the correct answer using the code given below:
(a) 1, 2, 3 and 4 (b) 1, 2 and 4 only
(c) 2 and 3 only (d) 3 and 4 only
- Consider the following statements about visible light, UV light and X-rays:
(1) The wavelength of visible light is more than that of X-rays.
(2) The energy of X-ray photons is higher than that of UV light photons.
(3) The energy of UV light photons is less than that of visible light photons.
Which of the statements given above is/are correct?
(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 only
- Which one of the following waves is used for detecting forgery in currency notes?
(a) Ultraviolet waves (b) Infrared waves
(c) Radio waves (d) Microwaves
- Which one among the following waves carries the maximum energy per photon?
(a) X-rays (b) Radio waves
(c) Light waves (d) Microwaves
- Which one of the following statements is **not** correct?
(a) The Kelvin scale of temperature is called the Absolute scale.
(b) Visible light radiation has wavelength range of 400-700nm.
(c) The capacity to do work is called power.
(d) The wavelength of Gamma rays is less than that of X-rays.

13. The waves used in radar system are :
(a) ultraviolet (b) infrared
(c) micro (d) radio
14. Which of the following properties of EMW does not change?
(a) frequency (b) Speed
(c) wavelength (d) All of these
15. Which of the following statement is NOT true about the properties of electromagnetic waves?
(a) These waves do not require any material medium for their propagation
(b) Both electric and magnetic field vectors attain the maxima and minima at the same time
(c) The energy in electromagnetic wave is divided equally between electric and magnetic fields
(d) Both electric and magnetic field vectors are parallel to each other.
16. If $\vec{E}$ and $\vec{B}$ represent electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along
(a) $\vec{E} \cdot \vec{B}$ (b) $\vec{B} \perp \vec{E}$
(c) $\vec{B} \times \vec{E}$ (d) $\vec{E} \times \vec{B}$
17. In electromagnetic waves, the phase difference between magnetic and electric field vectors is
(a) zero (b) π (c) $\pi/2$ (d) $\pi/4$
18. From Maxwell's hypothesis, a changing electric field gives rise to
(a) an electric field (b) an induced emf
(c) a magnetic field (d) a magnetic torque
19. The phenomenon which shows quantum nature of electromagnetic radiation is
(a) Piezoelectric effect (b) Photoelectric effect
(c) Hall effect (d) Tyndall effect
20. The electromagnetic radiations used for water purification and eye surgery is
(a) Infrared (b) Microwave
(c) X-rays (d) None of these
21. Electromagnetic wave having frequency 5×10^{11} Hz is
(a) Ultraviolet wave (b) Radio wave
(c) Microwave (d) X-rays
22. Proper arrangement of Gamma rays, Microwave, IR wave and UV rays in ascending order of frequency is
(a) Gamma rays > UV rays > IR rays > Microwave
(b) Microwave > IR rays > UV rays > Gamma rays
(c) UV rays > Gamma rays > Microwave > IR rays
(d) IR rays > UV rays > Microwave > Gamma rays
23. In vacuum, the physical property which remains same for microwave of wavelength 1 mm and UV radiation 1600 Å is
(a) Wavelength (b) Frequency
(c) Speed (d) None of these
24. In vacuum, the wavelength of the electromagnetic wave of frequency 5×10^{19} Hz is
(a) 6×10^{-12} m (b) 3×10^{-8} m
(c) 1.6×10^{11} m (d) 15×10^{27} m

Answer Key

1. (d)	2. (a)	3. (d)	4. (a)	5. (c)	6. (a)	7. (c)	8. (c)	9. (b)	10. (a)
11. (a)	12. (c)	13. (c)	14. (a)	15. (d)	16. (d)	17. (a)	18. (c)	19. (b)	20. (d)
21. (c)	22. (a)	23. (c)	24. (a)						

Answers with Explanations

1. **Option (d) is correct.**
Explanation: Wavelengths of X-ray lie in the range of 0.1 nm-10 nm.
2. **Option (a) is correct.**
Explanation:
Intensity of EMW is given by $I = U/At$
So $I \propto 1/A$
Hence, when area is halved, intensity becomes double.
3. **Option (d) is correct.**
Explanation: X rays are the electromagnetic radiations that have an extremely short wavelength of less than 100 angstroms.
4. **Option (a) is correct.**
Explanation: EMW waves doesn't need a medium to travel, they can travel through vacuum. Other two are mechanical waves, requires medium in order to propagate. Statement 4 is wrong.
5. **Option (c) is correct.**
Explanation: In EMW spectrum, gamma rays are extremely high energy waves having highest frequency and lowest wavelength. Frequencies of X-rays are less and wavelengths are more than that of gamma rays.
6. **Option (a) is correct.**
Explanation: In a microwave oven, microwaves are generated. Energy from these microwaves gets absorbed by the water molecules present inside the food which causes them to vibrate. This vibration creates heat resulting in heating of the food.
7. **Option (c) is correct.**
Explanation: Radiations coming from the Sun is in the form of em wave. Both light and short-range infrared wavelength comes under emw. Infrared waves have high frequency and short wavelength.
8. **Option (c) is correct.**
Explanation: EMW produces due to oscillating electric and magnetic field and doesn't need any medium to travel. Their speed in vacuum is 30 crore meter per second.
9. **Option (b) is correct.**
Explanation:
Energy/frequency: X-ray > UV > Visible
Wavelength: X-ray < UV < Visible

10. Option (a) is correct.

Explanation: Currency notes have a strip which shines in UV rays which is not easy to forge.

11. Option (a) is correct.

Explanation: Energy X-rays > Light > Microwaves > Radio waves

12. Option (c) is correct.

Explanation: Capacity to do work is termed as energy not power.

13. Option (c) is correct.

Explanation: Microwaves are used in radar system to detect objects.

14. Option (a) is correct.

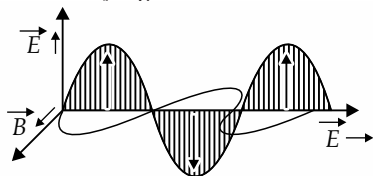
Explanation: In refraction, speed changes hence wavelength also changes. Frequency of EMW always remains same.

15. Option (d) is correct.

Explanation: For electromagnetic wave, electric and magnetic field vectors are perpendicular to each other.

16. Option (d) is correct.

Explanation: The direction of propagation of electromagnetic wave is perpendicular to $\vec{E} \times \vec{B}$ and $\vec{E} \times \vec{B}$ both and by right thumb rule.



The direction of propagation of electromagnetic wave is perpendicular to both the electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ magnetic field vector B , i.e., in the direction of $\vec{E} \times \vec{B}$.

Here, electromagnetic wave is along the z-direction which is given by the cross product of $\vec{E} \times \vec{B}$ and $\vec{E} \times \vec{B}$.

17. Option (a) is correct.

Explanation: Peaks of magnetic and electric waves of electromagnetic wave form at the same time. Hence, there is no phase difference between these two waves.

18. Option (c) is correct.

Explanation: A time varying electric field produces current which is a source of magnetic field.

19. Option (b) is correct.

Explanation: Photoelectric effect allows us to perceive the quantum nature of light and ultimately electromagnetic radiation.

20. Option (d) is correct.

Explanation: Ultraviolet rays are used for water purification and eye surgery.

21. Option (c) is correct.

Explanation: Microwave frequency ranges from 10^{13} to 10^9 Hz.

22. Option (a) is correct.

Explanation: Frequency range of Gamma rays: $10^{22} - 10^{19}$ Hz

Frequency range of UV rays: $10^{17} - 10^{15}$ Hz

Frequency range of IR rays: $10^{14} - 10^{12}$ Hz

Frequency range of Microwave: $10^{13} - 10^9$ Hz

23. Option (c) is correct.

Explanation: All types of electromagnetic waves travel with speed of light in vacuum.

24. Option (a) is correct.

Explanation: $\lambda = v/c = \frac{3 \times 10^8}{5 \times 10^{19}} = 6 \times 10^{-12}$ m

➤ Important Results and Formulae :

- Speed = Distance travelled / time taken
Velocity = displacement / Time
Acceleration = Change in velocity / time taken

2. Kinematic Equations

For accelerated motion	For decelerated motion
$v = u + at$	$v = u - at$
$s = ut + \frac{1}{2} at^2$	$s = ut - \frac{1}{2} at^2$
$v^2 = u^2 + 2as$	$v^2 = u^2 - 2as$
$s_n = u + \frac{a}{2} (2n - 1)$	$s_n = u - \frac{a}{2} (2n - 1)$

- The unit vector in the direction of $\vec{a}$ is given by $\frac{\vec{a}}{|\vec{a}|}$ and is represented by $\hat{a}$

4. Addition of Vectors :

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two vectors, then
 $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$

5. Scalar Multiplication :

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ then $\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$

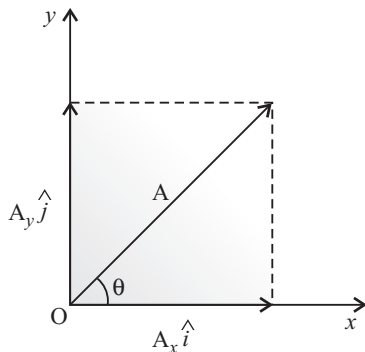
6. Vector Joining two points :

If $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ are any two points, then

$$\vec{P_1P_2} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\vec{P_1P_2}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

7. Resolution of Vectors :



$$A_x = A \cos \theta$$

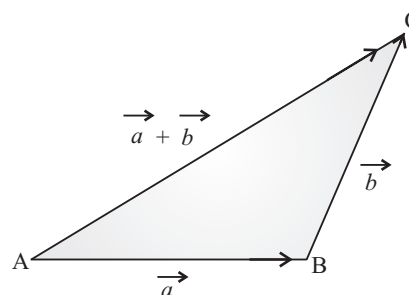
$$A_y = A \sin \theta$$

$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = \frac{A_y}{A_x}$$

8. Triangle Law of Vectors :

Let the two vectors $\vec{a}$ and $\vec{b}$ be so positioned that initial point of one coincides with terminal point of the other, $\vec{a} = \vec{AB}$, $\vec{b} = \vec{BC}$. Then, the vector is $\vec{a} + \vec{b}$ is represented by the third side of $\triangle ABC$ i.e.,



$$\vec{AB} + \vec{BC} = \vec{AC}$$

or

$$\vec{AC} - \vec{AB} = \vec{BC}$$

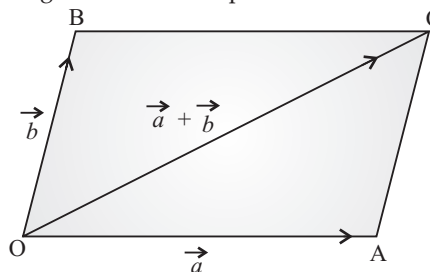
and

$$\vec{AC} - \vec{BC} = \vec{AB}$$

This is known as triangle law of vector addition.

9. Parallelogram law of vector addition :

If the two vectors $\vec{a}$ and $\vec{b}$ are represented by the two adjacent sides OA and OB of a parallelogram OACB, then their sum $\vec{a} + \vec{b}$ is represented in magnitude and direction by the diagonal OC of parallelogram through their common point O.

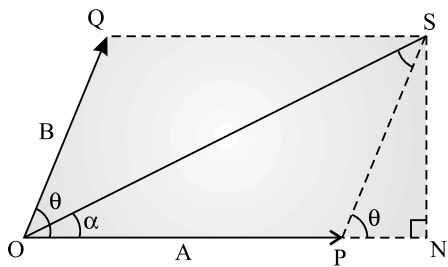


$$\text{i.e., } \vec{OA} + \vec{OB} = \vec{OC}$$

10. Magnitude and Direction of Resultant in Triangle Law and Parallelogram Law :

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$



11. Scalar or Dot Product

The scalar or dot product of two given vectors $\vec{a}$ and $\vec{b}$ having an angle θ between them is defined as

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two vectors

$$\text{Then } \vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$$

Properties of Scalar Product :

(a) $\vec{a} \cdot \vec{b}$ is a scalar quantity.

(b) When $\theta = 0^\circ$, $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos 0^\circ$
 $= |\vec{a}| |\vec{b}|$

(c) When $\theta = \frac{\pi}{2}$, $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \frac{\pi}{2} = 0$

$\Rightarrow$ When $\vec{a} \perp \vec{b}$, $\vec{a} \cdot \vec{b} = 0$

(d) When either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, $\vec{a} \cdot \vec{b} = 0$.

(e) $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

(f) $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$

12. Vector or Cross Product :

The cross product of two vectors $\vec{a}$ and $\vec{b}$ having angle θ between them is given as

$$\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \times \hat{n}$$

Where $\hat{n}$ is a unit vector perpendicular to the plane containing $\vec{a}$ and $\vec{b}$

If $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ are two vectors

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= (b_1c_2 - b_2c_1)\hat{i} - (a_1c_2 - a_2c_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$

Properties of Vector Product :

(a) $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \times \hat{n}$

(i) If $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, $\vec{a} \times \vec{b} = \vec{0}$.

(ii) $\vec{a} \parallel \vec{b}$, $\vec{a} \times \vec{b} = \vec{0}$ ($\sin \theta = 0^\circ$)

(b) $\vec{a} \times \vec{b}$ is a vector i.e., $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$$\therefore \vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$

$\Rightarrow \vec{a} \times \vec{b}$ is not commutative.

(c) When $\theta = \frac{\pi}{2}$, $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \times \hat{n}$

$$\text{or } |\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}|$$

(d) $\hat{i} \times \hat{j} = \hat{j} \times \hat{k} = \hat{k} \times \hat{i} = \hat{n}$

$$\text{and } \hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$$

$$\Rightarrow \hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$$

(e) $\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|}$

(f) If $\vec{a}$ and $\vec{b}$ represent adjacent sides of a parallelogram, then its area $= |\vec{a} \times \vec{b}|$.

(g) If $\vec{a}$, $\vec{b}$ represent the adjacent sides of a triangle, then its area $= \frac{1}{2} |\vec{a} \times \vec{b}|$.

13. Angle Between Two Vectors :

If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ are two vectors

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

14. Projectile Motion :

➤ Path followed by the projectile is parabola.

➤ Velocity of projectile at any instant t ,
 $v = [(u^2 - 2ugt \sin \theta + g^2 t^2)]^{1/2}$

➤ Horizontal range

$$R = \frac{u^2 \sin 2\theta}{g}$$

For maximum range $\theta = 45^\circ$,

$$R_{\max} = u^2/g$$

➤ Flight time, $T = 2u \sin \theta / g$

➤ Height,

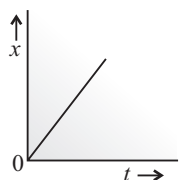
$$H = \frac{u^2 \sin^2 \theta}{2g}$$

For maximum height $\theta = 90^\circ$

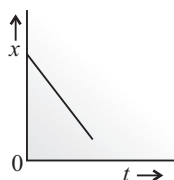
$$H_{\max} = u^2/2g$$

➤ Important Graphs :

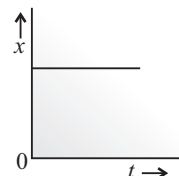
Position time Graph :



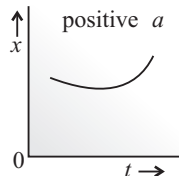
with positive velocity



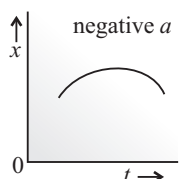
with negative velocity



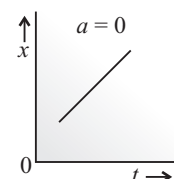
at rest



positive acceleration

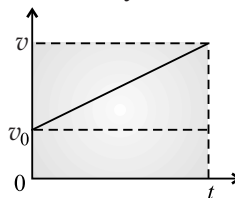
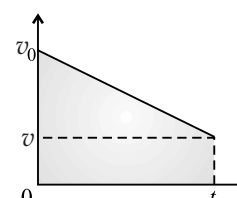
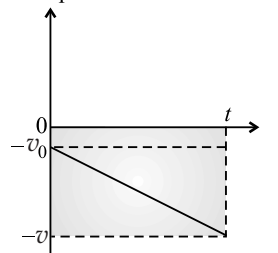
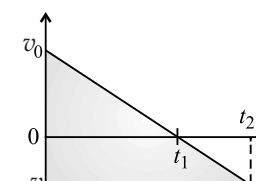


negative acceleration



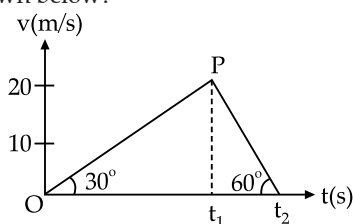
zero acceleration

Velocity time Graph

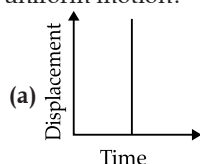
Motion in positive direction
with positive accelerationMotion in positive direction
with negative accelerationMotion in negative direction
with negative accelerationMotion of an object with
negative acceleration that
changes direction at time

EXERCISE

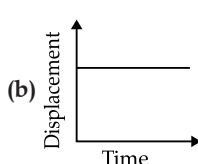
- The displacement (in meters) of a body varies with time t (in second) as $x = t^2 - 2t - 3$. The displacement is zero for a positive value of t which is equal to
(a) 1 s (b) 4 s (c) 3 s (d) 2 s
- A body starts from rest and travels with an acceleration of 5 m/s^2 . After t seconds its velocity is 10 m/s . Then t is
(a) 10 s (b) 15 s
(c) 20 s (d) None of these
- What the ratio of the acceleration during the time intervals 0 to t_1 and t_1 to t_2 in the velocity-time graph as shown below?



- (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 1 (d) $\sqrt{3}$
- Which of the following is not possible for a body in uniform motion?

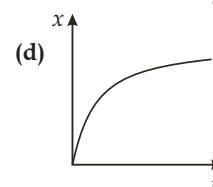
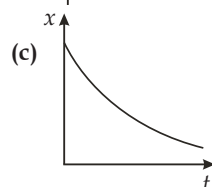
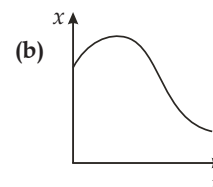
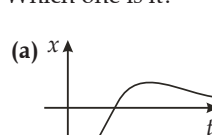


(a) Both (a) and (b)



(d) None of these

- When a body is dropped from a tower, then there is an increase in its
(a) weight
(b) acceleration
(c) velocity
(d) gravitational potential energy
- Among the four graphs shown below, there is only one graph for which average velocity over the time interval $(0, T)$ can vanish for a suitably chosen T . Which one is it?



- A lift is coming from 8th floor and is just about to reach 4th floor. Taking ground floor as origin and positive direction upwards for all quantities, which one of the following is correct?
(a) $x < 0, v < 0, a > 0$ (b) $x > 0, v < 0, a < 0$
(c) $x > 0, v < 0, a > 0$ (d) $x > 0, v > 0, a < 0$
- In one dimensional motion, instantaneous speed v satisfies $0 \leq v < v_0$

- (a) The displacement in time T must always take non-negative values.
 (b) The displacement x in time T satisfies $-v_0T < x < v_0T$.

- (c) The acceleration is always a non-negative number.
 (d) The motion has no turning points.

Answer Key

1. (c)	2. (d)	3. (b)	4. (c)	5. (c)	6. (b)	7. (a)	8. (b)		

Answers with Explanations

1. **Option (c) is correct.**

Explanation: $x = t^2 - 2t - 3$

For $x = 0$

$$0 = t^2 - 2t - 3$$

Or, $0 = (t + 1)(t - 3)$

$$\therefore t = 3, -1$$

Positive value of t is 3s.

2. **Option (d) is correct.**

Explanation: Putting $v = 10$ m/s, $u = 0$ and $a = 5$ m/s² in the equation $v = u + at$

$$10 = 0 + 5t$$

$$\therefore t = 2\text{s}.$$

3. **Option (b) is correct.**

Explanation: Acceleration during the time interval 0 to $t_1 = \text{slope of OC} = \tan 30^\circ$

Acceleration during the time interval t_1 to $t_2 = \text{slope of CB} = \tan 60^\circ$

$$\text{So, the required ratio} = \frac{\tan 30^\circ}{\tan 60^\circ} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$$

4. **Option (c) is correct.**

Explanation: In fig (a) time remaining constant displacement varies. This is not possible

In fig (b), displacement does not change with time. Hence, the body is not in motion. It is at rest.

5. **Option (c) is correct.**

Explanation: When a body is dropped it is subjected to a constant acceleration. Hence, its velocity increases.

6. **Option (b) is correct.**

Explanation: In (b) for the value of displacement, two timings are there. Therefore for one time, the average velocity is positive and for other time is equal but negative. Due to this average velocity can vanish.

$$\text{Retardation} = \frac{\text{Decrease in velocity}}{\text{Time difference}}.$$

7. **Option (a) is correct.**

Explanation: Lift is coming downwards and about to stop at 4th floor, so it's retarding. Thus, a is acting downwards, so $a > 0$ and the value of x becomes less here negative, i.e., $x < 0$, velocity is downwards (i.e., negative) so $v < 0$.

8. **Option (b) is correct.**

Explanation: Since, Maximum distance covered in time T is v_0t . That's why, for the object having one dimensional motion, the displacement x in time T satisfies $-v_0T < x < v_0T$.

UNITS AND DIMENSIONS

Physical quantity	Symbol	Dimension	Unit
Planck's constant	h	$[ML^2T^{-1}]$	Js
Stopping potential	V_0	$[ML^2T^{-3}A^{-1}]$	V
Work function	ϕ_0	$[ML^2T^{-2}]$	J; eV
Threshold frequency	ν_0	$[T^{-1}]$	Hz
de-Broglie wavelength	λ	$[L]$	m

IMPORTANT RESULTS AND FORMULAE

1. Energy of a photon:

$$E = h\nu = \frac{hc}{\lambda}$$

2. Momentum of the photon:

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

3. Work function:

$$W_0 = h\nu_0 = hc/\lambda_0.$$

4. Cut off potential:

$$eV_0 = \frac{1}{2}mv_{\max}^2$$

where, $v_{\max}$ = maximum velocity of photoelectrons

5. Einstein equation:

$$h\nu = W_0 + K_m.$$

$$K_m = h\nu - W_0 = h\nu - h\nu_0$$

6. The de-Broglie's wavelength of the particle of mass m and moving with velocity v is given by:

$$\lambda = \frac{h}{mv}$$

7. The de-Broglie wavelength of a particle of mass m and kinetic energy K is given by:

$$\lambda = \frac{h}{\sqrt{2mK}}$$

8. If a particle of mass m carrying charge q_0 is accelerated through potential V , then its de-Broglie wavelength is given by:

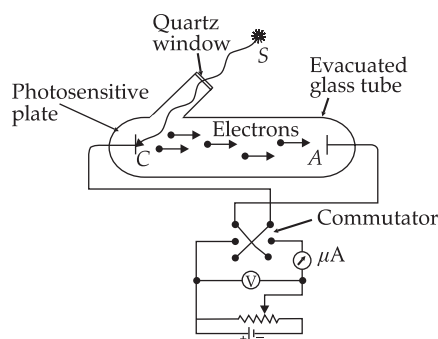
$$\lambda = \frac{h}{\sqrt{2mq_0V}}$$

9. The de-Broglie wavelength associated with the orbital electron in the n th orbit of hydrogen atom is given by:

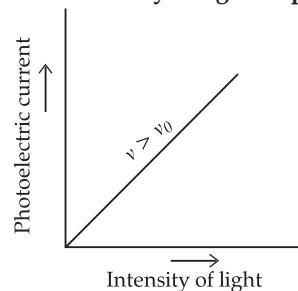
$$\lambda_n = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

IMPORTANT GRAPHS AND FIGURES

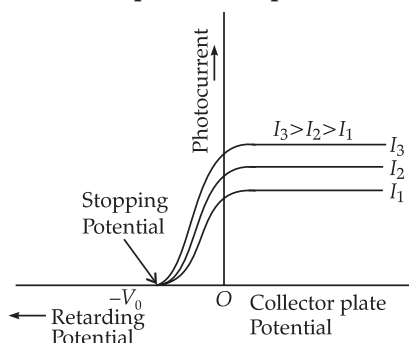
Photoelectric effect:



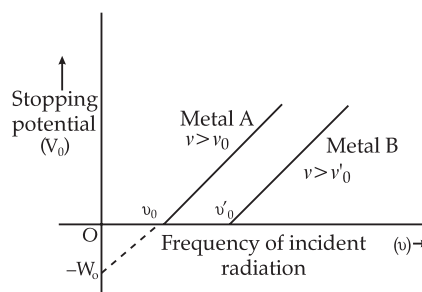
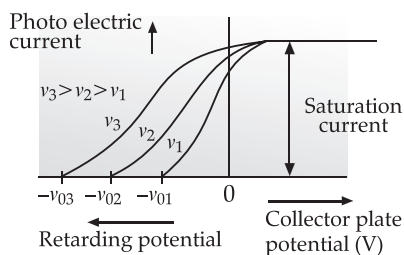
Effect of intensity of light on photocurrent:



Effect of potential on photocurrent:



Effect of frequency of incident radiation on stopping potential:



EXERCISE

- The work function for a metal surface is 4.14 eV. The threshold wavelength for this metal surface is
(a) 4125 Å (b) 2062.5 Å (c) 3000 Å (d) 6000 Å
- The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly
(a) 1.2 nm (b) 1.2×10^{-3} nm
(c) 1.2×10^{-6} nm (d) 1.2×10 nm
- Kinetic energy of electrons emitted in photoelectric effect is
(a) directly proportional to the intensity of incident light.
(b) inversely proportional to the intensity of incident light.
(c) independent of the intensity of incident light.
(d) independent of the frequency of light.
- Threshold wavelength of a photoelectric emission from a material is 600 nm. Which of the following illuminating source will emit photoelectrons?
(a) 400 W, infrared lamp
(b) 10 W, ultraviolet lamp
(c) 100 W, ultraviolet lamp
(d) Both (b) & (c)
- Photoelectrons emitted from a metal have
(a) different speeds starting from 0 to certain maximum.
(b) same kinetic energy.
(c) same frequency.
(d) Both (b) & (c)
- At stopping potential, the kinetic energy of emitted photoelectron is
(a) minimum. (b) maximum.
(c) zero. (d) cannot be predicted
- A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to
(a) H (b) $H^{1/2}$ (c) H^0 (d) $H^{-1/2}$
- A proton, a neutron, an electron and an α -particle have same energy. Then, their de-Broglie wavelengths compare as
(a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (b) $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$
(c) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (d) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$
- An electron is moving with an initial velocity $\mathbf{v} = v_0 \hat{i}$ and is in a magnetic field $\mathbf{B} = B_0 \hat{j}$. Then, its de-Broglie wavelength
(a) remains constant.
(b) increases with time.
(c) decreases with time.
(d) increases and decreases periodically.
- An electron (mass m) with an initial velocity $\mathbf{v} = v_0 \hat{i}$ ($v_0 > 0$) is in an electric field $\mathbf{E} = -E_0 \hat{i}$ ($E_0 = \text{constant} > 0$). Its de-Broglie wavelength at time t is given by
(a) $\frac{\lambda_0}{1 + \frac{eE_0 t}{m v_0}}$ (b) $\lambda_0 \left[1 + \frac{eE_0 t}{m v_0} \right]$
(c) λ_0 (d) $\lambda_0 t$
- The ratio of de-Broglie wavelength associated with two electrons accelerated through 25 V and 36 V is
(a) 25/36 (b) 36/25 (c) 5/6 (d) 6/5

Answer Key

1. (c)	2. (b)	3. (c)	4. (d)	5. (a)	6. (c)	7. ()	8. (b)	9. (a)	10. (a)
11. (d)									

Answers with Explanations

1. Option (c) is correct.

Explanation: Work function = 4.14 eV = $\frac{hc}{\lambda_0}$

$$\therefore \lambda_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.14 \times 1.6 \times 10^{-19}} \\ = 2.989 \times 10^{-7} = 3000 \text{ Å}$$

2. Option (b) is correct.

Explanation: Energy of the photon must be equal to the binding energy of proton.
So, energy of photon = 1 MeV

$$= 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-13}} = \frac{6.63 \times 3}{1.60} \times 10^{-26+13} \\ = \frac{19.89}{1.60} \times 10^{-13} = 12.4 \times 10^{-13} = 1.24 \times 10^1 \times 10^{-13} \\ = 1.24 \times 10^{-9} \times 10^{-3} = 1.24 \times 10^{-3} \text{ nm}$$

3. Option (c) is correct.

Explanation: $KE = h\nu - \phi$

So, KE is independent of intensity of incident light.

4. Option (d) is correct.

Explanation: The incident wavelength should be less than threshold wavelength for photoelectric emission. IR has a wavelength of more than 600 nm. UV has a wavelength of less than 600 nm. So, photoelectrons emitted when illuminated by UV lamp either 100 W or 10 W.

5. **Option (a) is correct.**

Explanation: When a photon strikes a metal surface, the surface electrons come out with maximum speed and maximum kinetic energy. But if the electron emission takes place from inner side of metal, then some energy of the electron is lost due to collision with other electrons and so their speed becomes less. So, ultimately the electrons come out with different speeds.

6. **Option (c) is correct.**

7. **Option (d) is correct.**

8. **Option (b) is correct.**

Explanation:

$$E_p = E_n = E_e = E_a$$

$$K.E. = K = \frac{1}{2}mv^2$$

$$2K = mv^2$$

$$2Km = m^2v^2$$

$$2mK = p^2$$

$$\sqrt{2mK} = p$$

$$\lambda = \frac{h}{p}$$

$$\lambda_d = \frac{h}{\sqrt{2mK}}$$

$$\text{or } \lambda = \frac{h}{\sqrt{2mK}}$$

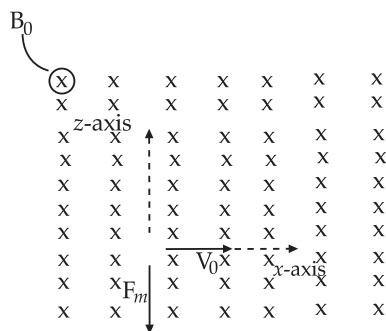
$$\therefore \lambda \propto \frac{1}{\sqrt{m}} [\text{as } h \text{ and } E \text{ (K.E.) are constant}]$$

$$m_\alpha > m_p = m_n > m_e$$

$$\therefore \lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$$

9. **Option (a) is correct.**

Explanation: Given, $v = v_0 \hat{i}$ and $B = B_0 \hat{j}$
Magnetic force on moving electron
 $= -e[v_0 \hat{i} \times B_0 \hat{j}] = -ev_0 B_0 \hat{k}$



So, the force is perpendicular to v and B both. So, the magnitude of v or mv ($p = mv$ momentum) will not change. So, the de-Broglie wavelength remains same.

10. **Option (a) is correct.**

Explanation: According to de-Broglie theory, the wavelength of de-Broglie wave is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$$

As initial velocity of the electron is $v_0 \hat{i}$, the initial de-Broglie wavelength of electron,

$$\lambda = \frac{h}{mv_0} \quad \dots(i)$$

Electrostatic force on electron in electric field is,

$$\vec{F}_e = -e\vec{E} = -e[-E_0 \hat{i}] = eE_0 \hat{i}$$

$$\text{Acceleration of electron, } \vec{a} = \frac{F}{m} = \frac{eE_0 \hat{i}}{m}$$

Velocity of the electron after time t ,

$$\vec{v} = v_0 \hat{i} + \left(\frac{eE_0 \hat{i}}{m} \right) t = \left(v_0 + \frac{eE_0}{m} t \right) \hat{i}$$

$$\Rightarrow \vec{v} = v_0 \left(1 + \frac{eE_0}{mv_0} t \right) \hat{i}$$

de-Broglie wavelength associated with electron at

$$\text{time } t \text{ is } \lambda = \frac{h}{mv}$$

$$\Rightarrow \lambda = \frac{h}{m \left[v_0 \left(1 + \frac{eE_0}{mv_0} t \right) \right]} = \frac{\frac{h}{mv_0}}{\left(1 + \frac{eE_0}{mv_0} t \right)}$$

$$\Rightarrow \lambda = \frac{\lambda_0}{\left(1 + \frac{eE_0}{mv_0} t \right)}, \text{ as } \lambda_0 = \frac{h}{mv_0}$$

11. **Option (d) is correct.**

Explanation: $\lambda \propto 1/\sqrt{V}$

$$\therefore \lambda_1/\lambda_2 = \sqrt{(V_2/V_1)} = 6/5$$

Reflection of light

- When light falls on an object, it bounces the light in the same medium from where the light comes. This is called the reflection of light.

Laws of reflection

- The angle of incidence is equal to the angle of reflection, and
- The incident ray, the normal to the surface at the point of incidence and the reflected ray, all lie in the same plane.

Plane mirror: Focal length of plane mirror is infinite.

- Image formed is virtual and erect.
- Image is laterally inverted
- Image forms behind the mirror
- Distance and size of image and object is same from the mirror.

Spherical mirror

- These are of two types:

- Concave mirror - Generally real and inverted images are formed by the concave mirrors.
- Convex mirror - forms virtual, erect and diminished images.

Refraction: Refraction is deviation of light when it obliquely travels from one medium to another medium. When light moves from denser medium to rarer medium it moves away from normal and when light moves from rarer to denser medium, it comes nearer to normal.

Laws of Refraction:

- The incident ray, the refracted ray and the normal, at the point of incidence, all lie in the same plane.
- Snell's law: For a given pair of medium, $\sin i / \sin r = \text{constant } (n_{21})$

Defect of eye

Myopia (short sightedness)	A short-sighted eye can see only nearer objects. Distant objects are not seen clearly. This defect can be removed by using a concave lens of suitable focal length.
Hypermetropia (long-sightedness)	A long-sighted eye can see distant objects clearly but nearer objects are not clearly visible. This defect can be removed by using a convex lens.
Presbyopia	In this defect both near and far objects are not clearly visible. It can be removed by using the bi-focal lens.
Astigmatism	In this defect eye cannot see horizontal and vertical lines clearly. The defect can be removed by using suitable cylindrical lenses.

IMPORTANT RESULTS AND FORMULAE

- Huygens' proposed wave theory of light: According to the theory, light travels in the form of longitudinal waves with uniform speed in a homogenous medium. Different wavelengths of light represent different colours of light.

Huygens' principle:

$$n_{21} \frac{\text{speed of light in vacuum (medium 1)}}{\text{speed of light in medium 2}} = \frac{v_1}{v_2} = \frac{c}{v}$$

$$\text{or } n_{21} = \frac{n_2}{n_1}$$

Total internal reflection: When light travels from an optically denser medium to a rarer medium at the interface, it is reflected into the same medium at a certain angle. This reflection is called total internal reflection. Critical angle is that value of incident angle for which angle of refraction is 90° .

Necessary conditions for total internal reflection:

- The light should travel from denser medium to the rarer medium.
- Angle of incidence should be larger than the critical angle.

Power of a lens (P):

The power of a lens is defined as the reciprocal of its focal length. It measures the converging or diverging power of a lens. SI unit of power of lens is diopter (D).

$$\text{It is given by, } P = \frac{1}{f}$$

Dispersion of light

- When a ray of white light passes through a prism, it splits into its constituents' color (VIBGYOR) Violet, Indigo, Blue, Green, Yellow, Orange, and Red.

Scattering of light

- Scattering of light is the phenomenon in which light rays get deviation from their straight path on striking an obstacle like dust or gas molecules, water vapors, etc.
- The colors we see in the sky are due to the scattering of light.

- Every point of a wavefront becomes secondary source of light.
- These secondary sources give their own light waves. Within small time, they produce their own wave called secondary wavelets. These secondary waves have same speed and wavelengths as waves by primary sources.

- At any instant, a common tangential surface on all these wavelets gives new wavefronts in forward direction.

2. **Coherent source:** Two sources of light are said to be coherent if the phase difference between the waves emitted by the sources remains constant with time.

3. **Young's double slit experiment:**

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

$$R = \sqrt{a^2 + b^2 + 2ab \cos \phi}$$

$$I \propto R^2$$

$$I \propto (a^2 + b^2 + 2ab \cos \phi)$$

For bright fringes:

$$\cos \phi = \max = +1$$

$$\phi = 0, 2\pi, 4\pi, \dots$$

$$\phi = 2n\pi$$

$$\text{Path difference} = \frac{xd}{D} = n\lambda$$

$$\text{where, } n = 0, 1, 2, 3, \dots$$

$$\text{Or, } x = n\lambda \frac{D}{d}$$

For dark fringes:

$$\cos \phi = \min = -1$$

$$\phi = \pi, 3\pi, 5\pi, \dots$$

$$\phi = (2n-1)\pi$$

$$\text{Path difference} = \frac{dx}{D} = (2n-1)\frac{\lambda}{2}$$

$$\text{Or, } x = (2n-1)\frac{\lambda D}{2d}$$

4. **Ratio of maximum and minimum intensities:**

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

a_1 and a_2 = amplitudes of interfering waves

5. **Fringe width in Young's double slit experiment:**

$$\beta = \frac{\lambda D}{d}$$

6. **Single slit diffraction:**

Angular width of central maximum:

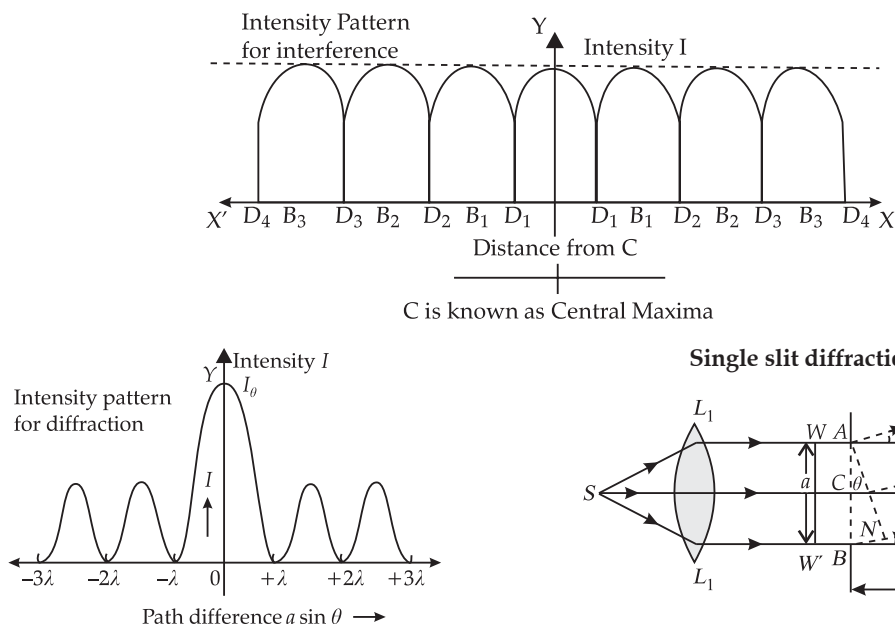
$$\theta = \frac{x}{D}$$

Width of central maximum:

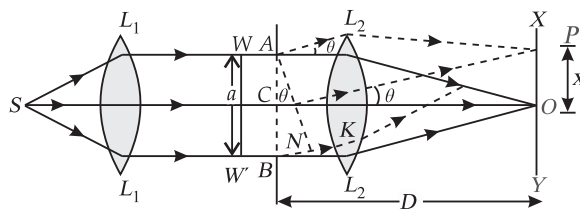
$$2x = \frac{2D\lambda}{a}$$

IMPORTANT GRAPHS AND FIGURES

Interference and diffraction:



Single slit diffraction:



EXERCISE

1. The image of an object formed by a plane mirror is :
 (a) erect, real and larger.
 (b) erect, virtual and same size.
 (c) inverted, virtual and same size.
 (d) inverted, real and smaller.
2. Light waves are incident on an air-glass boundary. Some of the light waves are reflected and some are refracted in the glass. Which one of the following

properties is the same for the incident wave and the refracted wave?

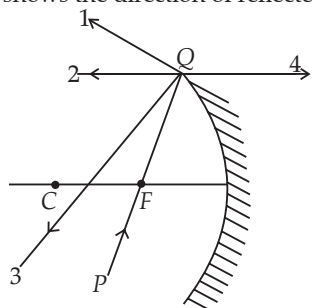
- (a) Speed
 - (b) Direction
 - (c) Wavelength
 - (d) Frequency
3. A ray of light when refracted suffers a change in velocity. In this context, which one among the following statements is correct?
 (a) Velocity increases as the ray passes from a rarer

- to a denser medium.
- (b) Velocity decreases as the ray passes from a denser to a rarer medium.
 - (c) Velocity decreases as the ray passes from a rarer to a denser medium.
 - (d) Change of velocity does not depend on the nature of medium.
4. The refractive indices of two media are denoted by n_1 and n_2 , and the velocities of light in these two media are respectively v_1 and v_2 . If $\frac{n_2}{n_1}$ is 1.5, which one of the following statements is correct?
 - (a) v_1 is 1.5 times v_2 .
 - (b) v_2 is 1.5 times v_1 .
 - (c) v_1 is equal to v_2 .
 - (d) v_1 is 3 times v_2 .
 5. If the image of an object, formed by a concave mirror is virtual, erect and magnified, then the object is placed :
 - (a) at the principal focus
 - (b) at the centre of curvature
 - (c) beyond the centre of curvature
 - (d) between the pole of the mirror and the principal focus
 6. Twinkling of stars is due to
 - (a) particular frequencies of the starlight.
 - (b) reflection of starlight from the oceanic surface.
 - (c) atmospheric refraction of starlight.
 - (d) magnetic field of Earth.
 7. When a beam of white light passes through a glass prism, the colour of light beam that deviates the least is
 - (a) Blue
 - (b) Red
 - (c) Green
 - (d) Violet
 8. Which of the following are the primary colours of light?
 - (a) Yellow, Red and Green
 - (b) Blue, Red and Green
 - (c) Violet, Red and Yellow
 - (d) Indigo, Violet and Green
 9. Two convex lenses have focal lengths of 50 cm and 25 cm, respectively. If these two lenses are placed in contact, then the net power of this combination will be equal to
 - (a) +2 dioptre
 - (b) +6 dioptre
 - (c) -6 dioptre
 - (d) +3 dioptre
 10. If the focal length of a convex lens is 50 cm, which one of the following is its power?
 - (a) +2 dioptre
 - (b) +0.25 dioptre
 - (c) -0.5 dioptre
 - (d) +0.5 dioptre
 11. Myopia is a defect in human vision where an image of a
 - (a) nearby object is focused beyond the retina.
 - (b) nearby object is focused before the retina.
 - (c) distant object is focused before the retina.
 - (d) distant object is focused beyond the retina.
 12. The part of the human eye on which the image is formed as :
 - (a) pupil
 - (b) cornea
 - (c) retina
 - (d) iris
 13. Light waves projected on oil surface show seven colours due to the phenomenon of :
 - (a) polarization
 - (b) refraction
 - (c) reflection
 - (d) interference
 14. Which one of the following is the natural phenomenon based on which a simple periscope works?
 - (a) Reflection of light
 - (b) Refraction of light
 - (c) Dispersion of light
 - (d) Total internal reflection of light
 15. What is the magnification produced by a concave lens of focal length 10 cm, when an image is formed at a distance of 5 cm from the lens?
 - (a) 2.0
 - (b) 1.0
 - (c) 0.5
 - (d) 0.33
 16. Wavefront generated from a line source is
 - (a) cylindrical wavefront
 - (b) spherical wavefront
 - (c) plane wavefront
 - (d) either (a) or (b)
 17. Phase difference between any two points of a wavefront is
 - (a) π
 - (b) $\pi/2$
 - (c) 0
 - (d) $\pi/4$
 18. In Huygens' theory, light waves
 - (a) are transverse waves and require a medium to travel.
 - (b) are longitudinal waves and require a medium to travel.
 - (c) are transverse waves and require no medium to travel.
 - (d) are longitudinal waves and require no medium to travel.
 19. Huygens' theory could not explain
 - (a) photoelectric effect.
 - (b) reflection of light.
 - (c) diffraction of light.
 - (d) interference of light.
 20. Which of the following statement is true?
 - (a) According to both Maxwell's electromagnetic theory and Huygen's wave theory light is treated as a wave in nature and require medium to travel.
 - (b) According to both Maxwell's electromagnetic theory and Huygen's wave theory light is treated as a particle in nature and require medium to travel.
 - (c) According to both Maxwell's electromagnetic theory and Huygen's wave theory light is treated as a wave in nature and does not require medium to travel.
 - (d) According to Maxwell's electromagnetic theory light is treated as a wave in nature and require no medium to travel. According to Huygens' theory light is treated as a wave in nature and require medium to travel.
 21. In a Young's double-slit experiment the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case,
 - (a) there shall be alternate interference patterns of red and blue.
 - (b) there shall be an interference pattern for red distinct from that for blue.
 - (c) there shall be no interference fringes.
 - (d) there shall be an interference pattern for red mixing with one for blue.
 22. In Young's double slit experiment, the distance between the slits is reduced to half and the distance between the slits and the screen is doubled. The fringe width
 - (a) will be double.
 - (b) will be half.
 - (c) will remain same.
 - (d) will be four times.
 23. A Young's double slit experiment is performed with blue (wavelength 460 nm) and green light (wave-

length 550 nm) respectively. If y is the distance of 4th maximum from the central fringe then

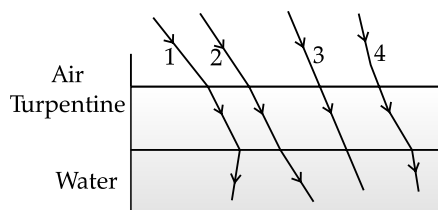
- (a) $y_B = y_G$ (b) $y_B > y_G$
(c) $y_G > y_B$ (d) $y_B/y_G = 550/460$

24. Angular width of central maxima of a single slit diffraction pattern is independent of
(a) slit width
(b) frequency of the light used
(c) wavelength of the light used
(d) distance between slit and screen
25. The main condition for diffraction to be observed is
(a) size of obstacle should be comparable to the wavelength of the wave
(b) size of obstacle should be much larger than the wavelength of the wave
(c) size of obstacle should be much smaller than the wavelength of the wave
(d) for any size of obstacle
26. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (Figure). Which of the four rays correctly shows the direction of reflected ray?



- (a) 1 (b) 2 (c) 3 (d) 4
27. When an object is placed between f and $2f$ of a concave mirror, the image formed is
(a) Real, diminished (b) Real, magnified
(c) Virtual, diminished (d) Virtual, magnified
28. In two positions concave mirror produces magnified image of given object. The positions are
(a) (i) At C, (ii) at F
(b) (i) Between F and C, (ii) Between P and F
(c) (i) Beyond C, (ii) Between P and F
(d) (i) At C, (ii) between P and F
29. The focal length of a concave mirror is f . An object is placed at a distance x from the focus. The magnification is
(a) $(f+x)/f$ (b) f/x (c) x/f (d) $f/(f+x)$
30. The optical density of turpentine is higher than that of water while its mass density is lower. Figure

shows a layer of turpentine floating over water in a container. For which one of the four rays incident on turpentine in Figure, the path shown is correct?



- (a) 1 (b) 2 (c) 3 (d) 4
31. A short pulse of white light is incident from air to a glass slab at normal incidence. After travelling through the slab, the first color to emerge is
(a) blue (b) green (c) violet (d) red
32. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will
(a) acts as a convex lens only for the objects that lies on its curved side.
(b) acts as a concave lens for the objects that lies on its curved side.
(c) acts as a convex lens irrespective of the side on which the object lies.
(d) acts as a concave lens irrespective of side on which the object lies.
33. Radius of curvature of human eye is 0.78 cm. For an object at infinity, image is formed at 3 cm behind the refracting surface. The refractive index of eye is
(a) 1.35 (b) 3 (c) 6.2 (d) 1
34. The relationship between angle of incidence i , prism of angle A and angle of minimum deviation for a triangular prism is
(a) $A + \delta_m = i$ (b) $A + \delta_m = 2i$
(c) $A + \delta_m/2 = i$ (d) $2A + \delta_m = i$
35. Magnifying power of a microscope depends on
(a) colour of light.
(b) focal length of objective and color of light.
(c) focal length of eyepiece and color of light.
(d) focal length of eyepiece and objective.
36. In a compound microscope, image produced by objective is _____ and the image produced by eyepiece is _____.
(a) Real, real (b) Virtual, virtual
(c) Real, virtual (d) Virtual, real
37. Reflecting telescope utilizes
(a) Convex mirror (b) Concave mirror
(c) Both (a) and (b) (d) Prism

Answer Key

1. (b)	2. (d)	3. (a)	4. (a)	5. (d)	6. (c)	7. (b)	8. (b)	9. (c)	10. (a)
11. (c)	12. (c)	13. (d)	14. (a)	15. (c)	16. (a)	17. (c)	18. (b)	19. (a)	20. (d)
21. (c)	22. (d)	23. (c)	24. (d)	25. (a)	26. (b)	27. (b)	28. (b)	29. (b)	30. (b)
31. (d)	32. (c)	33. (a)	34. (b)	35. (d)	36. (c)	37. (c)			

Answers with Explanations

1. Option (b) is correct.

Explanation: A plane mirror always forms a virtual, erect image of same size and at same distance but behind the mirror.

2. Option (d) is correct.

Explanation:

- Refraction is an effect that occurs when a light wave, incident at an angle away from the normal, passes a boundary from one medium into another in which there is a change in velocity of the light.
- Frequency is the property of light, and so it remains the same for incident and refracted wave.

3. Option (a) is correct.

Explanation: Since water is more optically dense than air so light would bend towards the normal.

4. Option (a) is correct.

Explanation:

$$\frac{n_2}{n_1} = \frac{v_1}{v_2} = 1.5$$

This indicates speed of light in medium 1 is 1.5 times more than its speed in medium 2.

5. Option (d) is correct.

Explanation: Image formed by a concave mirror is virtual and enlarged only when object is placed between focus point and pole of the mirror. In all other cases image formed by a concave mirror is real and inverted.

6. Option (c) is correct.

Explanation:

- The twinkling of stars is due to atmospheric refraction of star-light.
- The refraction of light caused by the earth's atmosphere having air layers of varying optical density is called atmospheric refraction.
- When a light of stars hits the earth's atmosphere from a distance then it begins to refract the light into any layer that reaches which results, we see the twinkling of stars.

7. Option (b) is correct.

Explanation:

- When white light undergoes dispersion from prism then red colour will deviate least and violet colour deviate the most.
- The light which has higher wavelength will scatter less and will have less dispersion.

8. Option (b) is correct.

Explanation:

- Primary colours are those which cannot be obtained by mixing of two or more colours.
- The concept of primary colour is, If the three colours of light can be mixed to produce white, they are called primary colours.
- Secondary colour are those which are complementary to a primary colour.

9. Option (c) is correct.

Explanation:

Given, $f_1 = -50$ cm and $f_2 = -25$ cm

Power of lens is given by,

$$P = \frac{1}{f}$$

$$P_1 = \frac{1}{f_1} = \frac{100}{-50} = -2 \text{ D}$$

$$P_2 = \frac{1}{f_2} = \frac{100}{-25} = -4 \text{ D}$$

$$P = P_1 + P_2 = -2 + (-4) = -6 \text{ D}$$

10. Option (a) is correct.

Explanation:

$$f = +\frac{50}{100} + 0.5 \text{ m}$$

$$P = \frac{1}{f} = \frac{1}{0.5} = +2 \text{ D}$$

11. Option (c) is correct.

Explanation: Myopia or near-sightedness occurs when light rays coming from a distant object, converge before the retina.

12. Option (c) is correct.

Explanation: The retina is a thin layer of tissue that lines the back of the eye on the inside. It is located near the optic nerve.

13. Option (d) is correct.

Explanation: Light waves projected on an oil surface shows 7 colors due to interference. In this phenomenon waves reflected from the top and the bottom layer of the oil surface superimpose to form a resultant wave of distributed energy.

14. Option (a) is correct.

Explanation: Periscope works on reflection of light. Multiple mirrors are placed in periscope, at an angle of 45° for straight reflection. When light falls on them it keeps on getting reflected and reaches at the other end.

15. Option (c) is correct.

Explanation: Given, $f = -10$ cm, $v = -5$ cm, $u = ?$

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\Rightarrow \frac{1}{-10} = \frac{1}{-5} - \frac{1}{u}$$

$$\Rightarrow \frac{1}{u} = \frac{1}{-5} + \frac{1}{10} = \frac{-1}{10}$$

$$\Rightarrow u = -10 \text{ cm}$$

Now, magnification $m = \frac{v}{u} = \frac{-5}{-10} = 0.5$

Alternative: $m = \frac{f - v}{f}$

$$= \frac{-10 - (-5)}{-10} = -\frac{5}{-10} = 0.5$$

16. Option (a) is correct.

17. Option (c) is correct.

Explanation: Wavefront is the locus of all points those are in same phase.

18. Option (b) is correct.

Explanation: According to Huygens, light waves are longitudinal waves and require a material medium to travel. For this reason Huygens assumed the existence of a hypothetical medium called luminiferous aether.

19. Option (a) is correct.

Explanation: Wave nature of light cannot explain the photoelectric effect. Particle nature of light can only explain it.

20. Option (d) is correct.

21. Option (c) is correct.

Explanation: For sustained interference, the source must be coherent and should emit the light of same frequency.

In this problem, one hole is covered with red and other with blue, which has different frequency, so no interference takes place.

22. Option (d) is correct.

Explanation: Fringe width $= \beta = \lambda D/d$

Initially, $\beta = \lambda D/d$

$$\text{Finally, } \beta' = \frac{\lambda \times 2D}{d/2} = 4 \times \frac{\lambda D}{d} = 4\beta$$

23. Option (c) is correct.

Explanation: $y_n = n\lambda D/d$

So, $y_n \propto \lambda$

Since, $\lambda_G > \lambda_B$

$\therefore y_G > y_B$

24. Option (d) is correct.

Explanation: Angular width $= 2\sin^{-1}\lambda/d$

So, it is independent of D (distance between slit and screen).

25. Option (a) is correct.

26. Option (b) is correct.

Explanation: Incidence ray PQ is coming through principal focus F so after reflection it must be parallel to principal axis, that is, either 2 or 4.

As it is a concave mirror so, ray cannot go behind the mirror so ray (4) is discarded.

So, ray 2 is the reflected ray that verifies answer (B).

27. Option (b) is correct.

Explanation: When an object is placed between f and 2f of a concave mirror, the image is formed beyond 2f. The image is real and magnified.

28. Option (b) is correct.

Explanation: When the object is placed between C and F in front of a concave mirror, the image is formed beyond C. Image is magnified, real and inverted.

When the object is placed between P and F in front of a concave mirror, the image is formed behind the mirror. Image is magnified, virtual and erect.

29. Option (b) is correct.

Explanation: $u = f + x$

Using mirror formula,

$$1/v + 1/u = 1/f$$

$$\text{Or, } 1/v - 1/(f+x) = -1/f$$

$$\therefore v = -f(f+x)/x$$

$$\text{So, the magnification} = |m| = v/u = f/x$$

30. Option (b) is correct.

Explanation: $\mu_A < \mu_T > \mu_W$

As incident ray passes from air to turpentine to water it means, from rarer to denser then denser to rarer. So, first it bends towards normal then away from normal so the path shown is correct for ray (2).

31. Option (d) is correct.

Explanation: As we know that the velocity of wave is :

$$v = v\lambda$$

When light ray goes from one medium to other medium, the frequency of light remains unchanged.

So, $v \propto \lambda$ i.e., greater the wavelength, greater the speed.

Light of red colour is of highest wavelength and therefore of highest speed. So, after travelling through the slab, the red colour emerges first.

32. Option (c) is correct.

Explanation: As we know the relation between f, μ , R_1 and R_2 is known as lens maker's formula :

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$R_1 = \infty, R_2 = -R$$

$$f = \frac{R}{(\mu - 1)}$$

Given that, $R = 20$ cm, $\mu = 1.5$

Put the values;

$$f = \frac{R}{\mu - 1} = \frac{20}{1.5 - 1} = 40 \text{ cm}$$

As $f > 0$, it means converging nature of the lens.

So, lens acts as a convex lens irrespective of the side on which the object lies.

33. Option (a) is correct.

$$\text{Explanation: } \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

Putting $v = 3$ cm

$$\mu_1 = 1$$

$$u = \infty$$

$$R = 0.78 \text{ cm}$$

$$\frac{\mu_2}{3} - \frac{1}{\infty} = \frac{\mu_2 - 1}{0.78}$$

$$\therefore \mu_2 = 1.35$$

34. Option (b) is correct.

Explanation: For refraction through prism,

$$i_1 + i_2 = \delta + A \text{ and } r_1 + r_2 = A$$

For minimum deviation,

$$i_1 = i_2 = i \text{ and } r_1 = r_2 = r$$

$$\text{So, } i = (A + \delta_m)/2$$

$$\therefore A + \delta_m = 2i$$

35. Option (d) is correct.

Explanation: Magnification $= m \propto 1/f_o f_e$

So, magnifying power of a microscope depends on focal length of eyepiece and objective only.

36. Option (c) is correct.

37. Option (c) is correct.

Explanation: Concave mirror is used as objective. A secondary convex mirror is used to reflect the light reflected by objective towards eyepiece.

IMPORTANT RESULTS AND FORMULAE

1. Distance of closest approach is given by,

$$r_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{K}$$

2. The orbital radius of the electron is given by,

$$r_n = 4\pi\epsilon_0 \cdot \frac{n^2 h^2}{4\pi^2 m e^2}$$

3. The orbital velocity of electron in
- n
- th orbit is given by,

$$v_n = \frac{1}{4\pi\epsilon_0} \cdot \frac{2\pi e^2}{nh}$$

4. Orbital frequency is given by,

$$f = \frac{1}{T} = \frac{v_n}{2\pi r} = \frac{m e^4}{4\epsilon_0^2 h^3 n^3}$$

5. The total energy of the orbital electron is given by,

$$E = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \left[\frac{2\pi^2 m e^4}{h^2}\right] \left[\frac{1}{n^2}\right]$$

$$K.E. = \frac{m e^4}{8n^2 h^2 \epsilon_0^2}$$

$$P.E. = -\frac{m e^4}{4n^2 h^2 \epsilon_0^2}$$

6. The kinetic, potential and total energies of the electron with
- r
- as the radius of the orbit are as follows:

$$K.E. = \frac{1}{2} \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r} \right]; P.E. = -\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r}$$

$$\text{and } E = -\frac{1}{2} \left[\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2}{r} \right]$$

Therefore they are related to each other as:

$$K.E. = -E \quad \text{and} \quad P.E. = 2E.$$

7. The energy of orbital electron in the
- n
- th orbit is given by,

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

So, energy associated with the different orbits is as follows:

$$E_1 = -\frac{13.6}{1^2} \text{ eV} = -13.6 \text{ eV}$$

$$E_2 = -\frac{13.6}{2^2} \text{ eV} = -3.4 \text{ eV}$$

$$E_3 = -\frac{13.6}{3^2} \text{ eV} = -1.51 \text{ eV}$$

$$\text{and } E_\infty = -\frac{13.6}{\infty^2} = 0$$

8. (i) For the Lyman series
- $n_i = 1$
- and
- $n_0 = 2, 3, 4, \dots$
-
- The wavelength of the radiations is given by,

$$\frac{1}{\lambda} = R_H \left[\frac{1}{1^2} - \frac{1}{n_0^2} \right]$$

They lie in the ultraviolet region.

- (ii) The Balmer series of the radiations corresponds to
- $n_i = 2$
- and
- $n_0 = 3, 4, 5, \dots$
-
- The wavelength of the radiations is given by,

$$\frac{1}{\lambda} = R_H \left[\frac{1}{2^2} - \frac{1}{n_0^2} \right]$$

They lie in the visible region.

- (iii) For Paschen series
- $n_i = 3$
- and
- $n_0 = 4, 5, 6, \dots$
-
- The wavelength of the radiations is given by,

$$\frac{1}{\lambda} = R_H \left[\frac{1}{3^2} - \frac{1}{n_0^2} \right]$$

They lie in the infrared region.

- (iv) For Brackett series
- $n_i = 4$
- and
- $n_0 = 5, 6, 7, \dots$
-
- The wavelength of the radiations is given by,

$$\frac{1}{\lambda} = R_H \left[\frac{1}{4^2} - \frac{1}{n_0^2} \right]$$

They lie in the far infrared region.

- (v) Pfund series of radiation corresponds to
- $n_i = 5$
- and
- $n_0 = 6, 7, 8, 9, \dots$
-
- The wavelength of the radiations, is given by,

$$\frac{1}{\lambda} = R_H \left[\frac{1}{5^2} - \frac{1}{n_0^2} \right]$$

They lie in infrared region.

9. Angular momentum of the electron in the
- n
- th orbit of hydrogen atom is given by,

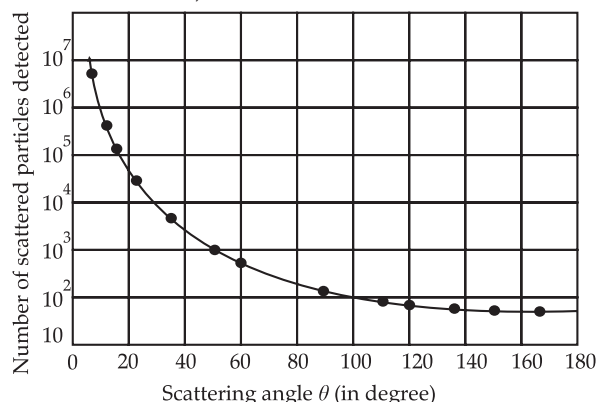
$$L = n \frac{h}{2\pi}$$

10. Impact parameter is given by,

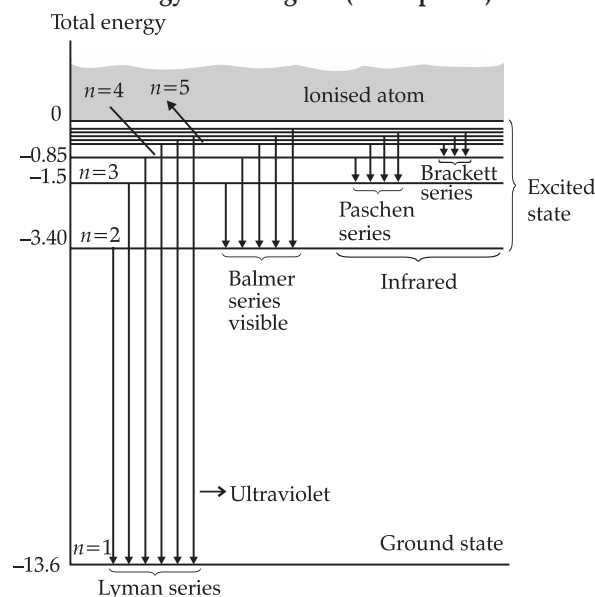
$$\frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2 \cot \theta / 2}{E}$$

IMPORTANT GRAPHS AND FIGURES

Result of Geiger and Marsden experiment (Rutherford model):



Energy level diagram (Line spectra):

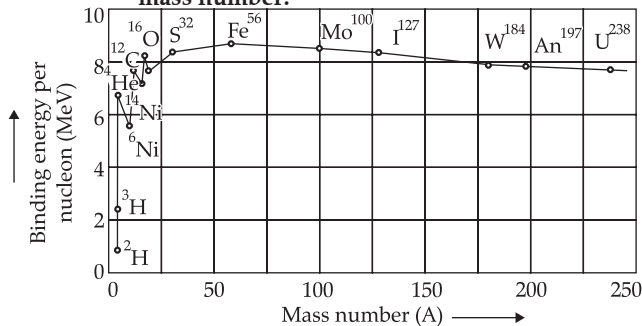


IMPORTANT RESULTS AND FORMULAE

- The volume of the nucleus is proportional to the mass number (A). Therefore, radius (R) of the nucleus is proportional to $A^{1/3}$.
i.e., $R \propto A^{1/3}$
Or, $R = R_0 A^{1/3}$
where, $R_0 \approx 1.1 \times 10^{-15} \text{ m}$.

IMPORTANT GRAPHS AND FIGURES

Binding energy per nucleon as a function of mass number:



- The difference in the sum of the masses of the neutrons and protons in the nucleus and the actual mass of the nucleus is called mass defect.

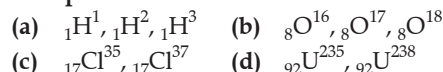
Consider nucleus ${}_Z^AX^A$. It has Z protons and $(A - Z)$ neutrons. Suppose its mass is M . Let the mass of the neutron be m_n and that of proton be m_p , then mass defect is given by,

$$\Delta m = [Zm_p + (A - Z)m_n] - M$$

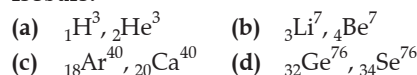
- If Δm be the mass defect of a nucleus, then its binding energy is given by Δmc^2 .

where, c = speed of light.

- Isotopes:



- Isobars:



- Isotones:



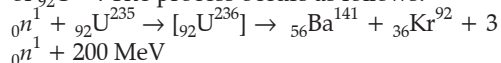
- 1 amu $\approx 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}$.

- Binding energy (B.E.) = $\Delta m \times 931.5 \text{ (MeV)}$
[where Δm = mass defect]

- Packing fraction: Mass defect per nucleon of nucleus.

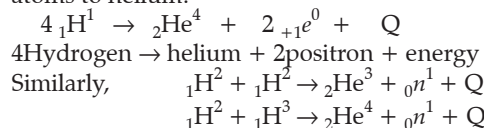
$$P.F. = \frac{\Delta m}{A}$$

- The process of splitting the heavy nucleus by bombarding it with neutrons is called **fission**. The most commonly discussed fission is that of ${}_{92}^{235}\text{U}$. The process occurs as follows:

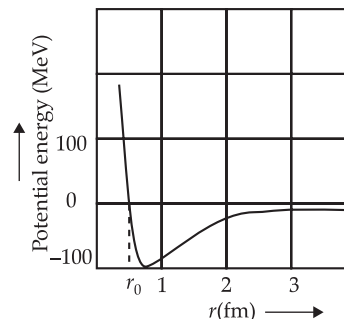


- The combination of lighter nuclei to form a heavy nucleus with the release of energy is called **fusion**.

Simplest example is the fusion of hydrogen atoms to helium:



Effect of separation on nuclear force:



EXERCISE

- O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms
 - is not important because nuclear forces are short-ranged.
 - is as important as electrostatic force for binding the two atoms.
 - cancels the repulsive electrostatic force between the nuclei.
 - is not important because oxygen nucleus have equal number of neutrons and protons.
- A set of atoms in an excited state decays
 - in general, to any of the states with lower energy.
 - into a lower state only when excited by an external electric field.
 - all together simultaneously into a lower state.
 - to emit photons only when they collide.
- Two H atoms in the ground state collide in elastically. The maximum amount by which their combined kinetic energy is reduced, is
 - 10.20 eV.
 - 20.40 eV.
 - 13.6 eV.
 - 27.2 eV.
- Taking the Bohr radius as $r_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about
 - 53 pm.
 - 27 pm.
 - 18 pm.
 - 13 pm.
- The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is

$$B = -\frac{me^4}{8n^2\epsilon_0^2h^2} \quad (m = \text{electron mass}).$$
 If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = -\frac{Me^4}{8n^2\epsilon_0^2h^2} \quad (M = \text{proton mass}).$$
 This last expression is not correct because
 - n would not be integral.
 - Bohr-quantisation applies only to electron.
 - the frame in which the electron is at rest is not inertial.
 - the motion of the proton would not be in circular orbits, even approximately.
- The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because
 - of the electrons not being subject to a central force.
 - of the electrons colliding with each other.
 - of screening effects.
 - the force between the nucleus and an electron will no longer be given by Coulomb's law.
- For the ground state, the electron in the H-atom has an angular momentum $= h$, according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,
 - because Bohr model gives incorrect values of angular momentum.
 - because only one of these would have a minimum energy.
 - angular momentum must be in the direction of spin of electron.
 - because electrons go around only in horizontal orbits.
- Which of the following statements about nuclear forces is not true?
 - The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres.
 - The nuclear force is much weaker than the Coulomb force.
 - The force is attractive for distances larger than 0.8 fm and repulsive if they are separated by distances less than 0.8 fm.
 - The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same.
- When two nuclei ($A \leq 10$) fuse together to form a heavier nucleus, the
 - binding energy per nucleon increases.
 - binding energy per nucleon decreases.
 - binding energy per nucleon does not change.
 - total binding energy decreases.
- In β^{-1} decay, a
 - neutron converts into a proton emitting antineutrino.
 - neutron converts into a proton emitting neutrino.
 - proton converts into a neutron emitting antineutrino.
 - proton converts into a neutron emitting neutrino.
- Heavy stable nucleus have more neutrons than protons. This is because of the fact that
 - neutrons are heavier than protons.
 - electrostatic force between protons is repulsive.
 - neutrons decay into protons through beta decay.
 - nuclear force between neutrons is weaker than that between protons.
- Tritium is an isotope of hydrogen whose nucleus Triton contains 2 neutrons and 1 proton. Free neutrons decay into $\bar{p} + e + \bar{\nu}$. If one of the neutrons in Tritium decays, it would transform into ${}_2He^3$ nucleus. This does not happen. This is because
 - tritium energy is less than that of a He^3 nucleus.
 - the electron created in the beta decay process cannot remain in the nucleus.
 - both the neutrons in triton have to decay simultaneously resulting in a nucleus with 3 protons, which is not a He^3 nucleus.
 - because free neutrons decay due to external perturbations which is absent in a triton nucleus.
- Nuclear force is a _____ and _____ force.
 - Strong, long-range
 - Strong, short-range
 - Weak, long-range
 - Weak, short-range

14. Two nuclei have mass number in the ratio 1 : 2. The ratio of their nuclear densities is
 (a) 1 : 2
 (b) 2 : 1
 (c) 1 : 1
 (d) Cannot be defined from mass number ratio
15. The mass of a nucleus in its ground state is
 (a) less than the total mass of neutrons and protons.
 (b) greater than the total mass of neutrons and protons.
 (c) equal to the total mass of neutrons and protons.
- (d) equal to the total mass of neutron, protons and electrons.
16. _____ has the mass closest to the mass of positron.
 (a) Proton (b) Neutron
 (c) Electron (d) Neutrino
17. X amount of energy is required to remove an electron from its orbit and Y amount of energy is required to remove a nucleon from the nucleus.
 (a) $X = Y$ (b) $X > Y$
 (c) $Y > X$ (d) $X \geq Y$

Answer Key

1. (a)	2. (a)	3. (a)	4. (c)	5. (c)	6. (a)	7. (a)	8. (b)	9. (a)	10. (a)
11. (b)	12. (a)	13. (b)	14. (c)	15. (a)	16. (c)	17. (c)			

Answers with Explanations

1. **Option (a) is correct.**

Explanation: The nuclear forces are too much stronger. Only attractive force as compared to electrostatic repulsive force and nuclear force decreases to zero on increasing distance.

So, in case of oxygen molecule, the distance between atoms of oxygen is larger as compared to the distances between nucleons in a nucleus. So that, the force between the nuclei of two oxygen atoms is not important as nuclear forces are short-ranged forces.

2. **Option (a) is correct.**

Explanation: A set of atoms in an excited state decays in general to any of the states with lower energy since lower energy state is more stable.

3. **Option (a) is correct.**

Explanation: Total energy of two H-atom in ground state = $2(-13.6) = -27.2$ eV.

The maximum amount by which their combined kinetic energy is reduced when any one H-atom goes into first excited state after the inelastic collision, that is, the total energy of two H-atom after inelastic collision :

$$\begin{aligned}
 E &= \frac{13.6}{n^2} + 13.6 \\
 &= \frac{13.6}{2^2} + 13.6 \quad [\text{For excited state } (n = 2)] \\
 &= 3.4 + 13.6 = 17.0 \text{ eV}
 \end{aligned}$$

So that the loss in kinetic energy due to inelastic collision will be,

$$= 27.2 - 17.0 = 10.2 \text{ eV}$$

4. **Option (c) is correct.**

Explanation: According to **Bohr's model of an atom**, radius of an atom in its ground state is

$$r = \frac{r_0}{Z}$$

where, r_0 is Bohr's radius and Z is atomic number.

As given that,

$r_0 = 53$ pm and atomic number of Lithium atom is 3

$$\text{so, } r = \frac{53}{3} = 17.67 \text{ pm} \approx 18 \text{ pm}$$

5. **Option (c) is correct.**

Explanation: In a hydrogen atom, electrons revolving around a fixed proton nucleus have some centripetal acceleration. So that, its frame of reference is non-inertial. In the frame of reference, where the electron is at rest, the given expression is not true as it forms the non-inertial frame of reference.

As the mass of an electron is negligible as compared to proton, so the centripetal force cannot provide the electrostatic force,

$$F_p = \frac{m_p v^2}{r}$$

So, the given expression is not true, as it forms non-inertial frame of reference due to $m_e \ll m_p$ or centripetal force on $F_e \ll F_p$.

6. **Option (a) is correct.**

Explanation: The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons because when we derive the formula for radius/energy levels, etc., we make the assumption that centripetal force is provided only by electrostatic force of attraction by the nucleus.

So that, this will only work for single electron atoms. In multi-electron atoms, there will also be repulsion due to other electrons. For this reason the simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons.

7. **Option (a) is correct.**

Explanation: According to Bohr's second postulate of atomic model, angular momentum of revolving

electron must be some integral multiple of $\frac{h}{2\pi}$. So,

the Bohr's model of atom does not give correct value of angular momentum.

8. **Option (b) is correct.**

Explanation: The nuclear force is much stronger than the Coulomb force acting between charges.

9. **Option (a) is correct.**

10. **Option (a) is correct.**

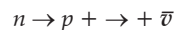
11. **Option (b) is correct.**

Explanation: Electrostatic force between proton-proton is repulsive which causes the instability of nucleus. So, number of neutrons is more than the number of protons.

12. **Option (a) is correct.**

Explanation: Tritium (${}^3_1\text{H}$) has 1 proton and 2 neutrons.

If a neutron decays as,



then nucleus will have 2 protons and 1 neutron, i.e., triton atom converts in ${}^3_2\text{He}$ (2 protons and 1 neutron). Binding energy of ${}^3_1\text{H}$ is much smaller than ${}^3_2\text{He}$, so, transformation is not possible energetically.

13. **Option (b) is correct.**

Explanation: Nuclear force is the strongest short-range force which binds the neutrons and protons in a nucleus.

14. **Option (c) is correct.**

Explanation: Nuclear density does not depend upon the mass number. Hence, density ratio will be 1 : 1.

15. **Option (a) is correct.**

Explanation: Since, the mass depends upon energy. Also, ground state energy is lesser than first excited energy therefore the mass of nucleus would be less than the mass of neutrons and protons.

16. **Option (c) is correct.**

Explanation: Anti-particle will have same characteristic because of its conjugate particle due to which they have closest mass.

17. **Option (c) is correct.**

Explanation: Nuclear force is greater than Coulomb force.

○○○

Chapter 48

METALS AND NONMETALS

Types of Alloy, Composition of element in Nature & their isolation

Alloys: These are homogeneous mixture of metals with metals and non—metals. For example,

- **Stainless steel:** Alloy of iron, nickel, chromium
- **Brass:** Alloy of copper and zinc
- **Bronze:** Alloy of copper and tin
- **Solder:** Alloy of lead and tin

Amalgam: If one of the metals is mercury, then the alloy is known as amalgam, Eg, Sodium amalgam and silver amalgam.

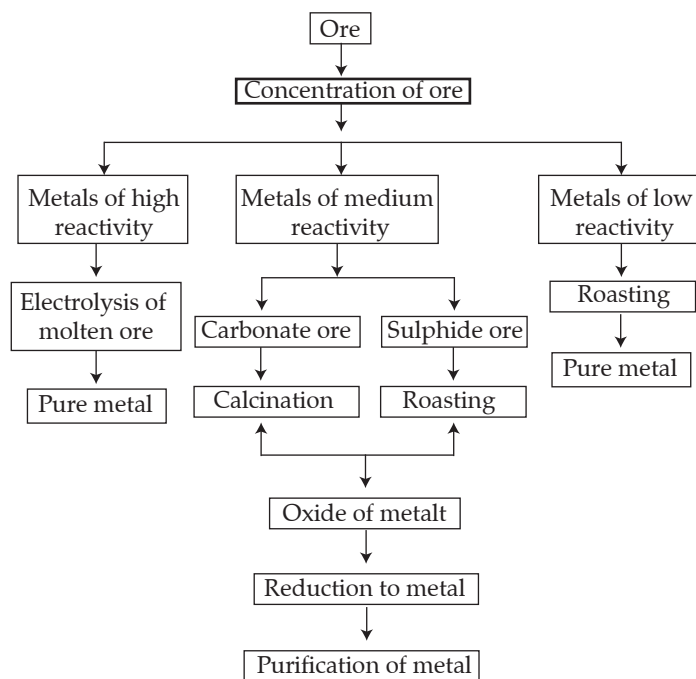
Occurrence of Metals

• **Minerals:** The elements or compounds which occur naturally in the earth's crust are called minerals.

• **Ores:** Minerals that contain very high percentage of particular metal and the metal can be profitably extracted from it, such minerals are called ores.

Extraction of metals: It is the process of obtaining pure metal from its ore. Extraction of metal can be classified into three steps :

1. Enrichment of ores or concentration of ores.
2. Extraction of metal from the concentrated ores.
3. Refining of metal.



Reactivity series of an element, Uses of Metal & Non metals

Reactivity or activity series of metals: All the metals do not react with the same rate. Some react very fast, some react moderately whereas others react very slowly. The series of metals in decreasing order of reactivity is called reactivity or activity series of metals. The metals at the top (K at the top most) are most reactive where as metals at the bottom (Pt at the extreme bottom) are least reactive.

Reactivity Series of Metals

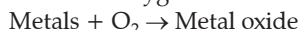
These metals are more reactive than hydrogen	Potassium	K	Most reactive metal
	Sodium	Na	
	Barium	Ba	
	Calcium	Ca	Decreasing chemical reactivity
	Magnesium	Mg	
	Aluminium	Al	
	Zinc	Zn	
	Iron	Fe	
	Nickel	Ni	
	Tin	Sn	
	Lead	Pb	
These metals are less reactive than hydrogen	Hydrogen	(H)	
	Copper	Cu	Least reactive metal
	Mercury	Hg	
	Silver	Ag	
	Gold	Au	
	Platinum	Pt	

Physical and chemical Properties of metal and non metal

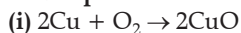
1.	Lustre	Metals have shining surface.	They do not have shining surface. • Except iodine
2.	Hardness	They are generally hard. • Except sodium, lithium and potassium which are soft and can be easily cut with knife.	Generally soft. • Except diamond, a form of carbon which is the hardest natural substance.
3.	State	Exist as solids. • Except Mercury that exists as liquid.	Exist as solids or gases • Except Bromine that exists as liquid.
4.	Malleability	Metals can be beaten into thin sheets. • Cold, Silver and Aluminium are the most malleable metals.	Non—metals are non—malleable. • They are brittle.
5.	Ductility	Metals can be drawn into thin wires.	They are non—ductile.
6.	Conductor of heat & electricity	Metals are good conductors of heat and electricity,	Non—metals are poor conductors of heat and electricity. • Except Graphite.
7.	Density and Melting point	Generally metals have high density and high melting point. • Except Sodium and Potassium	Non metals have low density and low melting point.
8.	Sonorous	Metals produce a sound on striking a hard	They are not sonorous.
9.	Oxides	Metallic oxides are basic in nature.	

- Chemical Properties of Metals :**

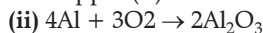
(A) Reaction of Metals with Air: Metals combine with oxygen to form metal oxide.



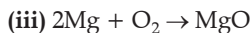
Examples:



Copper (II) oxide



Aluminium oxide



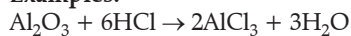
Magnesium oxide

Different elements show different reactivity towards O_2 .

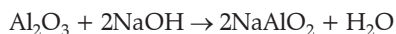
- Na and K react so vigorously with oxygen that they catch fire if kept in open. So, they are kept immersed in kerosene.
- Surfaces of Mg, Al, Zn and Pb are covered with a thin layer of oxide which prevents them from further oxidation.
- Fe does not burn on heating but iron filings burn vigorously.
- Cu does not burn but is coated with black copper (II) oxide.
- Au and Ag do not react with oxygen.

Amphoteric Oxide: Metal oxides which react with both acids as well as bases to produce salt and water are called amphoteric oxides.

Examples:

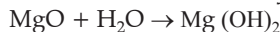


Aluminum chloride



Sodium aluminate

Reaction of metals with water: Metals react with water to produce metal hydroxide and hydrogen gas.



Magnesium hydroxide

- Sodium and Potassium react vigorously with water.
 - Magnesium metal reacts with hot water to produce magnesium hydroxide and hydrogen gas.
 $\text{Mg} + 2\text{H}_2\text{O} \rightarrow \text{Mg}(\text{OH})_2 + \text{H}_2$
 - Aluminium and zinc react with steam to produce metal oxide and hydrogen gas.
- Metals like silver, gold, copper and lead do not react with water.

(A) Reactions of Metals with Acid:

- Metal + Dil. Acid $\rightarrow$ Salt + Hydrogen gas
e.g., $\text{Mg} + \text{H}_2\text{SO}_4 \rightarrow \text{MgSO}_4 + \text{H}_2$
- Copper, mercury and silver don't react with dilute acids.

(B) Reaction of Metal with solutions of other Metal Salts: Metal A + Salt solution B $\rightarrow$ r Salt solution A + Metal B

- Reactive metals can displace less reactive metals from their compounds in solution form.
 $\text{Fe} + \text{CuSO}_4 \rightarrow \text{FeSO}_4 + \text{Cu}$

EXERCISE

1. Bronze is an alloy of :

- (a) Cu and Zn (b) Cu and Sn
(c) Zn and Mg (d) Fe and Cu

2. In electrolytic refining of copper, the electrolyte is a solution of :

- (a) acidified copper chloride.

- (b) acidified copper sulphate.
 (c) potassium chloride
 (d) sodium sulphate.
3. Solder is an alloy of :
 (a) Cu and Sn. (b) Fe and Zn.
 (c) Pb and Sn. (d) Ag and Zn.
4. Which one of the following elements forms highest number of compounds?
 (a) Oxygen (b) Chlorine
 (c) Hydrogen (d) Carbon
5. Which one of the following is the colour of hydrogen gas?
 (a) Light yellow (b) Orange
 (c) Black (d) Colourless
6. To help deep-sea divers breathe, they carry cylinders of oxygen mixed with :
 (a) chlorine (b) helium
 (c) nitrogen (d) ozone
7. Which one of the following metals is alloyed with sodium to transfer heat in a nuclear reactor?
 (a) Potassium (b) Calcium
 (c) Magnesium (d) Strontium
8. Which one of the following metals is used in the filaments of photo-electric cells that convert light energy into electric energy?
 (a) Tungsten (b) Copper
 (c) Rubidium (d) Aluminium
9. Which one of the following elements' isotopes is used in the treatment of cancer?
 (a) Iodine (b) Sodium
 (c) Cobalt (d) Uranium
10. In an incandescent electric bulb, the filament of the bulb is made up of which metal?
 (a) Aluminium (b) Copper
 (c) Tungsten (d) Silver
11. Which one of the following is a non-metallic mineral?
 (a) Iron (b) Mica
 (c) Copper (d) Bauxite
12. White phosphorus glows in the dark due to:
 (a) amorphous character
 (b) slow oxidation
 (c) high ignition temperature
 (d) good conducting property of electricity
13. Match List I with List II and select the correct answer using the code given below the Lists:

List I (Noble gas)		List II (Use)	
(A)	Argon	(1)	In lights for advertising display
(B)	Neon	(2)	Airport landing lights and in light houses
(C)	krypton	(3)	Light in photographer's flash gun
(D)	(D) Xenon	(4)	In tungsten filament to last long

Code:

- A B C D
 (a) 3 1 2 4

- (b) 3 2 1 4
 (c) 4 2 1 3
 (d) 4 1 2 3

14. Match the list -I with list-II and select the answer using the correct code given below

List I Element		List II Use	
(a)	Li	(1)	Time keeper in atomic clock
(b)	Na	(2)	batteries
(c)	K	(3)	Transfer of nerve impulses
(d)	Cs in the blood	(4)	Control of the water content in the blood

Code:

- A B C D
 (a) 2 3 4 1
 (b) 1 2 3 4
 (c) 2 4 3 1
 (d) 1 3 2 4

15. Which of the following is/are amphoteric?
 (a) $\text{Al}(\text{OH})_3(\text{s})$ and $\text{Fe}(\text{OH})_3(\text{s})$
 (b) $\text{Al}(\text{OH})_3(\text{s})$ and HCO_3
 (c) $\text{Ba}(\text{OH})_2(\text{s})$ and $\text{NaOH}(\text{aq})$
 (d) $\text{Al}(\text{OH})_3(\text{s})$ only
16. Which one of the following is the correct reactivity series with water?
 (a) Zinc > Iron > Lead > Copper
 (b) Copper > Lead > Zinc > Iron
 (c) Copper > Zinc > Iron > Lead
 (d) Zinc > Copper > Iron > Lead
17. Which one of the following metals does NOT react with cold water?
 (a) Calcium (Ca) (b) Potassium (K)
 (c) Magnesium (Mg) (d) Sodium (Na)
18. Which one of the following will NOT produce carbon dioxide on reacting with an aqueous solution of hydrochloric acid?
 (a) Limestone (b) Quick Lime
 (c) Chalk (d) Marble
19. Which one of the following represents the correct order of electron releasing tendency of metals?
 (a) $\text{Zn} > \text{Cu} > \text{Ag}$ (b) $\text{Ag} > \text{Cu} > \text{Zn}$
 (c) $\text{Cu} > \text{Zn} > \text{Ag}$ (d) $\text{Cu} > \text{Ag} > \text{Zn}$
20. Which one of the following elements is least reactive with water?
 (a) Lithium (b) Sodium
 (c) Potassium (d) Cesium
21. Which one of the following metals floats in cold water?
 (a) Magnesium (b) Calcium
 (c) Potassium (d) Copper
22. Which one of the following element's isotope is used in the treatment of cancer?
 (a) Uranium (b) Cobalt
 (c) Sodium (d) Iodine
23. Which one of the following alkali metals has lowest melting point?
 (a) Sodium (b) Potassium
 (c) Rubidium (d) Caesium

24. Which one of the following is called dry ice?
 (a) Solid carbon dioxide
 (b) Liquid carbon dioxide
 (c) Liquid nitrogen
 (d) Liquid ammonia
25. Silver artefacts get tarnished in air due to the formation of :
 (a) silver chloride (b) silver oxide
 (c) silver sulphide (d) silver sulphate
26. To protect steel and iron from rusting, a thin layer of which one of the following metals is applied?
 (a) Magnesium (b) Zinc
 (c) Aluminium (d) Lead
27. On exposure to moist air, copper gains a green coat on its surface due to formation of which one of the following compounds?
 (a) Copper carbonate (b) Copper oxide
 (c) Copper sulphate (d) Copper nitrate
28. Which one of the following elements corrodes rapidly?
 (a) Aluminium (b) Iron
 (c) Zinc (d) Silver
29. Zinc is used to protect iron from corrosion because zinc is :
 (a) more electropositive than iron
 (b) cheaper than iron
 (c) a bluish white metal
 (d) a good conductor of heat and electricity

Answer Key

1. (a)	2. (b)	3. (b)	4. (d)	5. (d)	6. (b)	7. (b)	8. (c)	9. (c)	10. (c)
11. (a)	12. (b)	13. (b)	14. (c)	15. (b)	16. (a)	17. (c)	18. (b)	19. (a)	20. (a)
21. (b)	22. (b)	23. (d)	24. (a)	25. (c)	26. (c)	27. (a)	28. (d)	29. (a)	

Answers with Explanations

1. **Option (a) is correct.**
Explanation: Bronze, alloy traditionally composed of copper and tin. Bronze was one of the earliest metals to be discovered, around 3500 BC (the Bronze Age), and before chemical parameters had been put in place for the alloying of this metal.
2. **Option (b) is correct.**
Explanation: The process of refining a metal especially Copper through Electrolytic refining is called electrolysis. So, by an electrolytic process, the purest copper is obtained, undertaken using a slab of impure copper as the anode and a thin sheet of pure copper as the cathode. The electrolyte is an acidic solution of copper sulphate.
3. **Option (b) is correct.**
Explanation: Pb and Sn are mainly used to form solder which is an alloy and it melts at lower temperature.
4. **Option (d) is correct.**
Explanation: Due to the tetravalency and high bond dissociation energy of carbon atom. Large number of carbon compounds are found in the nature.
5. **Option (d) is correct.**
Explanation: Hydrogen is colourless gas.
6. **Option (b) is correct.**
Explanation: Helium is used as a diluent for oxygen in modern diving apparatus because of its very low solubility in blood. The main reason for adding helium to the breathing mixture is to reduce the proportions of nitrogen and oxygen below those of air, to allow the gas mixture to be breathed safely on deep dives.
7. **Option (a) is correct.**
Explanation:
 Potassium reacts vigorously with water to liberate H_2
 $2K + 2H_2O \rightarrow 2KOH + H_2$
 It forms an alloy with Na to limit the reaction of potassium with water.
8. **Option (c) is correct.**
Explanation: Photoelectric cell is a device using a photoelectric effect to generate current using light is called as a photoelectric cell. Photo emission takes place and the current flows in the circuit. Rubidium is used in the filament of photoelectric cell that convert light energy into electric energy. The basic reasons behind this are high ohmic resistance and less current leak. Rubidium is silver - white, chemical element. It is used in photoelectric shells because it is highly electropositive element. When rays of light strike its surface it ejects number of electrons. Rubidium is also used in the filament of photoelectric shells that convert light energy into electric energy.
9. **Option (c) is correct.**
Explanation: Cobalt — 60 is mainly used in the treatment of cancer.
10. **Option (c) is correct.**
Explanation: Tungsten is a rare metal found naturally on Earth almost exclusively as compounds with other elements. It was identified as a new element in 1781 and first isolated as a metal in 1783.
11. **Option (a) is correct.**
Explanation:
 - Non-metallic materials are those which lack of metallic characteristics like good electric and thermal conductivity, luster, rigidity, and malleability.
 - If non-metallic minerals are melted, then no new product can be generated because they are a special group of chemical elements.
 - For example:** Sand, gravel, limestone, clay, and marble.
12. **Option (b) is correct.**
Explanation: Phosphorus glows in dark due to its

slow oxidation. The process is known as Phosphorescence.

13. **Option (d) is correct.**

14. **Option (c) is correct.**

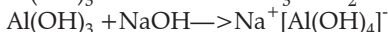
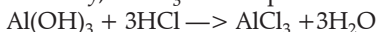
Explanation:

- Lithium (Li) - Batteries
- Sodium (Na) - Control of the water content in the blood
- Potassium (K) - Transfer of nerve impulses
- Cesium (Cs) - Time keeper in atomic clocks

15. **Option (b) is correct.**

Explanation: $\text{Al}(\text{OH})_3$ is amphoteric as it can react with both acid as well as base.

Similarly, HCO_3^- also amphoteric in nature.



Sodium tetrahydroxoaluminate (III)

16. **Option (a) is correct.**

Explanation: Order of reactivity of metal: $\text{Na} > \text{Ca} > \text{Mg} > \text{Al} > \text{Zn} > \text{Fe} > \text{Sn} > \text{Pb} > \text{Cu} > \text{Ag}$. The reactivity of elements (metals) towards water decreases towards the right in a period. It also increases down the group. But zinc is more reactive towards water than iron.

17. **Option (c) is correct.**

Explanation: Mg being less reactive, does not react with cold water. It reacts with hot water to form metal hydroxide and liberate H_2 . $\text{Mg} + 2\text{H}_2\text{O} \rightarrow \text{Mg}(\text{OH})_2 + \text{H}_2$

18. **Option (b) is correct.**

Explanation:

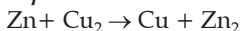
- In the given options marble, limestone is all either metal carbonates or bicarbonates.
- All of these when react with a dilute acid then they produce carbon dioxide.
- Whereas lime which is calcium oxide (CaO) when reacts with a dilute acid then it does not produce carbon dioxide.

For example: $\text{CaO} + 2\text{HCl} \rightarrow \text{CaCl}_2 + \text{H}_2\text{O}$

- Lime or calcium oxide (CaO) on reaction with a dilute acid such as hydrochloric acid (HCl) gives calcium chloride and water but not carbon dioxide.
- Thus, lime when reacts with an acid then it produces salt and water but it does not produce carbon dioxide

19. **Option (a) is correct.**

Explanation: Zinc releases electrons to copper:



Copper releases electrons to silver: $\text{Cu(s)} + 2\text{Ag}_2 + (\text{aq.}) \rightarrow \text{Cu(s)} + 2\text{Ag(s)}$

20. **Option (a) is correct.**

Explanation: Among alkali metals Lithium is the least reactive element. It reacts slowly with water due to its high melting point, while remaining element reacts violently with water and can cause explosion.

21. **Option (b) is correct.**

Explanation: Metals float on water because of two reasons: they are very lightweight metals and they react very because the bubbles of hydrogen gas formed stick to the surface of the metal.

22. **Option (b) is correct.**

Explanation: Cobalt-60 has been used in the treatment of cancer for over 60 years within an instrument known as the aggressively with water to form hydrogen gas which helps them to float in water. Metals react with water to give hydroxides and hydrogen gas. Calcium reacts less vigorously with water to form hydrogen gas. The metal starts floating

23. **Option (d) is correct.**

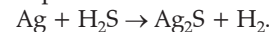
Explanation: Among alkali metal, the lowest melting point is of Caesium. In a periodic table, down the group of melting point of alkali metal decreases. $\text{Li} > \text{Na} > \text{K} > \text{Rb} > \text{Cs}$ Decreasing order of melting point

24. **Option (a) is correct.**

Explanation: Dry ice is also known solid CO_2 .

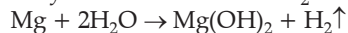
25. **Option (c) is correct.**

Explanation: The phenomenon is known as corrosion. It is called silver tarnishing, particularly for silver. The black substance obtained is known as silver sulphide.



26. **Option (c) is correct.**

Explanation: Mg being less reactive, does not react with cold water. It reacts with hot water to form metal hydroxide and liberate H_2 .



27. **Option (a) is correct.**

Explanation:

- Copper metal when exposed to air turns green in colour due to corrosion.

28. **Option (d) is correct.**

Explanation:

- Sugarcane is used for the production of sugar.
- Uttar Pradesh has the highest sugarcane production in India.

29. **Option (a) is correct.**

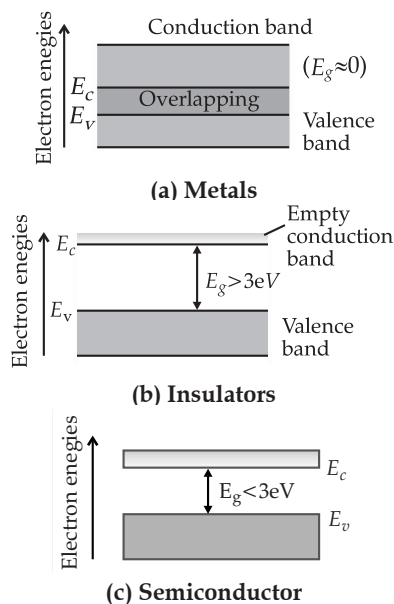
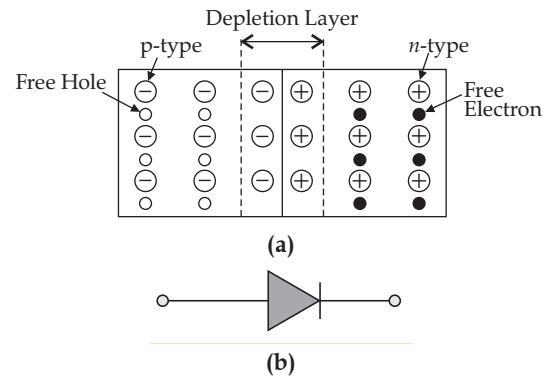
Explanation: In reactivity series, Zinc is placed above the iron, because Zinc is more electropositive than iron and it protect iron from corrosion.

IMPORTANT RESULTS AND FORMULAE

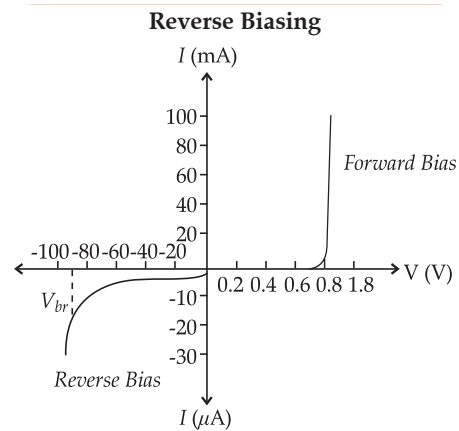
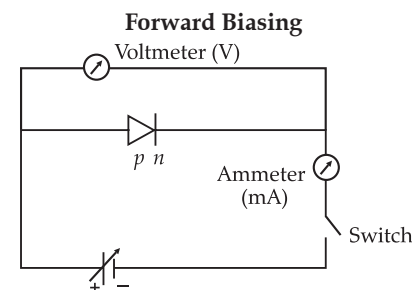
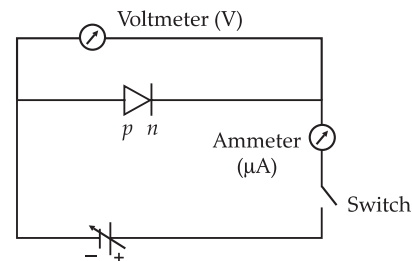
- (i) **Metals:** They possess very low resistivity (or high conductivity).
 $\rho \sim 10^{-2} - 10^{-8} \Omega - \text{m}$
 $\sigma \sim 10^2 - 10^8 \Omega - \text{m}^{-1}$
- (ii) **Semiconductors:** They have resistivity or conductivity intermediate to metals and insulators.
 $\rho \sim 10^{-5} - 10^6 \Omega - \text{m}$
 $\sigma \sim 10^5 - 10^{-6} \Omega - \text{m}^{-1}$
- (iii) **Insulators:** They have high resistivity (or low conductivity).
 $\rho \sim 10^{11} - 10^{19} \Omega - \text{m}$
 $\sigma \sim 10^{-11} - 10^{-19} \Omega - \text{m}^{-1}$
- The majority (charge) carriers in the n -type semiconductor are electrons and that in p -type semiconductor are holes.
- For intrinsic semiconductor: $n_e = n_h = n_i$
For doped semiconductor: $n_e \times n_h = n_i^2$
For n -type semiconductor: $n_e \gg n_h$
For p -type semiconductor: $n_e \ll n_h$
- Group V element is doped as impurity with Si or Ge to form n -type semiconductor. Electrons are majority charge carriers here.
Group III element is doped as impurity with Si or Ge to form p -type semiconductor. Holes are majority charge carriers here.
- To form a p - n junction acceptor impurity is doped from one side of a pure semiconductor and donor impurity is doped from other side.
- At p - n junction, electrons and holes diffuse and ultimately form a depletion layer.

IMPORTANT GRAPHS AND FIGURES

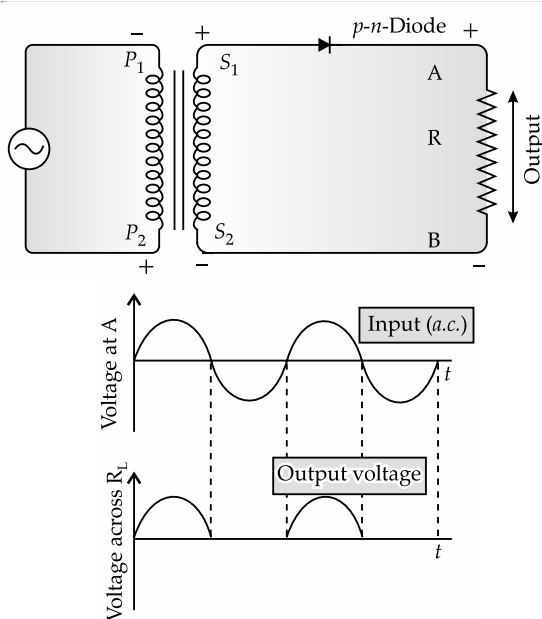
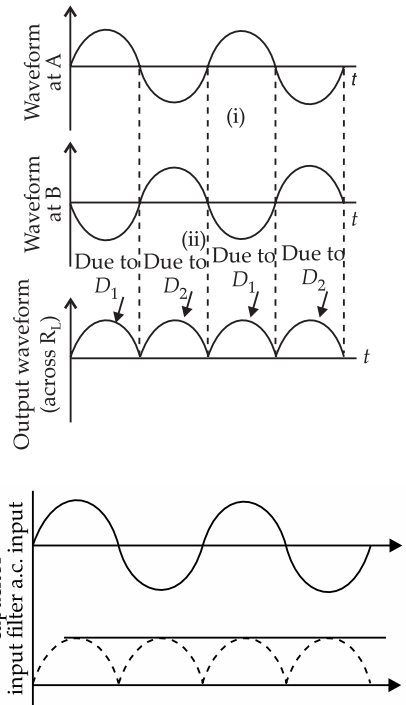
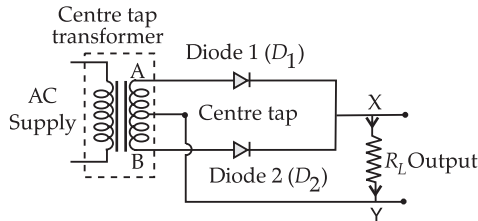
Band theory:

 p - n Junction diode:

Biasing and V-I characteristics:



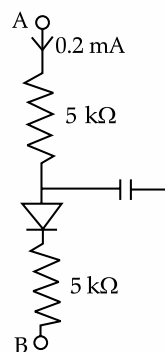
V - I Characteristics

Half wave rectifier:**Full wave rectifier:****EXERCISE**

- In an n -type silicon, which of the following statement is true?
 - Electrons are majority carriers and trivalent atoms are the dopants.
 - Electrons are minority carriers and pentavalent atoms are the dopants.
 - Holes are minority carriers and pentavalent atoms are the dopants.
 - Holes are majority carriers and trivalent atoms are the dopants.
- The conductivity of a semiconductor increases with increase in temperature because
 - number density of free current carriers increases.
 - relaxation time increases.
 - both number density of carriers and relaxation time increase.
 - number density of current carriers increases; relaxation time decreases but decrease in relaxation time is much less than the increase in number density.
- Hole is
 - an anti-particle of electron.
 - a vacancy created when an electron leaves a covalent bond.
 - absence of free electrons.
 - an artificially created particle.
- Semiconductors behave like insulators at ____
 - 0°C
 - 0 K
 - 273 K
 - None of the these
- Tetra valent semiconductor is to be doped with ____ valent element to achieve ____ type extrinsic semiconductor.
 - penta, n
 - tri, p
 - penta, p
 - both (A) and (B)
- In an unbiased p - n junction, holes diffuse from the p -region to n -region because
 - free electrons in the n -region attract them.
 - they move across the junction by the potential difference.
 - hole concentration in p -region is more as compared to n -region.
 - All of the above.
- When a forward bias is applied to a p - n junction, it
 - raises the potential barrier.
 - reduces the majority carrier current to zero.
 - lowers the potential barrier.
 - None of the above
- In Figure, assuming the diodes to be ideal,

 - D_1 is forward biased and D_2 is reverse biased and hence current flows from A to B

- (b) D_2 is forward biased and D_1 is reverse biased and hence no current flows from B to A and vice versa.
- (c) D_1 and D_2 are both forward biased and hence current flows from A to B.
- (d) D_1 and D_2 are both reverse biased and hence no current flows from A to B and vice versa.
9. In the circuit shown in Figure, if the diode forward voltage drop is 0.3 V, the voltage difference between A and B is



- (a) 1.3 V (b) 2.3 V (c) 0 V (d) 0.5 V

Answer Key

1. (c)	2. (b)	3. (b)	4. (d)	5. (c)	6. (b)	7. (c)	8. (b)	9. (b)	

Answers with Explanations

1. **Option (c) is correct.**

Explanation: In an n -type silicon the holes are the minority carriers. An n -type semiconductor is obtained when pentavalent atoms, such as phosphorus, are doped in silicon atoms.

2. **Option (d) is correct.**

Explanation: In semiconductor, the density of charge carriers (electron, holes) are very small, so its resistance is high. When temperature increases, the charge carriers (density) increases which increases the conductivity. As temperature of semiconductor increases, the speed of free electrons increases which decreases the relaxation time. As the density of charge carrier is small, so, there is small effect on decrease of relaxation time.

3. **Option (b) is correct.**

Explanation: Atoms of semiconductor are bound by covalent bonds between the atoms of same or different type. Due to thermal agitation when an electron leaves its position and becomes free, it leaves a vacancy of electron and this vacancy in the bond (covalent) is called hole.

4. **Option (b) is correct.**

Explanation: At 0 K temperature, all electrons of semiconductor are immovable from their shell as they do not have sufficient energy. So, no free electron is available as charge carrier. This makes the semiconductors to behave like insulators.

5. **Option (d) is correct.**

Explanation: When a semiconductor having 4 valence electrons is doped with an element having 3 valence electrons, then an extra hole is generated and the semiconductor becomes p -type extrinsic semiconductor.

When a semiconductor having 4 valence electrons is doped with an element having 5 valence electrons, then an extra electron becomes available and the semiconductor becomes n -type extrinsic semiconductor.

6. **Option (c) is correct.**

Explanation: The diffusion of charge carriers across a junction takes place from the regions of higher concentration to lower concentration. In this case, the p -region has greater concentration of holes than the n -region. Hence, in an unbiased p - n junction, holes diffuse from the p -region to the n -region.

7. **Option (c) is correct.**

Explanation: When a forward bias is applied to a p - n junction, it lowers the value of potential barrier. In the case of a forward bias, the potential barrier opposes the applied voltage. Hence, the potential barrier across the junction gets reduced.

8. **Option (b) is correct.**

Explanation: In circuit, A is at -10 V and B is at 0 V. So B is at higher voltage than A. So D_2 is in forward bias and D_1 is in reverse bias, so no current flows from A to B or B to A.

9. **Option (b) is correct.**

Explanation: In the middle right of the circuit the capacitor behaves like an open circuit for dc 0.2 mA current, so current will flow from A to B only. Let potential across A and B is V_{AB} , so by Kirchhoff's loop law,

$$V_{AB} = (5,000 \times 0.2 \times 10^{-3}) + 0.3 + 5,000 \times 0.2 \times 10^{-3}$$

$$V_{AB} = 1 \text{ V} + 0.3 \text{ V} + 1 \text{ V}$$

$$V_{AB} = 2.3 \text{ V}$$

Representation of organic molecule and Homologous series, Addition Reaction and Isomerism

- **Introduction:** The hydrides of carbon (hydrocarbons) and their derivatives are called organic compounds. The branch of chemistry which deals with these compounds is called organic chemistry.
- Berzilius (1808) defined organic chemistry as the chemistry of substances found in living matter and gave the vital force theory. Urea is the first organic compound synthesised in laboratory by E Wohlerin 1828.
- **Properties:** Organic compounds may be gases, liquids or solids. Being covalent in nature, these have low boiling point and melting point and soluble in organic solvents. These are generally volatile and inflammable. They do not conduct electricity because of the absence of free ions. They possess distinct colour and odour.
- Reasons for large number of organic compounds:
(i) **Catenation:** It is the tendency of self-combination and is maximum in carbon. A carbon atom can combine with the other carbon atoms by single, double or triple bonds. Thus, it forms more compounds than the others.

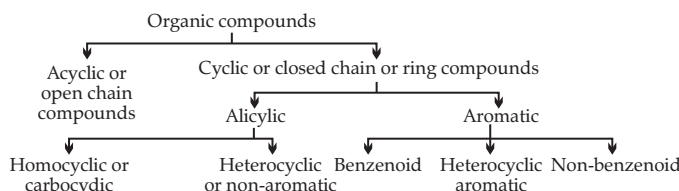
(ii) **Tetravalency:** Carbon being tetravalent is capable of bonding with four other C atoms or some other monovalent atoms.

Bond angles in sp^3 hybridised C is $109^\circ 28'$, in sp^2 hybridised C is 120° and in sp hybridised C is 180° .

The C-C, C = C and C $\equiv$ C bond lengths are 154 Å, 134 Å and 120 Å respectively {the shorter the bond, greater is its strength}. Also, greater the bond strength, greater is the bond energy,

(iii) **Small size:** Furthermore, these compounds are exceptionally stable because of the small size of carbon.

- The three dimensional structure of organic molecule can be represented on paper by using wedge-and-dash representation. In these formulas, the solid wedge (is used to show a bond coming out of the paper towards the observer and dashed wedge----- (is used to show a bond going away from the observer).
- **Classification of organic compound:** Organic compounds are broadly classified as follows:



- **Functional Group:** The atom or group of atoms e.g., -COOH, -CHO, which is responsible for the chemical properties of the molecule, is called functional group. Double and triple bonds are also functional groups, R- functional group
- **Homologous series:** Homologous series is defined as a family or group of structurally similar organic compounds all members of which contain the same functional group, show a gradation in physical and similarity in chemical properties and any two adjacent members of which differ by $-CH_2$ group. The individual members of this group are called homologous and the phenomenon is called homology.
- **Nomenclature of Organic Compounds : Common name (Common system):** Before the IUPAC system of nomenclature, organic compounds were named after the sources of origin, for example, urea was so named because it was obtained from the urine of mammals.

IUPAC (International Union of Pure and Applied Chemistry) System:

According to IUPAC system, the name of an organic compound contains three parts: (i) word root, (ii) suffix, (iii) prefix.

(i) **Word root:** Word root represents the number of carbon atoms present in the principal chain, which is the longest possible chain or carbon atoms.

(ii) **Suffix:** Suffix is of two types, primary suffix and secondary suffix.

(a) **Primary Suffix:** It indicates the type of bond in the carbon chain.

(b) **Secondary Suffix:** Secondary suffix is used to represent the functional group.

(iii) **Prefix:** Prefix is a part of IUPAC name which appears before the word root. Prefix is of two types:

(a) **Primary prefix:** For example, primary prefix cyclo is used to differentiate cyclic compounds.

(b) **Secondary prefix:** Some functional groups are considered and denoted by secondary prefixes. For example:

Substituted Group	Secondary prefix
F	Fluoro
Cl	Chloro
Br	Bromo
NO ₂	Nitro
NO	Nitroso
-CH ₃	Methyl
-OCH ₃	Methoxy

- **Naming of Compounds Containing Functional Groups :** (i) The longest chain of carbon atoms containing the functional group is numbered in such a manner that the functional groups attached at the carbon atoms possessing lowest possible number in the chain. In case of polyfunc-

tional compounds, one of the functional group is selected as principal functional group and the compound is named on that basis.

- (i) The choice of principal functional group is made on the basis of order of preference. The order of decreasing priority for the functional group is: -COOH, -SO₃H, -COOR (R-alkyl group), COCl₂, -CONH₂, -CN, -HC=O, >C=O, -OH, -NH₂, >C=C<, -C≡C-
- (ii) The number of carbon atoms in parent chain is denoted by proper word root.
- (iii) Name the groups attached to the parent chain and prefix with the number of carbon atoms to which they are attached in the parent chain.
- (iv) If the compound contains two or more functional groups, numerical prefixes like di, tri, tetra are used and the terminal e from the primary suffix is retained while writing the IUPAC name.

Some Functional Group and Classes of Organic Compounds

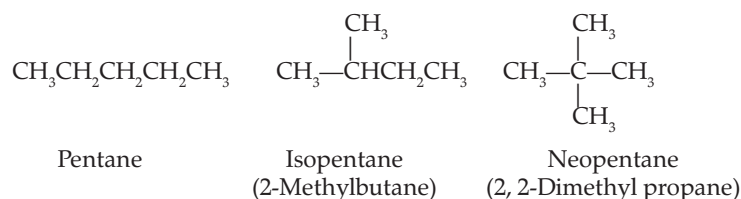
Class of compounds	Functional group structure	IUPAC group prefix	IUPAC group suffix	Example
Alkanes			-ane	Butane, CH ₃ CH ₂) ₂ CH ₃
Alkenes	>C=C<		-ene	Bul-1-ene, CH ₂ =CHCH ₂ CH ₃
Alkynes	-C≡C-		-yne	But-1-yne, CH≡CCH ₂ CH ₃
Arenes				Benzene,
Halides	-X (X=F,Cl,Br,I)	halo-		1-Bromobutane, CH ₃ (CH ₂) ₂ CH ₂ Br
Alcohols	-OH	hydroxy-	-ol	Bulan-2-ol, CH ₃ CH ₂ CHOHCH ₃
Aldehydes	-CHO	Formyl, or oxo	-al	Butanal, CH ₃ (CH ₂) ₂ CHO
Ketones	>C=O	oxo-	-one	Bulan-2-one, CH ₃ CH ₂ COCH ₃
Nitriles	-CN	cyano	nitrile	Pentanenitrile, CH ₃ CH ₂ CH ₂ CH ₂ CN
Ethers	-R-O-R-	alkoxy-		Ethoxyethane, CH ₃ CH ₂ OCH ₂ CH ₃
Carboxylic acids	-COOH	carboxy	-oic acid	Bulanoic acid, CH ₃ (CH ₂) ₂ CH ₂ H
Carboxylate ions	-COOR		-oate	Sodium butanoate, CH ₃ (CH ₂) ₂ CO ⁻ Na ⁺
Esters	-COOR	alkoxycarbonyl	-oate	Methyl propanoate, CH ₃ CH ₂ COOCH ₃
Acyl halides	-COX (X=F,Cl,Br,I)	halocarbonyl	-oyl halide	Bulanoyl chloride, CH ₃ (CH ₂) ₂ COCl
Amines	-NH ₂ , >NH, >N-	amino-	-amine	Bulan-2-amine, CH ₃ CHNH ₂ CH ₂ CH ₃
Amides	-CONH ₂ , -CONHR, -CONR ₂	-carbamoyl	-amide	Butanamide, CH ₃ (CH ₂) ₂ CONH ₂
Nitro compounds	-NO ₂	nitro		1-Nitrobutane, CH ₃ (CH ₂) ₃ NO ₂
Sulphonic acids	-SO ₃ H	sulpho	sulphonic acid	Methylsulphonic acid CH ₃ SO ₃ H

- **Isomerism:** Two or more compounds having the same molecular formula but different physical and chemical properties are called isomers and this phenomenon is called isomerism. It is of two types-
- (1) **Structural isomerism:** Structural isomerism is shown by compounds having the same molecular

formula but different structural formulae differing in the arrangement of atoms. Structural isomerism is of four types:

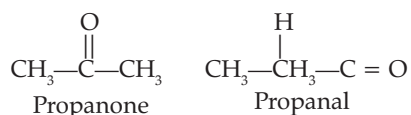
- (i) **Chain isomerism:** When two or more compounds have similar molecular formula but different carbon skeletons, these are referred to as chain isomers and the phenomenon is termed as chain isomerism. For

example, C_5H_{12} represents three compounds



(ii) **Position isomerism:** When two or more compounds differ in the position of substituent atom or functional group on the carbon skeleton, they are called position isomers and this phenomenon is termed as position isomerism. For example, the molecular formula C_3H_8O represents two alcohols:

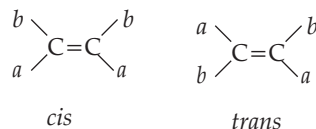
(iii) **Functional isomerism:** Two or more compounds having the same molecular formula but different functional groups are called functional isomers and this phenomenon is termed as functional group isomerism. For example, the molecular formula C_3H_6O represents an aldehyde and a ketone :



(iv) **Metamerism:** It arises due to different alkyl chains on either side of the functional group in the molecule. For example, $C_4H_{10}O$ represents methoxypropane ($CH_3OC_3H_7$) and ethoxyethane ($C_2H_5OC_2H_5$).

(2) **Stereoisomerism:** The compounds that have the same constitution and sequence of covalent bonds but differ in relative positions of their atoms or groups in space are called stereoisomers. This phenomenon is called as stereoisomerism

(i) **Geometrical or Cis-trans isomerism:** Isomerism in which the compounds possessing same molecular formula differ in their properties due to the difference in their geometry due to difference in the direction of attachment of same atoms or groups in their molecule. For example,



(ii) **Optical isomerism:** Optical isomers are two compounds which contain the same number and kinds of atoms, and bonds (i.e., the connectivity between atoms is the same), and different spatial arrangements of the atoms, but which have non-superimposable mirror images. Each non-superimposable mirror image structure is called an enantiomer. Molecules or ions that exist as optical isomers are called chiral.

Key Terms

- **Organic chemistry:** The branch of chemistry which deals with the study of carbon compounds is called organic chemistry.
- **Isomers:** Two or more compounds having the same molecular formula but different properties are called isomers.
- **Aliphatic compounds:** The compounds which have straight or branched chain are called aliphatic compounds.
- **Alicyclic compounds:** The compounds which have carbon atoms joined in the form of a ring are called alicyclic compounds.
- **Functional group:** It may be defined as an atom or group of atoms joined in a specific manner which is responsible for the characteristic chemical properties of the organic compounds.

Key Facts

- Buckminsterfullerene is a common name given to the newly discovered C_{60} cluster (a form of carbon) noting its structural similarity to the geodesic domes popularised by the famous architect R. Buckminster Fuller
- Acetic acid is the first organic compound synthesised in the laboratory from its elements.

EXERCISE

- Liquid vegetable oils are converted to solid margarine by the use of
 - hydrogen gas
 - chlorine gas
 - carbon dioxide gas
 - oxygen gas
- The number of structural isomers of pentane is
 - 5
 - 4
 - 2
 - 3
- C_4H_8 belongs to the homologous series of
 - alkanes
 - alkenes
 - alkynes
 - cycloalkanes
- How many covalent bonds are present in a Chloropropane molecule having molecular formula,



- 6
 - 8
 - 9
 - 10
- Which one of the following allotropes of carbon is isomorphous with crystalline silicon?
 - Coke
 - Diamond
 - Graphite
 - Coal
 - Bee sting leaves an acid which causes pain and irritation. The acid released is :
 - tartaric acid
 - citric acid
 - ethanoic acid
 - methanoic acid
 - Which one of the following materials is **not** an allotrope of carbon?

- (a) Diamond (b) Graphite
(c) Fly ash (d) Fullerene
8. What is the total number of covalent bonds in methanol?
(a) 3 (b) 4 (c) 5 (d) 6
9. Which one of the following statements is NOT correct?
(a) Buckminster fullerene is an allotrope of carbon.
(b) Diamond is a good conductor of electricity.
(c) Graphite is a good conductor of electricity.
(d) In graphite, each carbon atom is linked to three other carbon atoms.
10. How many covalent bonds are present in a Chloropropane molecule having molecular formula, C_3H_7Cl ?
(a) 6 (b) 8 (c) 9 (d) 10
11. Which one of the following is the largest composition in biogas?
(a) Carbon dioxide (b) Methane
(c) Hydrogen (d) Hydrogen sulphide
12. At nearly $70^\circ C$, sodium bicarbonate shows the property of gradually decomposing, which makes it usable in bakery products. The product of decomposition responsible for this use of sodium bicarbonate is
(a) Carbon dioxide (b) Hydrogen
(c) Water vapour (d) Oxygen
13. Which one of the following is termed as 'Dry ice'?
(a) Ice present in ice-cream
(b) Solid water at Antarctica
- (c) Solid state of carbon dioxide
(d) Solid water of ionosphere
14. How is carbon black obtained?
(a) By heating wood at high temperature in absence of air.
(b) By heating coal at high temperature in absence of air.
(c) By burning hydrocarbons in a limited supply of air.
(d) By heating coal at high temperature in presence of air.
15. Which one of the following properties is NOT true for graphite?
(a) Hybridisation of each carbon atom is sp^3 .
(b) Hybridisation of each carbon atom is sp^2 .
(c) Electrons are delocalized over the whole sheet of atoms.
(d) Each layer is composed of hexagonal rings.
16. Which one of the following is the purest form of Carbon?
(a) Charcoal (b) Coke
(c) Fullerene (d) Carbon black
17. The Poisonous nature of Carbon monoxide (CO) is due to its
(a) insolubility in water
(b) ability to form a complex with haemoglobin
(c) ability to reduce some metal oxides
(d) property of having one sigma bond

Answer Key

1. (a)	2. (d)	3. (b)	4. (d)	5. (b)	6. (d)	7. (c)	8. (c)	9. (b)	10. (d)
11. (b)	12. (a)	13. (c)	14. (c)	15. (b)	16. (c)	17. (b)			

Answers with Explanations

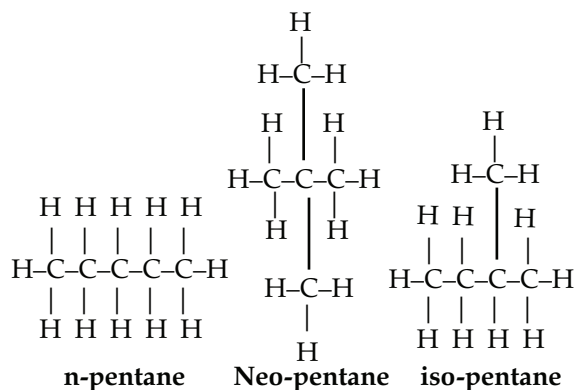
1. Option (a) is correct.

Explanation: Vegetable oils may be converted from liquids to solids by the hydrogenation reaction. Margarine and shortenings are "hardened" in this way to make them solid. Margarine has traditionally been manufactured by hydrogenation of vegetable oils (olive, palm, sunflower seed, etc.) by an industrial process for converting these oils into a solid, stable, and spreadable substance.

2. Option (d) is correct.

Explanation: Isomers have same molecular formula but different structure. Pentane C_5H_{12} . Therefore three structural isomers can be drawn from pentane.

C_3H_8 , C_4H_{10} and C_5H_{12} belong to the same homologous series. The melting and boiling points of C_5H_{12} are higher than that of C_4H_{10} .



3. Option (b) is correct.

Explanation: C_4H_8 belongs to alkynes homologous series.

4. Option (d) is correct.

Explanation: The Chloropropane molecule contains a total of 10 bond(s)

5. Option (b) is correct.

Explanation: Diamond and crystalline silicon are isomorphous because they both have three dimensional tetrahedral structure.

6. Option (d) is correct.

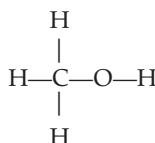
Explanation: Bee sting release methanoic acid or formic acid. The chemical formula of methanoic acid is HCOOH . When a bee stings, it injects methanoic acid into the skin which causes immense pain and irritation. We can get relief from this acid by rubbing baking soda on the affected area.

7. Option (c) is correct.

Explanation: Fly ash is not an allotrope of carbon. It is produced in small dark flecks by the burning of powdered coal or other material and carried into the air.

8. Option (c) is correct.

Explanation: The molecular formula of methanol is CH_3OH . The structural formula of methanol is as follows—



From the structure, it is clear that the number of covalent bond in methanol = 5

Short cut $\Rightarrow$ number of covalent bond = (Total number of atom) - 1

In methanol, total number of atoms = 6. So number of covalent bond = $6 - 1 = 5$

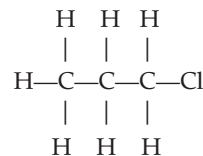
9. Option (b) is correct.

Explanation: In the 3-D structure of diamond, each carbon atom is covalently bonded to four other carbon atoms and thus, there are no free electrons and hence it does not conduct electricity.

10. Option (d) is correct.

Explanation: Number of covalent bond = (Total number of atom) - 1

$$11 - 1 = 10$$



11. Option (b) is correct.

Explanation: Methane is about 75% in biogas.

12. Option (a) is correct.

Explanation: Carbon dioxide at nearly 70 degree Celsius



13. Option (c) is correct.

Explanation: Dry ice is solid carbon dioxide.

14. Option (c) is correct.

Explanation: Carbon black is mainly produced by the incomplete combustion of heavy petroleum products including FCC tar, coal tar, ethylene, cracking tar, and some vegetable oils.

15. Option (b) is correct.

Explanation: In graphite each carbon combines with 3 other carbon atoms with 3 sigma bonds, so sp^2 hybridisation.

16. Option (c) is correct.

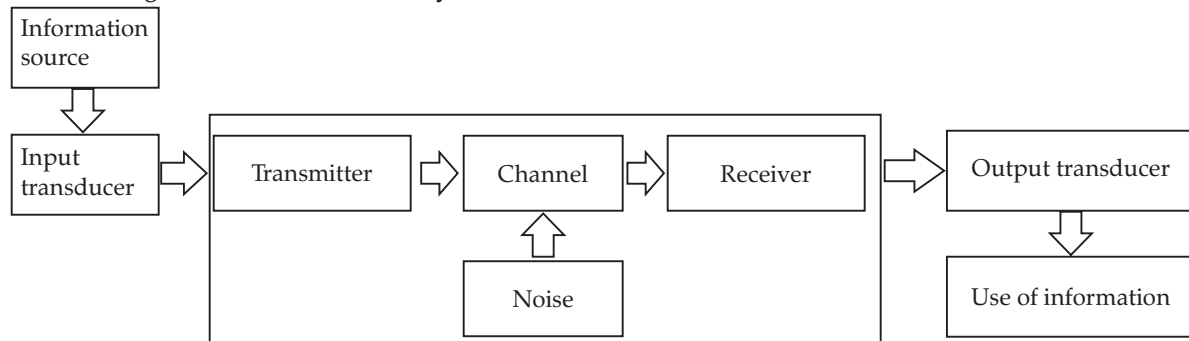
Explanation: Fullerene is the purest form of carbon. They do not have surface bonds that are to be attracted by other atoms without dangling bonds.

17. Option (b) is correct.

Explanation: This is because CO combines with the haemoglobin present in the blood cells of our body to form a stable compound called carboxyl-haemoglobin. This does not allow to absorb oxygen, thus depriving our body cells of oxygen. This causes obstruction in respiration and causes death.

○○○

➤ **Block diagram of a communication system:**



COMMUNICATION SYSTEM

Information source : The generator of information. It may be audio, video or data.

Input transducer : Converts the physical variables in electrical signal variable.

Transmitter : Converts the incoming electrical signal suitable for transmission through channel.

Channel : The medium through which the message signal travels from transmitter to receiver.

Receiver : Receives the signal from channel and reproduces the electrical signal.

Output transducer : Converts the electrical signal into physical signal (i.e., appropriate audio, video, data.)

Noise : Noise is an unwanted signal creeping in the channel between transmitter and receiver.

➤ **Bandwidth of different signals:**

Signal	Bandwidth
Speech	2800Hz
Music	20KHz
Video	4.2MHz
Audio & Video (TV)	6MHz
Digital data	3100 Hz - Infinity

➤ **Bandwidth of different channels:**

Channel	Bandwidth
Twisted pair cable	100MHz
Coaxial cable	750 MHz
Fibre optic cable	150 THz
Microwave	30GHz
Satellite	40GHz

➤ **Different types of Modes of propagation:**

- (i) Ground waves propagation
- (ii) Sky wave propagation
- (iii) Space wave propagation

➤ **Ground wave propagation:**

- (i) The radio waves travel through the atmosphere following the surface of the earth.
- (ii) The ground waves can travel around the curvature of the Earth and can go right around the globe.
- (iii) Intensity of ground wave falls with distance.
- (iv) The ground wave transmission becomes weaker with an increase in frequency due to absorption.
- (v) Suitable for low and medium frequency i.e., up to 2 MHz only.
- (vi) Generally used for local band broadcasting and is commonly known as a medium wave.

➤ **Sky wave propagation:**

- (i) The sky waves are the radio waves of frequency between 2 MHz to 30 MHz.
- (ii) These radio waves can propagate in the atmosphere and are reflected back by the ionosphere.
- (iii) Critical frequency (C.F) is the highest frequency of radio waves, which when sent straight (i.e., normally) towards the layer of ionosphere, returns to the earth. If the frequency of radio waves is more than critical frequency, it will not be reflected by the ionosphere. The critical frequency of a skywave for reflection from a layer of atmosphere is given by:

$$\nu_C = 9(N_{\max})^{1/2},$$

Where N is the number density of electrons/m³.

- (vi) The values of critical frequency are 4 MHz, 5 MHz, and 6 to 8 MHz for E , F_1 , and F_2 layers of the ionosphere which are at heights about 110 km, 180 km, and 300 to 350 km respectively from the surface of the earth.
- (v) Maximum Usable Frequency (MUF) is the highest frequency of radio waves which when sent at some angle towards the ionosphere, gets reflected, from that and returns to the earth.

- (vi) Skip distance is the distance over the earth surface between the points where a radio signal is transmitted and where it is received after reflection from the ionosphere.

➤ **Space wave propagation:**

- (i) The space waves are radio waves of very high frequency (*i.e.*, between 30 MHz to 300 MHz or more).
- (ii) The space waves travel through the atmosphere from transmitter antenna to receiver antenna. The space wave propagation is also known as tropospheric propagation.
- (iii) The range of space communication can be increased by increasing the height of the transmitting and receiving antenna.
- (iv) The space wave communication is utilized in television communication, radar communication, and microwave communication.
- (v) If h is the height of the transmitting antenna and d is the distance of receiving antenna from the transmitting antenna, then

$$d = \sqrt{2hR},$$

where, R is the radius of the earth.

For T.V. signals, the area covered = πd^2

➤ **Microwave propagation:**

- (i) Microwaves are electromagnetic waves of frequency 100 GHz to 300 GHz.
- (ii) Microwaves can be directed as beam signals in a particular direction, much better than radio waves.
- (iii) There is no spreading or bending of around the corners of any obstacle coming in their way.
- (iv) Microwave communication is commonly used to locate flying objects (like aeroplanes) in space and to estimate their distance from the radar station.

➤ **Satellite communication:**

- (i) A communication satellite is placed in an orbit around the earth.
- (ii) In satellite communication, a beam of modulated microwave from the transmitter is sent directly towards the communication satellite, which receives the coming signal, amplifies it, and returns it to earth at a different frequency to avoid interference between the uplink and downlink.
- (iii) A single geostationary satellite cannot cover the whole part of the earth for microwave communication. It is so because, a large part of the earth is out of sight due to the curvature of the earth.
In order to have the microwave communication link over the entire globe of earth, at least three geostationary satellites are required, which are 120° from each other.

- The various frequency ranges used in radio waves or microwave communication system are as follows:

- (a) Medium frequency band (M.F) — 300 kHz to 3000 kHz.
- (b) High frequency band (H.F) — 3 MHz to 30 MHz.
- (c) Very high frequency band (V.H.F) — 30 MHz to 300 MHz.
- (d) Ultra high frequency band (U.H.F) — 300 MHz to 3000 MHz.
- (e) Super high frequency band (S.H.F) — 3000 MHz to 30,000 MHz.
- (f) Extra high frequency band (E.H.F) — 30 GHz to 200 GHz.

➤ **Modulation and demodulation:**

In modulation, the original message signal is mixed with a carrier wave.

In demodulation, the combination of carrier and message signal are separated from each other to have the original message signal.

MODEM is a combination of modulator and demodulator.

➤ **Types of modulation:**

● **Amplitude Modulation**

In amplitude modulation, the amplitude of the carrier wave is varied in proportion to that of the message signal.

● **Frequency Modulation:**

In frequency modulation, the frequency of the carrier wave is varied in proportion to the instantaneous amplitude of the message signal.

➤ **Internet:**

- The internet is the global network composed of a large number of smaller interconnected networks of computers.
- ARPANET (Advanced Research Project Agency Network) was the first and small project run through internet technology, in 1969, by US defence department.
- **WWW (Web):**

It is a vast resource of information stored in form of trillions of interlinked web-pages and web-resources which can be accessed and shared through internet.

- o HTML (Hyper Text Mark-up Language) is a language which is used to design standard web pages so that the web content becomes accessible from any internet-connected computer across the world.
- o URL (Uniform Resource Locator) provides the mechanism and protocol to access the HTML resources.
- o HTTP (Hyper Text Transfer Protocol) is a set of rules which are used to retrieve linked web pages across the web.

● **Email (Electronic Mail):**

It is a procedure to send and receive messages through internet. There are several email ser-

vice providers like Google (Gmail), Microsoft (outlook), Yahoo (Yahoo mail) etc.

● **Chat:**

Chatting or Instant Messaging over internet is the process of world-wide communication in real time through text, audio and video messages. Service providers for Chat are Whats App, Yahoo Messenger, Facebook Messenger, Google Hangout etc.

● **VoIP (Voice over Internet Protocol):**

- o It is the voice transmission over computer network without using regular telephone network.
- o It is cheaper than regular telephone call.
- o Analog voice is converted into digital, modulated and transmitted through internet.

- o Quality depends on speed of internet connection.

➤ **Mobile Network:**

Mobile phone is a basic handset. But it cannot work without Mobile network.

Mobile network routes calls from one mobile phone to another and maintains the call when the mobile phones move from one place to another.

➤ **Global Positioning System (GPS):**

- Initially the system was design for military. In 1980 it is made accessible to civilians too.
- Global Positioning System is made up of 24 satellites which complete a circling round the earth in at a height 12000 km in 12 hours to provide continuous, real-time positioning, navigation and timing worldwide.

EXERCISE

1. Range of sky waves
(A) 3 MHz – 30 MHz (B) 3KHz – 30KHz
(C) 3GHz – 30GHz (D) 3THz – 30THz
2. Three different layers of ionosphere are
(A) A, B, C (B) E, F1, F2
(C) X, Y, Z (D) L1, L2, L3
3. RADAR communication uses
(A) Ground wave (B) Light wave
(C) Sky wave (C) Space wave
4. Which transmission channel has the highest bandwidth?
(A) Optical fibre
(B) Twisted pair cable
(C) Coaxial cable
(D) All transmission media has same bandwidth
5. The function of input transducer in a communication system is
(A) To convert electrical signal into appropriate non-electrical signal.
(B) To convert non-electrical electrical signal
(C) To amplify electrical signal
(D) To remove noise from transmitted electrical signal

Answer Key

1. (a)	2. (b)	3. (d)	4. (a)	5. (b)					

Answers with Explanations

1. **Option (a) is correct.**
Explanation: For sky wave propagation which involves reflection from ionosphere, the wave should be in high frequency range i.e. 3MHz to 30 MHz. Use of higher frequency of this will penetrate the ionosphere.
2. **Option (b) is correct**
Explanation: Due to solar flare, the ionosphere splits into several layers during day time. The layers are named – D, E, F₁ and F₂. D is towards to ground and F₂ is towards the sun.
3. **Option (d) is correct**
Explanation: RADAR communication uses line of sight communication. Space wave is for line of sight communication.
4. **Option (a) is correct.**
Explanation: Bandwidth of optical fibre in THz range.
5. **Option (b) is correct.**
Explanation: Transducer is an electronic device which converts signal of any form into appropriate electrical signal. So, to transmit a non-electrical signal, it is to be converted into electrical energy. Input transducer serves this purpose.

○○○

Balanced Diet:

A kind of food and provide adequate amounts of the nutrients required for good the body with essential nutrition: fluid, macronutrients such as protein, micronutrients such as vitamins, and adequate fiber and food energy.

Biomolecules:

In a living tissue identifies functional groups such as aldehydes, ketones and as biomolecules.

- Biomolecules have a turn over i.e., they are constantly being changed into some other biomolecules and also made from some other biomolecules.
- The entire set of biochemical reactions that take place in living organisms are known as metabolism.
- Each of the metabolic reactions results in the transformation of biomolecules.

Strategies in Food Production

Animal Husbandry

- Biological principles such as animal husbandry and plant breeding have a major role in increasing the food production.
- Animal husbandry is the agricultural practice of breeding and raising livestock.
- It deals with the care and breeding of livestock such as buffaloes, cows, pigs, horses, cattle, sheep, camels, goats, bees, silkworms, etc. and also poultry farming and fisheries.
- Fisheries include rearing, catching, selling, etc., of fish, molluscs like shell-fish and crustaceans like prawns, crabs, etc.
- Animals like bees, silk-worm, prawns, crabs, fishes, birds, pigs, cattle, sheep and camels have been used by humans for products like milk, eggs, meat, wool, silk, honey, etc. More than 70% of the world livestock population is in India and China.
- The contribution of farm produce to the world is only 25th number, i.e., the productivity per unit is very low.
- Hence new technologies have to be applied to achieve improvement in quality and productivity.

Management of Farms and Farm Animals

Management of farm and farm animals include traditional practices, procedures employed in various animal farm systems.

1. Dairy Farm Management (Dairying)

- Dairying is the management of animals for milk and its products for human consumption. It deals with the processes and systems that increase yield and improve quality of milk.
- The yield of milk depends on the quality of breeds in the farm.

Good breeds having high yielding potential and resistance to diseases are selected.

For the yield potential:

- (a) The cattle have to be looked after well.
- (b) They have to be housed well.
- (c) They should have adequate water.
- (d) They should be maintained disease free.
- (e) The feeding of cattle should be carried out in a scientific manner and with special emphasis on the quality and quantity of fodder.

However, these processes have been mechanised and so it has reduced the chance of direct contact of the produce with the handler.

To ensure these stringent measures there should be :

- (a) Regular inspections, with proper record keeping. It also helps to identify and rectify the problems.
- (b) Regular visits by a veterinary doctor.

2. Poultry Farm Management

- Poultry is the class of domesticated birds used for food or for their eggs.
- It includes chicken, ducks, turkey and geese.

Components of Poultry Farm Management

- (a) Selection of disease free and suitable breeds.
- (b) Proper and safe farm conditions.
- (c) Proper feed and water.
- (d) Hygiene and health care.

Animal Breeding

- A group of animals related by descent and similar in most characters like general appearance, features, size, configuration, etc., are said to belong to a breed. Breeding is the modification of genotype of an organism to make that organism more useful to humans.
- Animal breeding aims at increasing the yield of animals and improving the desirable qualities of the produce.

Types of Breeding

There are two types of breeding namely,

(a) Inbreeding

- It is the process of mating more closely related individuals within the same breed for 4 - 6 generations.

Process :

- Superior males and superior females of the same breed are identified and mated in pairs.
- The progeny obtained are evaluated and superior males and females among them are identified for further mating.
- In cattle, a superior female produces more milk per lactation whereas a superior male (bull) gives rise to superior progeny.

Advantages of Inbreeding

- (a) It increases homozygosity to evolve a pureline animal.
- (b) It exposes harmful recessive genes that are eliminated by selection.
- (c) It helps in accumulation of superior genes and elimination of less desirable genes. This approach increases the productivity of inbred population.

Disadvantage of Inbreeding

- (a) Continued inbreeding, especially close inbreeding, may reduce fertility and productivity. This is called inbreeding depression.
- (b) To solve this problem, selected animals of the breeding population should be mated with unrelated superior animals of the same breed which helps to restore fertility and yield.

(b) Out-breeding

- It is the breeding of the unrelated animals which may be between individuals of the same breed (but having no common ancestors), or between different breeds (cross-breeding) or different species (inter-specific hybridisation).
- It includes outcrossing, cross-breeding and inter-specific hybridization.

Out-crossing

- It is the practice of mating of animals within the same breed having no common ancestors on either side of their pedigree up to 4-6 generations.
- The offspring formed is known as an out-cross.
- It is the best breeding method for animals having low productivity in milk production, growth rate in beef cattle, etc.
- It helps to overcome inbreeding depression.

Cross-breeding

In this method, superior males of one breed are mated with superior females of another breed.

- It allows the desirable qualities of two different breeds to combine to form the progeny hybrid animals that may be used for commercial production or may be subjected to inbreeding and selection to develop new stable superior breeds.
e.g., Hisardale (sheep) developed in Punjab by crossing Bikaneri ewes and Marino rams.

Interspecific hybridization

- It is the mating of male and female animals of two different species.
- The progeny formed may combine desirable features of both the parents, and may be of considerable economic value.
e.g., Mule (male ass X female horse).

Controlled Breeding Experiments It includes,**(c) Artificial insemination**

- The controlled breeding experiments are carried out using artificial insemination.
- In this process, the semen collected from male parent is injected into the reproductive tract of selected female by the breeder.
- The semen may be used immediately or can be frozen and used later.
- It can also be transported in a frozen form to where the female is housed to obtain desirable hybrid.
- Success rate of crossing mature male and female animals is low even though artificial insemination is carried out.

(d) Multiple Ovulation Embryo Transfer Technology (MOET)

- It is a programme for herd improvement i.e., to improve the chances of successful production of

hybrids.

- In this process, a cow is administered with hormones like FSH to induce follicular maturation and super ovulation to produce 6-8 eggs per cycle instead of one egg.
- The animal is either mated with an elite bull or artificially inseminated.
- Then the fertilised eggs at 8—32 cells stages are recovered and transferred to surrogate mothers.
- This technology has been demonstrated for cattle, sheep, rabbits, buffaloes, mares, etc.
- High milk yielding breeds of females and high quality (lean meat with less lipid) meat-yielding bulls have been bred successfully to increase herd size in a short time.

Bee-keeping (Apiculture)

- It is the practice of maintenance of hives of honeybees for the production of honey and beeswax.
- Honey is a food of high nutritive and medicinal value.
- Honeybee produces beeswax which is used in the preparation of cosmetics, polishes etc.
- The increased demand of honey has led to large-scale beekeeping practices as income generating industry.
- Bee-keeping can be practiced in an area where there are sufficient bee pastures of some wild shrubs, fruit orchards and cultivated crops.
- Most common species that can be reared is *Apis indica*.

Important Features for Successful Bee-keeping

Bee-keeping though relatively easy does require some specialized knowledge as follows:

- (i) Knowledge of the nature and habits of bees.
 - (ii) Selection of suitable location for keeping beehives.
 - (iii) Catching and hiving of swarms (group of bees).
 - (iv) Management of beehives during different seasons
 - (v) Handling and collection of honey and of beeswax.
- Bees are the pollinators of many of our crop species such as sunflower, Brassics, apple and pear.
- Keeping beehives in crop fields during flowering period increases pollination which thereby improves crop and honey yield.

Fisheries

It deals with catching, processing or selling of fish, shellfish or other aquatic animals (prawn, crab, lobster, edible oyster etc).

The development and flourishing of the fishery industry is known as blue revolution. It includes,

- (a) **Freshwater fishes:** Catla, Rohu, common carp, etc.
 - (b) **Marine fishes:** Hilsa, Sardines, Mackerel, Pomfrets, etc.
- Fisheries provide income and employment to fishermen and farmers.
 - Techniques to Increase the Production of Aquatic Plants and Animals There are two main techniques namely :
(a) Aquaculture
(b) Pisciculture

EXERCISE

- To prevent heart problems, blood of a normal healthy person should have
(1) low cholesterol level. (2) high HDL level.
(3) high VLDL level. (4) high LDL level.
Select the correct answer using the code given below:
(a) 1 and 2 only (c) 3 and 4 only
(b) 1, 2 and 4 only (d) 1, 2 and 3 only
- Which one of the following statements is NOT correct?
(a) All proteins are enzymes
(b) Mostly enzymes are proteins
(c) All fats are energy rich compounds
(d) Glucose is a common carbohydrate
- Match List I with List II and select the correct answer using the code given below the Lists:

List-I (Molecule)		List-II (Product of digestion)	
A.	Proteins	(1)	Nitrogenous bases and pentose sugars
B.	Carbohydrates	(2)	Fatty acids and glycerol
C.	Nucleic acids	(3)	Monosaccharides
D.	Lipids	(4)	Amino acids

Code:

A	B	C	D
(a) 2	3	1	4
(b) 2	1	3	4
(c) 4	1	3	2
(d) 4	3	1	2

- Precursor of which one of the following vitamins comes from β - carotene?
(a) Vitamin A (b) Vitamin C
(c) Vitamin D (d) Vitamin K
- Which of the following statements about vitamins are correct?
(1) Vitamin C is essential to make connective tissue in body.
(2) Vitamin D is needed for synthesis of eye pigment.
(3) Vitamin B₁₂ helps in maturation of RBC.
(4) Vitamin C is required to make use of calcium absorbed from the intestine.
Select the correct answer using the code given below:
(a) 1 and 2 (b) 1 and 3
(c) 2 and 4 (d) 1 and 4
- Golden rice is a genetically-modified crop plant where the incorporated gene is meant for biosynthesis of
(a) Omega-3 fatty acids (b) Vitamin A
(c) Vitamin B (d) Vitamin C

Answer Key

1. (a)	2. (a)	3. (d)	4. (a)	5. (b)	6. (b)				

Answers with Explanations

1. Option (a) is correct.

Explanation: Cholesterol is required in the body of animals to maintain the elasticity of vein and arteries. If Cholesterol increases in blood, it can cause cardiovascular diseases. Cholesterol is a type of lipid that travel in the bloodstream as a spherical particle that is known as a lipoprotein. There are various types of cholesterol in animal body like HDL, LDL, IDL, VLDL.

LDL is considered as bad cholesterol because it contributes to fatty buildup in arteries, leading to narrowing of human of arteries thereby increasing the risk of heart attack. HDL has the tendency to absorb cholesterol and send it back to the liver, therefore it is also known as good cholesterol. Cholesterol and fats can move in water-based solutions with the help of IDL in the bloodstream. VLDL carries triglycerides and its elevation is harmful to humans.

2. Option (a) is correct

Explanation: There are lots of proteins in the body which have many functions in body. Such as fibrous like actin & myosin present in muscles, soluble proteins like albumin. Some proteins function as catalyst presenting various metabolic reaction these catalytic proteins are called enzymes. So, all enzymes are proteins but all proteins are not enzymes.

3. Option (d) is correct.

Explanation: Proteins are polymers of amino acids i.e., amino acids are the building blocks of proteins.

Monosaccharides are the basic units of carbohydrates. These monosaccharides polymerize to form polymeric carbohydrates like starch, cellulose, etc. Nucleic acids are polymers formed by nitrogenous bases and pentose sugar along with the phosphate group. Lipids contain fatty acids and glycerol and are soluble in organic solvents but are insoluble in non-polar solvents such as water.

Nucleic acids are polymers formed by the nitrogenous bases and pentose sugar along with phosphate group.

Lipids contains fatty acids and glycerol.

4. Option (a) is correct

Explanation: β - carotene is a pigment found in plants. It is predominantly found in carrot. It is an antioxidant.

It is required for the synthesis of Vitamin-A.

5. Option (b) is correct.

Explanation: Vitamin C is essential for the production of collagen. It helps in interaction of amino acids within the cell. vitamin B₁₂ is needed for the maturation of RBC.

6. Option (b) is correct

Explanation: Golden rice is named due to its golden color. It is genetically modified by incorporation of alpha gene which produces vitamin A precursor i.e., beta-carotene in its endosperm. Usually rice is poor in vitamin content so golden rice is an alternative.

Human Health and Disease

- Health is a state of complete physical, mental and social well-being.
- Health is affected by genetic disorders, infections, sedentary lifestyle (Junk food, lack of exercise, habits, etc).
- **Disease:** A disease can be defined as any condition that may lead to discomfort, distress, health problems or death of the affected person.
- **Congenital diseases:** These are diseases that are present since birth. For instance, a hole in the heart of an infant. They are caused by some genetic abnormalities or metabolic disorder or malfunctioning of an organ.
- **Acquired diseases:** These are diseases that may occur after birth during one's lifetime.
- Among non-infectious diseases, cancer is the major cause of death.
- Pathogens are disease-causing organisms,
- Parasites are pathogens as they harm the host by living in or on them.
- Pathogens have to adapt to live within the environment of the host.

Common Infectious Diseases in Human

1. BACTERIAL DISEASES

(a) Typhoid

- **Pathogen:** *Salmonella typhi*.
- **Mode of transmission:** It enters the small intestine through food and water and migrates to other organs through blood.
- **Symptoms:** Sustained high fever (39-40°C), weakness, stomach pain, constipation, headache and loss of appetite. Intestinal perforation and death may occur.
- **Confirmation:** The Widal test is used for confirmation of the disease.

(b) Pneumonia

- **Pathogen:** *Streptococcus* or *Diplococcus pneumoniae* & *Haemophilus influenzae*.
- **Mode of transmission:** Inhaling the droplets/aerosols released by an infected person. Sharing glasses and utensils with an infected person.
- **Symptoms:** Infects lung's alveoli. The alveoli get filled with fluid leading to respiratory problems. Fever, chills, cough, headache.
- **Severe cases:** Lips and fingernails turn grey to a bluish colour.
- Dysentery, plague, diphtheria are some other bacterial diseases in humans
- **Hemozoin:** It is a brownish black granular material within the red blood cell, which is generated during host haemoglobins degradation by the parasite.
- **Sporozoite:** It is a motile spore-like stage of some parasitic sporozoans, which act as an infective

agent,

2. VIRAL DISEASES

(a) Common cold

- **Pathogen:** Rhino viruses
- **Mode of transmission:** Inhaling droplets resulting from cough or sneezes through contaminated objects.
- **Symptoms:** Infects nose and respiratory passage. Nasal congestion and discharge, sore throat, hoarseness, cough, headache, tiredness, etc., Last for 3-7 days.
- *Aedes aegypti*, vector of a viral disease, dengue fever, is a daytime biting mosquito,
- National T.B programme was started in 1962,
- Hepatitis B is more dangerous than AIDS

3. PROTOZOAN DISEASES

(a) Malaria

- **Pathogen:** *Plasmodium* sp (*P. vivax*, *P. malariae*, *P. ovale*, and *P. falciparum*).
- **Mode of transmission:** Biting of *Anopheles* mosquito,
- **Symptoms:** **Haemozoin** causes chill and high fever recurring every 3-4 days,
- **Life cycle of Plasmodium:-** The life cycle of *Plasmodium* has three phases - Schizogony, gamogony and sporogony. Female *Anopheles* mosquito is the primary host while man is the secondary host.

(b) Amoebiasis (Amoebic dysentery) or Enteritis,

- **Pathogen:** *Entamoeba histolytica* found in the large intestine of humans.
- **Mode of transmission:** Houseflies (mechanical carriers) transmit **parasites** from the faeces of an infected person to food and water and thereby contaminating them.
- **Symptoms:** Constipation, abdominal pain and cramps, stools with excess mucous and blood clots,
- **Parasite:** An organism residing on organism of another species (it's host) and draws benefit by deriving nutrients from its host.

4. HELMINTH DISEASES

(a) Ascariasis

- **Pathogen:** *Ascaris lumbricoides* (Intestinal parasite).
- **Mode of transmission:** Soil, water, vegetables, fruits, etc., contaminated with faeces containing eggs of parasites.
- **Symptoms:** Internal bleeding, muscular pain, fever, anaemia and blockage of intestinal passage.

(b) Filariasis (Elephantiasis)

- **Pathogen:** Filarial worms or *Wuchereria* (*W. bancrofti* & *W. malayi*).
- **Mode of transmission:** Bite of female *Culex* mosquito

- **Symptoms:** Filarial worms live in lymphatic vessels (usually of lower limbs). It causes chronic **inflammation** of the organs, in which they live for many years. Limbs and genital organs may be deformed.

5. FUNGAL DISEASES

(a) Ring worms

- **Pathogens:** *Microsporum*, *Trichophyton* & *Epidermophyton*. They are seen in groin between the toes.
- **Mode of transmission:** From soil or by using towels, cloths, comb, etc. Heat and moisture help fungi to grow.
- **Symptoms:** Appearance of dry, scaly lesions on various body parts such as skin, nails and scalp. Intense itching,
- **Inflammation:** A process by which our body's WBC protect as from infection from outside invaders, such as

Medicinal use of Microorganisms are as follows

- Microorganisms are used in the production of Antibiotics for e.g., *Penicillin notatum* and *Streptomyces griseus*.
- These microorganism are used in the manufacturing of certain vaccines such as small pox, rabies, polio, hepatitis B and influenza.
- These organisms are also used as food supplements and helps in digestion of food.

Immunity

It is the ability of the immune system of the body to fight against the disease-causing organisms.

- It is of two types namely innate immunity and acquired immunity.

(b) Innate Immunity

- It is the *non-specific* defence present at the time of birth. It provides barriers to the entry of foreign agents into our body. It consists of four types of barriers:
- Mucus coating of the epithelium lining the respiratory, gastrointestinal and urogenital tracts also help in trapping microbes entering our body.

(ii) **Physiological Barriers:** Acid in the stomach, saliva in the mouth, tears from eyes-all prevent microbial growth.

(iii) **Cellular Barriers:** Certain types of leukocytes (WBC) of our body like polymorpho-nuclear leukocytes (PMNL- neutrophils) and monocytes and natural killer (type of lymphocytes) in the blood as well as macrophages in tissues can phagocytose and destroy microbes.

(iv) **Cytokine Barriers:** Virus-infected cells secrete proteins called *interferon* which protect non-infected cells from further viral infection.

(c) Acquired Immunity

- It is a pathogen-specific immunity.
- It is not present since birth but develops during the lifetime of an individual. The primary and secondary immune responses are carried out with B-lymphocytes and T-lymphocytes,
- B-lymphocytes (B-cells): Produce antibodies.
- T-lymphocytes: Help B-cells to produce antibodies.
- **Physical Barriers:** Skin on our body is the first and main barrier that prevents entry of the micro-organisms.
- It is the first line of defence.
- It is characterized by memory i.e., during the first encounter of a pathogen; our body produces a primary response in low intensity. The second encounter with the same pathogen produces a secondary (anamnestic) response in high intensity.

Human Circulatory System

During blood circulation, arteries carry oxygenated blood from the heart and branch out to form small arterioles that end in capillaries to reach the tissues.

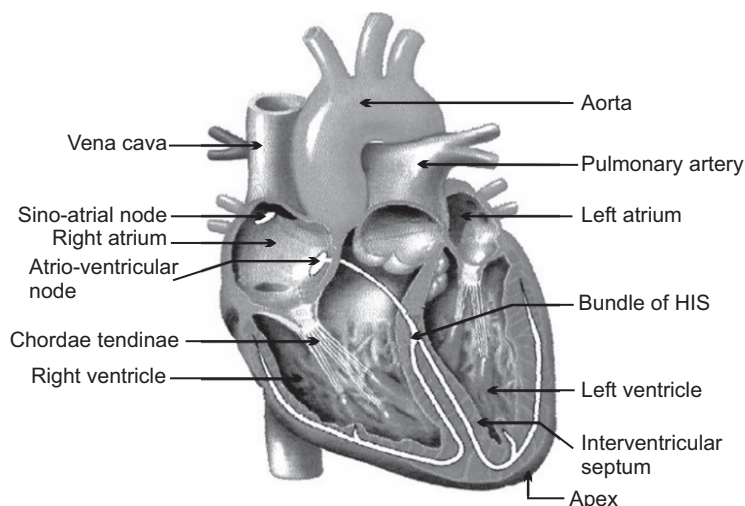
These capillaries collect deoxygenated blood from tissues and converge to form venules, which finally converge to form veins.

- However, there are two exceptions in which the artery and vein perform different functions – the pulmonary artery and the pulmonary vein.
- The human heart is about the size of a fist and lies in the thorax between the two lungs, slightly tilted towards the left lung. It is protected by the pericardium, which encloses the pericardial fluid.
- The inter-atrial septum separates the two atria, while the inter-ventricular septum separates the two ventricles and the atrio-ventricular septum separates the atria from their respective ventricles.
- The opening between the right atrium and the right ventricle is guarded by tricuspid valve whereas a bicuspid or mitral valve guards the opening between the left atrium and the left ventricle.

The openings of the right and the left ventricles into the pulmonary artery and aorta respectively are provided with semilunar valves.

- With the closure of the tricuspid and bicuspid valves, the first sound, 'lub' is produced and the closure of the semilunar valves produces the second sound, 'dub'.

Specific regions in the heart consist of specialised cardiac muscles called nodal tissues.



- The nodal tissues include Sino-atrial node (SAN) and atrio-ventricular node (AVN).
- AV bundle from the AVN branches into the right and left bundles in the ventricles to form minute Purkinje fibres.
- All these fibres together form the bundle of His.
- Here, the SAN can automatically generate around 70 to 75 action potentials or electrical impulses per minute.
- This action potential is transmitted to every other part of the heart through the AV node and bundle of His to bring about the rhythmic contraction of the heart. Hence, the SAN is called the pacemaker of the heart.
- The heart normally beats 70-75 times in a minute (average 72 beats per minute).

Nervous System

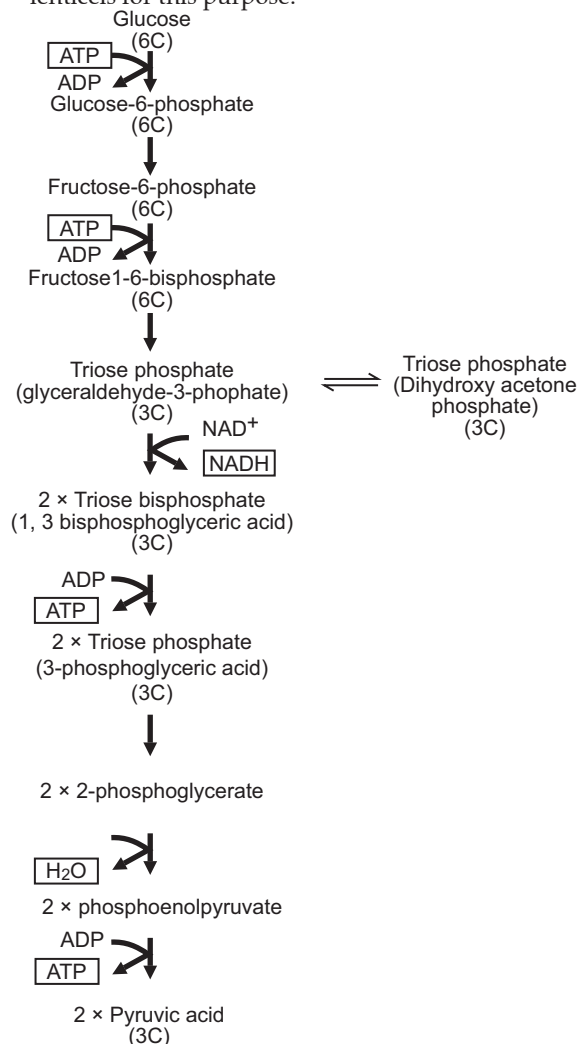
- The human neural system is divided into two parts namely – the central neural system (CNS) and the peripheral neural system (PNS).
- The brain and the spinal cord form the central neural system.
- It is the site of information-processing and control.
- The peripheral neural system comprises of cranial and spinal nerves.
- It controls the voluntary functions of the body.
- The nerve fibres of peripheral nerves are of two types namely – afferent and efferent nerve fibres.
- Afferent fibres conduct nerve impulses from the sense organs to the central nervous system.
- Efferent fibres conduct nerve impulses from the central nervous system to the involuntary organs.
- The peripheral neural system is divided into the somatic neural system and the autonomic neural system.
- The somatic neural systems relay impulses from the central neural system to the skeletal muscles.
- The autonomic neural system controls involuntary functions.
- It is further classified into the sympathetic neural system and parasympathetic neural system.

Respiration

- The breaking of the C-C bonds of complex compounds through oxidation within the cells, leading to release of considerable amount of energy is called respiration.

Respiratory Substrates:

- The compounds that are oxidised during this process are known as respiratory substrates.
- Plants, unlike animals have no specialised organs for gaseous exchange but they have stomata and lenticels for this purpose.



Steps of Glycolysis

Types of Respiration:

- (a) Aerobic respiration
- (b) Anaerobic respiration

Aerobic Respiration

- (a) It occurs in the presence of oxygen.
- (b) Respiratory substrate is glucose.
- (c) Glucose gets completely oxidised.
- (d) The end products formed are CO_2 , H_2O and 36 ATPs.
- (e) Energy is released in large quantities.
- (f) Cytoplasm and Mitochondria are the sites of breakdown.

Anaerobic Respiration

- (a) It occurs in the absence of oxygen.
- (b) Respiratory substrate is glucose.
- (c) Glucose gets partially oxidised.
- (d) The end products formed are ethyl alcohol/lactic acid, CO_2 , 2 ATPs.
- (e) Energy is released in lesser quantity.
- (f) Only cytoplasm is the site of breakdown.

Mechanism of Respiration

- Glycolysis – it is common to both aerobic and anaerobic respiration
- Citric acid cycle / Krebs cycle - Aerobic respiration in mitochondria
- Electron transport system – in the inner membrane of mitochondria
- Both aerobic and anaerobic respiration starts with glycolysis.
- In aerobic respiration glycolysis is followed by citric acid cycle and ETS (both occur in mitochondria).
- In anaerobic respiration glycolysis is followed by formation of ethyl alcohol / lactic acid in the cytoplasm.

Cardiac Cycle

To begin with, all four chambers are in relaxed state called joint diastole. The blood from pulmonary vein and vena cava flows to left and right ventricle through tricuspid and bicuspid valve. Semilunar valves are closed at this stage.

- SA node generates action potential that stimulate the contraction both atria (atrial systole). The action potential is passes to AV node and bundle of His transmit it to ventricular musculature to cause ventricular systole. At the same time atria undergoes relaxation (diastole) to close the bicuspid and tricuspid valve.
- Semilunar valves open into circulatory system that relax the ventricle and close the valves to prevent back flow of blood.
- As the pressure inside ventricles decreases the bicuspid and tricuspid valve open to repeat the process of cardiac cycle.
- During each cardiac cycle two sounds are produced. The first sound (lub) is due to closure of bicuspid and tricuspid valves and second heart sound (dub) is due to closure of semilunar valves.

Digestion of Food

- Carbohydrates, fats, proteins and nucleic acids occur in food in the form of large and complex insoluble macromolecules (polymers). These macromolecules are converted into small monomers by the action of enzyme.

In buccal cavity, teeth and tongue help in mastication

and mixing of food. Mucus in saliva mixes with masticated food to form bolus. Bolus is passed to pharynx and oesophagus by swallowing or deglutition.

Chemical digestion of food starts in oral cavity by the action of enzyme salivary amylase and lysozyme. Lysozyme acts as antibacterial agent in mouth to prevent infection.

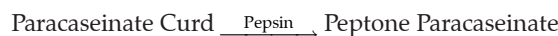
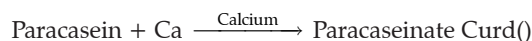
Mucosa of stomach have gastric glands having three types of cells- mucus neck cells that secrete mucus, peptic or chief cells that secrete proenzyme pepsinogen and parietal or oxyntic cells that secrete HCl.

Food mixes with gastric juice due to churning action of muscular wall to form chyme.

Hydrochloric acid [HCl] activates the pepsinogen to pepsin to digest protein.

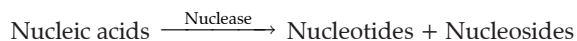
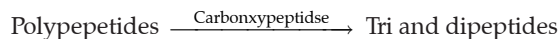
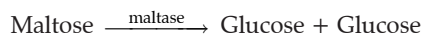
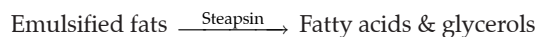
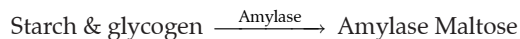
- Mucus and bicarbonates present in gastric juice play important role in lubrication and protecting inner wall of stomach from the action of hydrochloric acid [HCl].

Rennin is a proteolytic enzyme found in gastric juice of infants to digest milk protein.

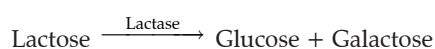
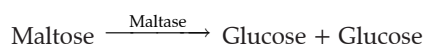
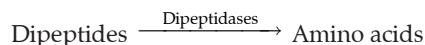


- The bile, pancreatic juice and intestinal juice are released in small intestine.
- Pancreatic juice contains inactive trypsinogen, chymotrypsinogen, procarboxypeptidases, amylases, lipases and nucleases.
- Trypsinogen is activated by enzyme enterokinase into trypsin, which further activates the other enzyme of intestinal juice.

Bile contains bile pigments (bilirubin and biliverdin), bile salts, cholesterol and phospholipids which help in emulsification of fats.



Secretion of brush border cells of mucosa and goblet cells contain enzyme succus entericus, containing variety of enzymes to complete the process of digestion.



Sucrose $\xrightarrow{\text{Sucrase}}$ Glucose + Fructose

Nucleotides $\xrightarrow{\text{Nucleotidase}}$

Nucleotides $\xrightarrow{\text{Nucleotidase}}$ Sugars + Bases

Di and monoglycerides $\xrightarrow{\text{Lipases}}$ Fatty acids + Glycerol

Excretory System

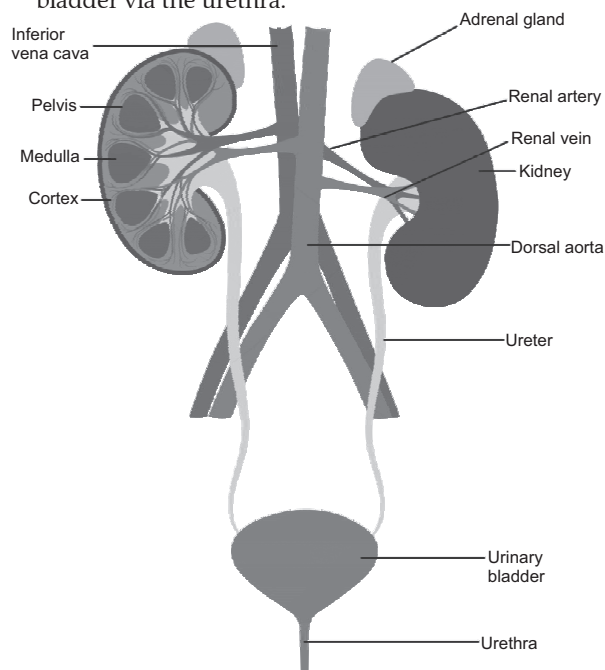
- The process by which the cellular nitrogenous wastes are eliminated is called excretion.

Three main nitrogenous wastes are ammonia, urea, and uric acid.

Ammonia-excreting animals are called ammonotelic animals and the process is called ammonotelism.

The human excretory system consists of a pair of kidneys, a pair of ureters, a urinary bladder and the urethra. The kidneys are compact, bean-shaped organs situated between the levels of last thoracic and third lumbar vertebra on either side of the vertebra.

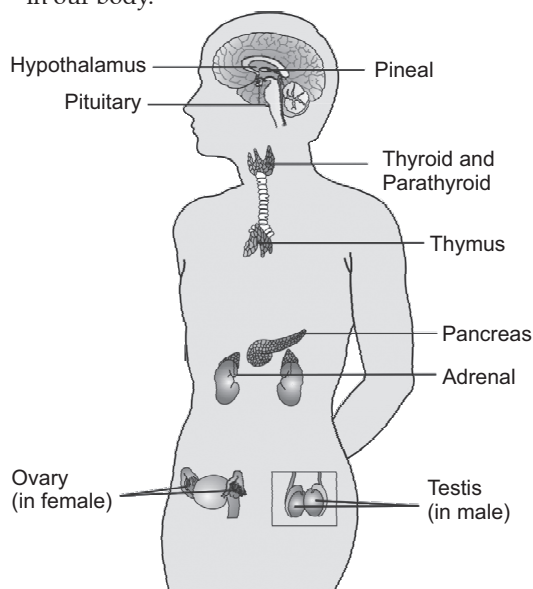
- Ureters:** These carry urine from the renal pelvis of the kidneys to the urinary bladder.
- Urinary bladder:** It temporarily stores urine till it is released from the body. Urine leaves the urinary bladder via the urethra.



Human Urinary System

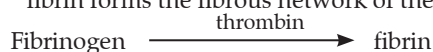
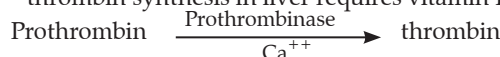
Human Endocrine System

- Pituitary, pineal, thyroid, adrenal, pancreas, and ovary in females) are the organised endocrine parathyroid, thymus and gonads (testis in males bodies in our body.



Coagulation of Blood

- A blood clot is a network of thread like protein fibres called fibrin that traps blood cells, platelets and fluid.
- The clotting depends on several proteins in the plasma. They are called coagulation factors. Normally these factors are in an inactive state.
- After injury they are activated to produce a clot.
- The activation takes place in three steps as follows:
- Step 1:** Formation of thrombokinase - Damaged tissues release a mixture of lipoproteins and phospholipids called tissue factor (TF) or thromboplastin. This factor in the presence of certain factors in the blood form a complex called prothrombinase or thrombokinase. This complex is formed by a series of enzymic reactions (Cascade process).
- Step 2:** Formation of thrombin - During this stage soluble plasma protein prothrombin is converted into the enzyme thrombin by prothrombinase. Prothrombin synthesis in liver requires vitamin K.
- Step 3:** The soluble plasma protein fibrinogen is converted to insoluble protein, fibrin by thrombin. The fibrin forms the fibrous network of the clot.



EXERCISE

- Which one of the following is caused by a bacterial pathogen?
 - AIDS
 - Dengue fever
 - COVID - 19
 - Typhoid fever
- Which one of the following viruses used to be responsible for highly infectious disease Smallpox ?
 - Adenovirus
 - Variola virus
 - Aichi virus
 - Coxsackie virus
- Which one of the following is a bacterium that causes disease in the human body?
 - Varicella zoster
 - Trypanosoma gambiense
 - Salmonella typhi
 - Plasmodium falciparum
- Which one of the following disease may be caused by the deficiency of vitamin C?
 - Rickets
 - Rabies
 - Hepatitis
 - Scurvy

5. Which of the following pairs of vector and disease is/are correctly matched?

	Vector	Disease
(1).	Anopheles	Malaria
(2).	<i>Aedes aegypti</i>	Chikungunya
(3).	Tsetse fly	Filariasis
(4).	Bed bugs	Sleeping sickness

Select the correct answer using the code given below:

- (a) 1, 2 and 3 only (c) 1 and 4 only
(b) 1 and 2 only (d) 2 only
6. Which one of the following is not a part of female reproductive system?
(a) Fallopian tube (b) Cervix
(c) Urethra (d) Vagina
7. Which one of the following is the major constituent of biogas?
(a) Carbon dioxide (b) Nitrous oxide
(c) Methane (d) Oxygen
8. Which one of the following statements about microbes is not correct?
(a) They are used in sewage treatment plants.
(b) They are used in industrial fermenters for the production of beverages
(c) No antibiotic has been obtained from microbe.
(d) They are used to get many bioactive molecules
9. The term 'Probiotic' is applied to
(a) Organic food
(b) Antacid
(c) Antibiotic
(d) Live microbial food supplement
10. Which one of the following microbes causes acidification and curdling of milk?
(a) Lactic Acid Bacillus
(b) *Clostridium botulinum*
(c) *Vibrio cholerae*
(d) *Saccharomyces cerevisiae*
11. In which one of the following, methanogens, i.e., methane producing bacteria are not present?
(a) Rumen of cattle
(b) Aerobic environment
(c) Anaerobic sludge
(d) Excreta of cattle
12. If human blood is placed in a 2% detergent solution, what will happen to the RBC?
(a) The RBC will shrink.
(b) The RBC will swell and become turgid.
(c) The RBC will swell and burst.
(d) The RBC will lyse.
13. Which one of the following depicts the correct circuit of a reflex arc?

- (a) Effector-sensory neuron-spinal cord-motor neuron-receptor
(b) Receptor-sensory neuron-spinal cord-motor neuron-effector
(c) Receptor-sensory neuron-brain-motor neuron-effector
(d) Sensory neuron-receptor-brain-effector-motor neuron
14. Intake of which one of the following food components should be minimized by patients having Gouty Arthritis due to elevated serum uric acid level?
(a) Food fibres (b) Nucleic acids
(c) Lipids (d) Carbohydrates
15. 'Lub-dup' sound is produced due to action of
(a) Large intestine (b) Lungs
(c) Heart (d) Oesophagus
16. Which one of the following statements regarding Electrocardiogram is correct?
(a) Electrocardiogram is graphical representation of electrical activity of cornea
(b) Electrocardiogram is graphical representation of activity of kidney
(c) Electrocardiogram is graphical representation of activity of brain
(d) Electrocardiogram is graphical representation of electrical activity of heart
17. Which one of the following parts of body does NOT take part in the process of breathing?
(a) Bronchi (b) Bowman's capsule
(c) Diaphragm (d) Trachea
18. The oxygenated blood from the lungs is received by the
(a) left auricle (b) left ventricle
(c) right auricle (d) right ventricle
19. Which endocrine gland requires iodine to synthesize a particular hormone whose deficiency may cause goitre disease?
(a) Hypothalamus (b) Pancreas
(c) Thymus (d) Thyroid gland
20. The protein-digesting enzyme secreted by the stomach wall in case of mammals is called
(a) chitinase (b) amylase
(c) pepsin (d) trypsin
21. The digestive enzymes are present in
(a) mitochondria (b) vacuoles
(c) lysosomes (d) ribosomes
22. Which one of the following hormones increases the heartbeat in mammals?
(a) Insulin (b) Melatonin
(c) Thyroxine (d) Adrenaline
23. Which one of the following hormones is responsible for the development of female sexual characters?
(a) Prolactin (b) Estrogen
(c) Oxytocin (d) Progesterone

Answer Key

1. (d)	2. (b)	3. (c)	4. (d)	5. (c)	6. (c)	7. (c)	8. (c)	9. (d)	10. (a)
11. (b)	12. (a)	13. (b)	14. (b)	15. (c)	16. (d)	17. (b)	18. (a)	19. (d)	20. (a)
21. (c)	22. (d)	23. (b)							

Answers with Explanations

1. Option (d) is correct.

Explanation: Typhoid also called enteric fever is a bacterial infection caused by *Salmonella typhi*. It can lead to high fever which is accompanied by diarrhea and vomiting. It is generally disseminated through the consumption of contaminated food and water.

2. Option (b) is correct.

Explanation: Variola virus causes small pox. Small pox is a contagious and deadly disease. It shows symptoms like fever, and appearance of flat, red spots on whole upper body which turns into small blisters filled with clear fluid and after sometime it gets filled with pus.

There is currently no cure, but only a vaccine could have prevented it.

3. Option (c) is correct.

Explanation:

- *Salmonella typhi* is a bacteria that causes typhoid infection in humans. The infection is transmitted by contaminated food and drinking water
- *Varicella - zoster virus* (VZV) causes chickenpox disease which is most commonly affecting children, teens, and young adults.
- *Trypanosoma gambiense* which is also called Trypanosoma rhodesiense is a sleeping sickness is transmitted to humans body which is caused by the bite of tsetse flies.
- *Plasmodium falciparum* is a unicellular protozoan parasite of humans and the deadliest species of Plasmodium that causes malaria in humans. The parasite is transmitted through the bite of a female anopheles mosquito which causes the most dangerous disease falciparum malaria.

Most common symptoms of bacterial disease include:

- Bloody urine and painful, frequent urination
- Diarrhoea
- Fatigue, fever, sore throat, headache, cough, aches and pains)
- Irritability
- Nausea and vomiting
- Pain such as joint, ear or abdominal pain
- Rashes, lesions and abscesses
- Stiff neck
- Weakness

4. Option (d) is correct.

Explanation: Rickets is caused due to the deficiency of vitamin D. Rabies is caused by infection of Rabies virus through the bite of dog, cat as this virus is present in their saliva.

Hepatitis is caused by Hepatitis virus.

Scurvy is caused due to the deficiency of vitamin C leading to gum disease and anemia.

5. Option (c) is correct.

Explanation: Anopheles is a mosquito that spreads of malaria through its bite and it carries protozoa known as *Plasmodium vivax*. It's the female anopheles mosquito.

Chikungunya is a viral disease transmitted by the bite of *Aedes aegypti* mosquito.

Filariasis is not transmitted by the tsetse fly, it is spread by culex mosquitoes. Filariasis is caused by roundworm and it is also known as Elephantiasis. Sleeping sickness is caused by tsetse fly, not by bed bugs.

6. Option (c) is correct.

Explanation: Female reproductive system consists of a pair of ovaries and fallopian tubes. And only one uterus, cervix, and vagina are present in the female reproductive system. The urethra is part of the excretory system. The urethra is a tube-like structure by which urine can pass from the urinary bladder to the outside of the body.

7. Option(c) is correct.

Explanation:

- Biogas is considered as a valuable renewable energy source which is produced by the breakdown of organic matter by certain bacteria under anaerobic conditions
- Biogas mainly comprises of three main gases such as it is a mixture of methane, hydrogen and carbon dioxide.
- Biogas is produced by agricultural waste, food waste, animal dung, manure, and sewage.
- The process of biogas production is also known as anaerobic digestion
- Methane comprises of 45-70% of biogas and thus biogas is inflammable. It produces a deep blue flame and can be used as an energy source

8. Option (c) is correct

Explanation: Microbes are used in sewage treatment to digest sewage aerobically and anaerobically also microbe are used in industries to produce beverages like beer, wine etc. through alcoholic fermentation.

penicillium is an antibiotic which is obtained from penicillium natatum fungus. streptomycin is obtained from streptomyces griseus.

Bioactive molecules are also obtained from microbes to cure disease like to cure cholesterol statins are obtained from *Monascus purpureus*.

9. Option (d) is correct

Explanation: Probiotics are live microorganism food supplements. They are claimed to provide health benefits upon consumption by restoring the gut flora. Probiotics contains live bacteria and yeast which are good for gut health.

10. Option (a) is correct

Explanation: *Lactobacillus* is the bacteria which causes curdling of milk into curd have many culinary uses. Lactic acid bacteria produces lactic acid leading to decrease in pH of milk thereby the casein protein gets coagulated and milk gets curdled. The bacteria feeds upon the lactose sugar present in milk and produces lactic acid.

11. Option (b) is correct.

Explanation: Methanogens are the group of archaeobacteria that are involved in the production of methane. They are present in the rumen of cattle where they help in digestion of cellulose. They are also found in the excreta of the wild animals.

Methanogens are anaerobic in nature so also found in anaerobic sludge but not found in aerobic environments.

12. Option (a) is correct.

Explanation:

- RBC (Transport of gases)
- Hb (Red Colour)
- O₂ and CO₂ is isotonic solution
- So, the concentrated solution is a hypertonic solution compared to red blood cells, hence if placed in it the cell will shrivel.

13. Option (b) is correct.

Explanation: Reflex action is the stimulation of a sensory organ resulting in the activity of some organ without the intervention of will. It is a spontaneous automatic mechanical response to a stimulus and without the will of animals.

A reflex action involve five things i.e., receptor, sensors nerve fibre, part of CNS, motor nerve fibre, effector muscle.

When an impulse travels a path during reflex action called as Reflex arc.

So, the correct path is Receptor-Sensory neuron- Spinal cord-motor neuron-effector muscle.

14. Option (b) is correct.

Explanation: Gouty arthritis is caused due to defect in purine metabolism leading to excess of uric acid and urate salts in body. Uric acid level increases in blood and it's crystals accumulate in the Joints resulting in gouty arthritis. So Food item contain high amount of nucleic acid should be avoided by it's patients like meat, Sea food, organ meat like liver which has high amount of nucleic acid.

15. Option (c) is correct.

Explanation: The heart produces "lub-dub" sound and this sound is produced during it's functioning in cardiac cycle. The "lub" is first sound of cardiac cycle produced due to the closure of by bicuspid and tricuspid valves. The dub is the second sound of cardiac cycle and is produced due to closure of semilunar valves.

16. Option (d) is correct.

Explanation:

- Electrocardiogram is a test which helps to check functioning of heart by measuring the electrical activity of heart.
- After every heart beat a electrical impulse travels through our heart.
- This wave causes muscle to squeeze and pump blood from the heart. Electrocardiography is the process of producing Electrocardiogram.

17. Option (b) is correct.

Explanation: Trachea is the wind pipe which divides into pair of primary Bronchi which enters into right

and left lungs. The diaphragm also helps in inspiration and expiration of gases.

Bowman's capsule is the part of nephron present in Kidney. It helps in filtration of blood to separate out urea through urine.

18. Option (a) is correct.

Explanation: The human heart contains four chambers ie., two auricle and two ventricles, one each on left and right side of it. It shows double circulation pathways the pulmonary circulation and the systemic circulation. Auricle always receives blood and ventricle always transfer blood. The right auricle receives deoxygenated blood from body and right ventricle pass it to lungs for oxygenation. The left auricle receives oxygenated blood from lungs and left ventricle pass it to body for utilization.

19. Option (d) is correct.

Explanation: Thyroid gland secretes three hormones Thyroxine, Tri-iodothyroxine and Thyro calcitonin. Thyroxine and Tri-iodothyroxine are collectively called as Thyroid hormones. Thyroid hormone deficiency causes Goitre. It occurs due to the dietary deficiency of Iodine. Iodine is needed to synthesize thyroid hormone. In Goitre, enlargement of thyroid occurs which causes myxoedema or cretinism.

20. Option (c) is correct.

Explanation: The gastric chief cells produce the precursor of pepsin known as pepsinogen. The hydrochloric acid secreted by the parietal cells of the stomach reduces the pH, which activates the enzyme pepsin that is required for digesting proteins.

21. Option (c) is correct.

Explanation: Lysosomes are membrane-bound cell organelles in the eukaryotic cells that are responsible for containing digestive enzymes. They contain around 50 different digestive enzymes, which can hydrolyze proteins, lipids, nucleic acids, and so on. Lysosomes are known as suicidal bags of the cells.

22. Option (d) is correct.

Explanation: Adrenaline is a hormone secreted by the adrenal glands, which are located over the kidneys. Adrenaline hormone is secreted in response to stress that result in an increase in heart rate. It increases the systolic blood pressure and makes the heart beat faster to increase the blood flow to the brain and muscles.

23. Option (b) is correct.

Explanation: Estrogen is a female sex hormone which helps in the development of female sexual characters. Estrogen makes the female body more feminine by making their hips and pelvis wider, development of breast and uterus. Estrogen is also responsible for the production of hairs in armpits and groin areas.

EXERCISE

- Which of the following is true for variable names in Python?
 - unlimited length
 - all private members must have leading and trailing underscores
 - underscore and ampersand are the only two special characters allowed
 - none of the mentioned
- Assertion (A):** Data types are used to identify the type of data.
Reason (R): Data types are two types as numbers and strings.
- What keyword would you use to add an alternative condition to an if statement?
 - else if
 - elseif
 - elif
 - None of these
- Assertion (A):** Python is portable and platform independent.
Reason (R): Portable and platform independent, means it can run on various operating systems and hardware platforms.
- To access a list which contains 5 elements, which of the following uses of range() would produce a list of the desired indexes?
 - range(1,5)
 - range(0,4)
 - range(5)
 - range(1,6)
- Which type of error occurs when rules of programming language are misused?
 - Syntax error
 - Semantic error
 - Run time error
 - Logical error
- What keyword would you use to add an alternative condition to an if statement?
 - else if
 - elseif
 - elif
 - None of these
- What will be the output for the following Python statement?


```
T=[10, 20, [30,40,50], 60, 70]
T[2][1] = 100
print (T)
```

 - (10, 20, 100, 60, 70)
 - (10, 20, [30,100,50], 60, 70)
 - (10, 20, [100,40,50], 60, 70)
 - Error
- Identify the output of the following Python statements:


```
L1, L2 = [10, 15, 20, 25], []
for i in range (len(L1)):
    L2.insert(i,L1.pop())
    print (L1, L2,sep="&")
```

 - []&[25, 20, 15, 10]
 - [10, 15, 20, 25]&[25, 20, 15, 10]
 - [10, 15, 20, 25]&[10, 15, 20, 25]
 - [25, 20, 15, 10]&[]
- What is the output of the following Python statements?


```
x = [[10.0, 11.0, 12.0],[13.0, 14.0, 15.0]]
y = x[1][2]
print(y)
```

 - 12.0
 - 13.0
 - 14.0
 - 15.0
- Which function calculates middle value of the arithmetic data in iterative order?
 - median()
 - mode()
 - mean()
 - None of these
- Which type of error causes abnormal termination of program while it is executing?
 - Syntax error
 - Logical error
 - Runtime error
 - None of the above
- Suppose a tuple T is declared as
 T = (10, 12, 43, 39), which of the following is incorrect?
 - print(T[1])
 - T[2] = -29
 - print(max(T))
 - print(len(T))
- Which of these about a dictionary is false?
 - The values of a dictionary can be accessed using keys.
 - The keys of a dictionary can be accessed using values.
 - Dictionaries aren't ordered.
 - Dictionaries are mutable.
- An error in a program is termed as
 - Bug
 - Debug
 - Both (A) and (b)
 - None of these
- Identify the output of the following Python statements:


```
D={}
T= ("ZEESHAN", "NISHANT", "GURMEET", "LISA")
for i in range (1, 5):
    D[i]=T[i-1]
    print (D)
```

 - ("ZEESHAN", "NISHANT", "GURMEET", "LISA")
 - "ZEESHAN", "NISHANT", "GURMEET", "4: "LISA"
 - {1:"ZEESHAN"}, {2:"NISHANT"}, {3:"GURMEET"}, {4:"LISA"}
 - {1:"ZEESHAN", 2:"NISHANT", 3:"GURMEET", 4:"LISA"}
- What is the output when the following code is executed?


```
print("\nhello")
```

 - a new line and hello
 - \nhello
 - the letter r and then hello
 - Error

18. What will be the output of the following Python code?
- ```
S="WELCOME"
def Change(T):
 T="HELLO"
 print(T, end='@')
Change(S)
print(S)
```
- (a) WELCOME@HELLO  
(b) HELLO@HELLO  
(c) HELLO@WELCOME  
(d) WELCOME@WELCOME
19. What will be the output of the following Python code?
- ```
def Funstr(S):
    T=""
    for i in S:
        if i.isdigit():
            T=T+i
    return T
X = "PYTHON 3.9"
Y = FunStr(x)
print (X, Y, sep="**")
```
- (a) PYTHON 3.9 (b) PYTHON 3.9*3.9
(c) PYTHON 3.9*39 (d) Error
20. Which arguments are also known as positional arguments?
- (a) Keyword argument
(b) Default argument
(c) Required argument
(d) Variable length argument
21. Suppose the content of "Rhymes.txt" is Good Morning Madam
What will be the output of the following Python code?
- ```
F = open("Rhymes.txt")
L = F.read().split()
for W in L:
 if W.lower() == W[::-1].lower():
 print(W)
```
- (a) Good                              (b) Morning  
(c) Madam                            (d) Error
22. Which function is used to split a line in columns?
- (a) split()                            (b) spt()  
(c) split\_line()                    (d) All of these
23. Which of these is the correct code for creating a list of names?
- (a) nameList = John, Harry, Jesse, John, Harry, Harry  
(b) nameList = ("John", "Harry", "Jesse", "John", "Harry", "Harry")  
(c) nameList = ["John", "Harry", "Jesse", "John", "Harry", "Harry"]  
(d) nameList = [John, Harry, Jesse, John, Harry, Harry]
24. Which piece of code below will add one more name "Kamal" to the end?
- ```
nameList = ["John", "Harry", "Jesse", "John", "Harry", "Harry"]
```
- (a) nameList.append(Kamal)
(b) append(nameList, "Kamal")
(c) nameList.append("Kamal", 7)
(d) nameList.append("Kamal")
25. What will be the output of following statement if it is applied into the original list.
del L[2][1:2]
- (a) Delete two elements (2,7,8) from sub-list of the list
(b) Delete one element (2,7) from sub-list of the list
(c) Generate error
(d) None of these
26. In a Python program, a control structure:
- (a) Defines program-specific data structures
(b) Directs the order of execution of the statements in the program
(c) Dictates what happens before the program starts and after it terminates
(d) None of the above
27. What keyword would you use to add an alternative condition to an if statement?
- (a) else if (b) elseif
(c) elif (d) None of these
28. Which one of the following is a valid Python if statement:
- (a) if x>=9: (b) if x >= 9)
(c) if (x >= 9) (d) if x >= 9
29. Which of following is not a decision-making or selection statement?
- (a) if-elif statement (b) for statement
(c) if-else statement (d) if statement
30. What does the following code print to console?
- ```
if True:
 print(10)
else:
 print(20)
```
- (a) 10                                (b) 20                                (c) True                                (d) False
31. If the user inputs: 10<ENTER>, what does the following code snippet print?
- ```
x = float(input())
if(x<10):
    print("Less than 10")
elif (x >= 10):
    print("Greater than 10")
else:
    print ("No")
```
- (a) Less than 10 (b) No
(c) Greater than 10 (d) Nothing is printed
32. In Python..... defines a block of statements.
- (a) Block (b) loop (c) indentation (d) {}
33. The..... statement terminates the execution of the whole loop.
- (a) continue (b) exit (c) jump (d) break
34. The for loop in Python is an_____
- (a) Entry Controlled Loop
(b) Exit Controlled Loop
(c) Both of the above
(d) None of the above
35. Which of the following loop is not supported by the python programming language?
- (a) for loop (b) while loop
(c) do...while loop (d) None of the above
36. What is another word for 'iteration' ?
- (a) Selection (b) Assignment
(c) Sequencing (d) Repetition

37. Which term describes a loop that continues repeating without a terminating (ending) condition?
 (a) Infinite Loop (b) Conditional Loop
 (c) Unlimited Loop (d) Sequence Loop
38. To access a list which contains 5 elements, which of the following uses of range() would produce a list of the desired indexes?
 (a) range(1,5) (b) range(0,4)
 (c) range(5) (d) range(1,6)
39. What is the output of the following Python statements?
`x = 2`

- ```
while x < 9:
 print(x, end="")
 x = x + 1
```
- (a) 12345678 (b) 123456789  
 (c) 2345678 (d) 23456789
40. What is the output of the following Python statements?  
`b=1`  
`for a in range(1, 10, 2):`  
 `b+= a + 2`  
`print(b)`  
 (a) 31 (b) 33 (c) 36 (d) 39

### Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (a)  | 2. (c)  | 3. (c)  | 4. (c)  | 5. (c)  | 6. (a)  | 7. (c)  | 8. (b)  | 9. (a)  | 10. (d) |
| 11. (a) | 12. (c) | 13. (b) | 14. (b) | 15. (a) | 16. (d) | 17. (b) | 18. (c) | 19. (c) | 20. (c) |
| 21. (c) | 22. (a) | 23. (c) | 24. (d) | 25. (b) | 26. (b) | 27. (c) | 28. (a) | 29. (b) | 30. (a) |
| 31. (c) | 32. (c) | 33. (d) | 34. (a) | 35. (c) | 36. (d) | 37. (a) | 38. (c) | 39. (c) | 40. (c) |

### Answers with Explanations

**1. Option (a) is correct.**

**Explanation:** Variable is a container object that stores a meaningful value that can be used throughout the program. Each variable has a specific type, which determines the size and layout of the variable memory, and the set of operations that can be applied to the variable.

**2. Option (c) is correct.**

**Explanation:** Data types are used to identify the type of data and associated operations to handle it. Python has five standard data types as number, strings, lists, Tuples and Dictionary.

**3. Option (c) is correct.**

**Explanation:** elif keywords is used to add an alternative condition to an if statement.

Syntax If (Condition 1):

Statement 1

elif (condition 2):

Statement 2

else:

Statement 3

**4. Option (c) is correct.**

**Explanation:** Python language is also a portable language. For example, if we have python code for windows and if we want to run this code on other platforms such as Linux, Unix, and Mac then we do not need to change it, we can run this code on any platform. Python programs are also platform independent. Once we write a Python program, it can run on any platform without rewriting once again.

Portable means you can run it anywhere without setting up any installation they are standalone and does not require other tools. While platform independent means the same code runs on all platforms be it various processors OS's or devices.

**5. Option (c) is correct.**

**Explanation:** The range() is a built-in function in Python. Syntax of range() function is:

range([start], stop[, step])

The start and step parameters are optional. If start value is not specified, by default the list starts from 0. If step is also not specified, by default the value increases by 1 in each iteration. All parameters of range() function must be integers. The step parameter can be a positive or a negative integer excluding zero.

**6. Option (a) is correct.**

**Explanation:** When a formal set of rules defined for writing a program in a particular language is not followed then error raised is known as syntax error. Syntax errors occur when syntax rules of any programming language are violated.

**7. Option (c) is correct.**

**Explanation:** elif keywords is used to add an alternative condition to an if statement.

Syntax If (Condition 1):

Statement 1

elif (condition 2):

Statement 2

else:

Statement 3

**8. Option (b) is correct.**

**Explanation:** t[2] returns the element present in the 2nd index which is a list [30,40,50].

t[2][1] returns the element present at index 1 from the above list which is 40,

Since list is a mutable type, we can modify this element to 100.

**9. Option (a) is correct.**

**Explanation:** For loop will iterate for the length of the list L1. during each iteration the last element of list L1 is removed and returned by L1. pop(). Now L2. insert() will insert this element in list L2. Thus at the end of the loop L2 will have all the elements of L1 in reverse order and L1 will be empty.

**10. Option (d) is correct.**

**Explanation:** x is a nested list, where each element of



the list is also a list. So `x[1][2]` will access element at index position 2 of the list that is at index position 1 of the outer list.

**11. Option (a) is correct.**

**Explanation:** Median function calculate middle value of the arithmetic data in iterative order. If these are an odd number of values, median () returns the middle value. If these are an even number of values it returns an average of two middle values.

**12. Option (c) is correct.**

**Explanation:** Runtime error is when the statement is correct syntactically, but the interpreter cannot execute it. Runtime errors do not appear until after the program starts running or executing.

**13. Option (b) is correct.**

**Explanation:** A tuple is a collection of Python objects separated by comma (). Tuples are immutable by design which means they cannot be changed after creation. Tuple holds a sequence of heterogenous elements.

**14. Option (b) is correct.**

**Explanation:** keys of a dictionary are unique while values can be duplicate. So values can be accessed by specifying a key but not vice versa. Values are accessed as `<dict_name>[key]`.

**15. Option (a) is correct.**

**Explanation:** There can be error in a program. These errors or bugs prevent a program from being compiled or to give correct output.

**16. Option (d) is correct.**

**Explanation:** This code will create a blank dictionary D. now its keys are defined with range function and each of these keys are given value from the elements of the tuple T.

**17. Option (b) is correct.**

**Explanation:** Given code will give output `\nhello` because `\nhello` is in double quotes preceded by `r` hence the actual meaning of `\n` is escaped and is treated as a character. Consider as string.

**18. Option (c) is correct.**

**Explanation:** The function `Change` will display `HELLO@` then `print (S)` will display `WELCOME`. Here `S` is declared outside any function so it is a global variable and can be accessed anywhere in the program. Now when it is passed as an argument to the function `Change()` its formal argument `T` also starts pointing to the same value as `S`, i.e. `WELCOME`. inside the function `T` is assigned a new value `HELLO`, but `S` continues to point to `WELCOME`. so `Print (T, end=@)` prints `HELLO` and `Print(s)` prints `WELCOME`.

**19. Option (c) is correct.**

**Explanation:** The function `FunStr()` copies the contents of the passed argument string to the string `T` only if they are a digit and then returns the new string. When `X="PYTHON 3.9"` is passed to this function it returns `39`. now print statement first prints `X` and the returned string i.e. `39` with a `*` in between.

**20. Option (c) is correct.**

**Explanation:** Required arguments are also known positional argument. These arguments are the

arguments passed to a function in correct positional order.

**21. Option (c) is correct.**

**Explanation:** The code here will read all the contents of the text file using `read()` and `split()` will split these contents word wise and store them in list `L`. Now the for loop will iterate over these words. if `W.lower() == W[::-1].lower()`: will check if that word is a palindrome and print it if it is.

**22. Option (a) is correct.**

**Explanation:** `split()` function splits a line in columns. It returns columns as item of a list.

**23. Option (c) is correct.**

**Explanation:** In creating a list, brackets used are `[ ]`, `not()` or `{ }`.

**24. Option (d) is correct.**

**Explanation:** The `append()` method is used to append an item to the list at its end. Thus, `nameList.append("Kamal")` adds `Kamal` to the end of the list.

**25. Option (b) is correct.**

**Explanation:** It will delete an item `2.7` from the sublist as the index mentioned in the command indicates `2.7`.

**26. Option (b) is correct.**

**Explanation:** In a Python program, a control structure directs the order of execution of the statements in the program. Various types of control structure are sequence (simple program), selection (if, if-else, if-elif-else) and iteration (for, while).

**27. Option (c) is correct.**

**Explanation:** `elif` keywords is used to add an alternative condition to an if statement.

Syntax If (Condition 1):

```
Statement 1
elif (condition 2):
Statement 2
else:
Statement 3
```

**28. Option (a) is correct.**

**Explanation:** Syntax of if statement is:

```
if condition:
 statement(s)
```

otherwise it will execute other part of the program. Here the given input is `10`, which satisfies the `elif` condition, so its statement will execute.

**29. Option (b) is correct.**

**Explanation:** A decision involves selecting from one of the two or more possible options. In programming, this concept of decision making or selection is implemented with the help of `if..else` statement.

**30. Option (a) is correct.**

**Explanation:** If the conditional test is true, compiler will execute if statement part of the program, otherwise it will execute other part of the program. Here if condition is written `True` always, so its if statement part will execute.

**31. Option (c) is correct.**

**Explanation:** If the conditional test is true, compiler will execute if statement part of the program,

**32. Option (c) is correct.**

**Explanation:** Python uses indentation for block as well as for nested block structures. Leading

whitespace (spaces and tabs) at the beginning of a statement is called indentation.

In Python, the same level of indentation associates statements into a single block of code. The interpreter checks indentation levels very strictly and throws up syntax errors if indentation is not correct.

**33. Option (d) is correct.**

*Explanation:* The break statement alters the normal flow of execution as it terminates the current loop and resumes execution of the statement following that loop.

**34. Option (a) is correct.**

*Explanation:* An entry controlled loop has a control over entry in the loop in the form of a test condition. 'for' and 'while' loop are entry controlled loop in python

**35. Option (c) is correct.**

*Explanation:* There are two looping constructs in Python for and while.

**36. Option (d) is correct.**

*Explanation:* Repetition of a set of statements in a program is made possible using looping constructs.

**37. Option (a) is correct.**

*Explanation:* When the condition becomes false,

the loop terminates. It is the responsibility of the programmer to ensure that this condition eventually does become false so that there is an exit condition and it does not become an infinite loop.

**38. Option (c) is correct.**

*Explanation:* The range() is a built-in function in Python. Syntax of range() function is:

`range([start], stop[, step])`

The start and step parameters are optional. If start value is not specified, by default the list starts from 0. If step is also not specified, by default the value increases by 1 in each iteration. All parameters of range() function must be integers. The step parameter can be a positive or a negative integer excluding zero.

**39. Option (c) is correct.**

*Explanation:* x is initialized at 2, while loop will print the value of x until it is less than 9. After printing 8 the value of x becomes 9, so the loop ends.

**40. Option (c) is correct.**

*Explanation:* The for loop will iterate from 1 to 10(excluding) skipping 2 numbers every time. 'a' will take values 1,3,5,7,9. All these will be accumulated in b which is initialized at 1.

○○○



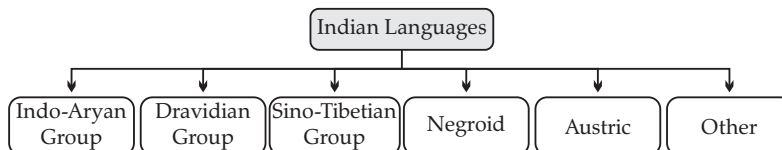
## CULTURE AND RELIGION

- Language in its literary meaning is a system of communication through speech, a collection of sounds that a group of people understand to have the same meaning.

A **language family** includes individual languages re-

lated through a common ancestor that existed before the recorded history.

**Dialect** is a **form of language spoken** in a local area. It should be noted that several dialects can be derived from a particular language.



## Important Mnemonics

| Indus Valley Sites |           |
|--------------------|-----------|
| Site               | River     |
| Harappa            | Ravi      |
| Mohenjodaro        | Indus     |
| Chanhudaro         | Indus     |
| Lothal             | Bhogava   |
| Kalibangam         | Ghaggar   |
| Dholavira          | Lumi      |
| Banawali           | Saraswati |

## UNESCO World Heritage Sites in India

- Ajanta Caves (Maharashtra)
- Ellora Caves (Maharashtra)
- Taj Mahal (Uttar Pradesh)
- Agra Fort (Uttar Pradesh)
- Sun Temple, Konark (Odisha)
- Group of Monuments at Pattadakal (Karnataka)
- Humayun's Tomb (Delhi)

## List of several places of cultural interest

- **Agra:** Situated on the banks of the Yamuna, this city is a medieval marvel. The Mughals made it their capital, which brought it to prominence. They constructed several monuments and gardens in the city.
- **Agartala:** It is the capital city of the state of Tripura and is situated on the banks of the Haora River, which provides the basic livelihood to them. The most important and old temple in the city is the Lakshmi Narayan Temple situated in the Ujjayanta Palace compound.
- **Guwahati:** It is one of the largest city in the state of Assam and the cultural capital of the North-East India. The city is also names as the 'Gateway of the North-east India.'
- **Goa:** Goa has become the most famous place for westerners to come to India. It is surrounded by Maharashtra on the north and Karnataka from the east and south. It is bounded by the Arabian Sea

on the west and this provides Goa with some of the best beaches in India.

- **Amritsar:** Initially named Ramdasapur, the modern day city of Amritsar is a historically important city in the state of Punjab. The Sri Harmandir Sahib or the Golden Temple is the most sacred Gurudwaras of the Sikh community.
- **Bodh Gaya:** It is situated in Bihar and it is one of the sacred places for those who believe in Buddhism. One of the most revered monuments in Bodh Gaya is the Mahabodhi Temple.
- **Hampi:** All the monuments and sculptures at Hampi were declared to be world heritage (sites) by UNESCO in 1986. The most iconic temple complex in Hampi is the Vittala Temple complex, which has the iconic stone chariot.
- **Khajuraho:** The Khajuraho group of monuments is situated in Madhya Pradesh. It is famous for the erotic temple structures. Most of the temples are constructed in the Nagara-style and were made by the Chandela dynasty that ruled over Bundelkhand.
- **Konark:** The famous Konark Sun temple is situated in the town of Konark in Odisha. This is one of the very few and famous Sun temples in India. It is also known as the 'Black Pagoda' because it is built out of black granite.
- **Rameshwaram:** The city of Ramerishwaram is one of the 'Char Dham' or four most important pilgrimages for Hindus. It is set on southernmost tip of the mainland of an island in the Gulf of Mannar.
- **Varanasi:** It has been historically known as 'Benaras' and 'Kashi' as it lies on the banks of the holy river Ganga. It is also known as the 'spiritual capital' of India as it is home to many Hindu temples and Buddhist sanctuaries. India has a long tradition of artistic excellence and painting is one of the major mediums that have been used to express it.

In the third century AD, Vatsyayana in his book Kamasutra mentioned about 6 main principles/limbs or shadanga of paintings.

| Styles of Paintings   | Major features            |
|-----------------------|---------------------------|
| <i>Cauka Pitaka</i>   | Isolated framed drawings  |
| <i>Dighala Pitaka</i> | Long scrolls of paintings |
| <i>Yama Pitaka</i>    | Isolated paintings        |

- **Makar Sankranti:** It is a festival dedicated to the Sun God, who is also considered to be the king of all planets. Makar Sankranti celebrates the movement of the Sun to the northern hemisphere.
- **Janmashtami:** It is the birth anniversary of Lord Krishna and is celebrated across the nation by the Hindu community.
- **Dussehra:** Dussehra is also known as 'Vijay-dashami' and is celebrated all over India in the honour of Lord Rama's victory over Ravana.
- **Eid-ul-Fitr:** This is one of the festivals celebrated by the Muslim community worldwide. The festival falls on the last day of the holy month of Ramadan (Ramzan), which is the ninth month of the Islamic calendar.
- **Muharram:** The festival of Muharram is a sad one, as it is associated with the death of Hussain, son of Ali.
- **Christmas:** This day is celebrated all across the world as the birth anniversary of Jesus Christ. It falls on the 25th of December every year.
- **Gurpurab:** The Sikh community all over the world celebrates it. Although Gurpurabs are celebrated for the birth anniversaries of all the 10 Sikh gurus but the most important are the Guru Nanak and Guru Gobind Singh.

#### Temples:

**Venkateswara Temple:** Tirumala at Tirupati in Chittoor District, Andhra Pradesh

**Sri Ranganthaswami Temple:** Nellore, Andhra Pradesh

**Umananda Devaloi:** Guwahati, Assam

**Mundeshwari Devi Temple:** Bihar

**Somnath Temple:** Gujarat

**Nageshwara Jyotirlinga:** Gujarat

**Virupaksha Temple:** Hampi, Karnataka

**Padmanabhaswamy Temple:** Thiruvananthapuram, Kerala

#### Harappan Art and Architecture:

A marked feature of this ancient civilisation was the vivid imagination. Bhairava form of Shiva is most prominent in the Brihadesvara Temple built by the Chola dynasty. The temple is one of the oldest and famous temples in India. It is situated in Thanjavur (Tanjore) in Tamil Nadu in India. Bhairava is the wandering form of Lord Shiva and it guards the cardinal points. There are 64 Bhairavas. These 64 Bhairavas are grouped under 8 categories and each category is headed by one major Bhairava. The major eight Bhairavas are called Astanga Bhairavas and artistic sensibilities exuded by the numerous sculptures, seals, potteries, jewellery found at the excavation sites. The towns were laid out in a rectangular grid pattern.

#### Mauryan Art and Architecture:

Mauryan art can be classified into Court Art & Popular

Art.

**Post Mauryan Art:** After the decline of the Mauryan Empire in 2nd century BC, small dynasties sprang up in various parts of India.

#### Inscriptions

- **Major Rock Inscriptions:** Minor Rock Edicts, the Queen's Edict, Barabar cave inscriptions and the Kandahar bilingual inscription. These inscriptions show Ashoka's devotion towards the Buddhist philosophy.
- **Prayag-Prashasti:** Prayag-Prashasti is the name given to the Allahabad Pillar. Prashasti means "in praise of someone" and is a eulogy. The Allahabad Prashasti was originally engraved on the Ashokan Pillar in Kausambhi near Allahabad. Later it was removed to the Allahabad fort.
- **Mehrauli Inscription:** The Mehrauli Iron Pillar is located in Delhi in the Qutub Minar complex. This pillar was established by Chandragupta-II of Gupta dynasty as Vishnupada in the honor of Lord Vishnu. A script is also known as writing system or orthography. The two ancient scripts in India comprise Brahmi script and Kharosthi script.

Most of the ancient and modern scripts in India are developed from Brahmi script be it Devanagari, Tamil, Telugu, Kannada, Odia, Assamese/Bengali, etc. Hence, it can very much be contended that Brahmi is the mother of scripts. Music is the soul of any culture and India has had a long tradition of musical ingenuity. It is said that Narada Muni (sage) introduced the art of music to earth. There are three main pillars of Indian classical music: Raga, tala and swara.

**Hindustani music:** practiced in the northern parts of India.

**Carnatic music:** practiced in the southern parts of India.

#### Gharana System

- **Gwalior Gharana:** It is one of the oldest and the most elaborate Khyal Gharana.
- **Kirana Gharana:** This gharana is named after the town named Kirana in Uttar Pradesh.
- **Agra Gharana:** Historians argue that Khuda Baksh established this Gharana in the 19<sup>th</sup> century but the musicologists argue that Haji Sujan Khan founded it.
- **Patiala Gharana:** Bade Fateh Ali Khan and Ali Baksh Khan started the Gharana in the 19<sup>th</sup> century.

#### Jainism

- Vardhamana Mahavira was the 24<sup>th</sup> Tirthankara of Jain tradition.
- He was born at Kundagrama near Vaishali to Kshatriya's parents Siddharta and Trisala.
- At the age of 30 years, he became ascetic and wandered for 12 years.
- Jainism is divided into two sects Svetambaras (white clad) by sthulababu and Digambaras (sky-clad or naked) by Bhadrababu.
- First Jain council held at Pataliputra in 3rd century BCE and was presided by Sthulbadhra.
- Second Jain Council held at Vallabhi is 512 CE and was presided by Devardhi kshmasramana.

**Buddhism:**

- Gautam or Siddharta, the founder of Buddhism, was born in 567 BCE in Lumbini Garden near Kapilvastu.
- He left home at the age of 29 in search of truth.
- Under a bodhi tree at Bodhgaya, he got enlightenment or nirvana, at the age of 35.
- He delivered his first Sermon at Sarnath near Benaras.
- He died at the age of 80 at Kushinagar.

- First Buddhism council was held at Rajagriha under the chairmanship of Mahakasyapa immediately after the death of Buddha.
- Second Buddhist council was convened at Vaishali around 383 BCE.
- Third Buddhist council held at Patliputra under the patronage of Asoka.
- Fourth Buddhist council was convened in Kashmir by Kanishka under the chairmanship of Vasumitra. Asvaghosa participated in council.

**EXERCISE**

- Early Buddhist sculptors did not show Buddha in human form. Through which of the following symbols was Buddha's presence shown by the early sculptors?
  - Empty Seat and Stupa only
  - Wheel and Tree only
  - Wheel, Tree and Stupa only
  - Empty Seat, Wheel, Tree and Stupa
- At which one of the following places was a Shiva temple **not** constructed under the patronage of the Chola rulers?
  - Chidambaram
  - Thanjavur
  - Gangaikonda Cholapuram
  - Naneghat
- Which one of the following is the correct meaning of *ziyarat*?
  - Pilgrimage to the tombs of sufi saints
  - The practice of revenue farming
  - The death anniversary of a sufi shaiikh
  - A form of Islamic divorce
- At which among the following Harappan sites are fire altars found?
 

|                 |                 |
|-----------------|-----------------|
| (a) Kalibangan  | (b) Harappa     |
| (c) Mohenjodaro | (d) Rakhi Garhi |
- Harappan crafts display an amazing degree of standardization. According to Kenoyer, what was the possible reason for such an achievement?
  - Availability of raw materials at local level
  - Centralized markets for crafts
  - Specialized training centres for craftsmen
  - State control
- The town of Chisht, from which the Sufi Chishti Silsila derives its name, is located in
 

|                    |                         |
|--------------------|-------------------------|
| (a) Western Turkey | (b) Central Afghanistan |
| (c) Eastern Iran   | (d) Eastern Iraq        |
- Match List-I with List-II and select the correct answer using the code given below the Lists:

| List-I (Temple) |                   | List-II (Town) |              |
|-----------------|-------------------|----------------|--------------|
| A.              | Kailasanathar     | 1.             | Bhubaneshwar |
| B.              | Lingaraj          | 2.             | Khajuraho    |
| C.              | Kandariya Mahadev | 3.             | Mount Abu    |
| D.              | Dilwara           | 4.             | Kanchipuram  |

**Code:**

- |     |           |          |          |   |
|-----|-----------|----------|----------|---|
|     | <b>AB</b> | <b>C</b> | <b>D</b> |   |
| (a) | 4         | 2        | 1        | 3 |
| (b) | 4         | 1        | 2        | 3 |

- |     |   |   |   |   |
|-----|---|---|---|---|
| (c) | 3 | 1 | 2 | 4 |
| (d) | 3 | 2 | 1 | 4 |
- Which of the following statements about the musical culture in 18<sup>th</sup> and 19<sup>th</sup> century South India is/are correct?
    - Musical developments were spearheaded by the Arcot court.
    - Tanjavur replaced Madras as the cultural capital of classical music in the second half of the nineteenth century.
    - Three great composers, Tyagaraja, Dikshitar and Syama Sastri experimented with the kriti form to set the foundations for modern Carnatic music.
 Select the correct answer using the code given below:
 

|                  |                |
|------------------|----------------|
| (a) 1 and 3 only | (b) 2 only     |
| (c) 3 only       | (d) 1, 2 and 3 |
  - Borobudur is the site of:
    - a huge temple of Vishnu and Shiva in Java, built in the 12<sup>th</sup> Century AD.
    - an enormous Stupa in Java, built in the 8<sup>th</sup> Century AD.
    - a magnificent palace of a Chola King in Tamil Nadu.
    - a Jain Monastery in Gujarat.
  - Which of the following statements about visual culture in 18<sup>th</sup> and early 19<sup>th</sup> century North India is/are correct?
    - Painters from Patna and Murshidabad flocked to Calcutta and produced water colours in the English mode.
    - Landscape and portraiture became extremely important at this time.
    - The artists Zayan-al-Din, Bhawani Das and Ram Das were adopted by the English East India Company to produce albums of Indian birds and animals.
    - While Indians were influenced by European artistic styles, European artists did not visit regional courts.
 Select the correct answer using the code given below:
 

|                |                  |
|----------------|------------------|
| (a) 1, 2 and 3 | (b) 2 and 3 only |
| (c) 1, 3 and 4 | (d) 4 only       |
  - Consider the following statements about votive inscriptions in the second century BC:
    - They record gifts made to religious institutions
    - They tell us about the idea of transference of the meritorious results of the action of one person to another person.

- Which of the statements given above is/are correct?  
 (a) 1 only (b) 2 only  
 (c) Both 1 and 2 (d) Neither 1 nor 2
12. Which one of the following is the common element between the Kailasanatha Temple at Ellora and the Shore Temple at Mamalla-puram?  
 (a) Both are examples of Nagara architecture  
 (b) Both are carved out from solid rocks  
 (c) Both are Gupta period temples  
 (d) Both were built under the patronage of Pallava Kings
13. What form of Shiva is most prominent in the Brihadishvara Temple built by the Chola dynasty?  
 (a) Harihara (b) Bhairava  
 (c) Rudra (d) Tripurantaka
14. Which of the following is/are **not** depicted in the Rajput paintings?  
 1. The stories of Krishna  
 2. *Ragas and Raginis*  
 3. The deeds of Hamza  
 4. The deeds of Babur  
 Select the correct answer using the code given below:  
 (a) 1, 2 and 3 (b) 2, 3 and 4  
 (c) 3 and 4 only (d) 4 only
15. Who deciphered the *Brahmi* and *Kharoshthi* scripts?  
 (a) Piyadassi  
 (b) Colin Mackenzie  
 (c) Alexander Cunningham  
 (d) James Prinsep
16. Khasi language is included in:  
 (a) Munda branch of Austro-Asiatic sub-family  
 (b) Mon-Khmer branch of Austro-Asiatic sub-family  
 (c) North Assam branch of Sino-Tibetan family  
 (d) Assam-Myanmari branch of Sino-Tibetan family
17. In the Gandhara School of Art, initially blue schist and green phyllite were used. When did stucco completely replace stone as main material used by Gandhara School sculptors?  
 (a) 1<sup>st</sup> century CE (b) 2<sup>nd</sup> century CE  
 (c) 3<sup>rd</sup> century CE (d) 5<sup>th</sup> century CE
18. 'Naropa' is an annual festival of:  
 (a) Sikkim (b) Ladakh  
 (c) Arunachal Pradesh (d) Nagaland
19. The major part of Central Asia is dominated by which one of the following language families?  
 (a) Indo-European (b) Sino-Tibetan  
 (c) Austric (d) Altaic
20. Which one of the following is not among the principal languages of Jammu and Kashmir?  
 (a) Urdu (b) Gujarati  
 (c) Koshur (d) Monpa

### Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d)  | 2. (d)  | 3. (a)  | 4. (a)  | 5. (a)  | 6. (b)  | 7. (b)  | 8. (c)  | 9. (b)  | 10. (a) |
| 11. (c) | 12. (b) | 13. (b) | 14. (c) | 15. (d) | 16. (b) | 17. (c) | 18. (b) | 19. (d) | 20. (d) |

### Answers with Explanations

**1. Option (d) is correct.**

**Explanation:** Early sculptors did not depict the Buddha in human form but instead showed his presence through symbols.

- **The empty seat** – Meditation of the Buddha.
- **The Stupa** – Mahaparinirvana
- **The wheel** – Stood for the First Sermon of the Buddha delivered at Sarnath.
- **The tree** – Symbolizes an event (Tree of awakening) in the life of the Buddha.

**2. Option (d) is correct.**

**Explanation:** The Chola Dynasty was situated in the southern state of Tamil Nadu which was one of the longest-ruling dynasties in the world's history. (The three great Chola temples constructed between 11<sup>th</sup> and 12<sup>th</sup> century are listed into UNESCO's World Heritage Sites). These are:

1. The Brihadishvara temples of Thanjavur,
  2. Gangaikonda cholisvaram, and
  3. The Airatesvara temple at Darasuram.
- (These three temples are built in Dravidian style of temple architecture)

**3. Option (a) is correct.**

**Explanation:** Ziyarat is an Arabic word which literally means 'visit'. It is used to refer to a form of pilgrimage

to sites associated with Muhammad, his family members and descendants, his companions and other venerated figures in Islam such as the prophets, Sufi Saints and Islamic scholars.

**4. Option (a) is correct.**

**Explanation:** The Indus Valley Civilization is one of the oldest civilizations in the world. Kalibangan in Rajasthan was a major provincial capital of the Indus Valley Civilization. During the excavation in this Harappan site, archaeologists have found Yagvedi (Fire Alter). Kalibanga is known for its unique fire altars and modern farming system.

**5. Option (a) is correct.**

**Explanation:** Harappan crafts are impressively standardized. According to Kenoyer, the high level of standardization of crafts may have been a result of state control. Numerous sculptures, seals, potteries, and jewelleryes found at excavation sites demonstrate the vivid imagination and artistic sensibility of this ancient civilisation. This was made possible due to the availability of raw materials at the local level

**6. Option (b) is correct.**

**Explanation:**

- The city of Chisht, from which the Sufi Chishti Silsila derives its name, is found in the central Islamic State of Afghanistan. The Chishti Order is a tariqa, an order or school within the mystic



- Sufi tradition of Sunni Islam.
- The Chishti Order is primarily followed in Afghanistan and the Indian subcontinent.
- It was the first of the four main Sufi orders (Chishti, Qadiri, Suhrawardi and Naqshbandi) to be established in this region.
- Khwaja Muinuddin Chishti introduced the Chishti Order in Ajmer, Rajasthan.

**7. Option (b) is correct.**

*Explanation:*

|    |                   |    |                                                                                                                   |
|----|-------------------|----|-------------------------------------------------------------------------------------------------------------------|
| A. | Kailasanathar     | 4. | The Kailasanathar temple is a Pallava - era historic Hindu temple in Kanchipuram, Tamil Nadu.                     |
| B. | Lingaraj          | 1. | Lingaraj Temple is a rich ornamental shrine in Bhubaneswar.                                                       |
| C. | Kandariya Mahadev | 2. | It is the largest and most ornate Hindu temple in the medieval temple group found at Khajuraho in Madhya Pradesh. |
| D. | Dilwara           | 3. | The Dilwara Temples or Delvada Temples are a group of svetambara Jain temples located near Mount Abu.             |

**8. Option (c) is correct.**

*Explanation:* From the above options, only statement (3) is correct regarding the musical culture in 18<sup>th</sup> and 19<sup>th</sup> century in South India. In 18<sup>th</sup> and 19<sup>th</sup> century in, the foundation of modern music was laid in South India, especially in Karnataka. Three composer-musicians named, Tyagaraja, Muthuswami Dikshitar and Syama Sastri laid the foundation of modern music in this regime. These three musicians are known in the music culture as the Trimurti/Trinity of Carnatic music. The people of Karnataka also affectionately promote them as the three Jewels of Carnatic music.

**9. Option (b) is correct.**

*Explanation:* Borobudur is the site of an enormous Stupa in Java built in the 8<sup>th</sup> Century AD for the devotees of Mahayana Buddhism during the reign of the Shailendra dynasty. The temple's design is based on Gupta's Gupta architecture which reflects India's influence on the region. UNESCO designated Borobudur a World Heritage site in 1991 as 'Borobudur Temple Compounds' in recognition of its unique and magnificent craftsmanship.

**10. Option (a) is correct.**

*Explanation:* Water colours were produced in the English mode as painters from Patna and Murshidabad flocked to Calcutta to do this. Besides, landscape and portraiture became extremely important at this time. The three great Indian artists Zaynaldin, Bhawani Das and Ram Das -adept in Mughal paintings – were adopted by the English East India

Company to produce albums of Indian birds and animals.

**11. Option (c) is correct.**

*Explanation:* Both the statements about the votive inscriptions in the second century BC is correct. Votive inscriptions denote an object as a votive offering to a deity (or deities) by one or more person. These are among the oldest inscriptions. They may express gratitude for victory in a battle or for a merchant ship that has returned home safely. Votive inscriptions were often made because of a vow taken in a moment of danger.

**12. Option (b) is correct.**

*Explanation:* The common element between the Kailasanatha Temple at Ellora and the Shore Temple at Mamallapuram is that both are carved out from solid rocks. Kailasanatha Temple was constructed in 8<sup>th</sup> century by Rashtrakuta king Krishna I in 756 - 773 CE. Shore Temple at Mamallapuram was built by Narasimhavarman II in the 8<sup>th</sup> century. It is the earliest significant free-standing stone temple in Tamil Nadu.

**13. Option (b) is correct.**

*Explanation:* Bhairava form of Shiva is most prominent in the Brihadesvara Temple built by the Chola dynasty. The temple is one of the oldest and famous temples in India. It is situated in Thanjavur (Tanjore) in Tamil Nadu in India. Bhairava is the wandering form of Lord Shiva and it guards the cardinal points. There are 64 Bhairavas. These 64 Bhairavas are grouped under 8 categories and each category is headed by one major Bhairava. The major eight Bhairavas are called Astanga Bhairavas.

**14. Option (c) is correct.**

*Explanation:* The deeds of Hamza and the deeds of Babur are not depicted in the Rajput paintings. Rajput paintings depict a number of themes, events of epics like the Ramayana and the Mahabharata, Krishna's life, Ragas and Raginis, beautiful landscapes, and humans. Miniatures were the preferred medium of the Rajput paintings, and these paintings were even done on the walls of the palaces, inner chambers of the forts, havelis, particularly, the havelis of Shekhawati.

**15. Option (d) is correct.**

*Explanation:* James Prinsep deciphered the Brahmi and Kharoshthi scripts. James prinsep was an English scholar, orientalist and antiquary. He was the founding editor of the journal of the Asiatic society of Bengal. The first evidence for the use of Brahmi script in India comes from around 250 B.C. It was used to write several different languages including Sanskrit and prakrit. Over 200 modern Indian scripts are based on Brahmi script. Brahmi script is written from left to right and was used to write on a variety of materials such as a stone, terracotta and metal. Kharosthi was a script used in the north-western region of India between about the 5<sup>th</sup> century B.C and the third century A.D. It was used to write many different local languages like Prakrit.

**16. Option (b) is correct.**

**Explanation:**

- Khasi language is included in Mon-Khmer branch of Austro-Asiatic sub-family.
- It is an Austro-Asiatic language spoken primarily in the Meghalaya state by the Khasi people.
- It is the official language of Meghalaya since 2005.
- It is written using the Latin and Bengali-Assamese scripts.

**17. Option (c) is correct.**

**Explanation:** Initially, grey-blue schist and green phyllite were predominantly used for making the sculptors. However, by the 3<sup>rd</sup> century CE stucco completely replaced stone as material used for sculptors.

**18. Option (b) is correct.****Explanation: Naropa Festival**

- It is also known as Kumbh of Himalayas
- It is celebrated in Ladakh at the famous Hemis Monastery

- It is celebrated in remembrance of the renowned Buddhist philosopher and teacher Naropa.

**19. Option (d) is correct.****Explanation:**

**The Central Asia comprises five countries:**

- Kazakhstan
- Kyrgyzstan
- Tajikistan
- Turkmenistan
- Uzbekistan.

Most of these countries speak Turkic language which is considered to be the part of Altaic language family.

**20. Option (d) is correct.**

**Explanation:** The official languages of Jammu and Kashmir are - Kashmiri, Dogri, Urdu, Hindi, and English. Koshur or Kashmiri is a widely spoken local language in Jammu and Kashmir. More than 50% can speak this language. Gujari or Gojri language is also spoken by people of Jammu & Kashmir. The Takpa or Dakpa language which is known in India as Tawang Monpa is an East Bodish language which is spoken in the Tawang district of Arunachal Pradesh.

○○○

## Soil, Mountains, Ports, Inland Harbours

### Physical Feature

- The physiographic divisions of India are – The Himalayan Range of Mountain
- The Peninsular Plateau
- The Great Plains of India
- The Coastal Plains
- The Islands of India

### The Himalayas

- Himalayas are young fold mountains of tertiary period, which were folded over Tethys Sea due to inter-continental collision.
- They are one of youngest fold mountain ranges in the world and comprise mainly sedimentary rocks.
- The Himalayas stretch across the north-eastern portion of India.
- It covers approximately 1,500 mi (2,400 km) and passes through India, Pakistan, Afghanistan, China, Bhutan and Nepal.

### They consist of three parallel ranges

1. Himadri (Greater Himalayas)
2. Himachal (Lesser Himalayas)
3. Shivalik (Outer Himalayas)

### The Great Plains

- The Great plains of northern India, also known as Ganga–Sutlej Plains, are transitional belt between the Himalayas and Peninsular India.

### Climatic Regions of India

| Type                              | Area                                                                       | Characteristic                                                                                                   |
|-----------------------------------|----------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| Tropical Rain Forests             | Western Ghats/ West Coastal Plains, Parts of Assam                         | High temperature throughout the year, heavy seasonal rainfall, annual rainfall 200 cm annually (Mar to November) |
| Tropical Savana Climate           | Most of Peninsular region (except leeward side of Western Ghats)           | Dry winters, annual rainfall varies from 76 cm to 150 cm.                                                        |
| Tropical Semi-Arid-Steppe Climate | Rain shadow belt running Southward from Central Maharashtra to Tamil Nadu, | Low rainfall varies from 38 cm to 80 cm and temperature from 20° to 30°C.                                        |
| Tropical and Sub-tropical Steppes | Punjab, Haryana and Kachchh region.                                        | Temperature varies from 120-350C.                                                                                |

| Season of India              |                           |
|------------------------------|---------------------------|
| Winter Season                | Mid December to Mid-March |
| Summer Season                | Mid-March to May          |
| Rainy Season                 | June to September.        |
| Season of Retreating Monsoon | October to Mid-December.  |

tween the Himalayas and Peninsular India.

- The Great plains cover an area of 7,74,000 km<sup>2</sup> (3,00,000 square miles) having west–east length of 2400 km and north-south width of 144 km.

### South to North Doabs

| Doab            | Region                    |
|-----------------|---------------------------|
| Bist Doab       | Between Beas and Sutlej   |
| Bari Doab       | Between Beas and Ravi     |
| Rechna Doab     | Between Ravi and Chenab   |
| Chaj Doab       | Between Chenab and Jhelim |
| Sind Sagar Doab | Between Jhelim and Indus  |

### The Coastal Plains

- On the basis of location and active geomorphological processes, it can be divided into
  1. Eastern Coastal Plains
  2. Western Coastal Plains
- Archipelago is the group of islands.

### Climate

**Monsoon:** A type of wind system, in which there is almost complete reversal of prevailing wind direction.

### Types

1. South West Monsoon (June to September)
1. North East Monsoon (September to December)



**Natural Vegetation of India**

| Name                     | Climatic Requirement                                                       | Feature                                                                                            | Important Species                                      | Found in Area                                                                                                      |
|--------------------------|----------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|--------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|
| Tropical Wet Evergreen   | Rainfall > 250 cm<br>Temperature 25-27°C<br>Humidity 80% or more           | Dense forest, tall trees                                                                           | Mesa, Dhup, White Cedar, Jamun, Bamboo, Agar and Hopes | North-East India, Western slopes of Western Ghats, Andaman and Nicobar Islands                                     |
| Tropical Semi-Evergreen  | Rain > 200-250 cm. Temperature 24-27°C Humidity 80%                        | Evergreen mixed with deciduous, Height 24-36 cm                                                    | Semul, Rosewood, Indian Chestnut, Kustun, Mesua        | Lower slopes of Eastern Himalayas. Odisha Coast                                                                    |
| Tropical Dry Evergreen   | Areas receive rain from North-East Monsoon, Temperature 28°C, Humidity 74% | Presence of canopy, low height, about 9-12 m                                                       | Kliimi, Jamun, Tamarind, Neem, Cane                    | Coromandel Coast of Tamil Nadu                                                                                     |
| Tropical Moist Deciduous | Moderate rainfall of 150-200 cm. Temperature 26-27°C, Humidity 60-80%      | Trees shed their leaves in the dry season                                                          | Sal, Teak, Sandalwood, Ebony, Mahua, Shisham           | Western, Ghats, Eastern coastal plains, Eastern Plateau                                                            |
| Tropical Dry Deciduous   | Rainfall < 150 cm, Dry season                                              | Undergrowth is shrubby and grassy, trees shed their leaves in the dry season                       | Sal, Teak, Khair, Palash, Tendu, Laurel                | Uttar Pradesh, Tamil Nadu, Western Ghats, Rajasthan and West Bengal                                                |
| Tropical Thorny          | Rainfall 50-70 cm, Temperature 25-27°C, Humidity < 47%                     | Trees are stunted (6-9 m), trees have long roots, sharp spines and glossy leaves to conserve water | Babul, Acacia, Kirair, Khejri                          | South-Western Punjab, Western Haryana and Uttar Pradesh, Western Madhya Pradesh, Kachchh and Saurashtra, Rajasthan |
| Tidal/Littoral Mangrove  | Rainfall > 200 cm, high water salinity and areas are flooded regularly     | Trees are evergreen, breathing roots called pneumatophores                                         | Keora, Amur, Sundari, Agar, Bhendi, Nipa               | Delta regions of Ganga, Mahanadi, Godavari and Krishna                                                             |

**Soils in India**

| Types                   | States where Found (Occurrence)                                                                       | Composition                                                                                     | Crop Grown                                                                                  |
|-------------------------|-------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| Alluvial                | Punjab, Haryana, Uttar Pradesh, Bihar and Jharkhand                                                   | Rich in potash and lime but deficient in nitrogen and phosphorus.                               | Large variety of rabi and kharif crops such as wheat, rice, sugarcane, cotton and jute etc. |
| Black soil (Regur soil) | Deccan Plateau, Valleys of Krishna and Godavari, Andhra Pradesh, Madhya Pradesh and Tamil Nadu.       | Rich in iron, lime, aluminium, magnesium, calcium, but lacks in nitrogen, phosphorus and humus. | Cotton sugarcane, jowar, tobacco, wheat and rice.                                           |
| Red                     | Eastern parts of Deccan Plateau, Tamil Nadu, Goa, Odisha and Meghalaya.                               | Rich in iron and potash, but deficient in lime, nitrogen, phosphorus and humus.                 | Wheat, rice, cotton sugarcane and pulses.                                                   |
| Laterite                | Summits of Eastern and Western Ghats, Assam hills, Andhra Pradesh, Karnataka, West Bengal and Odisha. | Rich in iron but poor in silica, lime phosphorus, potash and millets. humus.                    | Tea, coffee, rubber, cashew and millets                                                     |

|                                                      |                                                                                             |                                                                                              |                                                                                                                                                       |
|------------------------------------------------------|---------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| Desert                                               | West and North-West India, Rajasthan, North Gujarat and Southern Punjab.                    | Rich in soluble salts, but deficient in organic matter.                                      | Generally unsuitable for cultivation, but with irrigation useful for cultivation of drought-resistant lime; millets, barley, cotton, maize and pulse. |
| Mountain                                             | Hills of Jammu and Kashmir, Uttarakhand and Assam hills.                                    | Rich in iron and humus, but deficient in lime.                                               | With fertilisers, tea, fruits and medicinal plants can be grown.                                                                                      |
| Saline (Reh, Kallar, Usar, Thur, Rukar) and Alkaline | Drier Parts of Bihar, Jharkhand, Uttar Pradesh, Haryana, Punjab, Rajasthan and Maharashtra. | Many salts such as sodium, magnesium and calcium.                                            | Unfit for agriculture.                                                                                                                                |
| Peaty and Marshy                                     | Kerala, coastal regions of Odisha, Tamil Nadu and Sundarbans of West Bengal.                | Contain large amount of soluble salts and organic matter, but lack in potash and phosphates. | Useful for rice and jute cultivation.                                                                                                                 |

- Water drains in two directions of the main water divide line of India. Ninety percent of the rivers drain into Bay of Bengal and the rest drains into Arabian Sea.

**In India, the rivers can be divided into two main groups.**

1. Himalayan rivers.
2. Peninsular rivers.

#### **The Himalayan River System**

The Himalayan River system is divided into three major river systems.

#### **The Indus System**

Indus, also known as Sindhu, originates from a glacier near Bokhar Chu in the Tibetan region near Mansarovar Lake. It's most important tributaries are Jhelum, Chenab, Ravi, Beas and Sutlej.

#### **The Ganga System**

Ganga rises from the Gangotri glacier near Gaumukh (3900 m) in the Uttarakashi district of

#### **Important Rivers of India**

Uttarakhand. The left bank of tributaries of Ganga is Ramganga, Gomti, Kali or Sarda, Gandak, Kosi, Ghaghara. The right bank tributaries of Ganga are Yamuna and Son.

#### **The Brahmaputra System**

- It is one of the largest rivers of the world.
- Brahmaputra forms large number of riverine islands. Majuli is the largest riverine island in the world.

#### **The Peninsular River System**

- Peninsular rivers can be divided into two groups:
- East flowing Rivers (or Delta forming rivers) East flowing rivers drain into Bay of Bengal.
- Except some small rivers, East flowing rivers do not form estuaries.
- West flowing Rivers (or Estuaries forming rivers) West flowing rivers form Estuaries.
- West flowing rivers drain into Arabian Sea.

| Name        | Originates from                         | Falls into      |
|-------------|-----------------------------------------|-----------------|
| Ganges      | Gangotri Glacier                        | Bay of Bengal   |
| Sutlej      | Manasarovar, Rakas Lakes                | Chenab          |
| Indus       | Near Mansarovar Lake                    | Arabian Sea     |
| Ravi        | Kullu Hills near Rohtang Pass           | Chenab          |
| Beas        | Near Rohtang Pass                       | Sutlej          |
| Jhelum      | Verinag in Kashmir                      | Chenab          |
| Yamuna      | Yamunotri                               | Ganga           |
| Chambal     | Singar Chouri Peak, Vindhyan escarpment | Yamuna          |
| Ghaghara    | Matsatung Glacier                       | Ganga           |
| Kosi        | Near Gosai Dham Peak                    | Ganga           |
| Betwa       | Vindhyan                                | Yamuna          |
| Son         | Amarkantak                              | Ganga           |
| Brahmaputra | Near Mansarovar Lake                    | Bay of Bengal   |
| Narmada     | Amarkantak                              | Gulf of Khambat |
| Tapti       | Betul District in Madhya Pradesh        | Gulf of Khambat |
| Mahanadi    | Raipur District in Chhattisgarh         | Bay of Bengal   |

|             |                                   |                 |
|-------------|-----------------------------------|-----------------|
| Luni        | Aravallis                         | Rann of Kachchh |
| Ghaggar     | Himalayas                         | Near Fatehabad  |
| Sabarmati   | Mewar hill, Aravallis             | Gulf of Khambat |
| Krishna     | Western Ghats                     | Bay of Bengal   |
| Godavari    | Nasik district in Maharashtra     | Bay of Bengal   |
| Cauveri     | Brahmagiri Range of Western Ghats | Bay of Bengal   |
| Tungabhadra | Western Ghats                     | Krishna         |

## EXERCISE

- In India, rigid and stable elevated lands, denuded rocks and series of scarps are the features of which of the following?
  - Northern mountains
  - Peninsular plateau
  - Northern plains
  - Coastal plains
- A person had visited a region in India and found trees, such as Khair, Neem, Khejri and Palas. Which one of the following regions is she/he expected to have visited?
  - Malabar Coast
  - Garro Hills
  - Sunderban Delta
  - Desert Region
- If you want to observe moraines, eskers and outwash plains, which one of the following Union Territories you may have to visit?
  - Andaman and Nicobar Islands
  - Lakshadweep
  - Puducherry
  - Ladakh
- Consider the following statements:
  - The Sharavati River creates Jog Falls.
  - The Brahmani River is formed by the confluence of the Kosi and Sankh rivers.
  - The Tamraparni is a river of Tirunelveli district of Tamil Nadu.
 How many of the above statements is/are correct?
  - 1
  - 2
  - 3
  - None
- Which one of the following rivers is not a tributary of river Brahmaputra?
  - River Manas
  - River Kameng
  - River Mahananda
  - River Subansiri
- Basket-of-eggs topography is related to:
  - Drumlins.
  - Eskers.
  - Cirques.
  - Moraines.
- The natural vegetation which covers the maximum geographical areas of India is:
  - Tropical deciduous forests.
  - Tropical thorn forests.
  - Montane forests.
  - Tropical evergreen forests.
- River Beas, flowing from Himachal and Punjab, joins the river
  - Indus.
  - Satluj.
  - Chenab.
  - Ravi.
- Which one of the following is a crater lake in India?
  - Lonar Lake
  - Sambhar Lake
  - Chilika Lake
  - Vembanad Lake
- Fine-grained bed of ephemeral lake in a desert is also known as:
  - Playa
  - Oasis
  - Drumlin
  - Natural levee
- Which one of the following factors does **not** affect the distribution of groundwater?
  - Amount of precipitation
  - Rate of evaporation
  - Ability of the ground surface to allow water to infiltrate into the groundwater system
  - Distance from the sea
- Which one of the following statements about biodiversity is **not** correct?
  - The term 'biodiversity' was coined by Walter G. Rosen in 1986.
  - The term 'biodiversity hotspots' was coined by Norman Myers in 1988.
  - The regions having richest biodiversity are called 'biodiversity hotspots'.
  - More than 100 hotspots of biodiversity are identified in the world.
- Which one of the following is not a soil forming factor?
  - Parent material
  - Topography
  - Climate
  - Human habitation
- Match List I with List II and select the correct answer using the code given below the lists:
 

| List-I (Dam) |          | List-II (State) |           |
|--------------|----------|-----------------|-----------|
| A.           | Hirakund | (1)             | Gujarat   |
| B.           | Panchet  | (2)             | Bihar     |
| C.           | Kosi     | (3)             | Odisha    |
| D.           | Ukai     | (4)             | Jharkhand |

Code:

|     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 3 | 4 | 2 | 1 |
| (b) | 3 | 2 | 4 | 1 |
| (c) | 1 | 2 | 4 | 3 |
| (d) | 1 | 4 | 2 | 3 |

- Which one of the following features is the result of erosion and deposition work of a river?
  - Pothole
  - Oxbow lake
  - Levee
  - Rapid
- Which one of the following Indian state does NOT share international border with two or more countries?
  - Arunachal Pradesh
  - Assam
  - Mizoram
  - Tripura
- The duration of monsoon in India extends for an average period of:
  - 80 - 140 days
  - 100 - 120 days
  - 90 - 130 days
  - 100 - 140 days

18. Which one of the following is NOT a reason of decrease in biodiversity?  
 (a) Large scale deforestation  
 (b) Exploitation of forest produces  
 (c) Maintaining sacred groves  
 (d) Encroachment in forest areas
19. Which one of the following is NOT a cause of depletion in groundwater?  
 (a) Afforestation  
 (b) Loss of forests  
 (c) Excessive pumping of groundwater  
 (d) Construction of large scale concrete buildings
20. According to the Koppen climatic classification, the letter code Cfa denotes:  
 (a) Tropical wet climate  
 (b) Humid subtropical climate  
 (c) Tundra climate  
 (d) Tropical wet and dry climate
21. Consider the following places of India:  
 (1) Itanagar (2) Imphal  
 (3) Agartala (4) Aizawl  
 Which one of the following is the correct chronological order of the above places in terms of sunrise time?  
 (a) 3-2-1-4 (b) 2-1-4-3  
 (c) 1-4-3-2 (d) 4-3-2-1
22. Consider the following Wildlife Sanctuaries of India:  
 (1) Shikari Devi (2) Bhadra  
 (3) Simlipal (4) Pachmarhi  
 Which one of the following is the correct order of the above Wildlife Sanctuaries in terms of their location from south to north?  
 (a) 1-2-3-4 (b) 2-4-3-1  
 (c) 2-3-4-1 (d) 3-1-2-4
23. Spruce and Cedar are tree varieties of:  
 (a) Equatorial forest  
 (b) Temperate coniferous forest  
 (c) Monsoon forest  
 (d) Temperate deciduous forest
24. 'Sal' tree is a:  
 (a) Tropical evergreen tree  
 (b) Tropical semi-evergreen tree  
 (c) Dry deciduous tree  
 (d) Moist deciduous tree
25. In Thar region, the shifting sand dunes are locally known as:  
 (a) Dhrian (b) Dauris  
 (c) Dhoros (d) Dhaya
26. Match List I with List II and select the correct answer using the code given below the Lists:

| List-I<br>(River) |        | List-II<br>(Mouth of the river) |           |
|-------------------|--------|---------------------------------|-----------|
| A.                | Danube | (1)                             | North sea |

|    |       |     |                   |
|----|-------|-----|-------------------|
| B. | Rhine | (2) | Black sea         |
| C. | Rhone | (3) | Bay of Biscay     |
| D. | Loire | (4) | Mediterranean sea |

Code :

- |     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 2 | 1 | 4 | 3 |
| (b) | 3 | 4 | 1 | 2 |
| (c) | 2 | 4 | 1 | 3 |
| (d) | 3 | 1 | 4 | 2 |
27. Which one of the following Himalayan Rivers does not originate from across the Himalayas?  
 (a) Indus (b) Satluj  
 (c) Ganga (d) Brahmaputra
28. Rudraprayag is situated at the confluence of rivers Alaknanda and:  
 (a) Bhagirathi (b) Mandakini  
 (c) Nandakini (d) Dhauliganga
29. The place located at the confluence of Alaknanda and Bhagirathi rivers is:  
 (a) Badrinath (b) Rishikesh  
 (c) Rudraprayag (d) Devprayag
30. Which one of the following rivers is not a tributary of river Brahmaputra?  
 (a) River Manas (b) River Kameng  
 (c) River Mahananda (d) River Subansiri
31. Krishna Raja Sagara Dam/Reservoir is developed on:  
 (a) River Krishna (b) River Tungabhadra  
 (c) River Godavari (d) River Kavery
32. Paradeep Port is located on the delta of river:  
 (a) Rihand (b) Ganga  
 (c) Mahanadi (d) Baitarani
33. Which one of the following is **not** a sea port?  
 1 Paradeep 2 Haldia  
 3 Diamond Harbour 4 Dhamra  
 Select the correct answer using the code given below:  
 (a) 1 and 3 only (b) 2 only  
 (c) 3 only (d) 1, 2 and 3
34. Bharatmala Pariyojana is related to:  
 (a) interlinking of Northern and Southern Indian rivers in a garland shape.  
 (b) networks of National Highways in India.  
 (c) interlinking of all cities of India through Railways.  
 (d) interlinking of all industrial regions of India through pipelines.
35. Sea water is saltier than rain water because:  
 (a) rivers wash away salts from earth and pour them into the sea.  
 (b) sea beds have salt producing mines.  
 (c) sea animals are salt producing.  
 (d) the air around the sea is saltish.

| Answer Key |         |         |         |         |         |         |         |         |         |
|------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)     | 2. (d)  | 3. (d)  | 4. (b)  | 5. (c)  | 6. (a)  | 7. (a)  | 8. (b)  | 9. (d)  | 10. (c) |
| 11. (a)    | 12. (b) | 13. (c) | 14. (a) | 15. (c) | 16. (d) | 17. (b) | 18. (c) | 19. (a) | 20. (b) |
| 21. (b)    | 22. (c) | 23. (b) | 24. (d) | 25. (a) | 26. (a) | 27. (c) | 28. (b) | 29. (d) | 30. (c) |
| 31. (d)    | 32. (c) | 33. (c) | 34. (b) | 35. (a) |         |         |         |         |         |

### Answers with Explanations

**1. Option (b) is correct.**

**Explanation:** Peninsular plateau

It was formed due to the breaking and drifting of the Gondwana land and therefore is a part of the oldest landmass. The peninsular plateau is composed of metamorphic and igneous rocks with rising hills and wide valleys.

**2. Option (d) is correct.**

**Explanation:** Species of tree found in Tropical Thorn Forests – Babool, ber, and wild date palm, khair, neem, khejri, palas. These forests are found in the areas/regions, which receive rainfall less than 50 cm (example – deserts). States where Tropical Thorn Forests are found – South west Punjab, Madhya Pradesh, Rajasthan, Gujarat, and Uttar Pradesh.

**3. Option (d) is correct.**

**Explanation:**

- Moraines, eskers, outwash plains, and drumlins are the depositional landforms commonly found in glaciated areas.
- Ladakh stretches from the Siachen Glacier in the Karakoram range (North) to the main Great Himalayas (South).
- Because of various glaciers in Ladakh, are these features are found here.

**4. Option (b) is correct.**

**Explanation:** From the above, two statement, (1) and (3) is correct but statement (2) is incorrect. Jog Falls is second-highest plunge waterfall in India formed by the Sharavati River. The Brahmani is formed by the confluence of the Shankh and Dakshin Koel rivers, and flows through the districts of Sundergarh, Deogarh, Angul, Dhenkanal, Cuttack, Jajapur and Kendrapara. Tamirabharani is the only perennial river in Tamil Nadu. Thus, only second statement is incorrect.

**5. Option (c) is correct.**

**Explanation:** Brahmaputra is a Trans' boundary river which flows through Tibet (China), India, and Bangladesh. It is known by various names:

- Yarlung Tsangpo in Tibet

**6. Option (a) is correct.**

**Explanation:**

- The term 'basket of eggs' topography refers to Drumlins which is a depositional landform formed by glaciers.
- These baskets of eggs are basically forms of rounded hummocks resulting from the deposition of glacial till which look like inverted boat or spoon.
- They vary in size from a few metres to 60-100 metres in height and from a few hundred metres to one-two kilometres in length.

- When they occur in a cluster, they look like a basket of eggs.

**7. Option (a) is correct.**

**Explanation:** Tropical Deciduous Forests are also known as Monsoon forests and covers about 65.6 percent of the total forest area in India. In India these forests are found in the states of Madhya Pradesh, Uttar Pradesh, Odisha, Some parts of Maharashtra and Chhattisgarh.

**8. Option (b) is correct.**

**Explanation:**

- The Beas River which is a north India river originates from the Himalayas in central Himachal Pradesh and flows for some 470 kilometres to the Sutlej River in Punjab.
- Its total length is 470 kilometres and its drainage basin is 20,303 square kilometres large.

**9. Option (d) is correct.**

**Explanation:** Lonar Lake

- It located at Lonar in Buldhana district of Maharashtra.
- It is also known as Lonar crater.
- It is a notified National Geo-heritage Monument. A geographical area of national importance and heritage, as notified by the Geological Survey of India (GSI).
- It is also known as saline and Soda Lake.
- The crater is considered to be the result of a meteorite impact. It contains water which is both saline and alkaline.

**10. Option (c) is correct.**

**Explanation:** Playas

- Playas are shallow basins in deserts that periodically fill with rainwater.
- The water in playas is retained only for short duration due to evaporation and quite often the playas contain good deposition of fine-grained sediment and salts. The playa plain which is covered up by salts is called alkali flats.
- Other names for playas – pan, flat and dry lake, ephemeral ponds or lakes.

**11. Option (a) is correct.**

**Explanation:**

**Following are the factors affecting the distribution of groundwater:**

**1. Precipitation**

The most important factor controlling infiltration is the amount and characteristics of precipitation that falls as rain or snow. Precipitation infiltrates into the ground often seeps into streambeds over an extended period, thus a



stream will often continue to flow when it has not rained for a long time and where there is no direct runoff from recent precipitation.

**2. Infiltration**

It is defined as the flow of water from ground into the subsurface. It is meters per unit time of water entering into the soil regardless of the types or values of forces or gradients.

**3. Base flow**

The water in streams has a sustained flow, even during periods of lack of rain.

**4. Soil characteristic**

Soils, such as clays, absorb less water at a slower rate than sandy soils.

**5. Soil saturation**

Like a wet sponge, soil already saturated from previous rainfall cannot absorb much more so more rainfall will become surface runoff.

**6. Land cover**

Land covers have a crucial impact on infiltration and rainfall runoff.

Vegetation can slow the movement of runoff. It allows more time for rain water to seep into the ground.

**7. The slope of the land**

Water falling on steeply-sloped land runs off more quickly and infiltrates less with respect to water falling on flat land.

**8. Evapotranspiration**

Plants need this shallow groundwater to grow, and, by the process of evapotranspiration, water is returned back into the atmosphere.

Thus, Distance from the sea is not a direct factor affecting the distribution of groundwater.

**12. Option (b) is correct.**

**Explanation:** Biodiversity is the occurrence of wide variety of organisms on earth. Biodiversity term was coined by Walter G. Rosen in 1986. Biodiversity is nowadays conserved in biodiversity hotspots. Hotspot term was coined by Norman Myers in 1988. Biodiversity Hotspots are the areas having richest biodiversity along with high degree of endemism. Worldwide there are total 36 Hotspots identified. In India, there are total 4 Hotspots present.

**13. Option (c) is correct.**

**Explanation:** Soil is the mixture of organic materials and debris of rock which develop on the surface of Earth. The major factors affecting the formation of soil are:

- Relief or topography
- Parent material
- Climate
- Vegetation and other life-forms
- Time

**14. Option (a) is correct.**

**Explanation:**

**Hirakud Dam:** It is located in Sambalpur, Odisha. It is built across the Mahanadi River and is the longest earthen dam in the world.

**Panchet Dam:** It is located in Dhanbad District, Jharkhand and is built across the Damodar River.

**Kosi Dam:** It is also known as Saptakosi High Dam.

It is constructed on the Saptakoshi River of Nepal. Its objective was to control floods in south-east Nepal and northern Bihar.

**Ukai Dam:** It is the largest reservoir of Gujarat. It is constructed across the Tapi River and is also known as Vallabh Sagar.

**15. Option (c) is correct.**

**Explanation:** The River twists and turns as it enters the plain, forming huge bends known as meanders. The ends of meander loop come closer and closer due to constant erosion and deposition along the sides of meander. Consequently in due course of time, the meander loop cuts off from the river and forms a cut-off lake which is known as ox-bow Lake.

**16. Option (d) is correct.**

**Explanation:**

- Tripura- Bangladesh
- Arunachal Pradesh- China & Myanmar
- Assam – China & Bangladesh
- Mizoram – China and Myanmar

**17. Option (b) is correct.**

**Explanation:**

- The duration of monsoon or rainy season is from early June to mid-September which extends an average period of 100-120 days.
- The south-west monsoon also known as summer monsoon brings most of the rainfall during a year in the country.
- The north-east monsoon flows from land to sea and is commonly known as winter monsoon

**18. Option (c) is correct.**

**Explanation:** As part of the forest ecosystems conserved by local communities, sacred groves are reinforced by religious sentiments. In addition to preserving forest biodiversity, sacred groves provide refuge to many species of plant and animal that are important for conservation.

**19. Option (a) is correct.**

**Explanation:** Afforestation is the establishment of a forest or planting of trees in a previously barren environment.

**20. Option (b) is correct.**

**Explanation:** According to Koppen classification system Cfa denotes humid sub-tropical climate

- The humid sub-tropical climate occurs usually in the 20's and 30's degree latitude of the eastern coasts of the continent.
- Cfa denotes zones in which summers are sweltering and humid with frequent thunderstorms whereas Winters are milder in comparison and precipitation occurs due to mid-latitude cyclones in the Cfa zones.
- Tropical Wet Climate is denoted by Af and precipitation in this type of climate occurs all year round.
- Tundra Climate is denoted by ET
- Tropical Wet and Dry Climate is denoted by Aw. This climate has an extended dry season during winter.

**21. Option (b) is correct.**

**Explanation:**

Imphal (Capital of Manipur)

- Itanagar (Capital of Arunachal Pradesh)  
 Aizawl (Capital of Mizoram)  
 Agartala (Capital of Tripura)

**22. Option (c) is correct.**

**Explanation:** Bhadra wildlife sanctuary - Chikkamagaluru, Karnataka.

Simlipal wildlife sanctuary - Baripada, Odisha.

Pachmarhi wildlife sanctuary - Satpura, M.P.

Shikari devi wildlife sanctuary - Kalidoghri, Himachal Pradesh.

**23. Option (b) is correct.**

**Explanation:** Spruce and Cedar are two varieties of temperate coniferous forest.

Temperate coniferous forests are found predominantly in areas with warm summers and cool winters, and vary in their kinds of plant life.

**24. Option (d) is correct.**

**Explanation:** The sal tree is known also as sakhua in northern India, especially found in Madhya Pradesh, Odisha and Jharkhand.

- Moist deciduous tree forests are the mixture of trees and grasses.

These forests are found in areas of moderate rainfall of 100 to 200 cm per annum, mean annual temperature of about 27°C and the average annual relative humidity of 60 to 75%. They are basically found in Jharkhand, Orissa, Chhattisgarh, West Bengal and Andaman Nicobar Islands.

- Tropical Evergreen tree are a dominant part of the Natural vegetation in India. They flourish in the areas having more than 200 cm of rainfall.
- Tropical semi-evergreen tree, A distinct formation type, dominated by both evergreen and deciduous broad-leaved trees, flanking the rain forest in areas that have a marked dry season.
- Dry deciduous "forest" trees have lower height to diameter ratios, produce leaves with lower specific leaf areas, and have higher wood densities. Dry deciduous "forests" occur in areas with less rainfall than evergreen forests, although the two forest types overlap at the better end of the dry deciduous zone.

**25. Option (a) is correct.**

**Explanation:** In the Thar desert, the shifting sand dunes are locally known as Dhrian.

**26. Option (a) is correct.**

**Explanation:**

- The Danube is the second-longest river in Europe. The longest river of Europe is Volga in Russia. It flows through much of Central and South eastern Europe, from the Black Forest and fall into the Black Sea.
- Rhine is the second-longest river in Central and Western Europe (after the Danube). The river flows in a mostly northerly direction through the German Rhineland and the Netherlands and eventually empties into the North Sea.
- The Rhone is a major river in France and Switzerland, it arises in the Alps and discharges into the Mediterranean Sea.
- The Loire is the longest river in France. It is also the third longest river in Europe, after the Dan-

ube and the Volga. It empties into the Bay of Biscay (Atlantic Ocean) at Saint-Nazaire.

**27. Option (c) is correct.**

**Explanation:**

- **Ganga River**

This river rises in the western Himalayas in the Indian state of Uttarakhand, and flows south and east through the Gangetic Plain of North India into Bangladesh, where it empties into the Bay of Bengal. It is the third largest river on Earth by discharge

- **Indus River**

It is also known as Sindhu River, Originating in the Tibetan plateau in the vicinity of Lake Mansarovar. The river runs a course through Jammu and Kashmir in both Indian Kashmir and Northern Areas, flowing through the North in a southerly direction along the entire length of country, to merge into the Arabian Sea near the city of Pakistan's National City Karachi. The total length of the river is 3200 km.

- **Satluj River**

The Bhakra Dam is built around the river Sutlej to provide irrigation and other facilities to the states of Punjab, Rajasthan and Haryana.

It rises on the north slope of the Himalayas in Lake La'nga in southwestern Tibet, at an elevation above 15,000 feet (4,600 metres). Flowing northwestward and then westsouthwestward through Himalayan gorges.

- **Brahmaputra River**

The river originates from the Kailash ranges of Himalayas at an elevation of 5300 M. After flowing through Tibet it enters India through Arunachal Pradesh and flows through Assam and Bangladesh before it joins Bay of Bengal.

**28. Option (b) is correct.**

**Explanation:** Rudraprayag is a town located in Uttarakhand. It is considered as one of the five confluences of River Alaknanda. It is situated at the confluence of rivers Alaknanda and Mandakini. Mandakini is a tributary of the Alaknanda River.

**29. Option (d) is correct.**

**Explanation:**

- Devprayag is a town in Tehri Garhwal District in the state of Uttarakhand.
- It is the final one of the Panch Prayag (five confluences) of Alaknanda River.
- It is the place where Alaknanda meets the Bhagirathi River and both rivers thereafter flow on as the Ganges River or Ganga.
- It's believed that a third river, the mythical Saraswati River is underground and meets these two rivers at the confluence.

**30. Option (c) is correct.**

**Explanation:** Brahmaputra is a Tran's boundary river which flows through Tibet (China), India, and Bangladesh. It is known by various names. Yarlung Tsangpo in Tibet.

**31. Option (d) is correct.**

**Explanation:** The Krishna Raja Sagara Dam (KRS Dam) was built across river Kaveri in 1924 in Karnataka. The dam is the main source of water for irrigation in Mysore and Mandya cities. The reservoir



is also the main source of drinking water for entire Mysore city and almost the whole of Bangalore city.

**32. Option (c) is correct.**

*Explanation:* Paradip Port is located at confluence of river Mahanadi in Bay of Bengal in Odisha. It was the first major port on East Coast commissioned in independent India.

**33. Option (c) is correct.**

*Explanation:*

- Diamond Harbour, also known as Haldia Sea-port, is a city and a municipality of the 24th South Pargana district in West Bengal. It is situated on the eastern bank of Hooghly River.
- **Paradip Port** – It is a natural, deep-water port in Odisha. It is situated at confluence of the Mahanadi River and the Bay of Bengal.
- **Haldia Port** – Haldia port or also known as Haldia Dock Complex, is built at the meeting place of the Haldi River and Hooghly River.
- **The Dhamra Port** – It is located in Bhadrak district of Odisha, on the shore of the Bay of Bengal, about seven kilometres from the old Dhamra port.

**34. Option (b) is correct.**

*Explanation:*

- On 31 July 2015, Bharatmala Pariyojana was launched which is a centrally sponsored project for Road and Highways.
- Bharatmala Pariyojana is an umbrella program for the highways sector envisaged by the Ministry of Road Transport and Highways.
- It aims to improve the quality of roads with Total Road construction in the tenure within a span of five years.
- Its target to connect 550 districts in the country through national highway linkages.

**35. Option (a) is correct.**

*Explanation:* Seawater is more salted than rainwater because rivers clean out salts from the earth and flow them into the sea. Seawater has salt (and additional minerals) disbanded in it. Water vaporizes from the sea to create clouds (which eventually cause rain), but the salt is gone behind in the ocean.

○○○

**Sports/Championship/Winners/Terms/No of Players  
Olympics**

- Olympics games were started in 776 BC on Mount Olympia in the Honour of Greek God. "Zens" The modern Olympic Games were started in Athens, the capital of Greece on 6th April 1896 with great efforts made by French nobleman, Baron Pierre de Coubertin.
- The Olympic Games are organized after every 4 years. The Olympic Flag is made up of white silk and contains five intertwined rings as the Olympic Emblem.
- The five interlaced rings are arranged in 3-2 patterns on a white background, with the blue ring to the extreme left, followed by yellow, black, green, and red, in the same order. Blue for Europe, Black for Africa, Red for Americas (North and South America). Yellow for Asia and Green for Oceania (Australia and New Zealand).
- The official Olympic Motto is Citius, Altius, Fortius, a Latin phrase meaning Swifter, Higher, Stronger. 2024, Summer Olympics will be held in Paris, France.

**Commonwealth Games**

- The first Commonwealth Games were held in 1930 in Hamilton, Canada.
- Since 1930, the games have been conducted every 4 years except for 1942 and 1946 due to World War II.
- The Commonwealth Games Federation (CGF) is the organization that is responsible for the direction and control of the commonwealth Games.
- There are currently 53 members in the Commonwealth of Nations.
- The 2018 Commonwealth Games (21st) were held in Gold Coast, Queensland, Australia. Most gold medals were won by Australia. 2022 Commonwealth Games will be held in Birmingham, England.

**The Asian Games**

- The Asian Games, also called the Asiad, are a multi-sport event held every 4 years among athletes from all over Asia.
- The games are regulated by the Olympic Council of Asia (OCA), under the organization of the International Olympic Committee (IOC). The first Asian Games were held in 1951 in New Delhi, India. 18<sup>th</sup> Asian Games of 2018 was held in Jakarta (Indonesia) in which India finished at eighth position. The next game is scheduled to be held in Hangzhou, China in 2022.

**Cricket World Cup**

- The first Cricket World Cup was organized in

**England in June 1975. List of Cricket World Cup.**

|      |                           |                            |
|------|---------------------------|----------------------------|
| 1975 | England                   | West Indies beat Australia |
| 1979 | England                   | West Indies beat England   |
| 1983 | England                   | India beat West Indies     |
| 1987 | India                     | Australia beat England     |
| 1992 | Australia                 | Pakistan beat England      |
| 1996 | Pakistan                  | Sri Lanka beat Australia   |
| 1999 | England                   | Australia beat Pakistan    |
| 2003 | South Africa              | Australia beat India       |
| 2007 | West Indies               | Australia beat Sri Lanka   |
| 2011 | India                     | India beat Sri Lanka       |
| 2015 | Australia and New Zealand | Australia beat New Zealand |
| 2019 | England                   | England beat New Zealand   |
| 2023 | India                     | Australia beat New Zealand |

**T-20 Cricket World Cup**

|      |              |                             |
|------|--------------|-----------------------------|
| 2007 | South Africa | India beat Pakistan         |
| 2009 | England      | Pakistan beat Sri Lanka     |
| 2010 | West Indies  | England beat Australia      |
| 2012 | Sri Lanka    | West Indies beat Sri Lanka  |
| 2014 | Bangladesh   | Sri Lanka beat India        |
| 2016 | India        | West Indies beat England    |
| 2021 | India        | Australia beats New Zealand |

- Australia won the final match against New Zealand by 8 wickets and became ICC Men's T20 World Cup 2021 Winner. This is the first time when the Australian cricket team won the T20 World Cup.

**IPL (Indian Premier League)**

- It was initiated by the Board of Control for Cricket in India (BCCI). It is Headquarter in Mumbai. The first IPL tournament was started in 2008. Seventh IPL Tournament held in Bangalore, Kolkata Knight Riders defeated King XI Punjab.

**Football World Cup**

- The Football World Cup is organized by FIFA (Federation of International Football Association).
- The headquarters of FIFA is located in Zurich, Switzerland.
- The World Cup is called 'Jules Rimet Cup' named after the name of FIFA President Jules Rimet.
- The first Football World Cup was organized in

Uruguay in 1930.

- In 1912 and 1946, the Football World Cup was not played because of the World War II.

#### Hockey World Cup

- The Hockey World Cup is organized by the International Hockey Federation (FIH) once in four years. The headquarters of FIH is located in Lausanne, Switzerland.
- The first Hockey World Cup was organized in Barcelona (Spain) was 1971 and the winner was Pakistan.
- Women's Hockey World Cup is being held since 1974.
- The 13<sup>th</sup> Women's Hockey World Cup in 2014, was held in The Hague, Netherlands. The Netherlands won the tournament for a record seventh time after defeating Australia. The 14<sup>th</sup> Women's Hockey World Cup of 2018, was held in London, England.
- The 14<sup>th</sup> Men's Hockey World Cup 2018, was held at Kalinga stadium in Bhubaneswar, India. Belgium became champion defeating Netherlands.
- The 15<sup>th</sup> Men's Hockey World Cup 2023, will be held at the Kalinga Stadium in Bhubaneswar and in Rourkela, Odisha.

#### National Sports of Some Countries

| Country   | National Sport        |
|-----------|-----------------------|
| Australia | Cricket               |
| Canada    | Lacrosse (Ice Hockey) |
| China     | Table Tennis          |
| England   | Cricket               |
| India     | Hockey                |
| Japan     | Sumo                  |
| Malaysia  | Badminton             |
| Scotland  | Rugby/Football        |
| Spain     | Bull Fighting         |
| USA       | Baseball              |

#### Nickname of Players

| Player            | Nickname              |
|-------------------|-----------------------|
| Major Dhyan Chand | Hockey ka Jadugar     |
| Milkha Singh      | Flying Sikh           |
| PT Usha           | Payyoli Express       |
| PT Usha           | Golden Girl           |
| Shoaib Akhtar     | Rawalpindi Express    |
| Rahul Dravid      | The Wall              |
| Harbhajan Singh   | Bhaji, The Turbanator |
| Javagal Srinath   | Mysore Express        |
| Paes and Bhupathi | Indian Express        |

#### Important Cups and Trophies

| Sport        | Cup and Trophy                                                                                                                                                              |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Cricket      | Irani Trophy, Duleep Trophy, Ranji Trophy, Vijay Hazare Trophy, Asia Cup, Deodhar Trophy, CK Naidu Trophy, Cooch-Bihar Trophy, Gandhi-Mandela Series, The Ashes Series etc. |
| Football     | Durand Cup, Nizam Gold Cup, Rovers Cup, Sanjay Gold Cup, Santosh Trophy, Subroto, Mukherjee Cup, Vittal Trophy, Nehru Gold Cup                                              |
| Hockey       | Agha Khan Cup, Azian Shah Cup, Nehru Trophy, Dhyan Chand Trophy, Beighton Cup, Scindia Gold Cup, Modi Gold Cup, Indira Gandhi Gold Cup,                                     |
| Golf         | Canada Cup, Muthian Gold Cup, Ryder Cup, Walker Cup                                                                                                                         |
| Table Tennis | Corbillion Cup (women), Jayalaxmi Cup (women), Swaything Cup (men)                                                                                                          |
| Lawn Tennis  | Davis Cup, Hamlet Cup, Australian Open, French Open, Wimbledon, US open                                                                                                     |
| Badminton    | Thomas Cup (men), Uber Cup (women), Narang Cup, Malaysian open                                                                                                              |
| Boxing       | Aspy Adjanja Trophy                                                                                                                                                         |
| Rowing       | Wellington Trophy                                                                                                                                                           |
| Bridge       | Ruia Trophy                                                                                                                                                                 |
| Polo         | Ezra Cup, Winchester Cup, Radha Mohan Cup, West Chester Cup                                                                                                                 |

#### Important Sports Terms

| Sport          | Terms                                                                         |
|----------------|-------------------------------------------------------------------------------|
| Basketball     | Dunk, Front count, Lay-up, Held ball, Pivot, Rebound, Steal                   |
| Cricket        | Bye, Draw, Googly, Topspin, Overthrow, Duck, Hit wicket, Cut, Pull, Full Toss |
| Football       | Bend dribble, Dissent, Dummy, Feint, Free kick, Header, Red card, Throwins    |
| Hockey         | Bully, Striking, Circle, Post back                                            |
| Chess          | Castle, Diagonals, Files, Pawns, Pieces, Promote, Gambit, Pawn                |
| Boxing         | Jab, laying on Knock, Second out rabbit punch, Uppercut                       |
| Badminton      | Lab, Let, Drive, Drop, Love                                                   |
| Polo           | Chukker, Bunker                                                               |
| Baseball       | Diamond, Homerun, Put out, Strike, Anti-rubber                                |
| Rifle Shooting | Target, Muzzle full, Bullseye                                                 |
| Wrestling      | Hold Sager, Rebuts, Hal, Nelson, Free Style                                   |

|           |                                                                           |
|-----------|---------------------------------------------------------------------------|
| Golf      | Fore some, Stymie, T, put a hole, Caddy, Niblick, Iron, The green, Bunker |
| Billiards | Jigger, Pot, Breast pot, In off, Cans, Bolting, Long, Hazard, Cue         |
| Swimming  | Breast Stroke, Twist, Butterfly, Crawl, Springboard                       |

|              |                                                                                           |
|--------------|-------------------------------------------------------------------------------------------|
| Volley Ball  | Antennae, Attack hit, Libero, Service, Set up, Blocking, Dribbling                        |
| Lawn Tennis  | Advantage, Ace, Dence, Volley, Foot Fault, Smash, Grand-Slam, Slice, Love                 |
| Table Tennis | End line, Flat hit, Foil, Service, Phnholder grip, Reverse, Top-spin, Counterhitting, Let |

## EXERCISE

- The Headquarters of the proposed National Sports University (As per the National Sports University Ordinance, 2018) will be set up in:
  - Chhattisgarh
  - Manipur
  - Kerala
  - West Bengal
- Who among the following were selected for the Rajiv Gandhi Khel Ratna Award 2019?
  - Vijay Kumar and Yogeshwar Dutt
  - Sakshi Malik and Jitu Rai
  - Virat Kohli and Mirabai Chanu
  - Bajrang Punia and Deepa Malik
- The Government of India programme regarding 'Stay in India and Study in India' is initiated by:
  - the Ministry of Youth Affairs and Sports
  - the Ministry of Culture
  - the Ministry of Education
  - the Ministry of Tourism
- Which one of the following teams won the Senior Women's Football Championship, 2021?
  - Manipur
  - Railways
  - West Bengal
  - Odisha
- Who among the following won the FIDE World Chess Championship, 2021?
  - Nepomniachtchi
  - Magnus Carlsen
  - Ding Liren
  - Anish Giri
- Avani Lekhara excels in which one of the following sports?
  - Table Tennis
  - Cycling
  - Shooting
  - Boxing
- Avani Lakhara became the first Indian woman to win two Paralympics medals at the Tokyo Paralympics 2020. Which one of the following was her discipline?
  - Badminton
  - Shooting
  - Table Tennis
  - Archery
- Which one of the following is the oldest football tournament of Asia?
  - Federation Cup
  - Santosh Trophy
  - Durand Cup
  - Rovers Cup
- Who among the following is the first Indian woman to win a medal in the Olympics?
  - Karnam Malleswari
  - Mirabai Chanu
  - P.T. Usha
  - P.V. Sindhu
- The Russian Federation participated in the Tokyo Olympics, 2020 under which of the following names?
  - Russian Olympic Committee
  - Russian Federal Committee
  - Russian Sports Committee
  - Russian Republican Committee
- Who among the following won the Best Men's Player Award of FIFA Football Awards, 2019?
  - Cristiano Ronaldo
  - Virgil van Dijk
  - Lionel Messi
  - Xavi
- Which one of the following States is planned to host the Khelo India Youth Games (4<sup>th</sup> Edition)?
  - Kerala
  - Haryana
  - Gujarat
  - Manipur
- Who among the following has won the Singles Title in Wimbledon Tennis Championship (Women) in the year 2019?
  - Karolina Pliskova
  - Simona Halep
  - Serena Williams
  - Naomi Osaka
- Who among the following is/are the recipient/ recipients of Rajiv Gandhi Khel Ratna Award, 2018?
  - Virat Kohli
  - S. Mirabai Chanu and Virat Kohli
  - Neeraj Chopra
  - Hima Das and Neeraj Chopra
- Which one of the following is the official mascot of Tokyo 2020 Olympic Games?
  - Soohorang
  - Vinicius de Moraes
  - The Hare, the Polar Bear and the Leopard
  - Miraitowa
- Who among the following is the winner of the World Food Prize (year 2015)?
  - Sanjaya Rajaram
  - Baldev Singh Dhillon
  - Sir Fazle Hasan Abed
  - Rajendra Singh Paroda
- In November 2017, India's MC Mary Kom won the gold medal at the Asian Boxing Championship, 2017 held at Ho Chi Minh City in Vietnam. In which one of the following categories was she declared winner?
  - 48 kg
  - 51 kg
  - 54 kg
  - 57 kg
- Who among the following is the winner of the National Badminton Championship (Men), 2017?
  - Kidambi Srikanth
  - H.S. Prannoy
  - Ajay Jayaram
  - Sai Praneeth
- Which of the following is **not** correct regarding the 2014 FIFA Football World Cup?
  - 'We Are One' is the official song
  - 'Dar um Jeito (We Will Find A Way)' is the official anthem
  - Brazil is the first country to host a World Cup for the second time
  - For the first time two consecutive World Cups are not hosted in Europe
- Name the Indian cricketer who is **not** inducted to the ICC Cricket Hall of Fame (till July 2018).
  - Rahul Dravid
  - Sunil Gavaskar
  - Sachin Tendulkar
  - Anil Kumble

## Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (d)  | 3. (c)  | 4. (a)  | 5. (b)  | 6. (c)  | 7. (b)  | 8. (c)  | 9. (a)  | 10. (a) |
| 11. (c) | 12. (b) | 13. (b) | 14. (b) | 15. (d) | 16. (c) | 17. (a) | 18. (b) | 19. (c) | 20. (c) |

## Answers with Explanations

## 1. Option (b) is correct.

*Explanation:***THE NATIONAL SPORTS UNIVERSITY ACT, 2018**

- It is an Act to establish and incorporate a National Sports University in Manipur.
- The University will be the first of its kind to promote sports education in the areas of sports sciences, sports technology, sports management and sports coaching.
- It will also function as the national training centre for select sports disciplines by adopting best international practices.

## 2. Option (d) is correct.

*Explanation:*

Rajiv Gandhi Khel Ratna Award is given to the most outstanding performance by a sportsperson in his field of sports.

- This Award is been given after a period of four years.
- Sh. Bajrang Punia was awarded for Wrestling.
- Ms. Deepa Malik was awarded for Para Athletics.

## 3. Option (c) is correct.

*Explanation:* Stay in India and Study in India is initiated by the Ministry of Education.

It aims to target foreign students by branding India as an attractive education destination and also to ensure more and more students stay in India.

## 4. Option (a) is correct.

*Explanation:* Manipur successfully defended their Senior Women's National Football Championship crown after a dramatic penalty shootout win over Railways in the final at the EMS Stadium in Kozhikode, Kerala.

## 5. Option (b) is correct.

*Explanation:* Magnus Carlsen of Norway defended his title and won the FIDE World Championship 2021 in Dubai.

## 6. Option (c) is correct.

*Explanation:***Avani Lekhara**

- She is a Paralympian and rifle shooter from India.
- She hails from Jaipur, Rajasthan.
- She had won India's first gold medal at the 2020 Summer Paralympics.
- She was awarded Padma Shri in 2022.

## 7. Option (b) is correct.

*Explanation:*

Avani Lekhara is an Indian Paralympian and rifle shooter.

- She won a Gold Medal in 10m air rifle standing and a Bronze Medal in 50m rifle 3 positions at Tokyo 2020 Paralympics.

- She is also the first Indian woman to win multiple medals in a single Paralympic Games.

## 8. Option (c) is correct.

*Explanation:*

- Durand Cup is the oldest soccer tournament in Asia. It is an annual domestic football competition in India which was first held in 1888 in Shimla.
- It is hosted by the Durand Football Tournament Society (DFTS) and All India Football Federation (AIFF).

## 9. Option (a) is correct.

*Explanation:* Weightlifter Karnam Malleswari is the first Indian woman to win the medal at Olympics. She won the bronze medal at 2000 Sydney Olympics. She is from Andhra Pradesh.

## 10. Option (a) is correct.

*Explanation:* Russia is not allowed to compete under the country name due to a controversial doping scandal that first broke out in 2015 and led to the country getting banned from taking part in the Olympics and other major international events. This is why, all the 335 athletes from Russia took part in the Tokyo Olympics 2020 under the acronym ROC (Russian Olympic Committee).

## 11. Option (c) is correct.

*Explanation:***Best FIFA Football Awards 2019**

- It was held at the Teatro alla Scala in Milan, Italy.
- Best FIFA Men's Player - Lionel Messi (Argentina)
- Best FIFA Men's Goalkeeper - Alisson (Brazil)

## 12. Option (b) is correct.

*Explanation:*The 4<sup>th</sup> edition of Khelo India Youth Games (KIYG) was hosted by Haryana at the Tau Devi Lal Stadium in Panchkula, Haryana.

- Khelo India programme's inaugural edition was held in New Delhi in 2018.
- Pune and Guwahati hosted 2nd and 3rd edition in 2019 and 2020 respectively.

## 13. Option (b) is correct.

*Explanation:*

The 2019 Wimbledon Championships took place at the All England Lawn Tennis and Croquet Club in Wimbledon, London, United Kingdom.

- Women's singles was won by Simona Halep.

## 14. Option (b) is correct.

*Explanation:* The Rajiv Gandhi Khel Ratna Award is the highest honour in the field of sports in India and is awarded to those who make the country proud on the international platform. In 2018, Saikhom Mirabai Chanu, and Virat Kohli were awarded the Rajiv Khel Ratna Award by the Government of India. Saikhom Mirabai Chanu, a weightlifter, has won



the World Championships and multiple medals at the Commonwealth Games. Virat Kohli became the third cricketer after Sachin Tendulkar and Mahendra Singh Dhoni to receive this award.

**15. Option (d) is correct.**

*Explanation:*

- 'Miraitowa' is the official mascot of Tokyo 2020 Olympic Games.
- Miraitowa is a combination of the Japanese words for future and eternity.
- The mascot of the Tokyo 2020 Olympic Games has the same indigo blue Ichimatsu pattern as the Emblem of the Tokyo 2020 Games on its head and body.
- The mascot's personality is derived from a traditional Japanese proverb that means to learn old things well and to acquire new knowledge from them

**16. Option (c) is correct.**

*Explanation:* The World Food Prize was started in the year 1986 by Jhon Ruan and his family. It is an international award recognizing the achievements of individuals who have advanced human development by improving the quality, quantity, or availability of food in the world. The World Food Prize, 2015 was formally awarded to Sir Fazle Hasan Abed of Bangladesh, during a ceremony at the magnificent Iowa State Capitol on 15<sup>th</sup> October, 2015.

**17. Option (a) is correct.**

*Explanation:*

In November 2017, India's MC Mary Kom won the gold medal at the Asian Boxing Championship, 2017 held at Ho Chi Minh City in Vietnam.

- She was declared winner in Light Flyweight (48kg) category.
- She beat North Korea's Kim Hyang Mi by a 5-0 margin to clinch the gold medal.

**18. Option (b) is correct.**

*Explanation:* H.S.Prannoy was the winner of the National Badminton championship (men), 2017. He won the men's singles title at the 82nd Senior National Championship Senior National Badminton Championships in Nagpur, Maharashtra on November 2017. He defeated higher ranked Kidambi Srikanth in the final. The India National Badminton Championships is a tournament organized to crown the best badminton players in India since 1934.

**19. Option (c) is correct.**

*Explanation:*

The 2014 FIFA World Cup was the 20th FIFA World Cup.

- It took place in Brazil from 12 June to 13 July 2014.
- The official song of the tournament was "We Are One (Ole Ola)".
- "Dar um Jeito (We Will Find a Way)" was the official anthem.
- The 1930 FIFA World Cup was the inaugural FIFA World Cup which was held in Uruguay from 13 to 30 July 1930.
- Brazil, France, Italy, Germany, and Mexico have each hosted twice.

**20. Option (c) is correct.**

*Explanation:*

- Sachin Tendulkar is not inducted into the ICC Cricket Hall of Fame (till July 2018).
- Rahul Dravid has been inducted into the ICC Cricket Hall of Fame in 2018.
- Rahul Dravid is only the fifth cricketer from India to be included in the Hall of Fame.
- The other four Indians are Bishan Singh Anil Bedi, Kapil Dev, Sunil Gavaskar and Anil Kumble.

○○○

| Great Personalities of the Nation |           |                                                                            |
|-----------------------------------|-----------|----------------------------------------------------------------------------|
| Mahatma Gandhi                    | 1869-1948 | The Father of India                                                        |
| Dr. B. R. Ambedkar                | 1891-1956 | Indian Social Reformer                                                     |
| Jawaharlal Nehru                  | 1889-1964 | The First Prime Minister of India, an Indian Independence activist         |
| Indira Gandhi                     | 1917-1984 | The First and only Female Prime Minister of India to date                  |
| Mother Teresa                     | 1910-1997 | Albanian - Indian Roman Catholic nun and missionary                        |
| Sri Aurobindo                     | 1872-1950 | Indian Philosopher, Poet and Nationalist                                   |
| Lal Bahadur Shastri               | 1904-1966 | Indian Politician, Second Prime Minister of India                          |
| Dr APJ Abdul Kalam                | 1931-2015 | Indian Aerospace Scientist and 11 <sup>th</sup> President of India         |
| Homi J. Bhabha                    | 1909-1966 | Indian Nuclear Physicist                                                   |
| Valabh bhai Patel                 | 1875-1950 | Indian Politician, First Deputy Prime Minister of India                    |
| Rabindranath Tagore               | 1861-1941 | Indian Poet, Writer, Philosopher, Composer and Painter                     |
| Kalpana Chawla                    | 1961-2003 | The first woman of Indian origin to go to space                            |
| Ratan Tata                        | 1937      | Indian Industrialist, Philanthropist and Former Chairman of the Tata Group |

### Some Important Religious Institution and Founder

|                             |         |                            |
|-----------------------------|---------|----------------------------|
| Brahmo Samaj                | 1828    | Raja Ram Mohan Roy         |
| Young Bengal Movement       | 1826-31 | H. L. Vivian Derozio       |
| Tattavabodhisini Sabha      | 1839    | Debendranath Tagore        |
| Wahabi Movement Rohilakhand | 1820    | Syed Ahmed of Rai Bareilly |
| Prarthana Samaj Bombay      | 1867    | Atmaram Pandurang          |
| Arya Samaj Bombay           | 1875    | Dayanand Saraswati         |
| Aligarh Movement            | 1875    | Syed Ahmed Khan            |
| Ram Krishna Mission Belur   | 1897    | Vivekananda                |
| Servants of Indian Society  | 1905    | Gopal Krishna Gokhale      |

### Great Personalities of the World

| Personalities      | Period    | Field                                                                                                                                      |
|--------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------|
| Abraham Lincoln    | 1809-1865 | The 16th President of the US; Leader of the American Civil War                                                                             |
| Charles Darwin     | 1809-1882 | English Scientist and Naturalist; Known for his contributions to the Science of Evolution                                                  |
| Thomas Alva Edison | 1847-1931 | American inventor and businessman, known as America's greatest inventor                                                                    |
| Walt Disney        | 1901-1966 | American motion picture and television producer and showman, Created characters like Mickey Mouse, Founder of Disney land and Disney world |
| John F. Kennedy    | 1917-1963 | 35th President of the US; the youngest person to be elected to the Office                                                                  |
| Albert Einstein    | 1879-1955 | German Theoretical Physicist, Renowned for developing the Theory of Relativity                                                             |
| Rosa Parks         | 1913-2005 | American Activist, Best known for her role in Montgomery Bus Boycott                                                                       |
| Coco Chanel        | 1883-1971 | French Fashion Designer and Business woman, Founder of the world - famous brand Chanel                                                     |
| Muhammad Ali       | 1942-2016 | American Boxer, Activist and Philanthropist                                                                                                |



|                 |           |                                                                                                                 |
|-----------------|-----------|-----------------------------------------------------------------------------------------------------------------|
| Michael Jackson | 1958-2009 | American Singer, Song writer and Dancer, Renowned as the King of Pop                                            |
| Satyajit Ray    | 1921-1992 | Oscar - award Winning Indian Filmmaker, writer and dillustrator; Known for films like Apu Trilogy and Jalsaghar |

**First in the World (Male)**

|                                                                                |                                                                          |
|--------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| First Asian to Head the International Cricket Council                          | Jagmohan Dalmiya                                                         |
| The First man to climb Mount Everest                                           | Sherpa Tenzing Norgay and Sir Edmund Hillary (29 <sup>th</sup> May 1953) |
| First Man to go into Space                                                     | Major Yuri Gagarin (USSR) (1961)                                         |
| First Man to walk into Space                                                   | Alexei Leonov (Russia)                                                   |
| First Person to give information about Planets and their motion around the Sun | Nicolaus Copernicus                                                      |

**First in the World (Female)**

|                                                           |                                |
|-----------------------------------------------------------|--------------------------------|
| First Woman President of a Country                        | Maria Estela Peron (Argentina) |
| First Woman in the world to cross the Strait of Gibraltar | Arti Pradhan (India)           |
| First Woman Cosmonaut in Space                            | Valentina Tereshkova (USSR)    |

|                                                           |                                          |
|-----------------------------------------------------------|------------------------------------------|
| First Woman Prime Minister                                | Sirimavo Bandaranaike (Sri Lanka)        |
| First Woman to have a Spacewalk                           | Svetlana Yevgenyevna Savitskaya          |
| First Woman Vice-President of United States of America    | Kamala Harris                            |
| First Woman to Climb Mount Everest                        | Junko Tabei (Japan)                      |
| First Woman Space Tourist                                 | Mrs. Anoushell Ansari (Iranian-American) |
| First Female Amputee to Climb Mount Everest               | Junko Tabei (Japan)                      |
| First Women CFO and MD of World Bank                      | Anshula Kant                             |
| First Women Chief Economist for IMF                       | Gita Gopinath                            |
| First Astronaut to complete historic all-female Spacewalk | Christina Koch Jessica Meir              |

**EXERCISE**

- In cricket match, while catching a fast-moving ball, a fielder in the ground gradually pulls his hands backwards with the moving ball to reduce the velocity to zero. the act represents:
  - Newton's first law of motion
  - Newton's second law of motion
  - Newton's third law of motion
  - law of conservation of energy
- The First Delimitation Commission in India was constituted in:
  - 1949
  - 1950
  - 1951
  - 1952
- Which among the following was the host country of the United Nations World Environment Day, 2022?
  - Canada
  - Sweden
  - South Africa
  - Brazil
- In which one of the following years was the first telegraphic connection established between Europe and India?
  - 1851
  - 1854
  - 1865
  - 1871
- In the field of tourism, which one of the following Indian States is described as 'One State Many Worlds'?
  - Assam
  - West Bengal
  - Karnataka
  - Rajasthan
- Where is-world's first Partition Museum inaugurated recently?
  - New Delhi
  - Lahore
  - Amritsar
  - Islamabad
- Which one of the following are the first and the oldest Government-owned coal mining company in India?
  - Neyveli Lignite Corporation
  - Singareni Collieries Company Limited
  - Coal India Limited
  - Mahanadi Coalfields Limited
- The electronic voting machines are developed jointly with:
  - Bharat Heavy Electricals Limited
  - Bharat Electronics Limited
  - Electronics Corporation of India Limited
  - Bharat Sanchar Nigam Limited
 Select the correct answer using the code given below:
  - 1, 2 and 3
  - 2 and 3 only
  - 2 and 4
  - 3 and 4
- Which one of the following sea ports gained significance for handling iron-ore exports to Japan?
  - Kandla
  - Ennore
  - Kochi
  - Mormugao
- Kailash Satyarthi is a .....
  - Social Reformer
  - Agriculture Scientist
  - Mathematician
  - Great innovator
- What is the original name of Nelson Mandela?
  - Rolihlahla Mandela.
  - Rohil Mandela
  - Carta Mandela
  - James Mandela
- Who among the following played a vital role in abolishment of slavery in America.
  - Nelson Mandela
  - Abraham Lincoln
  - Coco Chanel
  - Anne Frank
- What is the complete name of APJ Abdul Kalam?
  - Dr Avul Pakir Jainulabdeen Abdul Kalam
  - Dr Abdul Pakir Jainulabdeen Abdul Kalam
  - Dr Avul Ppamol Jainulabdeen Abdul Kalam
  - Dr Abul Pakir Jainulabdeen Abdul Kalam

14. Who is popularly known as a "master blaster"?  
 (a) Sachin Tendulkar (b) Kapildev  
 (c) M.S Dhoni (d) Yuvraj Singh
15. Who is author of "The Death of Ivan Ilyich"?  
 (a) Leo Tolstoy (b) Adam Smith

- (c) Karl Marks (d) Rosa Parks
16. President Jovenel Moise, who was the President of which one of the following countries?  
 (a) Dominican Republic (b) Haiti  
 (c) Guatemala (d) Cuba

### Answer Key

|         |         |         |         |         |         |        |        |        |         |
|---------|---------|---------|---------|---------|---------|--------|--------|--------|---------|
| 1. (b)  | 2. (d)  | 3. (b)  | 4. (c)  | 5. (c)  | 6. (c)  | 7. (c) | 8. (b) | 9. (d) | 10. (a) |
| 11. (a) | 12. (b) | 13. (a) | 14. (a) | 15. (a) | 16. (b) |        |        |        |         |

### Answers with Explanations

**1. Option (b) is correct.**

**Explanation:** While catching a ball, a player takes his hands backwards there by increasing the time for change in momentum of the ball. This reduces the effect of force imparted by the ball on the player

**2. Option (d) is correct.**

**Explanation: Delimitation commission**

- It is also called as Boundary commission of India.
- The primary task of the commission is to redraw the boundaries of the various assemblies and Lok Sabha constituencies based on a recent census.
- The present delimitation of constituencies has been done under the provisions of Delimitation Act, 2002 and on the basis of 2001 census.
- The first delimitation commission was set up in 1952.

**3. Option (b) is correct.**

**Explanation:** The United Nations Environment Program is organized every year on 5th June. Under the leadership of the United Nations, the United Nations Environment Program is rganized every year on 5th June, on the occasion of World Environment Day since 1973. World Environment Day is the largest global forum for environmental public outreach and is celebrated by millions of people around the world. Sweden is hosting this event in the year 2022.

**4. Option (c) is correct.**

**Explanation:** Telegraphic communication with India was first established in 1865 by overland telegraph lines from Europe to the top of the Persian Gulf and then by an undersea cable to Karachi.

**5. Option (c) is correct.**

**Explanation:** The "One State Many Worlds" was introduced by Department of Tourism, under Karnataka Government to improve Tourism in the state. The entire state of North Karnataka is a showcase of South India's architectural achievements. Because of its architectural significance, places like Hampi, Halebid, Gulbarga, Badami, Bidar, Bijapur, and Bangalore are well-known not just in India but around the world.

**6. Option (c) is correct.**

**Explanation:** The Partition Museum is located in the Town Hall in Amritsar, it was inaugurated on 17 August 2017. The museum aims to become the central repository of stories, materials, and documents related to the post-partition riots that followed the division of British India into two independent countries: India and Pakistan.

**7. Option (c) is correct.**

**Explanation:** Coal India Limited (CIL)

- It is a government-owned coal mining and refining corporation of India.
- Its headquarters are located in Kolkata, West Bengal.
- It was founded in 1975.
- It is a Maharatna public sector undertaking (PSU) and, is the largest coal-producer in the world.
- Neyveli India Limited (NLC) was formerly known as Neyveli Lignite Corporation Limited). It is a government-owned-fossil fuel miner and thermal power generator. It comes under the ownership of Ministry of Coal. It annually produces about 30 million tonnes of Lignite. It was incorporated in 1956.
- The Singareni Collieries Company Limited or SCCL comes under the ownership of Ministry of Energy, Government of Telangana. It was founded on 23 December 1920.

**8. Option (b) is correct.**

**Explanation:** The electronic voting machines (EVMs) are developed jointly by Bharat Electronics Limited and Electronics Corporation of India Limited. EVM was used on experimental basis for the first time in the states of Madhya Pradesh, Rajasthan and NCT of Delhi at the General Elections to the respective Legislative Assemblies held in November 1998.

**9. Option (d) is correct.**

**Explanation:** Marmagao Port, is situated at the entrance of the Zuari estuary. It is a natural harbor in Goa. Marmagao is one of the oldest ports on the west coast of India. Major ports of India.

| Name                              | State          |
|-----------------------------------|----------------|
| Deendayal Port Trust              | Gujarat        |
| Paradip Port Trust                | Odisha         |
| Jawaharlal Nehru Port Trust       | Maharashtra    |
| Visakhapatnam Port Trust          | Andhra Pradesh |
| Mumbai Port Trust                 | Maharashtra    |
| Syama Prasad Mookerjee Port Trust | WestBengal     |
| Chennai Port Trust                | Tamil Nadu     |
| New Mangaluru Port Trust          | Karnataka      |
| Kochi Port Trust                  | Kerala         |

**10. Option (a) is correct.**

*Explanation:* Kailash Satyarthi is a Indian social reformer working towards the elimination of child labour in India; Nobel Peace Prize Winner 2014.

**11. Option (a) is correct.**

*Explanation:* Born in a small village named Mvezo in South Africa, Nelson Mandela was originally named Rolihlahla Mandela.

**12. Option (b) is correct.**

*Explanation:* Abraham Lincoln was a self-taught lawyer and legislator, who went on to become one of the American heroes and great personalities of the American Civil War. He was appointed as the 16th US President shortly before the outbreak of the Civil War. He was played a vital role in abolishment of slavery in America

**13. Option (a) is correct.**

*Explanation:* People's favourite leader, Dr Avul Pakir Jainulabdeen Abdul Kalam, also known as APJ Abdul Kalam, was a millennial Aerospace scientist. Dr Kalam spent close to 4 decades in ISRO and DRDO as a scientist and administrator. He served as the 11th President of India from 2002 to 2007. After retiring, he became a visiting professor at IIMs, IITs, and other universities where he motivated students to innovate and compete with the world.

**14. Option (a) is correct.**

*Explanation:*

- Famed all across the world and carrying the nation's pride on his No. 10 jersey, Sachin Ramesh Tendulkar is prominent in the list of great personalities.
- Popularly known as a "master blaster" he is a true inspiration for all cricket lovers. This finest right-handed batsman in the history of international cricket started his cricket journey at the age of 11 and made his firstclass debut when he was 15 and his first test debut against Pakistan at the age of 16.

**15. Option (a) is correct.**

*Explanation:*

- With notable work in the realm of Literature, Leo Tolstoy, a Russian novelist and moral philosopher are regarded as one of the great personalities of bibliophiles.
- Ranked as one of the world's leading writers, Tolstoy has worked intensely on philosophical essays, short stories and numerous plays that turned out to be best sellers.
- Some of his renowned work includes The Death of Ivan Ilyich, Anna Karenina, Resurrection, War and Peace, Hadji Murad, and The Kingdom of God Is Within You, to name a few

**16. Option (b) is correct.**

*Explanation:* Jovenel Moise was a Haitian entrepreneur and politician. He had served as the president of Haiti from 2017 until his assassination in 2021.

○○○

- Operation Kaveri launched to evacuate Indians from Sudan on April 24, 2023 in response to the crisis in Sudan.
- The Indian Air Force (IAF) and the United States Air Force (USAF) kicked off the first phase of Exercise Cope India 23. The joint military exercise is being held at Air Force Stations Arjan Singh (Panagarh), Kalaikunda and Agra.
- Dalai Lama, the Tibetan spiritual leader was presented the 1959 Ramon Magsaysay Award at his residence personally after 64 years
- The World Bank report has ranked India at 38<sup>th</sup>, up from 44 in 2018, among 139 countries.
- The National Council for Education Research and Training (NCERT) has released India's first national assessment regulator, PARAKH.
- China's CRRC Corporation Ltd. launched a hydrogen urban train, and it is the first in Asia and the second such train in the world.
- RBI Governor Shaktikanta Das was chosen for the prestigious Governor of the Year Award 2023.
- India and Australia signed agreements in the field of migration and mobility partnership and on Green Hydrogen task force.
- The Indian Film Festival of Melbourne (IFFM) Awards once again took center stage, honouring the finest cinematic gems from the Indian film industry. The 14<sup>th</sup> Indian Film Festival of Melbourne (IFFM) Awards were held in Melbourne, Australia, on 11 August, honouring the best of film and OTT content from India globally. Filmmaker Karan Johar was honoured for completing 25 years in the Indian film industry as a Filmmaker. He received a huge round of applause for his immense contributions in the field of entertainment.
- The International Cricket Council (ICC) has revealed the latest set of international stars to be named ICC Players of the Month for July 2023. Australia's spin bowling all-rounder Ashleigh Gardner and England's seamer Chris Woakes have been honored as the International Cricket Council (ICC) Players of the Month for July 2023.
- The year so far has seen the decline in influence and severity of the COVID-19 pandemic, with the WHO reducing its global health emergency status in May. The Russian invasion of Ukraine, which began in 2022 continued, and a series of coups, an armed conflict, and political crises broke out in numerous African nations.
- **Chandrayaan-3 launched**
  - Indian Space Research Organisation (ISRO) launched the Chandrayaan-3 lunar mission to explore the Moon on 14<sup>th</sup> July 2023 at 2:35 p.m. IST.
  - It took off from the Satish Dhawan Space Centre, Andhra Pradesh on the back of a Geosynchronous Satellite Launch Vehicle Mark III (GSLV-MK III) heavy-lift rocket. It consists of a lander named Vikram and a rover

er named Pragyan similar to Chandrayaan-2, but does not have an orbiter. The Chandrayaan-3 mission has three major objectives:

- Demonstrate safe and soft landing on the surface of the Moon,
  - Conduct rover operations on the Moon, and
  - Conduct on-site experiments on the lunar surface.
- The Vikram lander of the mission is planned to soft-land on the surface of the South Pole region of the Moon on August 23 at 5:47 p.m.
  - Ritu Karidhal, a senior scientist at ISRO known as the 'Rocket Woman' of India, is the Mission Director for Chandrayaan-3, India's third lunar exploration mission. P Veeramuthuvel is the Project Director for Chandrayaan-3.
  - In 2019, Chandrayaan-2, captured the world's attention when it successfully placed the Vikram lander and Pragyan rover near the Moon's South Pole. But the mission experienced partial setbacks. The Vikram lander on Chandrayaan-2 had crashed on the lunar surface while attempting to land. Despite the mishap, it still marked a significant achievement in India's space history.

## Key Points

|                             |                        |
|-----------------------------|------------------------|
| ISRO's Chairman             | – S. Somanath          |
| ISRO's HQ                   | – Bengaluru, Karnataka |
| Total Cost of Chandrayaan-3 | – ₹615 crore           |

## ➤ World's largest private communications satellite 'Jupiter 3' launched

- SpaceX, Elon Musk's space exploration company, launched the world's largest commercial communications satellite named Jupiter 3 from LC-39A, Kennedy Space Center, Florida, USA. The Falcon Heavy rocket was used for the launch.
- JUPITER 3, also referred to as 'EcoStar XXIV', is a satellite of Hughes, an EchoStar company. It will provide gigabytes of connectivity to customers across North and South America.
- EchoStar Corporation is an American company and serves as a global provider of satellite communication and Internet services through its business segments, namely Hughes Network Systems and EchoStar Satellite Services.

## ➤ India became the first country to successfully soft land on Moon's South Pole.

- ISRO's Vikram lander successfully soft-landed on Moon's south pole on 23<sup>rd</sup> August 2023 at 6:04 p.m. This made India the first country to successfully soft land on Moon's South Pole.
- Widely known as the dark side of the Moon, it has been an uncharted territory that scientists believe could hold important reserves.
- The solar-powered rover Pragyan will explore the surface and transmit data to Earth over its two-week lifespan.

- The lander, Vikram, which means 'valour' in Sanskrit, detached from its propulsion module and has been sending images of the Moon's surface since entering lunar orbit on August 5, 2023.
- ISRO launched the Chandrayaan-3 lunar mission to explore the Moon on 14<sup>th</sup> July 2023. It took off from the Satish Dhawan Space Centre, Andhra Pradesh on the back of a Geosynchronous Satellite Launch Vehicle Mark III (GSLV-MK III) heavy-lift rocket. It consisted of a lander named Vikram and a rover named Pragyan similar to Chandrayaan-2, but does not have an orbiter. The cost of Chandrayaan-3 is merely ₹600 crores.
- Ritu Karidhal, a senior scientist at ISRO known as the 'Rocket Woman' of India, is the Mission Director for Chandrayaan-3. P Veeramuthuvel is the Project Director for Chandrayaan-3.
- In 2019, Chandrayaan-2, captured the world's attention when it successfully placed the Vikram lander and Pragyan rover near the Moon's South Pole. But the mission experienced partial setbacks. The Vikram lander on Chandrayaan-2 had crashed on the lunar surface while attempting to land. Despite the mishap, it still marked a significant achievement in India's space history.

#### Key Points

|                 |                        |
|-----------------|------------------------|
| ISRO's Chairman | – S. Somanath          |
| ISRO's HQ       | – Bengaluru, Karnataka |

#### ➤ Chandrayaan-3's Pragyan rover confirms the presence of Sulphur on the surface of the Moon

- Chandrayaan-3's Pragyan rover module has confirmed the presence of sulphur on the surface of the Moon. This was confirmed by the 'Laser-Induced Breakdown Spectroscopy (LIBS)' instrument which is onboard the Rover.
- ISRO shared the results of the first such in-situ recordings near the lunar south pole that will be crucial for understanding the Moon's elemental composition.
- Apart from Sulphur, the rover has detected elements including Aluminium (Al), Calcium (Ca), Iron (Fe), Chromium (Cr), Titanium (Ti), Manganese (Mn), Silicon (Si), and Oxygen (O).
- LIBS is a technique wherein the surface of a material, such as soil or rock bed, is exposed to high-energy laser pulses. This generates a high temperature, localised plasma which is then collected and spectrally resolved and detected by instruments such as Charge Coupled Devices.
- Each element has a unique set of wavelengths of light. It is based on this wavelength of light that the elemental composition of the surface is determined.
- LIBS was developed at the Laboratory of Electro-Optics Systems (LEOS), a unit of ISRO that deals with 4 OSWAAL Current Affairs the design, development and production of attitude sensors for all Low Earth Orbit (LEO) and Geostationary Equatorial Orbit (GEO) and interplanetary missions.

#### ➤ PM Modi announces August 23 as 'National Space Day', lauds ISRO scientists

- Prime Minister Narendra Modi announced August 23 as the 'National Space Day' as Chandrayaan-3 made a successful historical soft landing on the south pole of the Moon on this day.

- He further announced that the point on the moon where Chandrayaan-3 landed will be called 'Shivshakti'. The point where its predecessor, Chandrayaan-2 hit, will be known as the 'Tiranga' point.

#### ➤ Indian Air Force (IAF) inducts new Heron Mark 2 drones

- In a mega capability boost, the Indian Air Force has inducted its latest Heron Mark 2 drones, which have strike capability and can carry out surveillance along the borders of both China and Pakistan in a single sortie.
- These drones can be equipped with long-range missiles and other weapons systems, and have been deployed at a forward air base in the northern sector.
- The drones with satellite communication capability have given a long desired capability to the Indian Air Force, as its drones can operate at very long distances for close to 36 hours at a stretch and can also laser illuminate enemy targets from very long ranges to help fighter aircraft destroy them using their longrange weapons.
- These can provide  $24 \times 7$  surveillance of targets.

#### Key Points

Chief of Air Staff (CAS) – Vivek Ram Chaudhari

#### ➤ Indian Army connects last village on LoC in J&K with bridge as Independence Day gift

- The Indian Army linked this Last Danna village on LoC in Machhal Sector, Kupwara District, of Jammu and Kashmir (J&K) with the 115 Feet Long Bridge.
- The 115-foot-long Bridge has been named as Bhagat Bridge in memory of Late Major Bhagat Singh, Veer Chakra, who laid down his life defending this sector in the 1965 war. Danna Village is also popularly known as Bhagat Village in memory of the brave son of India.
- The dedication ceremony was attended by villagers, including children, women, and elders from seven nearby villages, who expressed gratitude for the infrastructure that would improve education, healthcare access, and tourism prospects in the region.

#### ➤ President Murmu launched INS 'Vindhyagiri' in Kolkata, the sixth ship under 'Project 17 Alpha' of the Indian Navy

- President Droupadi Murmu launched 'Vindhyagiri', at Garden Reach Shipbuilders and Engineers Ltd's (GRSE) facility on the banks of Hooghly river. Vindhyagiri has been named after the mountain range in Karnataka.
- INS 'Vindhyagiri' is an advanced stealth frigate, or warship. This is the sixth of seven ships built under 'Project 17 Alpha' for the Navy. The first five ships of the project were launched between 2019 and 2022.
- This is the third and last stealth frigate that the Kolkata-based warship maker was contracted to build for the Navy under the project.
- 75% of orders for equipment and systems of P17A ships are from indigenous firms, including micro, small and medium enterprises (MSMEs).
- The state-of-the-art ship will be fitted with the latest gadgets and undergo extensive trials before being handed over to the Indian Navy for commissioning into service.
- These warships are follow-ons of the Project 17 Class Frigates (Shivalik Class), with improved stealth fea-



tures, advanced weapons and sensors and platform management systems.

- P17A ships are guided missile frigates, each of which is 149 metres long, with a displacement of approximately 6,670 tonnes and a speed of 28 knots. These are capable of neutralising threats in all three dimensions of air, surface and sub-surface.

#### ☛ **FIFA Women's World Cup 2023: Spain wins the title with 1-0 victory over England**

- Spain secured their first-ever Women's World Cup title with a 1-0 win over England at the 2023 FIFA Women's World Cup final in Sydney.
- Captain Olga Carmona smashed the decisive goal as Spain defeated England 1-0 to lift the FIFA Women's World Cup title.
- This win has made Spain the first European team to win the Women's World Cup since Germany in 2007. In addition, Spain's victory makes them only the second nation to have won both the men's and the women's World Cups. The other nation is Germany, which won the men's World Cup in 1974 and 1990, and the women's World Cup in 2003 and 2007.

#### ☛ **About G-20**

- G20 was established in 1999 as a response to various global economic crises and the G20 member nations collectively account for approximately 85% of the world's GDP, more than 75% of global trade, and roughly two-thirds of the global population.
- The G20 group is made up of Argentina, Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, South Korea, Mexico, Russia, Saudi Arabia, South Africa, Turkey, the United Kingdom, the United States, and the European Union (EU). In 2023, the African Union (AU) was also added to the group.

#### ☛ **G20 Summit held in Delhi from September 9-September 10, 2023**

- The 2023 G20 summit event was held at Bharat Mandapam International Exhibition-Convention Centre, Pragati Maidan, New Delhi from September 9-10, 2023.
- It was held under the theme 'Vasudhaiva Kutumbakam' which means 'One Earth, One Family, One Future.'
- India hosted the G20 Summit for the first time after it received the Presidency from Indonesia in 2022
- Some of the important world leaders who participated in the 2023 summit were – US President Joe Biden, UK Prime Minister Rishi Sunak, Canada PM Justin Trudeau, French President Emmanuel Macron, Australian Prime Minister Anthony Albanese, German Chancellor Olaf Scholz, Japanese Prime Minister Fumio Kishida, Brazilian President Luiz Inacio Lula da Silva, Bangladesh Prime Minister Sheikh Hasina, UAE President HH Shaikh Mohamed Bin Zayed Al Nahyan, Prime Minister of Italy Giorgia Meloni, Ursula von der Leyen, President of European Commission, Pravin Kumar Jugnauth, Prime Minister of Mauritius; European Council President Charles Michel, China's Premier Li Qiang, Turkey President Recep Tayyip Erdoğan. Chinese President Xi Jinping and Russia's President Vladimir Putin didn't attend the event.
- Some of the other international diplomats who participated in the event were Mathias Cormann, Secretary

General of OECD, Ajay Banga, President of the World Bank, Kristalina Georgieva, Managing Director of the International Monetary Fund (IMF), C and Tedros Adhanom, Director General of World Health Organisation (WHO).

- The G20 leaders unanimously arrived at the consensus and adopted the 'New Delhi Declaration'. Also, the African Union (AU) was included in the G20 group.
- PM Modi handed over the G20 presidency to Brazil's Lula Da Silva, marking the end of the G20 Summit in Delhi.

#### ☛ **ISRO launches Aaditya L-1 mission to study the Sun**

- On September 2, 2023, ISRO launched Aditya-L1 from the Satish Dhawan Space Centre in Sriharikota, Andhra Pradesh.
- Aditya-L1 is the first space-based observatory-class Indian solar mission to study the Sun. It will study the outermost layers of the Sun. It aims to shed light on the dynamics of several solar phenomena by imaging and measuring particles in the sun's upper atmosphere.
- It had seven distinct payloads developed, all indigenously – five by ISRO and two by Indian academic institutes in collaboration with ISRO.
- The seven payloads will observe the photosphere, chromosphere and the outermost layers of the Sun, called the corona, using electromagnetic, particle and magnetic field detectors. Four payloads will directly view the Sun and the remaining three payloads will carry out in-situ studies of particles and fields, "providing important scientific studies of the propagatory effect of solar dynamics in the interplanetary medium."
- Initially, the spacecraft will be placed in a low earth orbit. The orbit is gradually made more elliptical, after which the spacecraft will be launched towards its destination: L1 refers or Lagrange point 1 (L1) of the Sun-Earth system, which is about 1.5 million km from the Earth.
- Lagrange points are the positions in space where the gravitational pull of the two large bodies equals the necessary centripetal force required for a small object to move with them – meaning a small object placed there will stay there.
- The total travel time from launch to L1 would take about four months.
- The project director of Aaditya L-1 is Nigar Shaji.

#### **Key Points**

|              |                                  |
|--------------|----------------------------------|
| Headquarters | – Mandaluyong, Philippines       |
| Formation    | – 19 <sup>th</sup> December 1966 |
| President    | – Masatsugu Asakawa              |

#### ☛ **Old Indian Parliament building to be known as 'Samvidhan Sadan': PM Modi**

- Prime Minister Narendra Modi suggested that the old building, where meetings of the Constituent Assembly were held to frame the Constitution, should be known as 'Samvidhan Sadan' (Constitution House).
- The old Parliament Building was designed by British architects Sir Edwin Lutyens and Herbert Baker and was completed in 1927.
- The new parliament building was inaugurated by PM Modi on May 28, 2023, on Kartavya Path, New Delhi.
- The Lok Sabha Chamber of the new building has a capacity of 888 members while the Rajya Sabha Cham-

ber holds a capacity of 300 people. For a joint sitting of both houses, 1,280 MPs can be accommodated in the Lok Sabha chamber.

- Each Member of Parliament will have a 40 sq.m office space in the redeveloped Shram Shakti Bhawan, which will be completed by 2024. The national archives will also move from the old building to the new one. The four-storey mammoth edifice has a built-up area of 64,500 square metres with three main gates -- Gyan Dwar, Shakti Dwar and Karma Dwar. Entrances are separate entrances for VIPs, MPs and visitors.
- **NCERT panel suggests replacing 'India' with 'Bharat'**
- The 2022 Social Science Committee of the National Council of Educational Research and Training (NCERT) has suggested that 'India' should be called 'Bharat' in all Social Science textbooks up to Class 12.
- The roots of "Bharat", "Bharata", or "Bharatvarsha" are traced back to Puranic literature, and to the epic Mahabharata. The Puranas describe Bharata as the land between the "sea in the south and the abode of snow in the north". Bharata is also the name of the ancient king of legend who was the ancestor of the Rig Vedic tribe of the Bharatas, and by extension, the progenitor of all peoples of the subcontinent.
- Bharata is also the name of the ancient king of legend who was the ancestor of the Rig Vedic tribe of the Bharatas, and by extension, the progenitor of all peoples of the subcontinent.

#### Key Points

|                    |                         |
|--------------------|-------------------------|
| NCERT Headquarters | – Delhi                 |
| NCERT President    | – Dharmendra Pradhan    |
| NCERT Director     | – Dinesh Prasad Saklani |

#### ➤ **Operation Al-Aqsa Flood-Hamas launched an air attack on Israel**

- The Palestinian armed group Hamas on October 7 launched a surprise attack on Israel in one of the most serious escalations in the Israel-Palestinian conflict in years.
- Hamas fighters entered Israeli territory from the Gaza Strip, killing more than 1,400 people and sending shockwaves across Israel. The gunmen crossed the border with a heavy barrage of rockets from the Gaza Strip.
- Israeli Prime Minister Benjamin Netanyahu launched a brutal military offensive in Gaza, killing more than 4,600 people as he promised to finish Hamas.
- Hamas stands for the Islamic Resistance Movement and in Arabic means "zeal".
- The group politically controls the Gaza Strip, a territory of about 365 sq km that is home to more than two million people but is blockaded by Israel.
- Hamas has been in power in the Gaza Strip since 2007 after a brief war against Fatah forces loyal to President Mahmoud Abbas, the head of the Palestinian Authority and Palestine Liberation Organization (PLO). The Hamas movement was founded in Gaza in 1987 by an imam, Sheikh Ahmed Yasin, and his aide Abdul Aziz al-Rantissi shortly after the start of the first Intifada, an uprising against Israel's occupation of the Palestinian territories.
- The movement started as an offshoot of the Muslim Brotherhood in Egypt and created a military wing, the

Izz al-Din al-Qassam Brigades, to pursue an armed struggle against Israel with the aim of liberating historic Palestine.

#### ➤ **Israel's Iron Dome missile system**

- Iron Dome is designed to protect against incoming short-range weapons. It operates in all weather conditions. It uses radar to track rockets and can differentiate between those that are likely to hit builtup areas and those that are not.
- Interceptor missiles are only fired at rockets expected to strike populated areas.
- The system consists of batteries located across Israel, each with three to four launchers that can fire 20 interceptor missiles. There are both fixed and mobile versions of the system.
- Iron Dome was developed after the 2006 conflict between Israel and Hezbollah, a militant group based in southern Lebanon.
- Iron Dome was created by Israeli firms Rafael Advanced Defense Systems and Israel Aerospace Industries.

#### ➤ **Israel's Iron Dome missile system**

- Iron Dome is designed to protect against incoming short-range weapons. It operates in all weather conditions. It uses radar to track rockets and can differentiate between those that are likely to hit builtup areas and those that are not.
- Interceptor missiles are only fired at rockets expected to strike populated areas.
- The system consists of batteries located across Israel, each with three to four launchers that can fire 20 interceptor missiles. There are both fixed and mobile versions of the system.
- Iron Dome was developed after the 2006 conflict between Israel and Hezbollah, a militant group based in southern Lebanon.
- Iron Dome was created by Israeli firms Rafael Advanced Defense Systems and Israel Aerospace Industries.

#### ➤ **Indian Navy gets the third guided missile stealth destroyer- Imphal**

- The third guided-missile stealth destroyer, named "Imphal," has been delivered to the Indian Navy by Mazagon Docks.
- With a displacement of 7,500 tons and a crew of 312, the 164-meter warship is equipped with surface-to-air missiles and BrahMos supersonic cruise missiles.
- Imphal is the third vessel of Project 15B that was built at Mazagon Dock Shipbuilders Limited (MDL).
- The Project is a follow-on of the Kolkata class (Project 15A) destroyers commissioned in the last decade. Imphal follows in the wake of her predecessors Indian Naval Ships Visakhapatnam and Mormugao commissioned in the past two years.

#### Key Points

Chief of Naval Staff – Admiral R. Hari Kumar  
Navy Day – 4 May

#### ➤ **Indian Air Force (IAF) to stop flying MiG-21 by 2025**

- Amid the efforts to replace its ageing Russian-origin aircraft, IAF is all set to replace all MiG-21 squadrons with the LCA Mark 1A by 2025.
- MiG-21, which has been in service with the IAF for more than 60 years now, has met with several accidents in the recent past.



**Key Points**

Air Chief Marshal – V.R. Chaudhari

➤ **IAF Deploys Three S-400 Missile Units Along China And Pakistan Borders**

- IAF has deployed its three S-400 air defense missile squadrons along borders with China and Pakistan.
- In 2018-19, India agreed with Russia to procure S-400 missiles worth Rs. 35,000 crore (approx). As per the agreement five squadrons of S-400 missiles were set to be delivered by Russia to India. Out of five squadrons, three were delivered. Whereas the delivery of the remaining two was hindered due to the Russia-Ukraine conflict.

➤ **Asian Games 2023**

- The Asian Games were held from September 23 to October 8, 2023 in Hangzhou, the People's Republic of China. A 655-member Indian contingent participated in the same.
- This was the 19th edition of the continental quadrennial multisport. It was originally scheduled for 2022 but was postponed by a year due to COVID-19.
- India finished their Asian Games 2023 campaign with a record haul of 107 medals – 28 gold, 38 silver and 41 bronze. This surpassed India's previous record tally achieved at the last edition at Jakarta 2018, where a 570-strong Indian squad racked up 70 medals – 16 gold, 23 silver and 31 bronze.
- China led the overall Asian Games 2023 medals tally with 201 gold, ahead of Japan (52) and the Republic of Korea (42).
- India won 22 medals in shooting which led the way at the games. This included seven gold.
- In Athletics, Neeraj Chopra won the gold with his successful title defence in the javelin throw. Besides, athletes won six gold along with 14 silver and nine bronze.
- The archery compound team won five gold medals in the category.
- The cricket and kabaddi teams also bagged two gold medals each while the men's hockey team's gold secured a spot at the Paris 2024 Olympics.
- India also earned their first Asian Games badminton gold medal courtesy of the men's doubles pair of Chirag Shetty and Satwiksairaj Rankireddy.
- Squash, tennis and equestrian completed the 28 gold medals for India in Hangzhou.
- In addition to medals, Hangzhou offered 74 Paris 2024 Olympic quotas – six in archery, 10 in artistic swimming, 34 in boxing, two in breaking, two in hockey, 10 in modern pentathlon, six in sailing, two in tennis and two in water polo.
- India secured six quotas in all – four in boxing (Nikhata Zareen, Preeti Pawar, Parveen Hooda, Lovlina Borgohain) and one each in athletics (Kishore Jena) and men's hockey.

➤ **India wins 111 Medals at the fourth Asian Para Games**

- Indian para-athletes created history as they ended their Hangzhou Asian Para Games campaign with an unprecedented 111 medals, the biggest haul for the country in any major international multi-sport event.
- The para-athletes won 29 gold, 31 silver and 51 bronze.

➤ **Lionel Messi wins record eighth Ballon d'Or for best player in the world**

- Argentina captain Lionel Messi won a record eighth Ballon d'Or for the best player in the world, beating Norway's UEFA player of the year and treble winner Erling Haaland of Manchester City to the prestigious prize.
- He played a pivotal role in leading Argentina to their first World Cup victory since 1986. This remarkable achievement places Messi three Ballons d'Or ahead of his rival Cristiano Ronaldo, who last won the award in 2017.
- Messi has now been a top-three finalist for this prestigious accolade a total of 14 times, with five instances as the runner-up.

➤ **Cricket approved by IOC for 2028 Los Angeles Olympics**

- The International Olympic Committee (IOC) recently took a significant step towards embracing the world's most popular sport, Cricket, by accepting the recommendation to include T20 cricket for both men and women at the 2028 Los Angeles Summer Olympics.
- This decision was made during the IOC's executive board meeting in Mumbai, heralding a new era for the sport on the global stage.
- The acceptance of cricket's inclusion in the 2028 Los Angeles Summer Olympics represents a pivotal moment in the sport's history.

➤ **Sachin Tendulkar named as the 'Global Ambassador' for ODI World Cup 2023**

- The International Cricket Council (ICC) has made an exciting announcement designating the legendary cricketer Sachin Tendulkar as the 'Global Ambassador' for the Cricket World Cup 2023.
- Sachin Tendulkar holds the record for participating in six 50-over World Cups.
- In addition to Sachin Tendulkar, the ICC has assembled an impressive group of cricketing legends to serve as ambassadors for the World Cup 2023. This eclectic mix includes West Indies' icon Vivian Richards, South Africa's cricketing maestro AB de Villiers, England's World Cup-winning captain Eoin Morgan, Australia's formidable Aaron Finch, Sri Lanka's spin wizard Muttiah Muralitharan, New Zealand's stalwart Ross Taylor, India's charismatic Suresh Raina, former captain Mithali Raj, and Pakistan's versatile all-rounder Mohammad Hafeez.

**EXERCISE**

- Which one of the following statements with respect to Global Positioning System (GPS) is not correct?
  - It is based on network of satellites orbiting above the Earth.
  - It is based on the system of triangulation.

- GPS receivers provide location in terms of latitude, longitude and altitude.
- It provides information exclusively for military operations.

2. Nisarga, Gati, Nivar, Tauktae and Yaas are names of
  - (a) new fighter aircraft
  - (b) tourist places
  - (c) weather stations
  - (d) cyclones
3. Which one of the following states/UTs was connected with the Indian Railways network in the year 2021?
  - (a) Mizoram
  - (b) Ladakh
  - (c) Manipur
  - (d) Sikkim
4. Which one of the following statements is not correct in respect of the 'Legion of Merit' award?
  - (a) This award is conferred by the President of the United States of America.
  - (b) This award was conferred to the Prime Minister of India in December, 2020.
  - (c) This was also awarded to the Prime Minister of Australia, Scott Morrison.
  - (d) It is the highest civilian award of the United States of America.
5. Recently a state of emergency has been declared in which one of the following countries?
  - (a) Maldives
  - (b) Bhutan
  - (c) Nepal
  - (d) Myanmar
6. Which country has replaced Maldives to host the 2023 Indian Ocean Island Games?
  - (a) Madagascar
  - (b) Sri Lanka
  - (c) India
  - (d) Mauritius
7. Which one of the following nations is not a permanent member of G7?
  - (a) India
  - (b) Canada
  - (c) France
  - (d) Italy
8. 'Belt and Road Initiative' is a foreign policy World Test initiative of:
  - (a) China
  - (b) United States of America
  - (c) Canada
  - (d) Japan
9. Which one of the following countries has hosted the Army Exercise TSENTR, 2019?
  - (a) Russia
  - (b) China
  - (c) Kazakhstan
  - (d) Kyrgyzstan
10. In January 2020, a passenger aircraft crashed in Iran soon after taking off from Tehran's Imam Khomeini airport killing about 170 people onboard. The airplane belongs to:
  - (a) Qatar Airways
  - (b) Ukraine International Airlines
  - (c) Singapore Airlines
  - (d) Cathay Pacific
11. Which one of the following cities is associated with Biosafety Protocol to the Convention on Biological Diversity (2000)?
  - (a) Geneva
  - (b) Nairobi
  - (c) Cartagena
  - (d) Rio de Janeiro
12. Which one of the following is a major environmental issue in eastern Canada?
  - (a) Acid precipitation
  - (b) Groundwater depletion
  - (c) Land degradation
  - (d) Desertification
13. Which one of the following countries does NOT have direct access to the sea/ocean?
  - (a) Syria
  - (b) Jordan
  - (c) Azerbaijan
  - (d) Armenia
14. Which one of the following countries has failed to qualify for the first time in 60 years for the FIFA World Cup to be held in Russia in the year 2018?
  - (a) Mexico
  - (b) Iran
  - (c) Saudi Arabia
  - (d) Italy
15. Which among the following is the theme of the International Day for Monuments and Sites (World Heritage Day) 2017?
  - (a) The Heritage of Sport
  - (b) Cultural Heritage and Sustainable Tourism
  - (c) Past and Present Heritage
  - (d) Heritage and Science
16. In April 2017, USA dropped MOAB (Massive Ordnance Air Blast popularly known as the Mother of All Bombs) in the suspected hideouts of militants in which one of the following countries?
  - (a) Iran
  - (b) Syria
  - (c) Afghanistan
  - (d) Somalia
17. Which one of the following is the theme of the World Health Day, 2017 celebrated by the World Health Organization?
  - (a) Diabetes
  - (b) Food Safety
  - (c) Depression: Let's Talk
  - (d) Ageing and Health
18. The European Space Agency (ESA) successfully landed a spacecraft on a speeding comet that lies 310 million miles away from Earth. As a result, the scientists may find out:
  - (a) the age of the Sun
  - (b) the age of the Universe
  - (c) new fundamental atomic particles
  - (d) the role of the comets in bringing organic matter and water to Earth
19. Which one among the following statements with regard to a climate deal adopted between USA and China in November 2014 is not correct?
  - (a) China committed to cap its output of carbon pollution by the year 2030 or earlier.
  - (b) China committed to increase its use of zero emission energy to 20% by the year 2030.
  - (c) USA agreed to reduce its emissions by 26%-28% below its year 2005 levels in the year 2025.
  - (d) Both the countries agreed to completely phase out coal technologies by the year 2020.
20. Which one among the following films was awarded the best motion picture (Drama) in the 72<sup>nd</sup> Golden Globe Awards?
  - (a) The Theory of Everything
  - (b) Boyhood
  - (c) Still Alice
  - (d) Birdman
21. Which one of the following books won the Man Booker Prize 2014?
  - (a) History of the Rain
  - (b) The Lives of Others
  - (c) The Narrow Road to the Deep North
  - (d) How to be Both
22. Consider the following pairs of vaccine and category/type:
  1. Covaxin : Inactivated pathogen-based vaccine
  2. Covishield : mRNA vaccine
  3. Sputnik V : Viral vector-based vacciner

- Which of the pairs given above is/are correctly match?  
 (a) 1 only (b) 2 and 3 only  
 (c) 1 and 3 only (d) 1, 2 and 3
23. Recently, India's first green hydrogen-based advanced fuel cell electric vehicle (FCEV) was launched. It has no tailpipe emissions other than  
 (a) Hydrogen peroxide (b) Nitrogen  
 (c) Oxygen (d) Water
24. Which one of the following is the latest addition to the AYUSH group of health care system?  
 (a) Unani (b) Siddha  
 (c) Sowa-Rigpa (d) Reiki
25. Which one of the following Indian institutes was approved by the Drugs Controller General of India for conducting human trials of the Oxford-AstraZeneca Covid-19 vaccine candidate?  
 (a) Bharat Biotech  
 (b) AIIMS  
 (c) Serum Institute of India  
 (d) National Institute of Epidemiology
26. Adar Poonawalla launched the Clean City initiative in Pune in the year \_\_\_\_\_.  
 (a) 2018 (b) 2012 (c) 2010 (d) 2015

### Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d)  | 2. (d)  | 3. (c)  | 4. (d)  | 5. (d)  | 6. (a)  | 7. (a)  | 8. (a)  | 9. (a)  | 10. (b) |
| 11. (c) | 12. (a) | 13. (d) | 14. (d) | 15. (b) | 16. (c) | 17. (c) | 18. (d) | 19. (d) | 20. (b) |
| 21. (c) | 22. (c) | 23. (d) | 24. (c) | 25. (c) | 26. (d) |         |         |         |         |

### Answers with Explanations

**1. Option (d) is correct.**

**Explanation:**

- The GPS is a satellite-based navigation system made up of a network of satellites placed in earth's orbit. It is owned by the United States government and operated by the United States Space Force.
- It is based on the system of Triangulation. It is a method for determining a position based on the distance from other points that have known locations.
- GPS receivers provide location in longitude, latitude and altitude and also provide the accurate time.

**2. Option (d) is correct.**

**Explanation:**

- Cyclone Nisarga – State of Maharashtra in June 2020.
- Cyclone Gati – Somalia, Yemen (2020)
- Cyclone Nivar – Tamil Nadu, Andhra Pradesh (November 2020)
- Cyclone Tauktae – Arabian Sea (affected many Indian cities and other countries too)
- Cyclone Yaas – Odisha, West Bengal (May 2021)

**3. Option (c) is correct.**

**Explanation:** The state of Manipur was connected with Indian Railways Network in 2021. A passenger train, the Rajdhani Express, arrived at the Vaingai-chunpao railway station in Manipur for a trial run from Assam's Silchar railway station.

**4. Option (d) is correct.**

**Explanation: Legion of Merit**

- It is a military award of the United States Armed Forces.
- It is also the first U.S. medal to be awarded to citizens of other countries.
- It is given for exceptional service, fidelity, and loyalty in combat or noncombat roles.
- Prime Minister Narendra Modi was awarded the 'Legion of Merit' by U.S. President Donald Trump in 2020 for his role in encouraging the India- U.S. relationship.

**5. Option (d) is correct.**

**Explanation:** State of Emergency was declared in Myanmar for a period of one year after State Counsellor Aung San Suu Kyi and President U Win Myint were detained in Early-morning Raid. The emergency was declared by Myanmar military.

**6. Option (a) is correct.**

**Explanation:**

- Madagascar will host the 11<sup>th</sup> edition of the Indian Ocean Islands Games in 2023 after replacing Maldives due to the concerns over the ongoing pandemic.
- It was been announced by the president of International Games Council, Seychellois Antonio Gopal.
- The Indian Ocean Island Games is a multi-sport event held every four years among athletes from Indian Ocean Island nations, namely Mauritius, Seychelles, Comoros, Madagascar, Mayotte, Reunion and the Maldives.

**7. Option (a) is correct.**

**Explanation:**

**Group of Seven (G7)**

- It comprises seven countries - Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States.
- It is not based on any treaty and does not have permanent secretariat or office.
- Its presidency rotates annually among the member states and for 2022 Germany is holding the presidency.
- Several democratic countries – India, Spain, Indonesia and Poland participate as observers.

**8. Option (a) is correct.**

**Explanation:**

**Belt and Road Initiative**

- It was formerly known as One Belt One Road.
- It is a global infrastructure development strategy adopted in 2013 by the Chinese government.
- It aims to invest in nearly 70 countries and international organizations.

**9. Option (a) is correct.****Explanation:** Military Exercise TSENTR 2019

It was conducted from September 9 to 23 at Don-guz training ranges, Orenburg Russia. Every year, Russia holds major military exercise at one of its four military Commands:

1. Vostok (East)
2. Zapad (West)
3. TSENTR (Centre)
4. Kavkas (South)

Participating countries of TSENTR 2019 - China, India, Kazakhstan, Kyrgyzstan, Tajikistan, Pakistan and Uzbekistan

**10. Option (b) is correct.**

**Explanation:** The aircraft belonged to Ukraine International Airlines. The Iranian Islamic Revolutionary Guards Corps (IRGC) shot down the Boeing 737- 800 flying the route shortly after take-off from Tehran Imam Khomeini International Airport on January 8, 2020. All 176 passengers and crew were killed in the incident

**11. Option (c) is correct.**

**Explanation:** The Cartagena Protocol on Biosafety is also known as the Biosafety Protocol. It was adopted in January 2000. Cartagena is a city in Colombia. It aimed to protect biological diversity from the possible risks posed by genetically modified organisms resulting from modern biotechnology.

**12. Option (a) is correct.**

**Explanation:** Acid deposition is a problem in many parts of Canada since emissions (sulphur dioxide and nitrogen oxide gases and the related acid particles) that contribute to acid rain can travel thousands of kilometres from their source. The majority of Canadian geology consists of vulnerable hard rock (granite) areas that have poor natural defenses against acid deposition.

**13. Option (d) is correct.**

**Explanation:** Armenia is a landlocked country. It is located in western Asia. It is surrounded by:

- Turkey to the west,
- Georgia to the north,
- Azerbaijan to the east
- Iran to the south

**14. Option (d) is correct.**

**Explanation:** Italy the four time world cup winner failed to qualify for first World Cup in 60 years after play-off defeat to Sweden in 2018 at Russia.

- FIFA (Federation of International Football Association).
- It was founded in 1930.
- It is simply called World Cup and It held after the gap of every 4 years except in 1942 and 1946 due to 2nd World War.
- Last World Cup held in 2018 (at Russia) - winning team was France.
- Next World Cup is scheduled in 2022-Qatar (Host Country).

**15. Option (b) is correct.**

**Explanation:** The world Heritage Day is observed every year on 18<sup>th</sup> April. The theme of this day in 2017 was Cultural Heritage & Sustainable Tourism. The day was approved by General Assembly of UNESCO in 1983. It aims to enhance awareness about the importance of cultural heritage, monuments and to conserve them.

**16. Option (c) is correct.**

**Explanation:** The GBU-43/B Massive Ordnance Air Blast (MOAB) is also known as the "Mother of All Bombs. It is a large-yield bomb. It was developed by Albert L. Weimorts, Jr. of the Air Force Research Laboratory for the United States military. It was first tested in 2003. It was considered as the most powerful non-nuclear weapon.

**2017 Nangarhar airstrike**

It refers to the American bombing of the Achin District located in Nangarhar Province, Afghanistan. The United State dropped the MOAB Bomb on 13 April 2017 in an airstrike with the goal of destroying tunnel complexes used by the Islamic State of Iraq and the Levant – Khorasan Province (ISIL-KP or ISIS-K).

**17. Option (c) is correct.**

**Explanation:** World Health Day is celebrated every year on April 7. It is organized by World Health Organisation (WHO). Every year a theme is selected by WHO highlighting a particular priority area of public health. The theme of World Health Day 2017 - 'Depression: Let's Talk'.

**18. Option (d) is correct.****Explanation:**

- Rosetta was a space probe built by the European Space Rosetta was a space probe. It was built by the European Space Agency, and was launched on 2 March 2004. Rosetta, along with Philae, its lander module, performed a detailed study of comet.
- Philae probe landed on a comet 310 million miles from Earth.
- Rosetta was the first mission ever to orbit, a comet's nucleus and land a probe on its surface.
- Comets change very little over time. Therefore, they contain a record of the conditions of the early solar system.
- Comet collisions may have brought a great deal of water to Earth, in the early history of the solar system. Perhaps, the building blocks of life, too.

**19. Option (d) is correct.****Explanation:**

- President Barack Obama and President Xi Jinping in Beijing made a historic U.S.-China Joint Announcement in November 2014.
- China decided to lower carbon dioxide emissions, per unit of GDP by 60% to 65% from the 2005 level by 2030.



- President Jinping pledged that clean energy sources like windmills and solar power would account for 20 percent of China's total energy production by 2030.
- United States also pledged to emit 26 percent to 28 percent less carbon in 2025 than it did in 2005

**20. Option (b) is correct.**

*Explanation:*

- Golden Globe Awards started in January 1944, recognizing excellence in film and television, both domestic and foreign. These Awards are accolades bestowed by the 93 members of the Hollywood Foreign Press Association.
- Boyhood was awarded the best motion picture (Drama) in the 72 Golden Globe Awards which was directed by Richard Linklater.
- The film depicts the childhood and adolescence of Mason Evans Jr. (Coltrane) from ages six to eighteen, as he grows up in Texas with divorced parents.

**21. Option (c) is correct.**

*Explanation:*

- The **Booker Prize** is a literary prize awarded each year for the best novel written in English and published in the United Kingdom or Ireland.
- The Narrow Road to the Deep North was the sixth novel by Richard Flanagan. It won the Man Booker Prize in 2014.

**22. Option (c) is correct.**

*Explanation:*

**Covaxin:**

- It is an inactivated pathogen-based vaccine.
- It is developed with Whole-Virion Inactivated Vero Cell-derived technology.

**Covishield:**

- It is a recombinant, replication-deficient chimpanzee adenovirus vector vaccine.
- It has been modified to enable it to carry the COVID-19 spike protein into the cells of humans.

**Sputnik V:**

- It is a viral vector vaccine designed to produce lasting immunity against COVID-19.
- It is based on adenovirus DNA.
- Adenovirus is used as a carrier to deliver the coronavirus gene to cells and synthesize the SARS-CoV-2 virus's envelope proteins, introducing the immune system to a potential enemy.

**23. Option (d) is correct.**

*Explanation:*

- In March 2020, Union Minister (Road Transport and Highways) Nitin Gadkari, launched the world's most advanced technology - developed Green Hydrogen Fuel Cell Electric Vehicle (FCEV), Toyota Mirai.
- Fuel Cell Electric Vehicle (FCEV) is powered by Hydrogen and, is one of the best Zero Emission solutions.
- It is completely environment friendly with no tail-pipe emissions other than water.
- Green Hydrogen can be generated by utilizing biomass that is widely available and renewable energy sources.

**24. Option (c) is correct.**

*Explanation:*

- AYUSH is the acronym based on the medical systems that are being practiced in India such as Ayurveda, Yoga & Naturopathy, Unani, Siddha and Homeopathy.
- The latest medical system added to it was Sowa-Rigpa.
- Sowa-Rigpa means "Science of healing".
- The system is based on indigenous medicine of Tibet enriched with Ayurveda, Greek and Chinese medicine.
- It is practised widely in Arunachal Pradesh, Sikkim, Darjeeling, Dharamshala, Lahaul and Spiti and Ladakh.

**25. Option (c) is correct.**

*Explanation:* In 2020, Drugs Controller General of India (DCGI) gave permission to Serum Institute of India to resume clinical trial of the Oxford– Astra-Zeneca COVID-19 vaccine candidate in the country.

**26. Option (d) is correct.**

*Explanation:* Adar Poonawalla launched the Clean City initiative in Pune in the year 2015. This was launched with the purpose of improving the waste and environment management in Pune. He pledged to fund 100 crores towards this initiative. Adar Poonawalla is the CEO of Serum Institute of India (SII). SII is the world's largest manufacturers of vaccine. It is headquartered in Pune and was founded by Cyrus Poonawalla in 1966.

○○○

# Chapter 60

## IMPORTANT NATIONAL FACTS ABOUT INDIA: HERITAGE, ARTS, DANCE, HISTORY

### Art & Culture (Bhakti Movement)

- *Sufism had its origin in Persia and spread into India in the eleventh century.*  
Some sufi saints:  
1. Shaikh Ismail of Lahore (first sufi saint)  
2. Khwaja Moinuddin Chishti (Ajmer)
- Bahauddin Zakariya came under the influence of another famous mystic, Shihabuddin Suhrawardi. The sufis believed service to humanity was tantamount to service to god. The sufis lay stress on inner purity.
- In the ninth century Shankaracharya gave a new orientation to Hinduism.
- Shankaracharya was born in Kaladi in Kerala.
- His doctrine of Advaita or Monism was too abstract to appeal to common man.
- There was a reaction against the Advaita concept of Nirguna Brahman (God without attributes) with the emergence of the idea of Saguna Brahman (God with attributes).
- In the twelfth century, Ramanuja, who was

born at Sriperumbudur near modern Chennai, preached Visishtadvaita.

- In the thirteenth century, Madhava from Kannada region propagated Dvaita or dualism or Jivatma and Paramatma.
- Nimbarka and Vallabhacharya were preachers of Vaishnavite Bhakti in modern Telangana region.
- Surdas was disciple of Vallabhacharya and he popularised Krishna cult in North India.
- Mirabai was great devotee of Krishna and she became popular in Rajasthan, for her Bhajans. Tulsidas was worshipper of Rama and composed the famous Ramcharitmanas, the Hindi version of Ramayan.
- In the 14<sup>th</sup> and 15<sup>th</sup> centuries, Ramananda, Kabir and Nanak remained great apostles of Bhakti cult.
- Ramananda was born at Allahabad, originally a follower of Ramanuja. Later founded his own sect and preached his principles in Hindi at Banaras and Agra.

### State & Folk Dances

|                   |                                                                                            |
|-------------------|--------------------------------------------------------------------------------------------|
| Uttar Pradesh     | Nautanki, Raslila, Kajri, Jhora, Chappeli, Jaita                                           |
| Uttarakhand       | Garhwali, Kumayuni, Kajari, Jhora, Raslila, Chappeli                                       |
| Andhra Pradesh    | Kuchipudi, Ghantamardala, Ottam Thedal, Veedhi Natakam                                     |
| Assam             | Bihu, Bichhua, Natpuja, Maharas, Kaligopal, Bagurumba, Khel Gopal, Canoe, Jhumura Hobjanai |
| Bihar             | Jata-Jatin, Bakho-Bakhain, Panwariya, Sama-Chakwa, Bidesia                                 |
| Gujarat           | Garba, Dandiya Ras, Tippani Juriun, Bhavai                                                 |
| Haryana           | Jhumar, Phag, Daph, Dhamal, Loor, Gugga, Khor, Gagor                                       |
| Himachal Pradesh  | Jhora, Jhali, Chharhi, Dhaman, Chhapeli, Mahasu, Nati, Dangi                               |
| Jammu and Kashmir | Rauf, Hikar, Mandjas, Kud Dandi Nach, Damali                                               |
| Karnataka         | Yakshagan, Huttari, Suggi, Kunitha, Karga, Lambi                                           |
| Kerala            | Kathakali (Classical), Ottam Thulal, Mohiniattam, Kaikottikali                             |
| Maharashtra       | Lavani, Nakata, Koli, Lezim, Gafa, Dahikala Dasavtar or Powada                             |
| Odisha            | Odissi (Classical) Savari Ghumara, Painka, Munari, Chhau                                   |
| West Bengal       | Kathi, Gambhira, Dhali, Jatra, Baul, Marasia, Mahal, Keertan                               |

### Classical Dances and Dancers of India

|           |                                                                                                                      |
|-----------|----------------------------------------------------------------------------------------------------------------------|
| Kathak    | Bharti Gupta, Birju Maharaj, Damayanti Joshi, Durga Das, Gopi Krishna, Kumudini Lakhia, Shambhu Maharaj, Sitara Devi |
| Kuchipudi | Josyula Saatharamaiah, Vempathi Chinna Sathyam                                                                       |
| Manipuri  | Guru Bipin Sinha, Jhaveri Sisters, Nayana Jhaveri, Nirmala Mehta, Savita Mehta                                       |
| Odissi    | Debaprasad Das, Dharendra Nath Patnaik, Indrani Rahman, Kelucharan Mahapatra, Priyambada Mohanty                     |

|               |                                                                                                                                                              |
|---------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Kathakali     | Mrinalini Sarabhai, Guru Shankaran, Namboodripad, Thottam Shankaran, Kutti Nayyar, Shankar Kump, KC Pannikar, TT Ram Kulti                                   |
| Mohiniattam   | Protima Devi, Sanyukta Panigrahi, Sonal Mansingh, Pankaj Charan Das, Kelucharan Mahapatra, Madhvi Mudgal, etc                                                |
| Bharatanatyam | Bala Saraswati, CV Chandrasekhar, Leela Samson, Mrinalini Sarabhai, Padma Subramanyam, Rukmini Devi, Sanyukta Panigrahi, Sonal Mansingh, Yamini Krishnamurti |

#### Musical Instruments & Instrumentalists

| Stringed     | Instruments                                                                                                                                      |
|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. Been      | Asad Ali Khan, Zia Moin-ud-din Khan                                                                                                              |
| 2. Santoor   | Shiv Kumar Sharma                                                                                                                                |
| 3. Sarod     | Buddhadev Dasgupta, Ali Akbar Khan, Amjad Ali Khan, Bahadur Khan, Sharan Rani, Zarin S Sharma                                                    |
| 4. Sarangi   | Ustad Binda Khan                                                                                                                                 |
| 5. Sitar     | Ravi Shankar, Hara Shankar Bhattacharya, Nikhil Banerjee, Vilayat Khan, Mustaq Ali Khan                                                          |
| 6. Surb Ahar | Sajjad Hussain, Annapurna                                                                                                                        |
| 7. Veena     | Doraiswamy Iyengar, Chittibabu, Emani Sankara Shastri, Dhanammal, S Bala Chandran, KR Kumaraswamy                                                |
| 8. Violin    | Gajanan Rao Joshi, MS Gopal Krishna, TN Krishnan, Baluswamy, Dikshitar, Dwaran Venkataswamy Naidu Lalyuli G Jayaraman, Mysore T Chowdiah, VG Jog |

#### Wind Instruments

|                |                                                                |
|----------------|----------------------------------------------------------------|
| 9. Flute       | TR Mahalingam, N Ramani, Hari Prasad Chaurasia, Pannalal Ghosh |
| 10. Nadaswaran | Sheikh Chinna Moula, Neeruswamy Pillai, Rajaratnam Pillai      |
| 11. Shehnai    | Bismillah Khan                                                 |

#### Percussion (Striking Thumping) Instruments

|               |                                                                                                           |
|---------------|-----------------------------------------------------------------------------------------------------------|
| 12. Mridangam | Palghat Mani Iyer, Karaikudi R Mani, Palghat Raghu                                                        |
| 13. Pakhawag  | Pt Ayodhya Prasad, Gopal Das, Babu Ram Shanker Pagaldas                                                   |
| 14. Tabla     | Zakir Hussain, Nikhil Ghosh, Kishan Maharaj, Alla Rakha Khan, Pandit Samta Prasad, Kumar Bose, Latif Khan |
| 15. Kanjira   | Pudukkotai Dakshinamurthi Pillai                                                                          |

#### Cultural Institutions Headquarters

| Institutions                                 | Headquarters    |
|----------------------------------------------|-----------------|
| Sahitya Academy, 1954                        | New Delhi       |
| Sangeet Natak Academy, 1953                  | New Delhi       |
| Library of Tibetan Works and Archives        | Dharmashala     |
| Science City                                 | Kolkata         |
| Victoria Memorial Hall                       | Kolkata         |
| Birla Industrial and Tech Museum             | Kolkata         |
| Central Institute of Buddhist Studies        | Leh             |
| Nava Nalanda Mahavihara                      | Nalanda (Birla) |
| National Gallery of Modern Art               | New Delhi       |
| Anthropological Survey of India, 1945        | Kolkata         |
| Archaeological Survey of India, 1861         | New Delhi       |
| Asiatic Society, 1784 (Sir William Jones)    | Kolkata         |
| Indira Gandhi National Centre for Arts, 1985 | New Delhi       |

|                                                          |           |
|----------------------------------------------------------|-----------|
| Lalit Kala Akademi (National Academy of Fine Arts), 1954 | New Delhi |
| National Archives of India, 1981                         | New Delhi |
| National School of Drama, 1959                           | New Delhi |

#### Jainism:

- Vardhamana Mahavira was the 24th Tirthankara of Jain tradition.
- He was born at Kundagrama near Vaishali to Kshatriya's parents Siddharta and Trisala.
- At the age of 30 years, he became ascetic and wandered for 12 years.
- Jainism is divided into two sects Svetambaras (white clad) by sthulababu and Digambaras (skyclad or naked) by Bhadrababu.
- First Jain council held at Pataliputra in 3rd century BCE and was presided by Sthulbadhra.
- Second Jain Council held at Vallabhi is 512 CE and was presided by Devardhi kshmasramana.

#### Buddhism:

- Gautam or Siddharta, the founder of Buddhism, was born in 567 BCE in Lumbini Garden near Kapilvastu.

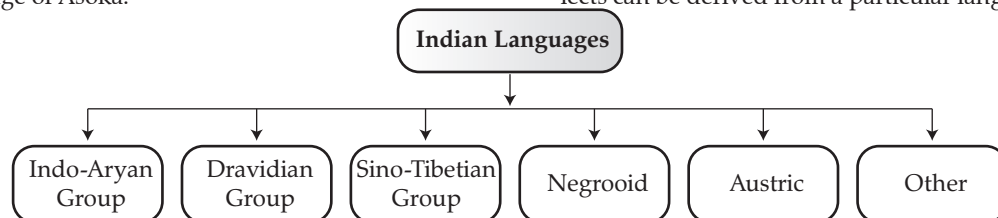


- He left home at the age of 29 in search of truth.
- Under a bodhi tree at Bodhgaya, he got enlightenment or nirvana, at the age of 35.
- He delivered his first Sermon at Sarnath near Benaras.
- He died at the age of 80 at Kushinagar.
- First Buddhism council was held at Rajagriha under the chairmanship of Mahakasyapa immediately after the death of Buddha.
- Second Buddhist council was convened at Vaishali around 383 BCE.
- Third Buddhist council held at Patliputra under the patronage of Asoka.

- Fourth Buddhist council was convened in Kashmir by Kanishka under the chairmanship of Vasumitra. Asvaghosa participated in council.

#### Language

Language in its literary meaning is a system of communication through speech, a collection of sounds that a group of people understand to have the same meaning. A language family includes individual languages related through a common ancestor that existed before the recorded history. Dialect is a form of language spoken in a local area. It should be noted that several dialects can be derived from a particular language.



#### List of several places of cultural interest

- **Agra:** Situated on the banks of the Yamuna, this city is a medieval marvel. The Mughals made it their capital, which brought it to prominence. They constructed several monuments and gardens in the city.
- **Agartala:** It is the capital city of the state of Tripura and is situated on the banks of the Haora River, which provides the basic livelihood to them. The most important and old temple in the city is the Lakshmi Narayan Temple situated in the Ujjayanta Palace compound.
- **Guwahati:** It is one of the largest city in the state of Assam and the cultural capital of the North-East India. The city is also names as the 'Gateway of the North-east India.'
- **Goa:** Goa has become the most famous place for westerners to come to India. It is surrounded by Maharashtra on the north and Karnataka from the east and south. It is bounded by the Arabian Sea on the west and this provides Goa with some of the best beaches in India.
- **Amritsar:** Initially named Ramdasapur, the modern day city of Amritsar is a historically important city in the state of Punjab. The Sri Harmandir Sahib or the Golden Temple is the most sacred Gurudwaras of the Sikh community.
- **Bodh Gaya:** It is situated in Bihar and it is one of the sacred places for those who believe in Buddhism. One of the most revered monuments in Bodh Gaya is the Mahabodhi Temple.
- **Hampi:** All the monuments and sculptures at Hampi were declared to be world heritage (sites) by UNESCO in 1986. The most iconic temple complex in Hampi is the Vittala Temple complex, which has the iconic stone chariot.
- **Khajuraho:** The Khajuraho group of monuments is situated in Madhya Pradesh. It is famous for the erotic temple structures. Most of the temples are constructed

in the Nagara-style and were made by the Chandela dynasty that ruled over Bundelkhand.

- **Konark:** The famous Konark Sun temple is situated in the town of Konark in Odisha. This is one of the very few and famous Sun temples in India. It is also known as the 'Black Pagoda' because it is built out of black granite.
- **Rameshwaram:** The city of Ramershwaram is one of the 'Char Dham' or four most important pilgrimages for Hindus. It is set on southernmost tip of the mainland of an island in the Gulf of Mannar.
- **Varanasi:** It has been historically known as 'Benaras' and 'Kashi' as it lies on the banks of the holy river Ganga. It is also known as the 'spiritual capital' of India as it is home to many Hindu temples and Buddhist sanctuaries. India has a long tradition of artistic excellence and painting is one of the major mediums that have been used to express it.

In the third century AD, Vatsyayana in his book Kamasutra mentioned about 6 main principles/limbs or shadanga of paintings.

| Styles of Paintings   | Major features            |
|-----------------------|---------------------------|
| <i>Cauka Pitaka</i>   | Isolated framed drawings  |
| <i>Dighala Pitaka</i> | Long scrolls of paintings |
| <i>Yama Pitaka</i>    | Isolated paintings        |

- **Makar Sankranti:** It is a festival dedicated to the Sun God, who is also considered to be the king of all planets. Makar Sankranti celebrates the movement of the Sun to the northern hemisphere.
- **Janmashtami:** It is the birth anniversary of Lord Krishna and is celebrated across the nation by the Hindu community.
- **Dussehra:** Dussehra is also known as 'Vijaydashami' and is celebrated all over India in the honour of Lord Rama's victory over Ravana.
- **Eid-ul-Fitr:** This is one the festivals celebrated by the Muslim community worldwide. The festival falls on

the last day of the holy month of Ramadan (Ramzan), which is the ninth month of the Islamic calendar.

- **Muharram:** The festival of Muharram is a sad one, as it is associated with the death of Hussain, son of Ali.
- **Christmas:** This day is celebrated all across the world as the birth anniversary of Jesus Christ. It falls on the 25th of December every year.
- **Gurpurab:** The Sikh community all over the world celebrates it. Although Gurpurabs are celebrated for the birth anniversaries of all the 10 Sikh gurus but the most important are the Guru Nanak and Guru Gobind Singh.

#### **Mahatma Gandhi**

The advent of Mahatma Gandhi was an indelible landmark in the history of the national movement. Mohandas Karamchand Gandhi was born on October 2, 1869 in Porbandar in the princely state of Kathiawar in Gujarat. His father was a diwan (minister) of the state. Having studied law in England, Gandhi, in 1898, went to South Africa in connection with a case involving his client, Dada Abdullah. In South Africa he witnessed the ugly face of white racism and the humiliation and contempt to which Asians, who had gone to South Africa as labourers, were subjected. He decided to stay in South Africa to organise the Indian workers to enable them to fight for their rights. He stayed there till 1914 after which he returned to India.

#### **Movements started by Mahatma Gandhi in South Africa**

- Satyagraha against Registration Certificates (1906)
- Campaign against Restrictions on Indian Migration
- Campaign against Poll Tax and Invalidation of Indian Marriages
- In the First World War (1914-1919), Britain allied with France, Russia, USA, Italy and Japan against Germany, Austria-Hungary and Turkey.
- This period saw the maturing of Indian nationalism.

#### **Home Rule League Movement**

The Home Rule Movement was the Indian response to the First World War in a less charged but a more effective way than the response of Indians living abroad which took the form of the romantic Ghadr adventure.

#### **Tilak's League**

- Tilak set up his Indian Home Rule League in April 1916.
- Tilak held his first Home Rule meeting at Belgaum.
- Poona was the headquarters of his league. His league was restricted to Maharashtra (excluding Bombay city), Karnataka, Central Provinces and Berar.

#### **Besant's League**

- Annie Besant set up her All-India Home Rule League in September 1916 in Madras (now Chennai) and covered the rest of India (including Bombay city).
- It had 200 branches, was loosely organised as compared to Tilak's league and had George Arundale as the organising secretary.
- Besides Arundale, the main work was done by

#### **Temples:**

**Venkateswara Temple:** Tirumala at Tirupati in Chittoor District, Andhra Pradesh

**Sri Ranganthaswami Temple:** Nellore, Andhra Pradesh

**Umananda Devaloi:** Guwahati, Assam

**Mundeshwari Devi Temple:** Bihar

**Somnath Temple:** Gujarat

**Nageshvara Jyotirlinga:** Gujarat

**Virupaksha Temple:** Hampi, Karnataka

**Padmanabhaswamy Temple:** Thiruvananthapuram, Kerala

B.W. Wadia and C.P. Ramaswamy Aiyar

#### **Governor-Generals of Bengal**

##### **Warren Hastings (AD 177-85)**

- Brought the dual government to an end by the Regulating Act, 1773.
- The Act of 1781 made a clear demarcation between the jurisdiction of the Governor General-in-Council and Supreme Court at Calcutta.
- Pitt's India Act (1784), Rohilla War (1774), First Maratha War (1778-1782), and the Treaty of Salbai with Marathas (1782) and Second Mysore war (1780-84). Foundation of Asiatic Society of Bengal (1784) in Calcutta by Sir William Jones.
- Charles Wilkins, in 1785, translated the Bhagavad Gita into English.

##### **Lord Cornwallis (AD 1786-93)**

- Third Mysore War (1790-92) and Treaty of Serigapatnam (1792).
- He introduced permanent settlement in Bengal and Bihar (1793).
- He is called the Father of Civil Services in India. He introduced judicial reforms by separating revenue administration from judicial administration and established a system of circles (thanas, headed by a Daroga (an Indian)).
- Translation of Abhigyan Shakuntalam in English by William Jones in 1789.

##### **Lord Wellesley (AD 1798-1805)**

- He introduced the subsidiary alliance in 1798. His first alliance was with Nizam of Hyderabad followed by Mysore, Tanjore, Awadhi, the Peshwa, the Bhonsle, and the Scindia.
- Treaty of Bassein (1805) and the second Maratha war

#### **Governor-Generals of India**

##### **Lord William Bentick (AD 1828-35)**

- The Charter Act of 1833 was passed. He was made the first Governor-General of India. Before him, the designation given was Governor-General of Bengal.
- He carried out social reforms like the prohibition of the Sati system (1829) and the elimination of thugs (1830).
- Macaulay's recommendations English was made the medium of higher education. He also suppressed female infanticide and child sacrifice.

##### **Lord Dalhousie (AD 1848-56)**

- Introduction of Doctrine of Lapse and annexation of Satara (1848), Jaipur and Sambalpur (1849),

Bhagat (1850), Udaipur (1852), Jhansi (1853), Nagpur (1854), and Awadh (annexed in 1856 on account of Maladministration).

- He laid down the first railway line between Bombay and Thane (1853). Telegraph line between Calcutta and Agra and Postal reforms (first issue of the Indian stamp in Karachi in 1854) with the Post Office Act.
- Widow Remarriage Act, 1856 (the main force being Ishwar Chand Vidyasagar).
- He started the public works department, Grand Trunk Road work, and harbor of Karachi, Bombay, and Calcutta developed.
- Charter Act, 1853–Selection to civil service through competitive examination.
- Started Engineering College at Roorkee made Shimla, the summer capital of India.

#### **Viceroy of India**

##### **Lord Canning (AD 1856–62)**

- He was the last Governor-General and first Viceroy. He withdrew Doctrine of Lapse.
- Revolt of 1857, Mutiny took place. Indian Penal Code 1860 was passed.
- Passed the Act, 1858, which ended the rule of the East India Company.
- The Universities of Calcutta, Bombay, and Madras were established in 1857.

##### **Lord Ripon (AD 1880–84)**

- The first factory Act of 1881 prohibited child labor under the age of Local Self-Government was introduced in 1882.
- Repealed the Vernacular Press Act in 1882. The finances of the center were divided.
- Lord Ripon is regarded as 'the founding father of local self-governance' in India.
- An education commission was appointed under Sir William Hunter in 1882 to improve primary and secondary education.
- The Ilbert Bill Controversy (1883) enabled Indian district magistrates to try European criminals.

##### **Lord Mountbatten (March to August 1947)**

- Last Viceroy of British India and the first Governor-General of free India.
- Partition of India decided by the 3rd June Plan or Mountbatten Plan.
- He retired in June 1948 and was succeeded by C. Rajagopalchari, the first and last Indian Government – General of Free India.
- Indian Independence Act was passed by the British Parliament on 4th July, by which India became independent on 15th August 1947.
- Indian society in the nineteenth century was caught in a vicious web created by religious superstitions and social obscurantism.
- Social conditions were equally depressing. The most distressing was the position of women. Attempts to kill female infants at birth were not unusual. Child marriage was another bane of society.
- Another debilitating factor was caste. This entailed a system of segregation, hierarchically ordained on the basis of ritual status.

- In the beginning, organisations such as the Social Conference, Servants of India Society and the Christian missionaries were instrumental in social reform along with many enlightened individuals like Jyotiba Phule, Gopalhari Deshmukh, K.T. Telang, B.M. Malabari, D.K. Karve, Sri Narayana Guru, E.V. Ramaswami Naicker and B.R. Ambedkar.

#### **Reforms:**

- **Abolition of Sati:** Influenced by the frontal attack launched by the enlightened Indian reformers led by Raja Rammohan Roy, the government declared the practice of sati illegal and punishable by criminal courts as culpable homicide.
- **Preventing Female Infanticide:** An Act passed in 1870 made it compulsory for parents to register the birth of all babies and provided for verification of female children for some years after birth, particularly in areas where the custom was resorted to in utmost secrecy.
- **Widow Remarriage:** The Brahmo Samaj had the issue of widow remarriage high on its agenda and did much to popularise it.

#### **The revolt of 1857 started at Meerut on 10th May 1857.**

- **Political causes:** The policy of Doctrine of Lapse.
- **Economic causes:** Heavy taxation, evictions, Discriminatory Tariff Policy against Indian products, and destruction of traditional handicrafts that hit pleasantries, artisans, and small zamindars.
- **Military Discrimination:** As Indian soldiers were paid low salaries; they could not rise above the rank of subedar and were racially insulted.
- **Grievances of Sepoys:** The introduction of the Enfield rifle, and its cartridge of which were greased with animal fat, provided the spark.
- A rebellion broke out among Sepoys of Meerut on 10th May 1857 which later spread to other parts of the country.
- British social reforms (widow remarriage, abolition of Sati, education for girls, Christian missionaries).

#### **Rowlatt Act (18th March 1919)**

- This gave unbridled powers to the government to arrest and imprison suspects without trial. This law enabled the government to suspend the right of Habeas Corpus, which had been the foundation of civil liberties in Britain.
- Rowlatt Satyagraha was started against the act. This was the first countrywide agitation by Gandhiji.

#### **Jallianwala Bagh Massacre (13th April 1919)**

- People were agitated over the arrest of Dr. Saifuddin Kitchlew and Dr. Satya Pal on 10th April 1919.
- General Dyer fired at people who assembled in the Jallianwala Bagh, Amritsar. Michael O'Dwyer was Lt. Governor of Punjab at that time. The Hunter Commission was appointed to enquire into it.
- Rabindranath Tagore returned his knighthood in protest.
- Sardar Udham Singh killed Michael O'Dwyer in Caxton Hall, London on March 13, 1940.

### Non-cooperation Movement (1920)

- In 1920 Congress adopted the new programme of non-violent Non-cooperation under the leadership of Gandhiji.
- The Non-cooperation movement aimed to redress the wrongs done to Punjab and Turkey. Another aim was to regain Swaraj.
- Many Indians resigned their job, foreign clothes were burnt, strikes all over the country.
- By the beginning of 1922 about 30,000 persons were in Jail.
- Early in February 1922 Gandhiji decided to launch a no-tax campaign in the Bardoli district in Gujarat.
- **The Ghadr:** The Ghadr Party was a revolutionary group organised around a weekly newspaper The Ghadr with its headquarters at San Francisco and branches along the US coast and in the Far East.
- The **Berlin Committee for Indian Independence was established in 1915 by Virendranath Chattopadhyay, Bhupendranath Dutta, Lala Hardayal and others with the help of the German foreign office under 'Zimmerman Plan'.**
- Right from the early nineteenth century, defence of civil liberties, including the freedom of the press, had been high on nationalist agenda.
- As early as 1824, Raja Rammohan Roy had protested against a resolution restricting the freedom of the press.
- Many newspapers emerged during these years under distinguished and fearless journalists. These included: The Hindu and Swadesamitran under G. Subramaniya Aiyar.
- The Bengalee under Surendranath Banerjee
- Voice of India under Dadabhai Naoroji
- Amrita Bazar Patrika under Sisir Kumar Ghosh and Motilal Ghosh
- It was a system devised by Lord Wellesley in 1798. All those who entered into such an alliance with the British had to accept certain terms and conditions:
- The British would be responsible for protecting their ally from external and internal threats to their power.
- In the territory of the ally, a British armed contingent would be stationed.
- The ally would have to provide the resources for maintaining this contingent.
- The ally could enter into agreements with other rulers or engage in warfare only with the permission of the British.
- In AD 1602, the Dutch East India Company was formed.
- In the Battle of Bedara, held in AD 1759, Dutch were defeated by the English. As per the agreement, the Dutch gained control over Indonesia and the British over India, Sri Lanka, and Malaya.
- They set up their first factory of Masulipatnam in 1605.
- A disastrous famine affected the lower Gangetic Plains of India including the regions of Bengal and Bihar between 1769 and 1773 where 1/3rd of the population perished.

- An estimated 10 million people died of starvation and famine-triggered epidemics that also affected the regions of Assam, Odisha, Jharkhand and Bangladesh.
- The territory was then ruled by the British East India Company.
- In 1599, the English East India Company was formed under a charter granted by Queen Elizabeth in 1600.
- Jahangir granted a Farman to Captain William Hawkins permitting the English to erect a factory at Surat (1613).
- In 1615, Sir Thomas Roe succeeded in getting an imperial Farman to trade and establish a factory in all parts of the Mughal Empire by ruler Jahangir.
- In 1690, Job Charnock established a factory at Sutanuti.
- In 1698, following the acquisition of Zamindari of three villages of Sutanuti, Kalikata, and Govindpur, the city of Calcutta was founded. Fort William was set up in 1700.
- In 1717, John Surman obtained a Farman from Farrukhsiyar, which gave large concessions to the company. This Farman has been called the Magna Carta of the company.
- Battle of Plassey, held in 1757, English defeated Sirajuddaula, the nawab of Bengal.
- Battle of Buxar, held in 1764, captain Munro, defeated joint forces of Mir Qasim (Bengal, Shujaudaula (Awadh) and Shah Alam II (Mughal).
- Auckland who came to India as the governor-general in 1836, advocated a forward policy.
- This implied that the Company government in India itself had to take initiatives to protect the boundary of British India from a probable Russian attack.
- This objective was to be achieved either through treaty with the neighbouring countries or by annexing them completely.
- The Amir of Afghanistan, Dost Mohammed, wanted British friendship but made it conditional on the British helping him to recover Peshawar from the Sikhs—a condition which the British government in India rejected.
- Dost Mohammed now turned to Russia and Persia for help.
- This prompted the British government to go ahead with the forward policy, and a Tripartite Treaty (1838) was entered into by the British, Sikhs and Shah Shuja.
- First Anglo-Afghan War (1839-1842)
- Second Anglo-Afghan War (1870-80)

### Sarla Devi Chaudhurani

- In 1910, Sarla Devi Chaudhurani convened the first meeting of the Bharat Stree Mahamandal in Allahabad.
- It is considered as the first major Indian women's organisation set up by a woman.
- Its objectives included promotion of education for women, abolition of the purdah system and improvement in the socio-economic and political status of woman all over India.



**Ramabai Ranade**

- She founded the Ladies Social Conference (Bharat Mahila Parishad), under the parent organisation National Social Conference, in 1904 in Bombay.

**Pandita Ramabai Saraswati**

- She founded the Arya Mahila Samaj to serve the cause of women.
- She pleaded for improvement in the educational syllabus of Indian women before the English Education Commission which was referred to Queen Victoria.

**EXERCISE**

- Which one of the following works was composed by Krishnadevaraya on statecraft in Telugu?
  - Amuktamalyada
  - Jambavati Kalyanam
  - Rayavachakamu
  - Satyavadu Parinaya
- Jhijhiya dance originated in the Indian state of:
  - Bihar
  - West Bengal
  - Rajasthan
  - Gujarat
- Teejan Bai, a recipient of the M.S. Subbulakshmi Centenary Award, 2016, is an exponent in :
  - Kannada classical vocal
  - Kajari dance
  - Bihu dance
  - Pandavani, a traditional performing art
- What were the 12 states of the Sikh confederacy called?
  - Misl
  - Gurmata
  - Sardari
  - Rakhi
- Which one of the following statements with regard to the National Rural Employment Guarantee Act, 2005 is correct?
  - It ensures 175 days of employment to needy person in rural areas .
  - It gives higher wages to women workers .
  - People are generally given both skilled and unskilled jobs.
  - This is implemented only in rural areas of India.
- The Government of India has recently constituted a civilian award in the name of Sardar Vallabhbhai Patel in the field of contribution to :
  - unity and integrity of India
  - art and culture
  - social work
  - entrepreneurship
- Which of the following statements about the philosopher Shankara is/are true?
  - Shankara espoused a form of Vedanta called Advaita.
  - He elaborated on the philosophy of Gaudapada.
  - Shankara tried to demonstrate that the Upanishads and Brahmasutras contain a unified, systematic philosophy.
  - Shankara founded the Amanaya Mathas.

Select the correct answer using the code given below:

  - 1 and 2 only
  - 1, 2 and 3
  - 3 and 4
  - 4 only
- The epic, *Silappadikaram* refers to the :
  - story of Rama
  - Jaina elements in the storyline
  - culture of Sri Lankan Buddhists
  - cult of *Shakti* worship
- The *Dashakumaracharita* or 'Tales of Ten Princes' was composed by
  - Bhatti
  - Banabhatta
  - Bhasa
  - Dandin
- Which one of the following is the common element between the Kailasanatha Temple at Ellora and the Shore Temple at Mamalla-puram?
  - Both are examples of Nagara architecture .
  - Both are carved out from solid rocks .
  - Both are Gupta period temples .
  - Both were built under the patronage of Pallava Kings.
- Tuareg is a pastoral nomad living in the desert of:
  - Kalahari
  - Sahara
  - Arabia
  - Patagonia
- Which one of the following is the common element between the Kailasanatha Temple at Ellora and the Shore Temple at Mamalla-puram?
  - Both are examples of Nagara architecture
  - Both are carved out from solid rocks
  - Both are Gupta period temples
  - Both were built under the patronage of Pallava Kings
- Legendary Kathak dancer Pandit Birju Maharaj died recently. He was the doyen of which one of the following Gharanas of Kathak?
  - Jaipur
  - Raigarh
  - Lucknow
  - Banaras
- At which one of the following places was a Shiva temple **not** constructed under the patronage of the Chola rulers?
  - Chidambaram
  - Thanjavur
  - Gangaikonda Cholapuram
  - Naneghat
- Who among the following was the first Indian to qualify for the Indian Civil Service ?
  - Satyendranath Tagore
  - Surendranath Banerjee
  - R.C. Dutt
  - Subhas Chandra Bose
- Arrange the following events chronologically starting from the earliest:
  - Ahmedabad Mill Strike
  - Champaran Satyagraha
  - Foundation of the Satyagraha Sabha
  - Jallianwala Bagh Massacre

Select the correct answer using the code given below:

  - 1, 2, 3, 4
  - 2, 1, 3, 4
  - 2, 4, 1, 3
  - 4, 2, 1, 3
- The year 1916 is important in Indian history for:
  - Lucknow Session of the Congress
  - Congress-League Pact

3. Formation of Home Rule League  
Select the correct answer using the code given below:  
(a) 1 and 3 only (b) 2 and 3 only  
(c) 1 and 2 only (d) 1, 2 and 3
18. Who among the following belonged to Serampore Mission?  
1. William Carey 2. Joshua Marshman  
3. William Ward 4. David Hare  
Select the correct answer using the code given below:  
(a) 1 and 2 only (b) 1 and 3 only  
(c) 1, 2 and 3 only (d) 2, 3 and 4 only
19. The provision for separate electorate for Muslims was given in:  
(a) Morley-Minto Reforms, 1909  
(b) Montagu-Chelmsford Reforms, 1919  
(c) Communal Award, 1932  
(d) Government of India Act, 1935
20. Which of the following was/were the founder member(s) of Hindustan socialist Republican Army established in September, 1928?  
1. Bhagat Singh  
2. Jatindranath  
3. Ajay Ghosh  
4. Phanindranath Ghosh  
Select the correct answer using the code given below:  
(a) 1 only (b) 2 and 3 only  
(c) 1, 2 and 3 only (d) 1, 2, 3 and 4
21. Who among the following did not belong to the group of 'No-Changeers'?  
(a) M. A. Ansari  
(b) Vithalbhai Patel  
(c) Chakravarti Rajagopalachari  
(d) Kasturi Ranga Iyengar
22. The headquarters of Ghadar Movement/Party were at:  
(a) San Francisco (b) Stanford University  
(c) Portland (d) Florida
23. Which one of the following pairs of Newspaper and Editor is **not** correctly matched?
- |    | Newspaper      | Editor                |
|----|----------------|-----------------------|
| a. | Navjivan       | Mahatma Gandhi        |
| b. | Mahratta       | Bal Gangadhar Tilak   |
| c. | Bengalee       | Surendranath Banerjea |
| d. | Voice of India | Gopal Krishna Gokhale |
24. Which one of the following was **not** included in the terms and conditions of Subsidiary Alliance System of Lord Wellesley?  
(a) The British would protect their ally.  
(b) The ally was free to enter into agreements with other rulers or engage in warfare.  
(c) In the territory of the ally, a British armed contingent would be stationed.  
(d) The ally would have to provide resources for the maintenance of the British armed contingent.
25. The Dutch were defeated by Marthanda Varma of Travancore Kingdom in the battle of:  
(a) Wandiwash (b) Rakshasi - Tangadi  
(c) Pullalur (d) Colachel
26. The Bengal Famine of 1769-70 was worsened by:  
(a) Locust attacks (b) Rat infestation  
(c) High levels of taxation (d) Smallpox epidemic
27. Consider the following statements about cavalry warfare in India:  
1. The iron stirrup made it possible for the armoured warrior to sit firmly on a horse without falling off.  
2. The iron stirrup also allowed a cavalry charge with lances held tightly to the body, without the rider being thrown off by the shock of impact.  
Which of the statements given above is/are correct?  
(a) 1 only (b) 2 only  
(c) Both 1 and 2 (d) Neither 1 nor 2
28. The British policy towards Afghanistan in the first half of the 19<sup>th</sup> century aimed at:  
(a) Strengthening Afghanistan as a buffer against Russia.  
(b) Weakening Russian influence in Afghanistan but not promoting a strong Afghanistan either.  
(c) Promoting Russian influence in Afghanistan to control insurgent tribal-leaders.  
(d) Annexing Afghanistan as a directly-controlled British Territory.
29. Who among the following was not a woman labour leader who organised workers in the strikes in the 1920s?  
(a) Sarala Devi Chaudhurani  
(b) Ushabai Dange  
(c) Prabhavati Devi  
(d) Anasuya Behn
30. In India, the first major public appearance of Mahatma Gandhi was in:  
(a) Champaran (1917)  
(b) Kheda (1918)  
(c) Inauguration of Banaras Hindu University (1916)  
(d) Rowlatt Satyagraha (1919)
31. In 1911, who among the following introduced a bill in the Imperial Legislative Council for introduction of compulsory and free primary education in India?  
(a) Dadabhai Naoroji  
(b) Bal Gangadhar Tilak  
(c) Sir Harcourt Butler  
(d) Gopal Krishna Gokhale
32. Which of the following statements about the Sanyasi and Faqir disturbances/rebellions are correct?  
1. Governor-General Warren Hastings faced the persistent Sanyasi and Faqir disturbances in Bengal and Bihar.  
2. There were a number of Shaivite Naga Sanyasis who formed into armed bands.  
3. Majnu Shah, who led bands into Bengal from 1771, was their prominent leader.  
Select the correct answer using the code given below:  
(a) 1 and 2 only (b) 1, 2 and 3  
(c) 2 and 3 only (d) 1 and 3 only

33. Who among the following founded the Marathi newspaper 'Kesari'?
- (a) Lokmanya Tilak (b) Vallabhbhai Patel  
(c) Lala Lajpat Rai (d) Mahatma Gandhi
34. When was the monopoly of China trade lost by East India Company?
- (a) 1813 (b) 1833 (c) 1838 (d) 1860
35. Who believed that the Russian designs were 'an imminent peril to the security and tranquility' of the Indian Empire in 1836?
- (a) Lord Auckland (b) Lord Palmerston  
(c) Lord Canning (d) Alexander Burnes
36. Which one among the following was demanded by the All India Depressed Classes Leaders' Conference at Bombay in 1931?
- (a) Universal adult suffrage  
(b) Separate electorates for untouchables  
(c) Reserved seats for the minorities  
(d) A unitary State in India
37. Which one of the following towns was not a centre of the Revolt of 1857?
- (a) Ayodhya (b) Agra  
(c) Delhi (d) Kanpur
38. During the 19<sup>th</sup> century, who among the following wrote Satapatra Series?
- (a) M. G. Ranade  
(b) B. G. Tilak  
(c) Bankim Chandra Chatterjee  
(d) G. H. Deshmukh
39. Who among the following wrote The Philosophy of the Bomb?
- (a) Sukhdev  
(b) Chandrashekhar Azad  
(c) Bhagwati Charan Vohra  
(d) Bhagat Singh
40. In 1921, during which one of the following tours, Gandhiji shaved his head and began wearing loin-cloth in order to identify with the poor?
- (a) Ahmedabad (b) Champaran  
(c) Chauri Chaura (d) South India

### Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (a)  | 2. (a)  | 3. (d)  | 4. (a)  | 5. (d)  | 6. (a)  | 7. (b)  | 8. (b)  | 9. (d)  | 10. (b) |
| 11. (b) | 12. (b) | 13. (c) | 14. (d) | 15. (a) | 16. (b) | 17. (d) | 18. (c) | 19. (a) | 20. (d) |
| 21. (d) | 22. (a) | 23. (d) | 24. (b) | 25. (d) | 26. (c) | 27. (b) | 28. (b) | 29. (c) | 30. (c) |
| 31. (d) | 32. (b) | 33. (a) | 34. (b) | 35. (b) | 36. (b) | 37. (b) | 38. (d) | 39. (c) | 40. (d) |

### Answers with Explanations

- Option (a) is correct.**  
*Explanation:* The most famous ruler of Vijayanagara was Krishnadevaraya and his work on statecraft composed in Telugu is known as the Amuktamalyada.
- Option (a) is correct.**  
*Explanation:* Jhijhiya is an artistic dance of Bihar in India. It is accomplished at the celebration of Dussehra. The dancers of Jhijhiya place lanterns constructed of clay on their heads, and this dance is executed by females only.
- Option (d) is correct.**  
*Explanation:* Teejan Bai is an exponent of Pandavani, a traditional performing art form, from Chhattisgarh, in which she enacts tales from Mahabharata, with music accompaniments.
- Option (a) is correct.**  
*Explanation:* The misls were the twelve sovereign states of the Sikh Confederacy, which rose during the 18<sup>th</sup> century in the Punjab region in the northern part.
- Option (d) is correct.**  
*Explanation:*
  - National Rural Employment Guarantee Act 2005 which was later renamed as Mahatma Gandhi National Rural Employment Guarantee Act (MGNREGA) is an act to provide for the enhancement of livelihood security of the households in rural areas of the country.
  - By providing at minimum 100 days of guaranteed wage employment in every financial year to every household.
  - They are adult members volunteer to do unskilled manual work and for matters connected therewith.
  - This scheme covers in all rural districts of the country.
- Option (a) is correct.**  
*Explanation:* On 20 September 2019, Sardar Patel National Unity Award was issued by Ministry of Home Affairs. It is highest civilian award in the field of contribution to the unity and integrity of India. The award will be announced on 31<sup>st</sup> October on the occasion of the National Unity Day, which is the birth anniversary of Sardar Patel.
- Option (b) is correct.**  
*Explanation:* In his pursuit to spread his philosophy, he travelled all over India and established four Mathas in the four corners of India. These are:
  - Uttaramnaya matha, or Northern matha at Joshi-math.
  - Purvamnaya matha, or Eastern matha, the Govardhana matha at Puri.
  - Dakshinamnaya matha, or the Sringeri Sharada Peetham, the Southern matha, at Sringeri.
  - Paschimamnaya matha, or the Dwarka Pitha, the Western matha, at Dwarka.
- Option (b) is correct.**  
*Explanation:* The epic Silappatikaram refers to the Jaina elements in the storyline.
  - Silappatikaram (literally translated, 'The Story of the Anklet') depicts the life of Kannagi, a chaste



woman who led a peaceful life with Kovalanin Puhar (Poompuhar), the then capital of Cholas.

- At the end of the Sangam age, the Tamil country was in political confusion. The kings and others encouraged the religions of Buddhism and Jainism. The author of Silappatikaram, probably lived in this period and was one of the vast numbers of Jain authors in Tamil poetry.
- These authors, perhaps influenced by their monastic faiths, wrote the futility of secular pleasures.

**9. Option (d) is correct.**

*Explanation:*

- Dandin composed the famous the "Dashakuntacharita" or 'Tales of Ten Princes'.
- It is a prose romance in Sanskrit and describes the adventures of ten young men, the Kumaras, all of whom are either princes or sons of royal ministers, as narrated by the men themselves.

**10. Option (b) is correct.**

*Explanation:*

- The common element between the Kailasanatha Temple at Ellora and the Shore Temple at Mamallapuram is that both are carved out from solid rocks.
- Kailasanatha Temple was constructed in 8<sup>th</sup> century by Rashtrakuta king Krishna I in 756- 773 CE. Shore Temple at Mamallapuram was built by Narasimhavarman II in the 8<sup>th</sup> century.
- It is the earliest significant free-standing stone temple in Tamil Nadu.

**11. Option (b) is correct.**

*Explanation:* The Tuareg people are a large Berber ethnic group that principally inhabits the Sahara Desert stretching from far southwestern Libya to southern Algeria, Niger, Mali, and Burkina Faso. Traditionally nomadic pastoralists (a form of animal husbandry), small groups of Tuareg are also found in northern Nigeria.

**12. Option (b) is correct.**

*Explanation:* The common element between the Kailasanatha Temple at Ellora and the Shore Temple at Mamallapuram is that both are carved out from solid rocks. Kailasanatha Temple was constructed in 8<sup>th</sup> century by Rashtrakuta king Krishna I in 756 - 773 CE. Shore Temple at Mamallapuram was built by Narasimhavarman II in the 8<sup>th</sup> century. It is the earliest significant free-standing stone temple in Tamil Nadu.

**13. Option (c) is correct.**

*Explanation:*

- Brij Mohan Nath Misra was popularly known as Pandit Birju Maharaj.
- He was an internationally acclaimed Kathak dancer.
- He belonged to the Lucknow Gharana and was responsible for making Kathak popular in Northern India.
- He was recipient of the Padma Vibhushan.

**14. Option (d) is correct.**

*Explanation:*

- The Chola Dynasty was situated in the southern state of Tamil Nadu which was one of the longest-ruling dynasties in the world's history

- (The three great Chola temples constructed between 11<sup>th</sup> and 12<sup>th</sup> century are listed into UNESCO's World Heritage Sites). These are:

1. The Brihadisvara temples of Thanjavur,
2. Gangaikonda cholisvaram, and
3. The Airatesvara temple at Darasuram.

- (These three temples are built in Dravidian style of temple architecture)

**15. Option (a) is correct.**

*Explanation:*

- Satyendranath Tagore was the first Indian to join the Indian Civil Service in June 1863.
- He completed his probationary training and returned to India in November 1864. He was posted as Judge, Satara after this examination where he served his entire career from 1864 to 1897.
- He was an author of various Bengali and English books and translated works from Sanskrit into Bengali
- Apart from this he was a song composer and linguist, and made a significant contribution towards the emancipation of women in Indian society during the British Raj.
- He was the second eldest brother of Rabindranath Tagore, the only Indian to be awarded a Nobel Prize in Literature.

**16. Option (b) is correct.**

*Explanation:*

- |                           |               |
|---------------------------|---------------|
| Champaran Satyagraha      | - 1916.       |
| Ahmedabad Mill Strike     | - 1917.       |
| Satyagraha Sabha          | - 1919 [Feb]  |
| Jallianwala Bagh massacre | - April 1919. |

**17. Option (d) is correct.**

*Explanation:*

The year 1916 is important in Indian history for:

- The Lucknow Session INC took place in 1916 and was presided by Ambica Charan Majumdar.
- The Lucknow Pact was an agreement between the Indian National Congress and the Muslim League regarding joint of both the parties held in Lucknow in December 1916.
- The Home Rule for India League was founded by Annie Besant in 1916.

**18. Option (c) is correct.**

*Explanation:* The Serampore Mission was the name given to three pioneering English missionaries, namely Joshua Marshman, William Carey and William Ward. Serampore Mission (1800-1845) was India's first Christian missionary organisation. William Carey and his two associates established this mission on 10 January 1800.

**19. Option (a) is correct.**

*Explanation:*

- The Minto-Morley Reforms of 1909 called for separate Muslim electorates.
- Communal Award extended the separate electorate to depressed Classes and other minorities.
- Government of India Act 1935 gave the provision for separate electorates for Anglo-Indians and Indian Christians.
- The separate electorate was introduced in Indian Councils Act 1909 for Muslims and extended to

Sikhs, Indian Christians, Anglo-Indians and Europeans by Government of India Act 1919.

**20. Option (d) is correct.**

**Explanation:** Hindustan Socialist Republican Association (HSRA), previously knew as the Hindustan Republican Army. Hindustan Republican Association (HRA), was an Indian revolutionary organisation founded by Ram Prasad Bismil, Bhagat Singh, Ashfaqulla Khan, Sachindra Nath Bakshi, Sachindranath Sanyal and Jogesh Chandra Chatterjee.

**21. Option (d) is correct.**

**Explanation:** Leaders like Vallabhbhai Patel, Jawaharlal Nehru and Rajendra Prasad, who became known as the No Changers as opposed to the Swarajist Changers. The working principle of the Swaraj Party they are known as No Changers. They are Vallabhbhai Patel, Rajendra Prasad, Dr. M.A. Ansari, C. Rajagopalachari.

**22. Option (a) is correct.**

**Explanation:**

- Headquarters of Ghadar Movement were situated at San Francisco.
- Baba Sohan Singh Bhakna was an Indian revolutionary, the founding president of the Ghadar Party.
- Lala Hardyal was the co-founder of Ghadar party.

**23. Option (d) is correct.**

**Explanation:**

- Navjivan newspaper started by Gandhi Ji.
- Mahratta newspaper started by Bal Gangadhar Tilak.
- Voice of India newspaper started by Dadabhai Naoroji.

**24. Option (b) is correct.**

**Explanation:**

The subsidiary alliance in India was planned by Lord Wellesley, but this term was introduced by French Governor Dupleix.

Following were the conditions of the subsidiary alliance:

- The Indian state was prohibited to keep any external relations, unless approved by the British.
- They were to give either money or some territory to the company for the maintenance of the British troops.
- They were to turn out from their states all non-english europeans whether they were employed in the army.
- They had to keep a British official called 'resident' at the capital of their respective states who would oversee all the negotiations and talks with the other states.

**25. Option (d) is correct.**

**Explanation:**

**The Battle of Colachel**

- It is also known as the Battle of Kulachal.
- It was fought in 1741.
- It was fought between the Indian kingdom of Travancore and the Dutch East India Company.
- During the Travancore-Dutch War, the forces of King Marthanda Varma's defeated the Dutch East India Company's forces, which were led by Admiral Eustachius De Lannoy.

- The Dutch, after this battle, never recovered from the defeat and no longer posed a large colonial threat to India.

**26. Option (c) is correct.**

**Explanation: The Bengal Famine of 1770**

- The Bengal Famine of 1770 affected the regions of Bengal and Bihar between 1769 and 1770 and affected 30 million people.
- It occurred during the period of dual governance in Bengal.
- People started dying due to Starvation.
- In order to make up for the loss in agricultural revenue brought on by the famine, the company continued to collect taxes from farmers, who could pay by further increasing the tax rate. The apathy of the company officials worsened the situation.

**27. Option (b) is correct.**

**Explanation:**

- In cavalry warfare, soldiers or warriors fought on horseback.
- A stirrup is a light frame or ring that holds the foot of a rider.
- The iron stirrup greatly increases the rider's ability to stay in the saddle and control the mount, increasing the animal's usefulness to humans in areas such as communication, transportation, and warfare.
- Stirrups enabled the horseman to use a lance (a spear designed to be used by a mounted warrior or cavalry soldier) more effectively.

**28. Option (b) is correct.**

**Explanation:**

- The British policy towards Afghanistan in the first half of the 19th century aimed at weakening Russian influence in Afghanistan but not promoting a strong Afghanistan either
- The First Anglo-Afghan War was one of the first major conflicts during the Great Game for power and influence in Central Asia between Britain and Russia.
- The Great Game was a political and diplomatic confrontation that existed for most of the 19th century and beginning of the 20th century between the British Empire and the Russian Empire over Afghanistan

**29. Option (c) is correct.**

**Explanation:** Woman labour leader who organised workers in the strikes in the 1920s were:

- Sarala Devi Chaudhurani
- Ushabai Dange
- Anasuya Behn

**30. Option (c) is correct.**

**Explanation:** Gandhi's first major public appearance was at the inauguration of Banaras Hindu University in 1916. Banaras Hindu University is an internationally acknowledged centre of learning, located in Varanasi, Uttar Pradesh.

**31. Option (d) is correct.**

**Explanation: Gopal Krishna Gokhale**

- He was an Indian 'moderate' political leader and also a social reformer.

- He was associated with Indian National Congress and the founder of the Servants of India Society.
  - Gokhale was elected to the Bombay Legislative Council In 1899.
  - On 20 December 1901, he was elected to the Imperial Council of the Governor-General of India.
  - In 1911, he had introduced a bill in the Imperial legislative Assembly in order to implement the principle of compulsory primary education for children of 6-10 years age.
- 32. Option (b) is correct.**  
*Explanation: Sanyasi Revolt (1763-1800)*
- It is also known as Fakir Rebellion or revolt.
  - A group of sanyasis in Eastern India (Bengal and Bihar) were forced to fight the British yoke after the devastating famine of 1770 and the strict economic regime of the British.
  - The revolt took place under the leadership of Majnoo Shah Fakir and Pandit Bhabani Charan Pathak.
  - These sanyasis were joined by a large number of dispossessed small zamindars, disbanded soldiers and naga sadhus.
  - They raided Company factories and the treasures, and fought the Company's forces.
  - It was only after a long battle that Warren Hastings could subdue the sanyasis.
- 33. Option (a) is correct.**  
*Explanation: Kesari is a Marathi newspaper which was founded in 4 January 1881 by Lokmanya Bal Gangadhar Tilak.*
- 34. Option (b) is correct.**  
*Explanation: The jealously-protected monopoly of the East India Company was abolished in 1833 and the China trade was made open to the competition to dozens of British companies, who had been petitioning the government for free and open trade for years.*
- 35. Option (b) is correct.**  
*Explanation: It was Lord Palmerston who believed that the Russian designs were 'an imminent peril to the security and tranquillity' of the Indian empire. He served twice as the Prime Minister of the United Kingdom in the mid-19th century and dominated the British foreign policy during the period 1830 to*
- 1865.*
- 36. Option (b) is correct.**  
*Explanation: The All-India depressed classes association was established by Bhimrao Ramji Ambedkar in 1930. He demanded for separate electorates and fair representation for untouchables in the decision-making bodies of India. The issue of separate electorate for the depressed classes was again raised by Ambedkar in the second Round Table Conference held in London in 1931.*
- 37. Option (b) is correct.**  
*Explanation: Agra was not the centre of the revolt on 1857. Other important cities of this revolt were Kanpur, Jhansi, Lucknow, Gwalior and Ayodhya. The Revolt of 1857 is also called the Sepoy Mutiny or India's First War of Independence.*
- 38. Option (d) is correct.**  
*Explanation: During the 19<sup>th</sup> century, the 'Satapatra Series' was written by G.H. Deshmukh.*
- The Satapatra was a series of patriotic literature that called for social reforms.
  - Gopal Hari Deshmukh was an Indian activist, thinker, social reformer and writer from Maharashtra.
- 39. Option (c) is correct.**  
*Explanation: In response to Gandhi's article, The Cult of Bomb, Bhagwati Charan Vohra in consultation with Azad, wrote an article entitled the Philosophy of Bomb. It appealed to youths to come forward and join them. Vohra died in Lahore on 28 May 1930 while testing a bomb on the banks of the Ravi. The device was required for the proposed rescue of Singh and others under trial in the Lahore Conspiracy Case but it exploded during the test and he was severely wounded*
- 40. Option (d) is correct.**  
*Explanation: In 1921, during his South India tour, Gandhiji shaved his head and began wearing a loincloth. This epoch-making decision was taken by Gandhiji in Madurai after he decided that he has to work for and with the poor people of India, and how he can identify with them if he wears clothes different from them.*

# Chapter 61

## NATIONAL LANGUAGE, BIRDS, ANIMAL, SONG, FLAG, MONUMENTS, CAPITAL AND CURRENCY

### Language

Language in its literary meaning is a system of communication through speech, a collection of sounds that a group of people understand to have the same meaning. The Indian languages belong to four language families:

- Indo-European
- Dravidian
- Mon-Khmer
- Sino-Tibetan

Among them, Indo-European and Dravidian languages are used by a large majority of India's population.

Based on geographic groups the language families are divided. Such as:

- Indo-European group are spoken mainly in northern and Central regions
- Dravidian group are the languages of Southern India
- Mon-Khmer group in Assam and other parts of eastern India speak languages
- Sino-Tibetan People in the northern Himalayan region and near the Burmese border speak Sino-Tibetan languages

### Important Sanctuaries and National Parks

| Name                           | Location          | Reserve For                                                           |
|--------------------------------|-------------------|-----------------------------------------------------------------------|
| Pachmarhi Sanctuary            | Madhya Pradesh    | Tiger, Leopard, Wild bear, Cheetah, Sambar, and Macaque               |
| Gandhi Sagar Sanctuary         | Madhya Pradesh    | Tiger, Panther, Boar, Sambar, Nilgai and Barking deer                 |
| Bandhavgarh National Park      | Madhya Pradesh    | Cheetah, Sambar, Chinkara, and Wild birds                             |
| Simlipal Sanctuary             | Odisha            | Tiger, Panther, Cheetah, Nilgai, and Wild boar                        |
| Ghana Bird Sanctuary           | Rajasthan         | Water birds, Blackbuck, Cheetah, and Sambar                           |
| Khangchendzonga National Park  | Sikkim            | Snow Leopard, Musk deer, and Himalayan boar                           |
| Vedanthangal, Bird Sanctuary   | Tamil Nadu        | Important bird sanctuary                                              |
| Chandraprabha Sanctuary        | Uttar Pradesh     | Gir lions, Cheetah, and Sambar                                        |
| Dudhwa National Park           | Uttar Pradesh     | Tiger, Panther, Sambar, Cheetah, Nilgai, and Barking deer             |
| Corbett National Park          | Uttarakhand       | Tiger, Leopard, Elephant, and Sambar (named in memory of Jim Corbett) |
| Jaldapara Sanctuary            | West Bengal       | Rhinoceros, Elephant                                                  |
| Sunderban Tiger Reserve        | West Bengal       | Tiger, Deer, Wild boar, Crocodile, and Gangetic dolphin               |
| Velvadore National Park        | Gujarat           | Wolf and Blackbuck                                                    |
| Wild Ass Sanctuary             | Gujarat           | Wild ass, Wolf, Nilgai, and Chinkara                                  |
| Gir Forest                     | Gujarat           | India's biggest wildlife sanctuary famous for Gir lions               |
| Michigan National Park         | Jammu and Kashmir | Kashmir Stag, Long-tailed marmot, Himalayan serow                     |
| Bannerghatta National Park     | Karnataka         | Elephant, Cheetah, Deer, and Grey Partridge, and Green Pigeon         |
| Bhadra Sanctuary               | Karnataka         | Elephant, Cheetah, Panther, Sambar, and Wild boar                     |
| Bandipur National Park         | Karnataka         | Elephant, Tiger, Panther, Sambar, Deer, and Birds                     |
| Tungabhadra Sanctuary          | Karnataka         | Tiger, Panther, Elephant, Cheetah, Sambar, and Wild boar              |
| Nagarhole National Park        | Karnataka         | Panther, Cheetah, Sloth bear, and Four-horned antelope                |
| Kaziranga National Park        | Assam             | One-horned rhinoceros, Gaur, Elephant, Leopard, and Wild buffalo      |
| Sonai Rupai Wildlife Sanctuary | Assam             | Elephant, Sambar, Wild boar, and One-horned rhinoceros                |
| Namdapha National Park         | Arunachal Pradesh | Elephant, Panther, Sambar, Tiger, Cheetah, and King cobra             |
| Gautam Buddha Sanctuary        | Bihar             | Tiger, Leopard, Sambar, Cheetah, and Barking deer (Indian Muntjac)    |
| Achanakmar Sanctuary           | Chhattisgarh      | Tiger, Boar, Cheetah, Sambar, and Bison                               |

**Important Monuments of Some Famous Countries**

| Monument                | Country | Monument                     | Country   |
|-------------------------|---------|------------------------------|-----------|
| Imperial Palace (Tokyo) | Japan   | Leaning Tower of Pisa        | Italy     |
| Eiffel Tower (Paris)    | France  | Pyramid (Giza)               | Egypt     |
| Great Wall of China     | China   | Opera House (Sydney)         | Australia |
| Kremlin Palace (Moscow) | Russia  | Statue of Liberty (New York) | USA       |
| Kinder Disk             | Denmark | Taj Mahal (Agra)             | India     |

**Countries with Capitals and Currencies**

| Country     | Capital      | Currency           |
|-------------|--------------|--------------------|
| Myanmar     | Naypyidaw    | Kyat               |
| Namibia     | Windhoek     | Namibian Dollar    |
| Nepal       | Kathmandu    | Nepalese Rupee     |
| Netherlands | Amsterdam    | Euro               |
| New Zealand | Wellington   | New Zealand Dollar |
| Nigeria     | Abuja        | Naira              |
| Norway      | Oslo         | Krone              |
| Pakistan    | Islamabad    | Rupee              |
| Philippines | Manila       | Peso               |
| Poland      | Budapest     | Zloty              |
| Portugal    | Lisbon       | Euro               |
| Kenya       | Nairobi      | Shilling           |
| North Korea | Pyongyang    | Won                |
| Kuwait      | Kuwait City  | Kuwait Dinar       |
| South Korea | Seoul        | Won                |
| Libya       | Tripoli      | Libyan Dinar       |
| Malaysia    | Kuala Lumpur | Ringgit            |
| Maldives    | Male         | Rufiyaa            |
| Mauritius   | Port Louis   | Rupee              |
| Mongolia    | Ulan Bator   | Tugrik             |
| Montenegro  | Podgorica    | Euro               |
| Belarus     | Minsk        | Ruble              |

|             |               |                   |
|-------------|---------------|-------------------|
| Belgium     | Brussels      | Euro              |
| Bhutan      | Thimphu       | Ngultrum          |
| Brazil      | Brasilia      | Cruzeiro Real     |
| Cambodia    | Phnom-Penh    | Kiel              |
| Canada      | Ottawa        | Canadian Dollar   |
| Chile       | Santiago      | Peso              |
| China       | Beijing       | Yuan, Renminbi    |
| Afghanistan | Kabul         | Afghani           |
| Albania     | Tirana        | T-Lek             |
| Algeria     | Algiers       | Algerian Dinar    |
| Angola      | Luanda        | Kwanza            |
| Argentina   | Buenos Aires  | Peso              |
| Australia   | Canberra      | Australian Dollar |
| Austria     | Vienna        | Euro              |
| Bangladesh  | Dhaka         | Taka              |
| Sweden      | Stockholm     | Krona             |
| Switzerland | Bern          | Swiss Franc       |
| Taiwan      | Taipei        | New Taiwan Dollar |
| Thailand    | Bangkok       | Baht              |
| Turkey      | Ankara        | Lira              |
| Uganda      | Kampala       | Uganda Shilling   |
| Ukraine     | Kiev          | Hryvnia           |
| UK          | London        | Pound Sterling    |
| US          | Washington DC | US Dollar         |

**EXERCISE**

- Consider the following statements about an Indian State :
  - It is well known globally for its variety of rocks and minerals.
  - It has the largest deposits of Chrysotile Asbestos in the country.
  - The Neelam Sanjiva Reddy Sagar Hydel Project is located in this State.
 Identify the State from the following :
  - Jharkhand
  - Rajasthan
  - Andhra Pradesh
  - Chhattisgarh
- The city of Cartagena, which is famous for Protocol on Biosafety, is located in:

- Colombia
  - Venezuela
  - Brazil
  - Guyana
- 'Rand/ZAR' is the currency of:
    - Burundi
    - Libya
    - Sudan
    - South Africa
  - Which one of the following fishing banks is located off the coast of the United Kingdom?
    - Great Fisher Bank
    - Grand Banks
    - Dogger Bank
    - Reed Bank
  - Which one of the following languages was added to the Eighth Schedule by a Constitutional Amendment in 1967?
    - Khasi
    - Kashmiri
    - Sindhi
    - Urdu



6. *Chandimangala* was composed in which one of the following languages during the 16th century CE?  
 (a) Sanskrit (b) Tamil  
 (c) Bengali (d) Oriya
7. Which one of the following is not among the principal languages of Jammu and Kashmir?  
 (a) Urdu (b) Gujarati  
 (c) Koshur (d) Monpa
8. In which one of the following types of connective tissues in animals does fat get stored?  
 (a) Adipocyte (b) Chondrocyte  
 (c) Osteocyte (d) Reticulocyte
9. Consider the following:  
 1. Tughlaqabad Fort  
 2. Bada Gumbad in Lodhi Garden  
 3. Qutub Minar  
 4. Fatehpur Sikri  
 Which one of the following is the correct chronological order of building the above monuments?  
 (a) 3, 1, 4, 2 (b) 3, 1, 2, 4  
 (c) 1, 3, 2, 4 (d) 1, 3, 4, 2
10. Consider the following:  
 1. Tughlaqabad Fort  
 2. Bada Gumbad in Lodhi Garden  
 3. Qutub Minar  
 4. Fatehpur Sikri  
 Which one of the following is the correct chronological order of building the above monuments?  
 (a) 3, 1, 4, 2 (b) 3, 1, 2, 4  
 (c) 1, 3, 2, 4 (d) 1, 3, 4, 2
11. What is, Unicorn Company often mentioned in Indian news?  
 (a) Any privately held Startup Company with a value of over \$ 1 billion.  
 (b) Any public sector company to be merged with another public sector company.  
 (c) Privatization of any loss-making State-owned company.  
 (d) Any foreign multinational company doing business in India in collaboration with an Indian company.
12. The terms 'Ethereum, Solana, Polkadot and Tether', sometimes mentioned in the news, refer to:  
 (a) Cryptocurrencies  
 (b) Earth Observation Satellites  
 (c) Hypersonic Cruise Missiles  
 (d) Virtual Private Network service providers
13. Which of the following statements about "delegation" is/are correct?  
 1. It is the abdication of responsibility.  
 2. It means conferring of specified authority by a lower authority to a higher one.  
 3. It is subject to supervision and review.  
 4. It is a method of dividing authority in the organization.  
 Select the correct answer using the code given below:  
 (a) 3 only (b) 2 and 4 only  
 (c) 3 and 4 (d) 1, 2 and 4
14. Which of the following will be the outcome if an economy is under the inflationary pressure?  
 1. Domestic currency heads for depreciation.
2. Exports become less competitive with imports getting costlier.  
 3. Cost of borrowing decreases.  
 4. Bondholders get benefitted.  
 Select the correct answer using the code given below:  
 (a) 1 and 2 (b) 2 and 3  
 (c) 1 and 3 only (d) 1, 3 and 4
15. Exchange rates state the value of one currency in terms of other currencies. Which one of the following statements with respect to the exchange rate of currency is correct?  
 (a) Floating exchange rates are rates in which the Governments interfere by buying or selling their currencies.  
 (b) Fixed exchange rates are rates set by Government decisions and maintained by Government actions.  
 (c) Under the Bretton Woods System, the exchange rates are floated in terms of rise or fall in price of gold.  
 (d) Under the classical gold standard, the exchange rates are fixed in terms of price of dollar.
16. Devaluation of currency will be more beneficial if prices of:  
 (a) domestic goods remain constant  
 (b) exports become cheaper to importers  
 (c) imports remain constant  
 (d) exports rise proportionately
17. Which one of the following biosphere reserve comprises islands with estuaries, beaches, coral reefs, salt marshes and mangroves?  
 (a) Sunderban biosphere reserve  
 (b) Gulf of Mannar biosphere reserve  
 (c) Nilgiri biosphere reserve  
 (d) Nandadevi biosphere reserve
18. Which of the following National Parks of India are declared as World Heritage by UNESCO?  
 1. Keoladeo National Park  
 2. Sundarbans National Park  
 3. Kaziranga National Park  
 4. Ranthambore National Park  
 Select the correct answer using the code given below:  
 (a) 1 and 2 only (b) 1, 2 and 3 only  
 (c) 3 and 4 only (d) 1, 2, 3 and 4
19. Arrange the following tiger reserves of India from North to South:  
 1. Indravati 2. Dudhwa  
 3. Bandipur 4. Simlipal  
 Select the correct answer using the code given below:  
 (a) 3 – 4 – 1 – 2 (b) 4 – 2 – 3 – 1  
 (c) 2 – 4 – 1 – 3 (d) 2 – 1 – 4 – 3
20. Which of the following about Khijadiya Bird Sanctuary is/are correct?  
 1. It is recently declared as a Ramsar Site (wetlands of international importance)  
 2. It is located in Uttar Pradesh  
 Select the correct answer using the code given below:  
 (a) 1 only (b) 2 only  
 (c) Both 1 and 2 (d) neither 1 nor 2

## Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (a)  | 3. (d)  | 4. (c)  | 5. (c)  | 6. (c)  | 7. (d)  | 8. (a)  | 9. (d)  | 10. (b) |
| 11. (a) | 12. (a) | 13. (c) | 14. (c) | 15. (b) | 16. (a) | 17. (b) | 18. (b) | 19. (d) | 20. (a) |

## Answers with Explanations

## 1. Option (c) is correct.

**Explanation:** Andhra Pradesh

- It is well known globally for a variety of rocks & minerals, and called "Ratna Garha".
- Neelam Sanjeeva Reddy Sagar (Srisailem) Project is a project of Andhra Pradesh.
- It has the largest deposits of Chrysotile Asbestos in the country. There are two asbestos mines in Kadapa district of Andhra Pradesh.

## 2. Option (a) is correct.

**Explanation:** The city of Cartagena is located in South America's country Colombia.

- It is a major port of Colombia and is located on its northern coast.
- The Cartagena Protocol on Biosafety to the Convention on Biological Diversity is an international agreement which aims to ensure the safe handling, transport and use of living modified organisms (LMOs).
- It was effective from 11 September 2003.

## 3. Option (d) is correct.

**Explanation:** "Rand/ZAR" is the currency of South Africa.

- The Burundian Franc is the currency of Burundi.
- The Libyan Dinar is the currency of Libya.
- The Sudanese Pound is the currency of Sudan.

## 4. Option (c) is correct.

**Explanation:** Dogger Bank is located off the coast of the United Kingdom. Dogger Bank is a large sand-bank in a shallow area of the North Sea about 100 kilometres off the east coast of England. During the last ice age, the bank was part of a large landmass connecting Europe and the British Isles, now known as Doggerland.

## 5. Option (c) is correct.

**Explanation:** The Twenty-first Amendment Act, 1967, amended the Eighth Schedule of the Constitution. It included Sindhi, as one of the languages, as a result the total number of languages listed in the schedule raised to fifteen.

## 6. Option (c) is correct.

**Explanation:** The Chandimangala is an important subgenre of mangalkavya, the most significant genre of medieval literature in Bengali. The texts show primarily some folk goddess like Chandi or Abhaya.

## 7. Option (d) is correct.

**Explanation:** The official languages of Jammu and Kashmir are - Kashmiri, Dogri, Urdu, Hindi, English

- Koshur or Kashmiri is a widely spoken local language in Jammu and Kashmir. More than 50% can speak this language
- Gujari or Gajri language is also spoken by people of Jammu & Kashmir.
- The Takpa or Dakpa language which is known in India as Tawang Monpa is an East Bodish lan-

guage which is spoken in the Tawang district of Arunachal Pradesh.

## 8. Option (a) is correct.

**Explanation:** In animals, the fat is stored in the adipose tissues. It is a fat storing tissue in animals. Chondrocytes are found in cartilage while osteocytes are present in bones. On the other hand, reticular cell or reticulocytes are found in lymphoid organs.

## 9. Option (d) is correct.

**Explanation:**

- Dandin composed the famous the "Dashakuntacharita" or "Tales of Ten Princes".
- It is a prose romance in Sanskrit and describes the adventures of ten young men, the Kumaras, all of whom are either princes or sons of royal ministers, as narrated by the men themselves.

## 10. Option (b) is correct.

**Explanation:**

- The Tughlaqabad fort was built by Ghiyas-ud-din Tughlaq in 1321.
- The Bara Gumbad was constructed in 1490 during the reign of Sikandar Lodi.
- Qutub Minar is a soaring, 73 m-high tower of victory, built in 1193 by Qutubud-din Aibak.
- Fatehpur Sikri was founded in 1571 by Emperor Akbar.

## 11. Option (a) is correct.

**Explanation:** A unicorn is a privately held startup company valued at over US\$1 billion.

The term was first popularised in 2013 by venture capitalist Aileen Lee, choosing the mythical animal to represent the statistical rarity of such successful ventures.

## 12. Option (a) is correct.

**Explanation:** A cryptocurrency is a type of digital or virtual currency, which uses encryption to protect it from counterfeiting or duplicate spending. Ethereum, Solana, Polkadot and Tether are some of the names of the cryptocurrencies.

## 13. Option (c) is correct.

**Explanation:** Statements (3) and (4) are correct about delegation. Delegation of powers in law, is the transfer of authority by one person or group to another person or group who is subordinate to them. It means transfer of certain responsibilities to the employees and giving them the required authority which is necessary for the discharge of the responsibility properly.

## 14. Option (c) is correct.

**Explanation:** Inflation is defined as a steady increase in the overall price level. Inflationary pressures are demand and supply-side factors that can lead to an increase in the overall price level. When actual GDP surpasses potential GDP, resulting in a positive output gap, demand-pull inflationary pressure is strong.



gest. Debtors benefit from inflation since they are repaying creditors with a currency that has lost its buying value

**15. Option (b) is correct.**

**Explanation:** This statement is correct regarding fixed exchange rate. A fixed exchange rate is a regime applied by any government or central bank that ties the country's official currency exchange rate to another country's currency or the price of gold. The purpose of a fixed exchange rate system is to keep a currency's value within a narrow band.

**16. Option (a) is correct**

**Explanation:**

Devaluation of currency will be more beneficial if the prices of domestic goods remain constant.

- Devaluation is a monetary policy tool used by the countries that have a fixed exchange rate or semi-fixed exchange rate.
- The one reason a country may devalue its currency is to combat trade imbalances.
- After devaluation, the new lower value of the domestic currency will make it less expensive for the foreign consumers to obtain local currency with which to buy locally produced export goods.

So, more exports will be sold helping domestic businesses.

**17. Option (b) is correct.**

**Explanation:** The Gulf of Mannar Biosphere Reserve contains islands with estuaries, beaches, coral reefs, salt marshes, and mangroves. The Gulf of Mannar is a large shallow bay forming part of the Laccadive Sea in the Indian Ocean. In 2019, UNESCO included Gulf of Mannar in the world network of biosphere reserves under the 'Man and the Biosphere Programme'

**18. Option (b) is correct.**

**Explanation:**

- Ranthambore National Park is not a world Heritage site designated by UNESCO.
- Kaziranga National Park (1985), Keoladeo National Park (1985), Sundarbans National Park (1984) were declared World Heritage site by UNESCO.

- The foremost attraction in Kaziranga National Park is the one-horned rhinoceros, characterized by its enormous body and massive horn.
- Keoladeo National Park, formerly known as the Bharatpur Bird Sanctuary, is acclaimed for being one of the world's most important bird breeding and feeding grounds.
- Sundarbans National park is most famous for the Royal Bengal Tiger, and is one of the largest reserves for the Bengal tiger.

**19. Option (d) is correct.**

**Explanation:**

|    |           |                                                                                                                                                                                                                                                 |
|----|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 2. | Dudhwa    | The Dudhwa Tiger Reserve is a protected area in Uttar Pradesh that stretches mainly across the Lakhimpur Kheri and Bahraich districts and comprises the Dudhwa National Park, Kishanpur Wildlife Sanctuary and Katarniaghat Wildlife Sanctuary. |
| 1. | Indravati | Indravati National Park is a national park located in Bijapur district of Chhattisgarh state of India.                                                                                                                                          |
| 4. | Simlipal  | Simlipal National Park is a national park and a tiger reserve in the Mayurbhanj district in the Indian state of Odisha covering 2,750 km <sup>2</sup> .                                                                                         |
| 3. | Bandipur  | Bandipur National Park is a national park covering 868.63 km <sup>2</sup> (335.38 sq mi) in Chamara Nagar district in the Indian state of Karnataka.                                                                                            |

**20. Option (a) is correct.**

**Explanation** – The Khijadiya Bird Sanctuary near Jamnagar in Gujarat and Bakhira Wildlife Sanctuary in Uttar Pradesh have been listed as Wetlands of International Importance by the Ramsar Convention.

○○○

COMMON NAMES, FULL  
FORMS AND ABBREVIATIONS**Common Names/Full Names/Abbreviations****Common names of cyclones in different countries:**

- North Atlantic (including Caribbean and Gulf of Mexico): Hurricanes.
- Arabian Sea/Northern Indian Ocean: Tropical Cyclones.
- South Indian Ocean: Tropical Cyclones
- Willy-Willy for southwest Australia.
- Eastern and Central North Pacific: Hurricanes.
- Western Northern Pacific: Typhoons.

**Ranks of Commissioned officers**

| Army          | Air Force         | Navy           |
|---------------|-------------------|----------------|
| General       | Air Chief Marshal | Admiral        |
| Lt. General   | Air Marshal       | Vice Admiral   |
| Major General | Air Vice Marshal  | Rear Admiral   |
| Brigadier     | Air Commodore     | Commodore      |
| Colonel       | Group Captain     | Captain        |
| Lt. Colonel   | Wing Commander    | Commander      |
| Major         | Squadron Leader   | Lt. Commander  |
| Captain       | Flight Lieutenant | Lieutenant     |
| Lieutenant    | Flying Officer    | Sub-Lieutenant |

**Geographical Epithets**

|                               |               |
|-------------------------------|---------------|
| Blue Mountains                | Nilgiri Hills |
| Beautiful City                | Chandigarh    |
| City of Golden Gate           | San Francisco |
| City of Magnificent Buildings | Washington    |
| City of Palaces               | Kolkata       |
| City of Seven Hills           | Rome          |
| Cockpit of Europe             | Belgium       |

**Geographical Discoveries**

| Discovery                                | Discoverer           | Discovery                 | Discoverer           |
|------------------------------------------|----------------------|---------------------------|----------------------|
| America                                  | Christopher Columbus | New Foundland             | John Cabot           |
| Sea Route to India via Cape of Good Hope | Vasco Da Gama        | Hudson Bay                | Henry Hudson         |
| Solar System                             | Copernicus           | Circumnavigation of World | Magellan             |
| Planets                                  | Kepler               | Mount Everest             | Edmund Hillary       |
| South Pole                               | Roald Amundsen       | Brazil                    | Pedro Alvares Cabral |
| North Pole                               | Robert Peary         | Tasmania Island           | Abel Tasman          |
| China                                    | Marco Polo           | Cape of Good Hope         | Bartolomeu Dias      |

|                          |                           |
|--------------------------|---------------------------|
| Continent of Birds       | South Africa              |
| City of Smoke            | Chicago                   |
| Dark Continent           | Africa                    |
| Forbidden City           | Lhasa (Tibet)             |
| Gift of the Nile         | Egypt                     |
| Granite City             | Aberdeen                  |
| Holy Land                | Palestine                 |
| Island Continent         | Australia                 |
| Island of Cloves         | Zanzibar                  |
| Isle of Pearls           | Bahrain                   |
| Key to the Mediterranean | Gibraltar                 |
| Land of Golden Fleece    | Australia                 |
| Land of Maple            | Canada                    |
| Land of Morning Calm     | Korea                     |
| Land of the Midnight Sun | Norway                    |
| Land of the Rising Sun   | Japan                     |
| Land of the Thunderbolt  | Bhutan                    |
| Land of Thousand Lakes   | Finland                   |
| Land of White Elephant   | Thailand                  |
| Mistress of Eastern Sea  | Sri Lanka                 |
| Pearl of the Antilles    | Cuba                      |
| Pearl of the Pacific     | Guayaquil Port of Ecuador |
| The Roof of the World    | The Pamirs, Central Asia  |
| Spice Garden of India    | Kerala                    |
| Sugar Bowl of the World  | Cuba                      |

**Important Monuments of Some Famous Countries**

| Monument                | Country | Monument                     | Country   |
|-------------------------|---------|------------------------------|-----------|
| Imperial Palace (Tokyo) | Japan   | Leaning Tower of Pisa        | Italy     |
| Eiffel Tower (Paris)    | France  | Pyramid (Giza)               | Egypt     |
| Great Wall of China     | China   | Opera House (Sydney)         | Australia |
| Kremlin Palace (Moscow) | Russia  | Statue of Liberty (New York) | USA       |
| Kinder Disk             | Denmark | Taj Mahal (Agra)             | India     |

**National Emblems of Major Countries**

| Country    | Emblem            | Country                  | Emblem                     |
|------------|-------------------|--------------------------|----------------------------|
| Australia  | Kangaroo          | Italy                    | White Lily                 |
| Bangladesh | Water Lily        | Japan                    | <i>Chrysanthemum</i>       |
| Belgium    | Lion              | Netherlands              | Lion                       |
| Canada     | White Lily        | New Zealand              | Southern Cross, Kiwi, Fern |
| Chile      | Candor and Huemul | Norway                   | Lion                       |
| France     | Lily              | Pakistan                 | Crescent                   |
| Germany    | Corn Flower       | Spain                    | Eagle                      |
| India      | Lion Capital      | United Kingdom           | Rose                       |
| Iran       | Rose              | United States of America | Golden Rod                 |

**Abbreviations**

Abbreviation is the short pronunciation of a complex and long names. The names of any place, department, organization and programme of most of the institutions or governments are usually very long, which are very impractical in pronunciation and transmission, hence their short name or abbreviation is used. It may consist of a group of letters or words taken from the full version of the word or phrase. Like NASA abbreviation is used for the National Aeronautics and Space Administration, which is an independent agency of the U.S. federal government.

| Abbreviations | Complete Name                                       |
|---------------|-----------------------------------------------------|
| NABARD        | National Bank for Agriculture and Rural Development |
| SISFS         | Startup India Seed Fund Scheme                      |
| PM AASHA      | Pradhan Mantri Annadata Aay SanraksHan Abhiyan      |
| NTTM          | National Technical Textiles Mission                 |
| ONORCS        | One Nation One Ration Card Scheme                   |
| AEC           | Atomic Energy Commission                            |
| BARC          | Bhabha Atomic Research Centre                       |
| MSS           | Ministry of State Security                          |
| ASIO          | Australian Security and Intelligence Organisation   |
| FBI           | Federal Bureau of Investigation                     |
| CIA           | Central Intelligence Agency                         |
| BCCI          | Board of Control for Cricket in India               |

|        |                                                                  |
|--------|------------------------------------------------------------------|
| SAARC  | South Asian Association for Regional Cooperation                 |
| WWF    | World Wildlife Fund                                              |
| ISO    | International Organization for Standardization                   |
| NATO   | North Atlantic Treaty Organization                               |
| OPEC   | Organization of Petroleum Exporting Countries                    |
| APEC   | Asia Pacific Economic Cooperation                                |
| ADB    | Asian Development Bank                                           |
| ASFAN  | Association of SouthEast Asian Nations                           |
| CIS    | Commonwealth of Independent States                               |
| OECD   | Organization for Economic Cooperation and Development            |
| UNODC  | United Nations Office on Drugs and Crime                         |
| UNESCO | United Nations Educational, Scientific and Cultural Organization |
| WHO    | World Health Organization                                        |
| IAEA   | International Atomic Energy Agency                               |
| IDA    | International Development Association                            |
| UNEP   | United Nations Environmental Programme                           |

|        |                                                        |
|--------|--------------------------------------------------------|
| UNDP   | United Nations Developmental Programme                 |
| WTO    | World Trade Organization                               |
| IOC    | Organisation of Islamic Cooperation                    |
| ITU    | International Telecommunication Union                  |
| ILO    | International Labour Organization                      |
| IMF    | International Monetary                                 |
| UNICEF | United Nations International Children's Emergency Fund |
| EVD    | Ebola Virus Disease                                    |
| BSF    | Border Security Force                                  |
| CISF   | Central Industrial Security Force                      |

| Terms | Banking Abbreviations       |
|-------|-----------------------------|
| ACF   | Auto Correlation Function   |
| AD    | Authorised Dealer           |
| ADB   | Asian Development Bank      |
| ADR   | American Depository Receipt |
| AFS   | Annual Financial Statement  |

|          |                                                                |
|----------|----------------------------------------------------------------|
| AIRCSC   | All India Rural Credit Survey Committee                        |
| ASSOCHAM | Associated Chambers of Commerce and Industry of India          |
| BIS      | Bank for International Settlements                             |
| BoP      | Balance of Payments                                            |
| FEMA     | Foreign Exchange Management Act                                |
| FI       | Financial Institution                                          |
| FICCI    | Federation of Indian Chambers of Commerce and Industry         |
| FII      | Foreign Institutional Investor                                 |
| FIMMDA   | Fixed Income Money Market and Derivatives Association of India |
| FISIM    | Financial Intermediation Services Indirectly Measured          |
| FLAS     | Foreign Liabilities and Assets Survey                          |
| FOF      | Flow of Funds                                                  |
| FPI      | Foreign Portfolio Investment                                   |
| FRA      | Forward Rate Agreement                                         |
| FRN      | Floating Rate Note                                             |

## EXERCISE

- Which of the following statements about Maulvi Ahmadullah Shah, who played an important part in the Revolt of 1857, is/are correct?
  - He was popularly known as Danka Shah or the Maulvi with a drum.
  - He fought in the famous Battle of Chinhat
  - He was killed by British troops under the command of Henry Lawrence.
 Select the correct answer using the code given below:
  - 1 only
  - 2 and 3
  - 1 and 3
  - 1 and 2
- Who among the following was popularly known as 'Lokahitwadi'?
  - Gopal Hari Deshmukh
  - Mahadev Govind Ranade
  - Gopal Krishna Gokhale
  - Jyotiba Phule
- Which of the following Gods are also known as Lokapalas or the Guardians of the Universe?
  - Yama, Indra, Varuna and Kubera
  - Indra, Varuna, Skanda and Kubera
  - Indra, Varuna, Yama and Brahma
  - Yama, Shiva, Kubera and Indra
- The Indian desert also known as *Marusthali* is believed to be under the sea during which one of the following eras?
  - Cenozoic era
  - Palaeozoic era
  - Mesozoic era
  - Archaean and Pre-Cambrian era
- A peripedium in a desert or semi-desert region is also known as:
  - Inselberg
  - Zeugen
  - Bajada
  - Playa
- Which of the following is the right expansion of ILO?
  - International Law and Order
  - Inter-State Lawful Ordinance
  - Indian Legal Orientation
  - International Labour Organization
- 'Terra rossa' is a Latin word which means –
  - Hot area
  - Red terrain
  - Lateritic region
  - Region near the poles
- The NITI Aayog serves as the apex public policy think tank of the Government of India. What is the full form of NITI Aayog.
  - National Institution for Training in India
  - National Institution for Transmission of India
  - National Institution for Transmuting India
  - National Institution for Transforming India
- The abbreviation LHC stands for which machine?
  - Light Heat Collider
  - Large Hadron Collider
  - Long Heavy Collider
  - Large High Collider
- The abbreviation CSIR stands for which Indian Institute?
  - College of Scientific and Information & Research
  - College of Science and Industrial Research
  - Council of Scientific and Industrial Research
  - Country's Scientific and Innovative Research

11. Which of the following is the right expansion of CBI?
  - (a) Central Bureau of Investigation
  - (b) Commercial Bureau of Investigation
  - (c) Command Bureau of Investigation
  - (d) Crucial Bureau of Investigation
12. IMF stands for:
  - (a) International Monetary Function
  - (b) International Monetary Fund
  - (c) Indian Manufacturing Firm
  - (d) Interest Minimum Firm Function
13. NSG stands for:
  - (a) National Security Guard
  - (b) National Service Guard
  - (c) National Service Guarantee
  - (d) National Scheme for Guarantee
14. Which of the following is the right expansion of IDL?
  - (a) International Date Line
  - (b) International Data Line
  - (c) International Data Link
  - (d) International Death Line
15. Which of the following is the right expansion of GHG?
  - (a) Greenhouse Gas
  - (b) Greisen Homogeneous Gas
  - (c) Green Homogeneous Gas
  - (d) Greenwich House Gas
16. Which of the following is the right expansion of NIN institute?
  - (a) National Institute of Nutrition
  - (b) National Institute of Nazism
  - (c) National Institute of Narcotics
  - (d) National Institute of Neutrons
17. IBM stands for:
  - (a) Indian Bureau of Mines
  - (b) Indian Bureau of Ministers
  - (c) Indian Bureau of Minerals
  - (d) Indian Bureau of Multimedia
18. GDP stands for:
  - (a) Gross Domestic Product
  - (b) Gross Domestic Profit
  - (c) Gross Direct Production
  - (d) Gross Depreciation in production
19. SAARC refers to which of the following international organization?
  - (a) South Asian Association for Regional Cooperation
  - (b) South African Association for Regional Cooperation
  - (c) South Asian Accreditation for Regional Cooperation
  - (d) South Indian Association for Regional Cooperation
20. NCC stands for:
  - (a) National Credit Corps
  - (b) National Cadet Corps
  - (c) National Care Corps
  - (d) National Command Corps

### Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (d)  | 2. (a)  | 3. (a)  | 4. (c)  | 5. (d)  | 6. (d)  | 7. (b)  | 8. (d)  | 9. (b)  | 10. (c) |
| 11. (a) | 12. (b) | 13. (a) | 14. (a) | 15. (a) | 16. (a) | 17. (a) | 18. (a) | 19. (a) | 20. (b) |

### Answers with Explanations

**1. Option (d) is correct.**

**Explanation:** Maulvi Ahmadullah Shah, who played an important part in the Revolt of 1857, could never be caught by the British alive. On 30th June, 1857, British officer Sir Henry Lawrence led his troop in the famous Battle of Chinhat.

**2. Option (a) is correct.**

**Explanation:**

Gopal Hari Deshmukh (1823-1892)

- He was a social reformer from Maharashtra.
- Under British raj, he held the post of a judge.
- He used to write for a weekly called Prabhakar on social reform issues under the pen name of Lokahitawadi.

**3. Option (a) is correct.**

**Explanation:**

- Yama, Indra, Varuna and Kubera are known as the Lokapalas or the Guardians of the Universe.
- There are total eight lokapalas, the remaining four being Agni, Pavana, Soma and Surya.
- They are also called the Guardians of the Directions and each one is associated with a particular cardinal point on the compass

**4. Option (c) is correct.**

**Explanation:**

In Sanskrit *Marusthali* means 'dead land'.

- Because of the arid climatic conditions and low vegetation cover in this region, the Great Indian Desert is called *marusthali*.
- It is believed that this region during the Mesozoic era was under the sea.
- And this belief is made strong by the pieces of evidence found at the wood fossils park at Aakal, marine deposits around Brahmsar, near Jaisalmer.

**5. Option (d) is correct.**

**Explanation:** Playa is a flat-bottom depression found in interior desert basins and adjacent to coasts in arid and semiarid regions. The playas are created by local cataclysmic disruptions of drainage that are volcanism, landslides and meteorite impacts.

**6. Option (d) is correct.**

**Explanation:** ILO means the International Labour Organization dealing with employment issues. In particular, the International Labor Standards titled "Protection and Employment Opportunities for All".

Founded in 1919 and headquartered in Geneva, Switzerland.

**7. Option (b) is correct.**

*Explanation:* 'Terra Rossa' is a Latin term that represents Red Landscape. When weathering limestone rock carries, the Clay retained in the stones is left-back, and this red clay soil is named 'Terra Rossa'. This kind of red soil is likewise made by weathering of dolomite rock.

**8. Option (d) is correct.**

*Explanation:* NITI Aayog or National Institution for Transforming India Aayog is a policy think-tank of Union Government of India that replaces Planning Commission of India and aims to involve the states in economic policy-making in India. It will be providing strategic and technical advice to the Central and the State Governments.

**9. Option (b) is correct.**

*Explanation:* LHC is the globe's most giant and most effective particle collider, the enormous, most difficult testing structure ever made, and a single giant machine on the earth.

**10. Option (c) is correct.**

*Explanation:* CSIR stands for Council of Scientific & Industrial Research, India. It was established by the Government of India in September 1942 as an autonomous body. Shanti Swarup Bhatnagar Awards is awarded annually by the Council of Scientific and Industrial Research for notable and outstanding research.

**11. Option (a) is correct.**

*Explanation:* CBI stands for Central Bureau of Investigation. It was established in 1963 and its objective is to investigate cases of misconduct by public servants, cases of cheating, embezzlement, and fraud.

**12. Option (b) is correct.**

*Explanation:* The IMF is the International Monetary Fund. It is an Organization of 188 countries dedicated to promote intergovernmental monetary cooperation, ensuring financial stability, slowing international trade, promote high employment and sustainable economic growth, and reducing global poverty. The headquarters are located in Washington, DC, USA.

**13. Option (a) is correct.**

*Explanation:* The NSG stands for National Security Guard. It was established in 1984 to counter

the surge of militancy in the country. It is a highly trained force that deals with the militants effectively.

**14. Option (a) is correct.**

*Explanation:* The IDL stands for International Date Line. It is the longitude where the date changes by exactly one day when it is crossed. 180° East and 180° West meridians is the same line, which is called the International Date Line.

**15. Option (a) is correct.**

*Explanation:* Greenhouse Gas is also abbreviated as GHG. It is a gas in the atmosphere that absorbs and emits radiation within the thermal infrared range. This process is the fundamental cause of the greenhouse effect.

**16. Option (a) is correct.**

*Explanation:* National Institute of Nutrition is also abbreviated as NIN. It is an Indian Public Health, Nutrition, and Translational study center. The location of NIN is Hyderabad. Beneath the Indian Council of Medical Research, it is one of the investigation centers in India.

**17. Option (a) is correct.**

*Explanation:* Indian Bureau of Mines is also abbreviated as IBM. The Indian Bureau of Mines (IBM) was launched in the year 1948, is a multi-disciplinary government association beneath the Department of Mines, Ministry of Mines. The headquarters of IBM is in Nagpur.

**18. Option (a) is correct.**

*Explanation:* GDP stands for Gross Domestic Product. GDP is the total money value of all final goods and services produced within the geographical boundaries of the country during a given period of time.

**19. Option (a) is correct.**

*Explanation:* The South Asian Association for Regional Cooperation (SAARC) is the regional intergovernmental organization and geopolitical union of states in South Asia. Its member states are Afghanistan, Bangladesh, Bhutan, India, Maldives, Nepal, Pakistan, and Sri Lanka.

**20. Option (b) is correct.**

*Explanation:* NCC stands for National Cadet Corps, established in 1948. Its main objective is to stimulate interest among the youth in the defense of the country to build Nation.



# DISCOVERIES, DISEASES AND NUTRITION

## Important Inventions & Discoveries

| Invention            | Inventor              | Year |
|----------------------|-----------------------|------|
| Automatic calculator | Wilhelm Schickard     | 1623 |
| Air conditioner      | Willis Carrier        | 1902 |
| Atom Bomb            | J.R. Oppenheimer      | 1945 |
| Airplane             | Wilber & Orvik Wright | 1903 |
| Bicycle              | K. Macmillan          | 1839 |
| Diesel Engine        | Rudolf Diesel         | 1895 |
| Dynamite             | Alfred B. Nobel       | 1867 |
| Gramophone           | Thomas Edison         | 1878 |

|                |                       |      |
|----------------|-----------------------|------|
| Helicopter     | Ig or Sikorsky        | 1939 |
| Locomotive     | George Stephenson     | 1804 |
| Light Bulb     | Thomas Edison         | 1854 |
| Microphone     | Alexander Graham Bell | 1876 |
| Oxygen         | Joseph Priestly       | 1774 |
| Printing Press | Jo Hannes Gutenberg   | —    |
| Polio Vaccine  | Jonas Edward Salk     | —    |
| Penicillin     | Alexander Fleming     | 1928 |
| Telephone      | Graham Bell           | 1874 |

## Some Important Discoveries

| Discovery                  | Made by                    | Country  |
|----------------------------|----------------------------|----------|
| Antibiotic                 | Alexander Flemming (1928)  | Scotland |
| Antiseptic                 | Joseph Lister (1869)       | Scotland |
| Blood circulation          | William Harvey (1628)      | Britain  |
| Blood transfusion          | Jean-Baptiste Denys (1625) | France   |
| Cholera and TB germs       | Robert Kock (1883)         | Germany  |
| Electrocardiogram (ECG)    | William Einthoven (1903)   | Dutch    |
| CT Scan                    | Godfrey Hounsfield (1973)  | England  |
| Sphygmomanometer           | Scipione Riva-Rocci (1898) | Italy    |
| Stethoscope                | Rene Laennec (1819)        | France   |
| Thermometer                | Sir Thomas Aelburt (1867)  | England  |
| Ultrasound                 | Ian Donald (1950)          | Ireland  |
| X-ray                      | WC Roentgen (1895)         | Germany  |
| Electroencephalogram (EEG) | Hans Berger (1929)         | Germany  |

| Disease                   | Pathogen            | Incubation            | Symptoms                                                                                       | Prevention Vaccine                                    |
|---------------------------|---------------------|-----------------------|------------------------------------------------------------------------------------------------|-------------------------------------------------------|
| Chickenpox (Varicella)    | Herpes zoster virus | 12-20 days            | Dark red colored rash or pox changing into vesicles, crusts and falling                        | Varicella vaccine                                     |
| Smallpox                  | Variola virus       | 12 days               | Appearance of rash changing into pustules, scabs and falling pock-marks are left               | Smallpox vaccine                                      |
| Poliomyelitis             | Poliovirus          | 7-14 days             | Damages motor neurons causing stiffness of neck, convulsion, paralysis of limbs generally legs | Salk vaccine and Oral Polio Vaccine (OPV)             |
| Measles (Rubella disease) | Rubella virus       | 10 days               | Rubella (skin eruptions), coughing, sneezing, etc                                              | Measles-mumps- rubella-Varicella Combo (MMRV vaccine) |
| Mumps                     | Mumps virus         | 12-26 days            | Painful enlargement of parotid and salivary glands                                             | Mumps-vaccine, isolation                              |
| Rabies (Hydrophobia)      | Rabies virus        | 10 days to 1-3 months | Spasm in throat and chest muscles, fears from water paralysis and death                        | Immunization of dogs                                  |



|                           |                                    |                 |                                                                                                                                 |                                                                                                             |
|---------------------------|------------------------------------|-----------------|---------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------|
| Tuberculosis              | <i>Mycobacterium tuberculosis</i>  | 2-10 weeks      | Coughing, chest pain and bloody sputum with tuberculin                                                                          | BCG vaccine                                                                                                 |
| Diphtheria                | <i>Corynebacterium diphtheriae</i> | 2-6 days        | Inflammation of mucosa of nasal chamber, throat, etc, respiratory tract blocked.                                                | DPT vaccine                                                                                                 |
| Cholera                   | <i>Vibrio cholera</i>              | 6 h to 2-3 days | Acute diarrhea and dehydration                                                                                                  | Sanitation, boiling of water and oral cholera vaccine                                                       |
| Leprosy                   | <i>Mycobacterium leprea</i>        | 2-5 years       | Skin hypopigmentation, nodulated skin, deformity of fingers and toes.                                                           | BCG also offers variable amount of protection against leprosy. Lepromin skin tests                          |
| Tetanus (Lockjaw)         | <i>Clostridium tetani</i>          | 3-21 days       | Degeneration of motor neurons, rigid jaw muscles, spasm and paralysis                                                           | ATS and DPT vaccines                                                                                        |
| Typhoid                   | <i>Salmonella Typhi</i>            | 2-6 days        | Bubonic plague affects, lymph nodes, pneumonic plague affects lungs and septicemic plague causes anemia                         | Killing of rats and rat fleas, plague vaccine                                                               |
| Gonorrhoea                | <i>Neisseria gonorrhoea</i>        | 2-10 days       | Inflammation of urinogenital tract                                                                                              | Avoid prostitution                                                                                          |
| Pneumonia                 | <i>Streptococcus pneumonia</i>     | 1-3 days        | Decrease in respiratory efficiency                                                                                              | PCV13                                                                                                       |
| Salmonellosis             | <i>Salmonella</i>                  | 48h             | Diarrhoea                                                                                                                       | RASV vaccine                                                                                                |
| Swine Flu                 | H1N1 flu virus (Orthomy)           | 1-4 days        | Fever with or without chill, sore throat, dyspnea, myalgia, diarrhea, vomiting and dizziness                                    | Oseltamivir (Tamiflu), Zanamivir (Relenza) are antiviral drugs vaccines are available against this disease. |
| Ebola Virus Disease (EVD) | Ebola virus (Filo-virus)           | 2-21 days       | Haemorrhagic fever, muscle pain, headache, sore throat, diarrhea, kidney and liver dysfunction, internal and external bleeding. | No licensed vaccine available, immune therapies are done currently.                                         |
| Dengue                    | RNA virus of genus Flavivirus      | 3-14 days       | Muscle pain, swollen lymph nodes, fever, headache and rash                                                                      | No specific antiviral drug is available, however symptoms based treatment is done.                          |

#### Important Vaccines Discoverer

| Vaccine                | Discovered By                                   |
|------------------------|-------------------------------------------------|
| Smallpox               | Edward Jenner (1786)                            |
| Cholera                | Louis Pasteur (1880)                            |
| Diphtheria and Tetanus | Emil Adolf Von Behring and Shibasaburo Kitasato |
| Tuberculosis           | Leon Calmette and Camille Guerin (1992)         |
| Polio                  | Jonas E Salk (1954)                             |
| Oral polio             | Albert Bruce Sabin (1995)                       |
| Measles                | John F Enders (1960)                            |
| Rabies                 | Charles Nicolle (1969)                          |

#### Nutrition in human beings:

- The human digestive system comprises of alimentary canal and associated digestive glands.
- The alimentary canal is a long muscular tube extending from the mouth to the anus. Various regions are specialised to perform different

functions.

- Associated glands include **salivary gland**, gastric gland, Liver and Pancreas. Most of the digestive disorders take place due to improper eating of non-nutritious food.
- Irritable bowel syndrome, hiatal hernia, Gastro-esophageal reflux disease, cancer, and lactose intolerance are the most often digestive and nutritional disorders.
- Bleeding, constipation, diarrhoea, bloating, nausea, heartburn, vomiting, and pain are some of the most common symptoms of digestive disorders.

#### Food and Nutrition, diseases and prevention, vitamins and their uses

##### Balanced Diet

- A diet consisting variety of different kinds of food and provide adequate amounts of the nutrients required for good health. A healthy diet provides the body with essential nutrition: fluid, macronutrients such as protein, micronutrients such as vitamins, and adequate fiber and food energy.

**Biomolecules**

- Analysis of organic constituents in a living tissue identifies functional groups such as aldehydes, ketones and aromatic compounds, also known as biomolecules.
- Biomolecules have a turn over i.e., they are constantly being changed into some other biomolecules and also made from some other biomolecules.
- The entire set of biochemical reactions that take place in living organisms are known as metabolism.
- Each of the metabolic reactions results in the transformation of biomolecules.

**Human Diseases and Disease**

- Health is a state of complete physical, mental and social well-being.
- Health is affected by genetic disorders, infections, sedentary lifestyle (Junk food, lack of exercise, habits, etc).
- **Disease:** A disease can be defined as any condition that may lead to discomfort, distress, health problems or death of the affected person.
- **Congenital diseases:** These are diseases that are present since birth. For instance, a hole in the heart of an infant. They are caused by some genetic abnormalities or metabolic disorder or malfunctioning of an organ.
- **Acquired diseases:** These are diseases that may occur after birth during one's lifetime.
- Among non-infectious diseases, cancer is the major cause of death.
- Pathogens are disease-causing organisms.
- Parasites are pathogens as they harm the host by living in or on them.
- Pathogens have to adapt to live within the environment of the host.

**COMMON INFECTIOUS DISEASES IN HUMAN BACTERIAL DISEASES****(a) Typhoid**

- **Pathogen:** *Salmonella typhi*.
- **Mode of transmission:** It enters the small intestine through food and water and migrates to other organs through blood.
- **Symptoms:** Sustained high fever (39°- 40°C), weakness, stomach pain, constipation, headache and loss of appetite. Intestinal perforation and death may occur.
- **Confirmation:** The Widal test is used for confirmation of the disease.

**(b) Pneumonia**

- **Pathogen:** *Streptococcus* or *Diplococcus pneumoniae* & *Haemophilus influenzae*.
- **Mode of transmission:** Inhaling the droplets/aerosols released by an infected person. Sharing glasses and utensils with an infected person.
- **Symptoms:** Infects lung's alveoli. The alveoli get filled with fluid leading to respiratory problems. Fever, chills, cough, headache.
- **Severe cases:** Lips and fingernails turn grey to a bluish colour.
- Dysentery, plague, diphtheria are some other bacterial diseases in humans
- **Hemozoin:** It is a brownish black granular material within the red blood cell, which is generated during host haemoglobins degradation by the parasite.

- **Sporozoite:** It is a motile spore-like stage of some parasitic sporozoans, which act as an infective agent.

**2. VIRAL DISEASES****(a) Common cold**

- **Pathogen:** Rhino viruses
- **Mode of transmission:** Inhaling droplets resulting from cough or sneezes through contaminated objects.
- **Symptoms:** Infects nose and respiratory passage. Nasal congestion and discharge, sore throat, hoarseness, cough, headache, tiredness, etc., Last for 3-7 days.
- *Aedes aegypti*, vector of a viral disease, dengue fever, is a daytime biting mosquito.
- National T.B. programme was started in 1962.
- Hepatitis B is more dangerous than AIDS.

**3. PROTOZOAN DISEASES****(a) Malaria**

- **Pathogen:** *Plasmodium* sp. (*P. vivax*, *P. malariae*, *P. ovale* and *P. falciparum*).
- **Mode of transmission:** Biting of *Anopheles* mosquito.
- **Symptoms:** Haemozoin causes chill and high fever recurring every 3-4 days.
- **Life cycle of Plasmodium:-** The life cycle of *Plasmodium* has three phases - *Schizogony*, *gamogony* and *sporogony*. Female *Anopheles* mosquito is the primary host while man is the secondary host.

**(b) Amoebiasis (Amoebic dysentery) or Enteritis.**

- **Pathogen:** *Entamoeba histolytica* found in the large intestine of humans.
- **Mode of transmission:** Houseflies (mechanical carriers) transmit **parasites** from the faeces of an infected person to food and water and thereby contaminating them.
- **Symptoms:** Constipation, abdominal pain and cramps, stools with excess mucous and blood clots.
- **Parasite:** An organism residing on organism of another species (it's host) and draws benefit by deriving nutrients from its host.

**4. HELMINTH DISEASES****(a) Ascariasis**

- **Pathogen:** *Ascaris lumbricoides* (Intestinal parasite).
- **Mode of transmission:** Soil, water, vegetables, fruits, etc., contaminated with faeces containing eggs of parasites.
- **Symptoms:** Internal bleeding, muscular pain, fever, anaemia and blockage of intestinal passage.

**(b) Filariasis (Elephantiasis)**

- **Pathogen:** Filarial worms or *Wuchereria* (*W. bancrofti* & *W. malayi*).
- **Mode of transmission:** Bite of female *Culex* mosquito.
- **Symptoms:** Filarial worms live in lymphatic vessels (usually of lower limbs). It causes chronic **inflammation** of the organs, in which they live for many years. Limbs and genital organs may be deformed.

**5. FUNGAL DISEASES****(a) Ring worms**

- **Pathogens:** *Microsporum*, *Trichophyton* & *Epidermophyton*. They are seen in groin between the toes.
- **Mode of transmission:** From soil or by using towels, cloths, comb, etc. Heat and moisture help fungi to grow.
- **Symptoms:** Appearance of dry, scaly lesions on various body parts such as skin, nails and scalp. Intense itching.

- **Inflammation:** A process by which our body's WBC protect as from infection from outside invaders, such as bacteria and viruses. It can be either short-lived (acute) or long-lasting (chronic).

#### Antibiotics

- **Immunity:** It is the ability of the immune system of the body to fight against the disease-causing organisms. It is of two types namely innate immunity and acquired immunity.

#### (b) Innate Immunity

- It is the non-specific defence present at the time of birth.
- It provides barriers to the entry of foreign agents into our body.
- It consists of four types of barriers:
  - (i) Physical Barriers:** Skin on our body is the first and main barrier that prevents entry of the micro-organisms. It is the first line of defence.
  - (ii) Physiological Barriers:** Acid in the stomach, saliva in the mouth, tears from eyes—all prevent microbial growth.
  - (iii) Cellular Barriers:** Certain types of leukocytes (WBC) of our body like polymorpho-nuclear leukocytes (PMNL neutrophils) and monocytes and natural killer (type of lymphocytes) in the blood as well as macrophages in tissues can phagocytose and destroy microbes.
  - (iv) Cytokine Barriers:** Virus-infected cells secrete proteins called interferon which protect non-infected cells from further viral infection.

#### (c) Acquired Immunity

##### Vitamin

- It is a pathogen-specific immunity.
- It is not present since birth but develops during the lifetime of an individual.
- It is characterized by memory i.e., during the first encounter of a pathogen; our body produces a primary response in low intensity. The second encounter with the same pathogen produces a secondary (anamnestic) response in high intensity.
- The primary and secondary immune responses are carried out with B-lymphocytes and T-lymphocytes.
- (a) B-lymphocytes (B-cells):** Produce antibodies.
- (b) T-lymphocytes:** Help B-cells to produce antibodies.

#### Structure of an Antibody Molecule

- Each antibody has 4 polypeptide chains namely, 2 small light chains and 2 large heavy chains (H2L2).
- In our body, different types of antibodies such as IgG, IgA, IgM, IgE and IgD are produced.
- Acquired immune response is of two types namely humoral mediated response and cell-mediated response.
- (a) Humoral or Antibody-Mediated Response/ Antibody-Mediated Immunity (AMI)**
  - Antibodies are found in blood plasma. So, it is called as humoral immune response.
  - It includes B-lymphocytes and T-lymphocytes. The latter help the former to produce antibodies.
- (d) Cell-Mediated Response/Cell-Mediated Immunity (CMI)**
  - It is T-lymphocytes (T-cells) mediated (CMI).
  - CMI causes graft rejection.
  - The body can differentiate 'self' and 'non self'.
  - Tissue matching and blood group matching are essential before undertaking any graft / transplant. After this, the patient has to take immune-suppressants for all his life.

| Vitamins                | What it does                                                                                                                                                                                                           | Where is it found                                                                                                                                                                                                                                                                           | Daily Value |
|-------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|
| Vitamin A               | <ul style="list-style-type: none"> <li>• Growth and development</li> <li>• Immune function</li> <li>• Reproduction</li> <li>• Red blood cell formation</li> <li>• Skin and bone formation</li> <li>• Vision</li> </ul> | <ul style="list-style-type: none"> <li>• Cantaloupe</li> <li>• Carrots</li> <li>• Dairy products</li> <li>• Eggs</li> <li>• Fortified cereals</li> <li>• Green leafy vegetables (e.g., spinach and broccoli)</li> <li>• Pumpkin</li> <li>• Red peppers</li> <li>• Sweet potatoes</li> </ul> | 5,000 IU    |
| Vitamin B <sub>6</sub>  | <ul style="list-style-type: none"> <li>• Immune function • Nervous system function</li> <li>• Protein, carbohydrate, and fat metabolism</li> <li>• Red blood cell formation</li> </ul>                                 | <ul style="list-style-type: none"> <li>• Chickpeas</li> <li>• Fruits (other than citrus)</li> <li>• Potatoes</li> <li>• Salmon</li> <li>• Tuna</li> </ul>                                                                                                                                   | 2 mg        |
| Vitamin B <sub>12</sub> | <ul style="list-style-type: none"> <li>• Conversion of food into energy</li> <li>• Nervous system function</li> <li>• Red blood cell formation</li> </ul>                                                              | <ul style="list-style-type: none"> <li>• Dairy products</li> <li>• Eggs</li> <li>• Fortified cereals</li> <li>• Meats</li> <li>• Poultry</li> <li>• Seafood (e.g., clams, trout, salmon, haddock, tuna)</li> </ul>                                                                          | 6 mcg       |

| Vitamins                                            | What it does                                                                                                                                                                                                                | Where is it found                                                                                                                                                                                                                                                                                                                    | Daily Value |
|-----------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|
| Vitamin C                                           | <ul style="list-style-type: none"> <li>• Antioxidant</li> <li>• Collagen and connective tissue formation</li> <li>• Immune function</li> <li>• Wound healing</li> </ul>                                                     | <ul style="list-style-type: none"> <li>• Broccoli</li> <li>• Brussels sprouts</li> <li>• Cantaloupe</li> <li>• Citrus fruits and juices (e.g., oranges and grapefruit)</li> <li>• Kiwifruit</li> <li>• Peppers</li> <li>• Strawberries</li> <li>• Tomatoes and tomato juice</li> </ul>                                               | 60 mg       |
| Vitamin C<br>Nutrient of concern for most Americans | <ul style="list-style-type: none"> <li>• Blood pressure regulation</li> <li>• Bone growth</li> <li>• Calcium balance</li> <li>• Hormone production</li> <li>• Immune function</li> <li>• Nervous system function</li> </ul> | <ul style="list-style-type: none"> <li>• Eggs</li> <li>• Fish (e.g., herring, mackerel, salmon, trout, and tuna)</li> <li>• Fish liver oil</li> <li>• Fortified cereals</li> <li>• Fortified dairy products</li> <li>• Fortified margarine</li> <li>• Fortified orange juice</li> <li>• Fortified soy beverages (soymilk)</li> </ul> | 400 IU      |
| Vitamin E                                           | <ul style="list-style-type: none"> <li>• Antioxidant</li> <li>• Formation of blood vessels</li> <li>• Immune function</li> </ul>                                                                                            | <ul style="list-style-type: none"> <li>• Fortified cereals and juices</li> <li>• Green vegetables (e.g., spinach and broccoli)</li> <li>• Nuts and seeds</li> <li>• Peanuts and peanut butter</li> <li>• Vegetable oils</li> </ul>                                                                                                   | 30 IU       |
| Vitamin K                                           | <ul style="list-style-type: none"> <li>• Blood clotting</li> <li>• Strong bones</li> </ul>                                                                                                                                  | <ul style="list-style-type: none"> <li>• Green vegetables (e.g., broccoli, kale, spinach, turnip greens, collards, Swiss chard, mustard greens)</li> </ul>                                                                                                                                                                           | 80 mcg      |

**Vitamins Deficiency**

Vitamin "A": Night blindness

- Vitamin "B<sub>1</sub>": Beriberi- Vitamin "B<sub>2</sub>": Riboflavinosis- Vitamin "B<sub>3</sub>": Pellagra- Vitamin "B<sub>5</sub>": Paresthesia- Vitamin "B<sub>6</sub>": Anemia- Vitamin "B<sub>7</sub>": Dermatitis and enteritis- Vitamin "B<sub>9</sub>" - "B<sub>12</sub>": Megaloblastic anemia- Vitamin "B<sub>17</sub>": Cancer

0- Vitamin "C": Scurvy and swelling of gums

1- Vitamin "D": Rickets and Osteomalacia

2- Vitamin "E": Less fertility

3- Vitamin "K": Non-Clotting of blood

**EXERCISE**

- Bleeding of gums, falling of teeth, fragile bones and delayed wound healing occur due to the deficiency of which one of the following vitamins?  
(a) Vitamin C (b) Vitamin K  
(c) Vitamin D (d) Vitamin B
- Excessive use of which of the following fertilizers may be responsible for the presence of a toxic substance in groundwater?  
(a) Nitrogen  
(b) Phosphate only  
(c) Potassium only  
(d) Phosphate and potassium
- Rutherford's alpha particle scattering experiment on thin gold foil was responsible for the discovery of:  
(a) electron (b) proton  
(c) atomic nucleus (d) neutron
- Mission Indradhanush aims at:  
(a) reducing child deaths due to pneumonia.  
(b) reducing the impact of rotavirus.  
(c) elimination of maternal tetanus.  
(d) full immunisation of children
- Consider the following pairs of vaccine and category / type:  
1. Covaxin : Inactivated pathogen-based vaccine  
2. Covishield : mRNA vaccine  
3. Sputnik V : Viral vector-based vacciner Which of the pairs given above is/are correctly matched?  
(a) 1 only (b) 2 and 3 only  
(c) 1 and 3 only (d) 1, 2 and 3
- Which one of the following Indian institutes was approved by the Drugs Controller General of India for conducting human trials of the Oxford AstraZeneca Covid-19 vaccine candidate?  
(a) Bharat Biotech  
(b) AIIMS  
(c) Serum Institute of India  
(d) National Institute of Epidemiology
- Match List-I with List-II and select the correct answer using the code given below the lists :

| List-I (Institute) |                                | List-II (Location) |         |
|--------------------|--------------------------------|--------------------|---------|
| A.                 | National Institute of Ayurveda | (1)                | Chennai |

|    |                                      |     |           |
|----|--------------------------------------|-----|-----------|
| B. | National Institute of Homoeopathy    | (2) | Bengaluru |
| C. | National Institute of Unani Medicine | (3) | Kolkata   |
| D. | National Institute of Siddha         | (4) | Jaipur    |

Code :

| A     | B | C | D |
|-------|---|---|---|
| (a) 1 | 2 | 3 | 4 |
| (b) 1 | 3 | 2 | 4 |
| (c) 4 | 3 | 2 | 1 |
| (d) 4 | 2 | 3 | 1 |

8. Which of the following variable(s) is/are displayed in population pyramids?
  1. Age-groups
  2. Sex indicators (male and female)
 Select the correct answer using the code given below:
 

|                  |                     |
|------------------|---------------------|
| (a) 1 only       | (b) 2 only          |
| (c) Both 1 and 2 | (d) Neither 1 nor 2 |
9. Which one of the following is the special type of milk produced by a lactating mother, essential for the development of immune response of newborn baby in human?
  - (a) Breast milk produced after a month of childbirth
  - (b) Transitional milk
  - (c) Colostrum
  - (d) Mineralized milk
10. The term soil impoverishment relates to which one of the following?
  - (a) Soil erosion
  - (b) Soil deposition
  - (c) Soil getting very deficient in plant nutrients
  - (d) Soil getting enriched with plant nutrients
11. Kwashiorkor, a form of malnutrition, is caused by the deficiency of:
 

|              |              |
|--------------|--------------|
| (a) Minerals | (b) Vitamins |
| (c) Fats     | (d) Proteins |
12. A child receives a tall beautiful plant as a birthday gift from his father with a quiz. The father asked her how she would verify whether this tall plant was the progeny of both the tall parents or one tall and one short parent plant. She could verify this through:
  - (a) cross-pollination
  - (b) self-pollination
  - (c) tissue culture
  - (d) negative propagation
13. Cobalt is associated with:
 

|                    |                             |
|--------------------|-----------------------------|
| (a) growth hormone | (b) vitamin B <sub>12</sub> |
| (c) hemoglobin     | (d) intestinal enzymes      |
14. Fertilizers are used to obtain higher yields of crops. However, all nutrients are usually not available in fertilizers. Which one of the following nutrients is usually **not** available in fertilizers?
 

|              |                |
|--------------|----------------|
| (a) Iron     | (b) Potassium  |
| (c) Nitrogen | (d) Phosphorus |
15. Which one of the following types of pesticides is convenient to control stored grain pests?
 

|                           |                     |
|---------------------------|---------------------|
| (a) Systematic pesticides | (b) Fumigants       |
| (c) Contact Poisons       | (d) Stomach Poisons |

16. Which one of the following cattle breeds produces highest amount of milk?
 

|                  |              |
|------------------|--------------|
| (a) Brown Swiss  | (b) Holstein |
| (c) Dutch belted | (d) Blaarkop |
17. The main constituent of Gobar gas is:
 

|             |               |
|-------------|---------------|
| (a) Ethane  | (b) Methane   |
| (c) Propane | (d) Acetylene |
18. Atmospheric conditions are well-governed by humidity. Which one among the following may best define humidity?
  - (a) Form of suspended water droplets caused by condensation.
  - (b) Deposition of atmospheric moisture.
  - (c) Almost microscopically small drops of water condensed from and suspended in air.
  - (d) The moisture content of the atmosphere at a particular time and place.
19. Which of the following structures of a plant is responsible for transpiration?
 

|             |          |
|-------------|----------|
| (a) Xylem   | (b) Root |
| (c) Stomata | (d) Bark |
20. The 'eye' of the cyclone has:
  - (a) abnormally high temperature and lowest pressure
  - (b) abnormally low temperature and pressure
  - (c) clear sky and lowest temperature
  - (d) dense cloud cover and low pressure
21. Which one of the following is caused by a bacterial pathogen?
 

|              |                   |
|--------------|-------------------|
| (a) AIDS     | (b) Dengue fever  |
| (c) COVID-19 | (d) Typhoid fever |
22. Which one of the following viruses used to be responsible for highly infectious disease Smallpox ?
 

|                 |                     |
|-----------------|---------------------|
| (a) Adenovirus  | (b) Variola virus   |
| (c) Aichi virus | (d) Coxsackie virus |
23. Which one of the following is a bacterium that causes disease in the human body?
  - (a) *Varicella zoster*
  - (b) *Trypanosoma gambiense*
  - (c) *Salmonella typhi*
  - (d) *Plasmodium falciparum*
24. Which one of the following diseases may be caused by the deficiency of vitamin C?
 

|               |            |
|---------------|------------|
| (a) Rickets   | (b) Rabies |
| (c) Hepatitis | (d) Scurvy |
25. Which of the following pairs of vector and disease is/are correctly matched?
 

| Vector                   | Disease             |
|--------------------------|---------------------|
| (1) Anopheles            | : Malaria           |
| (2) <i>Aedes aegypti</i> | : Chikungunya       |
| (3) Tsetse fly           | : Filariasis        |
| (4) Bed bugs             | : Sleeping sickness |

 Select the correct answer using the code given below:
 

|                     |                  |
|---------------------|------------------|
| (a) 1, 2 and 3 only | (b) 1 and 2 only |
| (c) 1 and 4 only    | (d) 2 only       |
26. Which one of the following is **not** a part of female reproductive system?
 

|                    |            |
|--------------------|------------|
| (a) Fallopian tube | (b) Cervix |
| (c) Urethra        | (d) Vagina |



27. Which one of the following statements about microbes is not correct?  
 (a) They are used in sewage treatment plants.  
 (b) They are used in industrial fermenters for the production of beverages.  
 (c) No antibiotic has been obtained from any microbe.  
 (d) They are used to get many bioactive molecules for the treatment of diseases.
28. The term 'Probiotic' is applied to:  
 (a) Organic food  
 (b) Antacid  
 (c) Antibiotic  
 (d) Live microbial food supplement
29. Which one of the following microbes causes acidification and curdling of milk?  
 (a) Lactic Acid *Bacillus*  
 (b) *Clostridium botulinum*  
 (c) *Vibrio cholerae*  
 (d) *Saccharomyces cerevisiae*
30. In which one of the following, methanogens, i.e., methane producing bacteria are **not** present?  
 (a) Rumen of cattle  
 (b) Aerobic environment  
 (c) Anaerobic sludge  
 (d) Excreta of cattle

### Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (a)  | 3. (c)  | 4. (d)  | 5. (c)  | 6. (c)  | 7. (c)  | 8. (c)  | 9. (c)  | 10. (c) |
| 11. (d) | 12. (b) | 13. (b) | 14. (a) | 15. (b) | 16. (b) | 17. (b) | 18. (d) | 19. (c) | 20. (a) |
| 21. (d) | 22. (b) | 23. (c) | 24. (d) | 25. (b) | 26. (c) | 27. (c) | 28. (d) | 29. (a) | 30. (b) |

### Answers with Explanations

**1. Option (b) is correct.**

**Explanation:** Vitamin K is one of the main components of the proteins that are required for the formation of the blood clot and it is also required by the teeth and bones for the binding of proteins that are required for their formation. The deficiency of vitamin K can cause bleeding gum because of the improper or insufficient blood clotting. It can also cause weakness in bone or fragile bone and falling of teeth because their deficiency leads to improper holding of calcium in the cells and tissues.

**2. Option (a) is correct.**

**Explanation:** Excessive use of fertilizers is one of the main causes of soil pollution and ground water pollution. Excessive use of nitrogen can lead to increase the presence of nitrate in soil as fewer proportion of nitrogenous fertilizers are utilized by the plant. Excess nitrate results in its runoff with surface water and its leaching into the groundwater sources. This increases the proportion of anthropogenic nitrogen and causes groundwater pollution.

**3. Option (c) is correct.**

**Explanation:** Due to Rutherford's alpha scattering experiment on thin gold foil, the discovery of atomic nucleus takes place. He concluded that proton and neutron remain concentrated in the center of an atom called nucleus.

**4. Option (d) is correct.**

**Explanation:**

**Mission Indradhanush**

- It was launched on 25 December 2014
- The final goal of Mission Indradhanush is to ensure that children up to two years of age and pregnant women get fully immunized with all available vaccines.

**Vaccination is provided against eight vaccine-preventable diseases nationally. These are –**

1. Diphtheria
2. Whooping Cough
3. Tetanus
4. Polio

5. Measles

6. Severe form of Childhood Tuberculosis

7. Hepatitis B and meningitis

8. Pneumonia caused by *Haemophilus influenza* type B  
 Also, in some selected states and districts vaccines against Rotavirus Diarrhea and Japanese Encephalitis are also provided.

**5. Option (c) is correct.**

**Explanation:**

**Covaxin:**

- It is an inactivated pathogen-based vaccine.
- It is developed with Whole-Virion Inactivated Vero Cell-derived technology.

**Covishield:**

- It is a recombinant, replication-deficient chimpanzee adenovirus vector vaccine.
- It has been modified to enable it to carry the COVID-19 spike protein into the cells of humans.

**Sputnik V:**

- It is a viral vector vaccine designed to produce lasting immunity against COVID-19.
- It is based on adenovirus DNA.
- Adenovirus is used as a carrier to deliver the coronavirus gene to cells and synthesize the SARS-CoV-2 virus's envelope proteins, introducing the immune system to a potential enemy.

**6. Option (c) is correct.**

**Explanation:** In 2020, Drugs Controller General of India (DCGI) gave permission to Serum Institute of India to resume clinical trial of the Oxford– Astra-Zeneca COVID-19 vaccine candidate in the country.

**7. Option (c) is correct.**

**Explanation:**

The National Institute of Ayurveda (NIA) is the apex institute for training and research in Ayurveda in India, established in 1976 at Jaipur, by the Ministry of Health and Family Welfare, Government of India.

- National Institute of Homeopathy (NIH) is an autonomous organisation under the Ministry of AYUSH.

- It was established on 10 December 1975 under the Ministry of Health and Family Welfare.
  - It was located at Amherst Street, Kolkata and later shifted to the present location which is at Salt Lake, Kolkata.
  - National Institute of Unani Medicine (NIUM) is an autonomous organization for research and training in Unani medicine in India. It was established in 1984 at Bengaluru under the Department of Ayurveda, Yoga and Naturopathy, Unani, Siddha and Homoeopathy (AYUSH), Ministry of Health and Family Welfare, Govt. of India in a joint venture with Government of Karnataka.
  - National Institute of Siddha is an institute for study and research of Siddha medicine. It was established in 2005 at Tambaram, Chennai, India. It is one of the eight national institutes established across nation, for training and research in "Indian Systems of Medicine and Homeopathy education", by Department of AYUSH, Government of India.
8. **Option (c) is correct.**  
*Explanation:* Both statements are correct.
- A population pyramid is a scale consist of two variables first is age and second is sex.
  - They are used by demographers, who study population.
  - A population pyramid is a graph that shows the distribution of ages across a population divided down the center between male and female members of the population.
9. **Option (c) is correct.**  
*Explanation:* Colostrum is the first formed milk of a lactating mother which is produced by her mammary glands after the delivery of a child. The Colostrum is highly rich in nutrients, antibodies such as IgA which are essential in building up the immune system of new born child. Colostrum is thicker and yellow in colour than normal breast milk.
10. **Option (c) is correct.**  
*Explanation:* Soil impoverishment relates to the removal of nutrients from the soil which is of great importance to the plants. It is a strategy frequently employed in the forestry sector to encourage the growth of one species over others.
11. **Option (d) is correct.**  
*Explanation:* Kwashiorkor is an acute form of malnutrition caused by a lack of protein. This is a serious illness that can occur when a person does not consume enough protein. A severe protein deficiency can lead to fluid retention, which causes stomach bloating. It is more common in children, especially if they do not have access to adequate nutrition immediately after stopping breastfeeding.
12. **Option (b) is correct.**  
*Explanation:* When the pollen from the anther of the flower gets transferred to the stigma of same flower (autogamy), or from different flower on the same plant (geitonogamy), the process is referred to as self-pollination. If all the progenies that are formed are tall, it means the tall progeny has been formed by both tall parents but if some plants are tall while others are small, it would establish the fact that one of the parent plants might have been small. Cross pollination occurs when the pollen from a plant gets transferred to the stigma of the other plant by means of insects or wind. It would not be possible to establish the progeny's height by means of cross pollination.
13. **Option (b) is correct.**  
*Explanation:* Vitamin B<sub>12</sub> [Cyanocobalamin] is a complex compound of Cobalt but required in very fewer amounts.
14. **Option (a) is correct.**  
*Explanation:* Iron is a micronutrient that is required in minuscule amount by the plants. Hence, it is not available in the fertilizers. Nitrogen, phosphorus, and potassium are the macronutrients that are required by the plant for their optimum production and are generally supplied as nutrients in the fertilizers.
15. **Option (b) is correct.**  
*Explanation:* Fumigants are also referred to as the gaseous pesticides. They usually are in the solid form such as a tablet and they releases a fume over a period of time that kills the pests or keep them away. They are best to protect the stored food grains from the pests as they are usually not absorbed by the grains. Some examples of fumigants are methyl bromide and hydrogen phosphide.
16. **Option (b) is correct.**  
*Explanation:* Holstein is one of the most popular dairy breeds of the cows and it is the highest milk producing cow. It can produce up to 40 liters of milk in one day. This breed was originated in Holland, the Netherlands in the 1850s.
17. **Option (b) is correct.**  
*Explanation:* Gobar gas is formed by the anaerobic decomposition of the dung from the dairy animals like Cow. Its main constituent is methane (around 75%). Apart from methane, it also contains carbon dioxide, nitrogen, oxygen, and hydrogen sulfide in small quantities.
18. **Option (d) is correct.**  
*Explanation:* Atmospheric humidity is the amount of moisture present in the air at a particular time and place. It is a major climatic factor.
- All forms of precipitation, like drizzle, rain etc, are produced by the condensation of atmospheric moisture that forms clouds. After attaining sufficient size of drops due to condensation, it falls on the surface of the earth.
19. **Option (c) is correct.**  
*Explanation:* Stomata are pore like structures that are present on the surface of leaves and green stems. The pore of stomata is surrounded by two guard cells that regulate its opening and closing. Plant can lose water in the form of vapours through stomata by the process termed as transpiration.
20. **Option (a) is correct.**  
*Explanation:* The 'eye' of the cyclone due to high temperature and the lowest pressure. In case of a cyclone, 'eye' is the place where all the winds coming



inside in a spiral motion converge.

- In strong tropical cyclones, the eye is characterized by light winds and clear skies, surrounded on all sides weaker tropical cyclones, the eye is less well defined and can be covered by the central dense overcast, an area of high, thick clouds.
- In all storms, the eye is the location of the storm's minimum barometric pressure - where the atmospheric pressure at the sea level is the lowest.

**21. Option (d) is correct.**

**Explanation:** Typhoid also called enteric fever is a bacterial infection caused by *Salmonella typhi*. It can lead to high fever which is accompanied by diarrhea and vomiting. It is generally disseminated through the consumption of contaminated food and water.

**22. Option (b) is correct.**

**Explanation:** Variola virus causes small pox. Small pox is a contagious and deadly disease. It shows symptoms like fever, and appearance of flat, red spots on whole upper body which turns into small blisters filled with clear fluid and after sometime it gets filled with pus. There is currently no cure, but only a vaccine could have prevented it.

**23. Option (c) is correct.**

**Explanation:**

- *Salmonella typhi* is a bacteria that causes typhoid infection in humans. The infection is transmitted by contaminated food and drinking water
- *Varicella-zoster virus* (VZV) causes chickenpox disease which is most commonly affecting children, teens, and young adults.
- *Trypanosoma gambiense* which is also called Trypanosoma rhodesiense is a sleeping sickness is transmitted to human body which is caused by the bite of tsetse flies.
- *Plasmodium falciparum* is a unicellular protozoan parasite of humans and the deadliest species of *Plasmodium* that causes malaria in humans. The parasite is transmitted through the bite of a female anopheles mosquito which causes the most dangerous disease falciparum malaria. Most common symptoms of bacterial disease include:
  - (i) Bloody urine and painful, frequent urination.
  - (ii) Diarrhoea
  - (iii) Fatigue, fever, sore throat, headache, cough, aches and pains)
  - (iv) Irritability
  - (v) Nausea and vomiting
  - (vi) Pain such as joint, ear or abdominal pain
  - (vii) Rashes, lesions and abscesses
  - (viii) Stiff neck
  - (ix) Weakness

**24. Option (d) is correct.**

**Explanation:**

- Rickets is caused due to the deficiency of vitamin D.
- Rabies is caused by infection of Rabies virus

through the bite of dog, cat as this virus is present in their saliva.

- Hepatitis is caused by Hepatitis virus.
- Scurvy is caused due to the deficiency of vitamin C leading to gum disease & anemia.

**25. Option (b) is correct.**

**Explanation:** Anopheles is a mosquito that spreads of malaria through its bite and it carries protozoa known as *Plasmodium vivax*. It's the female anopheles mosquito. Chikungunya is a viral disease transmitted by the bite of *Aedes aegypti* mosquito. Filariasis is not transmitted by the tsetse fly, it is spread by culex mosquitoes. Filariasis is caused by roundworm and it is also known as Elephantiasis. Sleeping sickness is caused by tsetse fly, not by bed bugs.

**26. Option (c) is correct.**

**Explanation:** Female reproductive system consists of a pair of ovaries and fallopian tubes. And only one uterus, cervix, and vagina are present in the female reproductive system. The urethra is part of the excretory system. The urethra is a tube-like structure by which urine can pass from the urinary bladder to the outside of the body.

**27. Option (c) is correct**

**Explanation:** Microbes are used in sewage treatment to digest sewage aerobically and anaerobically also microbe are used in industries to produce beverages like beer, wine etc. through alcoholic fermentation. Microbes like *penicillium* from and antibiotic streptomycin is produced from streptomycin which is also a microbe. Bioactive molecules are also obtained from microbes to cure disease like to cure cholesterol statins are obtained from *Monascus purpureus*.

**28. Option (d) is correct**

**Explanation:** Probiotics are live microorganism food supplements. They are claimed to provide health benefits upon consumption by restoring the gut flora. Probiotics contains live bacteria and yeast which are good for gut health.

**29. Option (a) is correct**

**Explanation:** Lactic acid *Bacillus* is the bacteria which causes curdling of milk. Curdled milk have many culinary uses. Lactic acid bacteria produces lactic acid leading to decrease in pH of milk there by the casein protein gets coagulated and milk get curdled. The bacteria feeds upon the lactose sugar present in milk and produces lactic acid.

**30. Option (b) is correct.**

**Explanation:** Methanogens are the group of archaeobacteria that are involved in the production of methane. They are present in the rumen of cattle where they help in digestion of cellulose. They are also found in the excreta of the wild animals. Methanogens are anaerobic in nature so also found in anaerobic sludge but not found in aerobic environments.

# Chapter 64

## AWARDS AND AUTHORS

### Grammy Awards

| Awarded for             | United States                                  |
|-------------------------|------------------------------------------------|
| Awarded for             | Outstanding achievements in the music Industry |
| Presented by            | National Academy of Recording Arts & Science   |
| Country                 | United States                                  |
| 1 <sup>st</sup> Awarded | 1959                                           |

### Indian Winning Grammy Awards

|                     |                  |
|---------------------|------------------|
| Pandit Ravi Shankar | 1968, 1973, 2002 |
| Zakir Hussain       | 1992 and 2009    |
| Vikku Vinayak       | 1991             |
| Vishwa Mohan Bhatt  | 1993             |
| AR Rahman           | 2009             |
| Ricky Kej           | 2015             |
| Sandeep Das         | 2017             |

### Pulitzer Prize

|               |                                                                                    |
|---------------|------------------------------------------------------------------------------------|
| Awarded for   | Excellence in newspaper journalism, literary achievements, and musical composition |
| Presented by  | Columbia University                                                                |
| Country       | United States                                                                      |
| First awarded | 1917                                                                               |

### Indian Winning Pulitzer Prize

|                                                                                                       |      |
|-------------------------------------------------------------------------------------------------------|------|
| Gobind Behari Lal (for Reporting)                                                                     | 1937 |
| Jhumpa Lahiri                                                                                         | 2000 |
| Geeta Anand (for Journalism)                                                                          | 2003 |
| Dr. Siddhartha Mukherjee (for his book on cancer "The Emperor of All Maladies A Biography of Cancer") | 2011 |
| Vijay Seshadri Poet Category                                                                          | 2014 |
| Yasin Dar, Mukhtar Khan and Channi Anand (feature photography)                                        | 2020 |

### Man Booker Prize

|                       |                                                |
|-----------------------|------------------------------------------------|
| Awarded for           | Best full-length English novel                 |
| Presented by          | Man Group                                      |
| Country               | Commonwealth of Nations, Ireland, and Zimbabwe |
| 1 <sup>st</sup> Award | 1969-Indian Winner                             |

|                                                 |      |
|-------------------------------------------------|------|
| Arundhati Roy (Novel) - The God of small things | 1997 |
| Kiran Desai (Novel)                             | 2006 |
| Arvind Adiga (Novel) - The white tiger          | 2008 |

### Oscar Awards or Academy Awards

|               |                                             |
|---------------|---------------------------------------------|
| Awarded for   | Excellence in cinematic achievements        |
| Presented by  | Academy of Motion Picture Arts and Sciences |
| Country       | United States                               |
| First awarded | 16 <sup>th</sup> May 1929                   |

- A.R. Rahman and Gulzar also won the Oscar award in 2009.

### Golden Globe Awards

|               |                                     |
|---------------|-------------------------------------|
| Awarded for   | Excellence in film and television   |
| Presented by  | Hollywood Foreign Press Association |
| Country       | United States                       |
| First awarded | 1944                                |

- AR Rahman is the first Indian to win Golden Globe Award.
- 77<sup>th</sup> Golden Globe Best Film (2019) is Awarded to '1917' (Drama).

### Man Booker International

|               |                                                                  |
|---------------|------------------------------------------------------------------|
| Awarded for   | Best English (or available for translation into English) fiction |
| Presented by  | Man Group                                                        |
| Country       | United Kingdom                                                   |
| First awarded | 2005                                                             |

### Beauty Contest Miss Universe

- It is an annual international beauty contest that is run by the Miss Universe Organisation. The contest was founded in 1952, by California Clothing Company Pacific Mills. Its headquarters are in New York City (US).
- **Sushmita Sen** was the first Indian woman to win the Miss Universe contest in 1994.

### Miss World

- The Miss World pageant is the oldest surviving international beauty pageant. It was created in the United Kingdom by Eric Morley's 1951. Since he died in 2000, Morley's wife, Julia Morley, co-chairs the pageant.
- Its headquarter is in London (UK).
- **Reita Faria Powell** (born in Bombay) became the first Indian to win the Miss World title in 1966.

- India's Manushi Chhillar won the Miss World 2017 contest. She Crowned Vanessa Ponce of Mexico as Miss World 2018.

#### Miss India

- Miss India or Femina. Miss India is a national beauty pageant in India. It is organized by Femina, a women's magazine published by Bennett, Coleman and Co Ltd.
- Its headquarters is in Mumbai and it was formed in 1963.
- Miss India beauty contest started in 1947.
- Pramila was the first woman to win the Miss India contest in 1947.
- Suman Rao won the title of Femina Miss India 2019.
- The 2020 Miss India event was postponed due to COVID-19 Pandemic.

#### Ramon Magsaysay Award

Awarded for Outstanding contributions in six categories (Government Service, Public Service and other Fields)

Presented by Raman Magsaysay Award Foundation

Presentation Asian Individuals

Country India

First awarded 1998

#### Prominent Indian Magsaysay Award Winners

- Vinoba Bhave (1958)
- CD Deshmukh (1959)
- Verghese Kurien (1963)
- Jayaprakash Narayan (1965)
- Satyajit ray (1967)
- Mother Teresa (1962)
- Ms. Swaminathan (1971)
- Ela Bhatt (1977)
- Pramod Karan Sethi (1981)
- Kiran Bedi (1994)
- TN Seshan (1996)
- RK Laxman (1984)
- Ravi Shankar (1992)
- Arvind Kejriwal (2006)
- P Sainath (2007)
- Prakash and Mandakins Amte (2008)
- Neelima Mishra (2011)
- Kulandei Francis (2012)
- TM Krishna, Bezwada Wilson (2016)
- Bharat Vatwani (2018)
- Sonam Wangchuk (2018)
- Ravish Kumar (2019)

#### NATIONAL AWARDS

##### Bharat Ratna

- Bharat Ratna is India's highest Civilian Award. It was first awarded in 1954.
- The actual award is designed in the shape of a Peepal leaf with *Bharat Ratna* inscribed in the Devanagari script in the Sun Figure.
- The reverse side of the decoration *Satyamev Jayate* has been written in Hindi with an inscription of state emblem.
- The emblem, the Sun, and the rim are of platinum. The inscriptions are in burnished bronze.

- Indira Gandhi was the youngest person alive at the time of receiving the award and Rajiv Gandhi is the Youngest overall to receive the Bharat Ratna.
- DK Karve was the eldest person alive at the time of receiving the award and Vallabhbhai Patel is the eldest overall.
- First Naturalised Indian Citizen to be awarded Bharat Ratna is Mother Teresa (1980).
- Abdul Ghaffar Khan was the first foreigner to be Honoured with this award in 1987 and Later Nelson Mandela (1990).
- Mother Teresa (1962)
- Ms. Swaminathan (1971)
- Ela Bhatt (1977)
- Pramod Karan Sethi (1981)
- Kiran Bedi (1994)
- TN Seshan (1996)
- RK Laxman (1984)
- Ravi Shankar (1992)
- Arvind Kejriwal (2006)
- P Sainath (2007)
- Prakash and Mandakini Amte (2008)
- Neelima Mishra (2011)
- Kulandei Francis (2012)
- TM Krishna, Bezwada Wilson (2016)
- Bharat Vatwani (2018)
- Sonam Wangchuk (2018)
- Ravish Kumar (2019)

#### Winners of Bharat Ratna

| Year | Winners              |
|------|----------------------|
| 2019 | Pranab Mukherjee     |
| 2019 | Bhupen Hazarika      |
| 2019 | Nanaji Deshmukh      |
| 2015 | Madan Mohan Malaviya |
| 2015 | Atal Bihari Vajpayee |
| 2015 | CNR Rao              |
| 2014 | Sachin Tendulkar     |
| 2008 | Pandit Bhimsen Joshi |

#### NATIONAL AWARDS

##### Bharat Ratna

- Bharat Ratna is India's highest Civilian Award. It was first awarded in 1954.
- The actual award is designed in the shape of a Peepal leaf with *Bharat Ratna* inscribed in the Devanagari script in the Sun Figure.
- The reverse side of the decoration *Satyamev Jayate* has been written in Hindi with an inscription of state emblem.
- The emblem, the Sun, and the rim are of platinum. The inscriptions are in burnished bronze.
- Indira Gandhi was the youngest person alive at the time of receiving the award and Rajiv Gandhi is the Youngest overall to receive the Bharat Ratna.
- DK Karve was the eldest person alive at the time of receiving the award and Vallabhbhai Patel is the eldest overall

- First Naturalised Indian Citizen to be awarded Bharat Ratna is Mother Teresa (1980).
- Abdul Ghaffar Khan was the first foreigner to be Honoured with this award in 1987 and Later Nelson Mandela (1990).

|      |                                               |
|------|-----------------------------------------------|
| 2001 | Ustad Birmdish Khan                           |
| 2001 | Lata Dinmath Mangeshkar                       |
| 1999 | Lokpriya Gopinath Bordoloi                    |
| 1999 | Professor Amartya Sen                         |
| 1999 | Loknayak Jayprakash Narayan                   |
| 1999 | Pandit Ravi Shankar                           |
| 1998 | Smt Madurai Shanmukhavadiyu Subbulakshmi      |
| 1998 | Shri Chidambaram Subramaniam                  |
| 1997 | Shin (Dr) Avul Pakir Jainulabdeen Abdul Kalam |
| 1997 | Aruna Asal Al                                 |
| 1992 | J.R.D. Tata                                   |
| 1992 | Abul Kalam Azad                               |
| 1992 | Satyajit Ray                                  |
| 1991 | Morarji Desai                                 |
| 1991 | Rajiv Gandhi                                  |
| 1991 | Vallabhbbhai Patel                            |
| 1991 | Shri Morari Ranchhodji Desai                  |
| 1990 | Dr. Nelson Rolihlahin Mandela                 |
| 1990 | Dr. Bhimrao Ramji Ambedisar                   |
| 1988 | Shri Marudur Gopalan Ramchandran              |
| 1987 | Khan Abdul Ghaffar Khan                       |
| 1983 | Shri Acharya Vinoba Bhave                     |
| 1980 | Mother Teresa (Agnes Conxha Bojaxhiu)         |
| 1976 | Shri Kumaraswamy Kamraj                       |
| 1975 | Shri Varahagiri Venkata Giri                  |
| 1971 | Smt Indira Gandhi                             |
| 1966 | Shri Lal Bahadur Shastri                      |
| 1963 | Dr. Zakir Hussain                             |
| 1963 | Dr Pandurang Vaman Kane                       |
| 1962 | Dr. Rajendra Prasad                           |
| 1961 | Dr Bidhan Chandra Roy                         |
| 1961 | Shri Purushottam Das Tandon                   |
| 1958 | Dr Donde Keshav Karve                         |
| 1957 | Pt Govind Ballabn Pant                        |
| 1955 | Pandit Jawaharlal Nehru                       |
| 1955 | Dr. Mokshagundam Vivesvaraya                  |
| 1955 | Dr. Bhagwan Das                               |

|      |                          |
|------|--------------------------|
| 1954 | Sarvepalli Radhakrishnan |
| 1954 | C. Rajagopalachari       |
| 1994 | C. V. Raman              |

#### Padma Awards

- There are three Padma awards given on republic day (26<sup>th</sup> January)

#### Padma Vibhushan

- This is the second-highest National Award. Instituted on 2<sup>nd</sup> January 1954 by the Government of India.
- It is awarded to recognize exceptional and distinguished service to the nation in any field.

#### Padma Bhushan

- This is the third-highest National Award. Instituted on 2<sup>nd</sup> January 1954 by the Government of India.
- It is awarded to recognize distinguished service of a high order to the nation, in any field.

#### Padma Shri

- This is the fourth highest National Award instituted on 2<sup>nd</sup> January 1954 by the Government of India.
- In 1960 Dr. MG Ramachandran refused to accept the award as the wordings of the award are in Hindi.
- It recognizes the contribution of Indian citizens (generally) in various spheres of activity including Arts, Education, Industry, Literature, Science, Sports, Social Service, and Public life.

#### Mahatma Gandhi Peace Prize

- It was instituted in 1995 and awarded by the Government of India to encourage and promote Gandhian values worldwide.
- The award carries ₹ 1 crore in cash, convertible in any currency in the world, a plaque, and citation.
- First recipient (1995) Julius Nyerere
- Last recipient (2020) Sheikh Mujibur Rahman

#### Indira Gandhi Prize

- It was instituted in 1986 and awarded by Indira Gandhi Memorial Trust.
- It is awarded for peace, disarmament, and development.
- The prize carries a cash award of 25 lakh and a citation.
- First recipient (1986) Parliamentarians for Global Action (2017)
- Last recipient David Attenborough (2019)

#### Jawaharlal Nehru Award

- It was instituted in 1965 and awarded by the Government of India for international understanding, goodwill and friendship.
- The money constituent of this award is ₹ 25 Lakh.
- First recipient (1965)

#### Sangeet Natak Akademi Puraskar (Akademi Award)

- Awarded by the Sangeet Natak Akademi India's National Academy of Music, Dance, and Drama
- It is the highest Indian recognition given to prac-

ting artists in the categories of music, dance, theatre, other traditional/folk/tribal dance/music

**Books and Authors**

theatre and Puppetry and contribution/scholarship in performing arts.

| Author               | Book                                                                                                                                                  |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| V V Giri             | • Voice of Conscience                                                                                                                                 |
| Vinita Kamte         | • To the Last Bullet                                                                                                                                  |
| Vinod Mehta          | • Lucknow Boy (Autobiography)                                                                                                                         |
| Vinod Lokpally       | • Driven                                                                                                                                              |
| Yuvraj Singh         | • Test of My Life Autobiography                                                                                                                       |
| YV Reddy             | • Global Crisis Recession, and Uneven Recovery                                                                                                        |
| Navin Chawla         | • Every Vote Counts                                                                                                                                   |
| NR Narayan Murthy    | • A Better India A Better World                                                                                                                       |
| PM Nayar             | • The Kalam Effect: My Years with the President                                                                                                       |
| PVR Rao              | • Defence without Drift                                                                                                                               |
| Rabindra Nath Tagore | • Chitra<br>• The Post office<br>• Sadhana: The Realisation of Life                                                                                   |
| Ramachandra Guha     | • "Make of Modern India"<br>• "India after Gandhi: The History of the World's Largest Democracy"<br>• Patriots and Partisans<br>• Gandhi before India |
| Lal Krishna Advani   | • My Country My Life                                                                                                                                  |
| Mahadevi Verma       | • Yama                                                                                                                                                |
| Mahatma Gandhi       | • My Experiments with Truth<br>• Hind Swaraj                                                                                                          |
| Gopinath Mohanry     | • Moti Mahal                                                                                                                                          |
| Govind Mishra        | • Dhool Paudho Par                                                                                                                                    |
| Hamid Ansari         | • Travelling Through Conflict                                                                                                                         |
| IK Gujral            | • Matters of Discretion (Autobiography)                                                                                                               |
| Indira Gandhi        | • My Truth                                                                                                                                            |
| Imran Hashmi         | • The Kiss of Life                                                                                                                                    |
| Janardan Thakur      | • All the Prime Minister's Men                                                                                                                        |
| Jaswant Singh        | • Jinnah—India, Partition, Independence                                                                                                               |
| Jawaharlal Nehru     | • Bunch of Old Letters<br>• Glimpses of World History<br>• Letters from a Father to his Daughter<br>• The Discovery of India                          |
| JP Narayan           | • Prison Diary<br>• To all Fighters of Freedom. Why Socialism?                                                                                        |

## EXERCISE

- Which one of the following films was adjudged as the Best Feature Film in the 66<sup>th</sup> National Film Awards?  
(a) Bulbul Can Sing (b) Hellaro  
(c) Badhaai Ho (d) Uri
- Consider the following statements:  
(1) Mobile Seva, the National Mobile Governance Initiative of Department of Electronics and Information Technology of India, has won the

United Nations Public Service Award, 2014.

- The innovative initiative is aimed at mainstreaming mobile governance in the country. Which of the statements given above is/are correct?  
(a) 1 only (b) 2 only  
(c) Both 1 and 2 (d) Neither 1 nor 2
- Identify the correct reason, out of the following, about Claire Polosak for being in the news:  
(a) She has been honoured with the Pulitzer Prize.



- (b) She recently won a Grand Slam championship.  
 (c) She became the first female match official to conduct a men's cricket test match.  
 (d) She has been conferred with the Gandhi Peace Prize.
4. Who among the following is the winner of the French Open 2021 Men's Singles Final?  
 (a) Novak Djokovic (b) Rafael Nadal  
 (c) Stefanos Tsitsipas (d) Daniil Medvedev
5. Who among the following shared the Nobel Prize in 1962 along with Francis Crick and James Watson for their discoveries concerning the molecular structure of nucleic acids?  
 (a) Erwin Chargaff  
 (b) Maurice Hugh Frederick Wilkins  
 (c) Rosalind Franklin  
 (d) Phoebus Levene
6. Dr. Bindeshwar Pathak, who was awarded Nikkei Asia Prize, 2018 for Culture and Community, is the founder of  
 (a) Bachpan Bachao Andolan  
 (b) PRS Legislative Research  
 (c) Sulabh Sanitation and Social Reform Movement  
 (d) Smile Foundation
7. Who was the first American to win Nobel Peace prize?  
 (a) Abraham Lincoln (b) George Bush Sr  
 (c) Theodore Roosevelt (d) Franklin Roosevelt
8. Highest Civilian Honour received by Azim Premji is:  
 (a) Padma Shri (b) Padma Bhushan  
 (c) Padma Vibhushan (d) Bharat Ratna
9. Name the first female Nobel Prize winner from India  
 (a) Indira Gandhi (b) Mother Teresa  
 (c) Kavitha Krishnan (d) Sarojini Naidu
10. Bharat Ratna is designed like the leaf of:  
 (a) Banyan tree (b) Peepal tree  
 (c) Coconut tree (d) Sandalwood tree
11. Who among the following is the author of the book, *Tomb of Sand*?  
 (a) Arundhati Roy (b) Geetanjali Shree  
 (c) Chetan Bhagat (d) Jhumpa Lahiri
12. Match List-I with List-II and select the correct answer using the code given below the Lists:

| List-I (Author) |                     | List-II (Book) |                                      |
|-----------------|---------------------|----------------|--------------------------------------|
| A.              | Bal Gangadhar Tilak | (1)            | The Arctic Home in the Vedas         |
| B.              | Dadabhai Naoroji    | (2)            | Hind Swaraj                          |
| C.              | Mahatma Gandhi      | (3)            | The Discovery of India               |
| D.              | Jawaharlal Nehru    | (4)            | Poverty and Un-British Rule in India |

Code:

|     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 3 | 4 | 2 | 1 |
| (b) | 3 | 2 | 4 | 1 |
| (c) | 1 | 4 | 2 | 3 |
| (d) | 1 | 2 | 4 | 3 |

13. The book, *Kalila wa Dimna* is an Arabic translation of the  
 (a) *Hitopadesha* (b) *Panchatantra*  
 (c) *Suryasiddhanta* (d) *Kathasaritsagar*
14. Which Chinese traveller in ancient India wrote the diary called 'Records of the Travels to Middle India'?  
 (a) Wang Xuance (b) Xuanzang  
 (c) Yijing (d) Li Yibiao
15. The 13<sup>th</sup> Century text *Lekhapaddhati* gives us information on which one of the following topics?  
 (a) Art of writing (b) Essay writing  
 (c) Legal document (d) Epigraphic style
16. Which one of the following was composed by Harishena?  
 (a) Nashik Inscription of Gautami Balashri  
 (b) Prayaga Prashasti of Samudragupta  
 (c) Deopara Prashasti of Vijayasena  
 (d) Hathigumpha Inscription of Kharavela
17. Who among the following was considered to be the preceptor of Mirabai?  
 (a) Dadu (b) Raidas  
 (c) Ramanand (d) Surdas
18. Which one of the following statements about the *Ain-i-Akbari* is not correct?  
 (a) It was written by Abu'l Fazl.  
 (b) It is a part of a larger work called *Akbar Nama*.  
 (c) It describes the Mughal Empire as having a diverse population and a composite culture.  
 (d) It was later revised by Sadullah Khan on the orders of Shah Jahan.
19. Which one of the following statements about *Tattva-bodhini Patrika* is correct?  
 (a) It promoted a systematic study of India's past in the Bengali language.  
 (b) It promoted a systematic study of India's past through Sanskrit sources.  
 (c) It promoted a systematic study of India's past through Persian sources.  
 (d) It promoted a systematic study of India's past through Western sources.
20. Which one of the following pairs is correctly matched?
- |                     |                   |
|---------------------|-------------------|
| <b>Bhakti Saint</b> | <b>Philosophy</b> |
| (a) Shankara :      | Avadhuta          |
| (b) Ramananda :     | Kevaladvaita      |
| (c) Ramanuja :      | Vishishtadvaita   |
| (d) Chaitanya :     | Advaita           |

## Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (c)  | 3. (c)  | 4. (a)  | 5. (b)  | 6. (c)  | 7. (c)  | 8. (c)  | 9. (b)  | 10. (b) |
| 11. (b) | 12. (c) | 13. (b) | 14. (a) | 15. (c) | 16. (b) | 17. (b) | 18. (d) | 19. (a) | 20. (c) |

## Answers with Explanations

**1. Option (b) is correct.**

**Explanation:** The 66<sup>th</sup> National Film Awards ceremony was the award ceremony which took place in 2019 to honour the best films of 2018, in the Indian cinema.

List of 66<sup>th</sup> National film awards 2019 Winners

- Best Film : Hellaro
- Best Hindi Film : Andhadhun
- Best Direction : Aditya Dhar (Uri)
- Best Actor : Ayushmann Khurrana, Vicky Kaushal
- Best Actress : Keerthy Suresh

**2. Option (c) is correct.**

**Explanation:** Mobile Seva is funded and implemented by the Department of Electronics and Information Technology (DeitY). It was among the second-place winners of the 2014 UN Public Service Awards. The vision of Mobile Seva is to mainstream mobile governance in India.

**3. Option (c) is correct.**

**Explanation:** Australia Claire Polosak becomes the first female umpire at a men's ODI for Border-Gavaskar Trophy.

**4. Option (a) is correct.**

**Explanation:** 2021 French Open

- Novak Djokovic defeated Stefanos Tsitsipas in the final to win the men's singles tennis title at the 2021 French Open. It was his second French Open title.

**5. Option (b) is correct.**

**Explanation:** Francis crick and James watson along with maurice Hugh Fredrick william shared and received noble prize for their discovery of molecular structure of DNA by giving the famous double helical model of DNA.

**6. Option (c) is correct.**

**Explanation:**

Bindeshwar Pathak

- He is the founder of Sulabh International.
- Sulabh International is an India-based social service organisation. It works to promote human rights, environmental sanitation, waste management through education.
- He is also the Brand Ambassador for Swachh Rail Mission of Indian Railways.
- He was conferred with Padma Bhushan in 1991.
- He was honoured with the Nikkei Asia prize for culture and community by Nikkei inc in Tokyo, Japan in June 2018. He was honoured for his notable work in tackling poor hygiene and discrimination.

**7. Option (c) is correct.**

**Explanation:** 'Theodore Roosevelt' was the foremost American to succeed Nobel Peace Prize in the year 1906. He was the 26th president. He won the prize for having negotiated peace in the Russo-Japanese war in 1904 -5.

**8. Option (c) is correct.**

**Explanation:** Azim Premji was awarded Padma Vibhushan, the second-highest civilian award of India (in 2011), for his excellent work in trade and com-

merce in 2005, the government also awarded him with the title of Padma Bhushan.

**9. Option (b) is correct.**

**Explanation:** The first female to win Nobel Prize from India was Mother Teresa. She won the prize in 1979. She was a great social worker, who gave her whole life to social work.

**10. Option (b) is correct.**

**Explanation:** Bharat Ratna Award is designed like a leaf of Peepal tree and carries with the words "Bharat Ratna" (inscribed in Devanagiri Script). The reverse flank of the trophy maintains the circumstances logo and slogan.

**11. Option (b) is correct.**

**Explanation:** Tomb of Sand (Reet Ka Maqbara) is a Hindi-language novel written by Geetanjali Shree. This novel is the story of a sad and depressed 80 year old woman. For the first 100 pages of this novel, the reader can only sees her back. This Novel was also translated into English by Daisy Rockwell.

**12. Option (c) is correct.**

**Explanation:**

|                        |                                            |
|------------------------|--------------------------------------------|
| Bal Gangadhar Tilak    | The article home in the vedas-1903         |
| Dada Bhai Naroji       | Poverty and Un-British Rule in India -1901 |
| Hind Swaraj            | Mahatma Gandhi-1909                        |
| The discovery of India | Jawahar lal Nehru-1946                     |

**13. Option (b) is correct.**

**Explanation:** The book, *Kalila wa Dimna* is the Urdu translation of the book Panchatantra. 'Abdullah Ibn al-Muqaffa' translates Panchatantra into Urdu.

**14. Option (a) is correct.**

**Explanation:** The book Zhong Tianzhu Guo Xingji (Travel Notes of Central India) was written by Wang Xuance. Wang Xuance was a guard officer and diplomat of Tang Dynasty. This book contains a wealth of geographical information.

**15. Option (c) is correct.**

**Explanation:** **Lekhapaddhati (Models of Written Documents)**

- Collection of Sanskrit documents written between the 8th and 15th centuries.
- Written during the Chaulukya rule in Gujarat
- It was a legal document containing rules for drafting various type of documents. These documents were related to land grants, treaties between the rulers and rule of administration.

**16. Option (b) is correct.**

**Explanation:** Harishena was the court poet of king Samudragupta in the Gupta Empire. He was considered a high – ranking Gupta poet. He composed Prashati in praise of Samudragupta. It is the famous Allahabad Prashasti which describes Samudragupta as a great warrior and a great king.



**17. Option (b) is correct.****Explanation:** Mirabai (1498-1546)

- The most famous woman of Bhakti cult was Mirabai
- She was born into a royal family of Pali, Rajasthan.
- She was an ardent devotee of Lord Krishna and considered her as to be her lover.
- She composed her bhajans in Braj bhasa and Rajasthani language.
- It is believed that Mirabai was a disciple of Saint Ravidas (Raidas), however, there is no corroborating historical evidence for this.

**18. Option (d) is correct.****Explanation:** Ain-i-Akbari (Administration of Akbar)

- This document is written by Akbar's court historian, Abu'l Fazl, in the Persian language.
- The document gives the detailed view of Mughal administration during Akbar's reign.
- It was the final part (Volume III) of the much larger document, the Akbarnama (Account of Akbar).

- The *Ain-i-Akbari* document is divided into five books. The fourth book of *Ain-i-Akbari* contains information on Hindu philosophy, science, social customs and literature.

**19. Option (a) is correct.****Explanation:** Tattvabodhini (truth-searching) Patrika was established by Debendranath Tagore on 16 August 1843. It got published in Bengali and was devoted to the systematic study of India's past with a rational outlook and to the propagation of Raja Ram Mohan's ideas.**20. Option (c) is correct.****Explanation:** Ramanuja (1060-1118 AD).

- He was a Tamil Brahmin.
- He was one of the most important exponents of Vaishnavism.
- He was the founder of 'Vishist Advaita' philosophy (qualified monoism).
- He believed that salvation can be attained through Karma, Gyan or Bhakti.
- comprises ₹ 1.5 lakh and a certificate.

○○○

- The individuals in charge of transmitting and receiving a code or signal are commonly referred to as the sender and receiver, respectively. Decoding refers to the action taken by the receiver to interpret the transmitted codes or signals. Coding-decoding questions typically involve encoding a word, usually a fundamental term, in a specific way and then requiring test takers to encode other words using the same rule. The goal of coding and decoding exams is to assess a candidate's understanding of the encoding rule applied to a specific word or message, as well as their ability to decipher the code and decode the message.

## Letter Coding -Decoding

- In this category, we deal with the questions where the letters of a word are swapped out particularly for other letters in accordance with a predetermined pattern or rule to create a code. You must recognise the coding pattern and, using that coding pattern, answer the subsequent question(s).

- **Example 1:** If in a certain language MYSTIFY is coded as NZTUJGZ, how is NEMESIS coded in that language?

- (a) MDLHRDR (b) OFNFTJT  
(c) ODNHTDR (d) PGOKUGU

- Sol. (b)** It is obvious that each letter in the term MYSTIFY is advanced one space to correspond to a letter in the code.

MYSTIFY  
NZTUJGZ

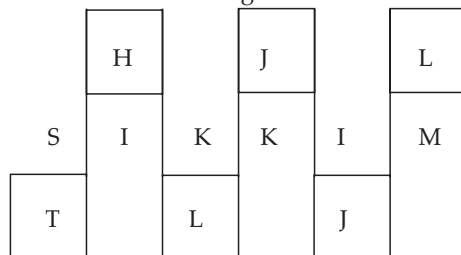
So, NEMESIS will be coded as OFNFTJT.

- **Example 2:** In a certain code, SIKKIM is written as THLJLL. How is TRAINING written in that code?

- (a) SQBHOHOH (b) UQBHOHOF  
(c) UQBJOHHO (d) UQBJOHOH

- Sol. (b)** Clearly, it is obvious that to obtain the letters of the code, the letters in the word SIKKIM are alternately moved one step forward and one step backward.

We can see in this figure



So, TRAINING will be coded as UQBHOHOF.

## NUMBER CODING-DECODING

- In these questions, either a word is given a numerical code value or the numbers are given alphabetical code letters. The candidate must analyse the code in accordance with the instructions.

- **Example 3:** If in a certain language A is coded as 1, B is coded as 2, and so on, how is BIDDIC coded in that code?

- (a) 294493 (b) 284563  
(c) 375582 (d) 394492

- Sol. (a)** As given, the letters are coded as

|   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | I |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

So, in BIDDIC, B is coded as 2, I as 9, D as 4, and C as 3. So, BIDDIC = 294493

- **Example 4:** If D = 4 and COVER = 63, then BASIS = ?

- (a) 49 (b) 50  
(c) 54 (d) 55

- Sol. (b)** Clearly, in the given code,

A = 1, B = 2, C = 3, D = 4 ....

So, that COVER = 3 + 15 + 22 + 5 + 18 = 63

Now, BASIS = 2 + 1 + 19 + 9 + 19 = 50

## MIXED CODING-DECODING

- In mixed coding-decoding problems, you are typically given a set of rules or patterns that govern the coding and decoding of information. These rules define how certain elements or attributes of the information are represented or transformed using symbols. Your task is to analyse the given information and apply the rules to decode or encode new pieces of information.

- **Example 5:** In a certain code, 2 is coded as P, 3 as N, 9 as Q, 5 as R, 4 as A and 6 as B. How is 599423 coded in that code?

- (a) EIIDBC (b) RQPANB  
(c) EIMDBC (d) RQQAPN

- Sol. (d)** Clearly, as given 5 is coded as R, 9 as Q, 4 as A, 2 as P and 3 as N. So, RQQAPN = 599423

- **Example 6:** In a certain code, 3456 is coded as ROPE, 15526 is coded as APPLE, then how is 54613 coded?

- (a) RPPEO (b) ROPEA  
(c) POEAR (d) PAREO

- Sol. (c)** We can see, the number are coded as

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| 3 | 4 | 5 | 6 | 1 | 2 |
| R | O | P | E | A | L |

So, 54613 = POEAR

## EXERCISE

- If in a certain code, BAT = 23 and CAT = 24, then how will you code BALL?  
(a) 27 (b) 28  
(c) 32 (d) 120
- In a certain code, ROAD is written as URDG. How is SWAN written in that code?  
(a) VXDQ (b) VZDQ  
(c) VZCP (d) UXDQ
- If A = 2, M = 26, Z = 52, then RATE = ?  
(a) 78 (b) 88  
(c) 64 (d) 50
- If FLOWER is coded as 14 and DISTASTE is coded as 18, then how will BUREAUCRAT be coded as?  
(a) 22 (b) 18  
(c) 20 (d) 28
- If DEER = 12215 and HIGH = 5645, how will you code HEEL?  
(a) 2328 (b) 3449  
(c) 4337 (d) 5229
- If A = 26, RUN = 28, Then RAT = ?  
(a) 24 (b) 40  
(c) 42 (d) 38
- In a certain code language, STRAIGHT is written as TSARGITH. How will THURSDAY be written as in that language?  
(a) UHTDRSYA (b) HTRUDSYA  
(c) HTRUDSAY (d) AYS DURTH
- If in a certain code, LUTE is as MUTE and FATE is written as GATE, then how will BLUE be written in that code?  
(a) CLUE (b) GLUE  
(c) FLUE (d) SLUE
- In a certain code, RUMOUR is written as QVLPTS. How is DANGER written in that code?  
(a) EBDHFS (b) CBMHDS  
(c) BEHOSF (d) BCHMSD
- If each of the letters in the English alphabet is assigned odd numerical value beginning A = 1, B = 3 and so on, what will be the total value of the letters of the word INDIAN?  
(a) 86 (b) 88  
(c) 89 (d) 96  
(e) None of these
- In a certain code language, SERVANT is coded as 195182211420. How will MAGNIFY be coded as in that language?  
(a) 1426693625 (b) 1316143522  
(c) 1317149625 (d) 1417139625
- In a certain code, PRODUCTIONS is written as QQPCVEUHPMT. How is ORIENTATION written in that code?  
(a) PQJDOVBSJNO (b) PQJDOUBUJPO  
(c) PSJFOVBSJNO (d) NSHFMVBSJNO  
(e) None of these
- If FISH is written as EHRG in a certain code, how would JUNGLE be written in that code?  
(a) ITMFKD (b) ITNFKD  
(c) KVOHMF (d) TIMFKD
- If AT = 20, CAT = 60, then MAT will be equal to?  
(a) 300 (b) 500  
(c) 260 (d) 270
- In a certain code language, LARVAE is coded as 3-1-9-4-1-5, how will INSECT be coded as in that language?  
(a) 9-4-8-4-6-7 (b) 3-4-8-2-6-8  
(c) 9-4-8-2-6-7 (d) 9-5-10-5-3-2
- In code language 43720 is coded as 52811. What will be the code for 75163 in that language?  
(a) 75340 (b) 85643  
(c) 84254 (d) 85374
- If SUMMER is coded as RUNNER, the code for WINTER will be?  
(a) SUITER (b) VIOUER  
(c) WALKER (d) SUFFER
- If, in a code, MODE becomes KMBC and RIGHT becomes PGEFR, then what will CREATE be in that code?  
(a) APCYRC (b) PAYARC  
(c) DSFBUF (d) BQDZSD
- If REQUEST is written as S2R52TU, then how will ACID be written?  
(a) 1394 (b) IC94  
(c) BDJE (d) B3J4  
(e) None of these
- In certain code, TWINKLE is written as SVHOJKD, then how would FILTERS be written in the same code?  
(a) EHKSDQR (b) EHKUDQR  
(c) EGKUDQR (d) GJMSFST  
(e) None of these
- In a certain code, the word DIAL is coded as 4 - 9 - 1 - 12. Following the same rule of coding, what should be the code for the word CODE?  
(a) 10 - 4 - 3 - 25 (b) 3 - 15 - 4 - 5  
(c) 9 - 1 - 5 - 23 (d) 3 - 14 - 4 - 5
- If GO = 32, SHE = 49, then SOME will be equal to?  
(a) 56 (b) 58  
(c) 62 (d) 64
- If TIP = 234 and TAP = 282, then how will you code MAP?  
(a) 198 (b) 324  
(c) 250 (d) 300
- If in a certain language, MADRAS is coded as NBESBT, how is BOMBAY coded in that language?  
(a) CPNCBX (b) CPNCBZ  
(c) CPOCBZ (d) CQOCBZ
- In a certain 'code language, OPERATION is written as NODQBUJPO. How is INVISIBLE written in that code?  
(a) JOWJTJCMF (b) JOWJTHAKD  
(c) HMUHTJCMF (d) HMUHTHAKD  
(e) None of these

## Answer Key

|         |         |         |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (a)  | 2. (b)  | 3. (b)  | 4. (a)  | 5. (d)  | 6. (c)  | 7. (b)  | 8. (a)  | 9. (b)  | 10. (d) |
| 11. (c) | 12. (a) | 13. (a) | 14. (c) | 15. (d) | 16. (c) | 17. (b) | 18. (a) | 19. (e) | 20. (b) |
| 21. (b) | 22. (a) | 23. (b) | 24. (b) | 25. (c) |         |         |         |         |         |

## Answers with Explanations

## 1. Option (a) is correct.

**Explanation:** The code for CAT is one more than the code for BAT, and C is one step ahead of B. As a result, the letters of the English alphabet are coded using numbers.  $A = 1, B = 2, Z = 26$ .

So, BALL =  $B + A + L + L = 2 + 1 + 12 + 12 = 27$

## 2. Option (b) is correct.

**Explanation:** To obtain the corresponding letter of the code, the word's fetch letter is advanced three steps.

## 3. Option (b) is correct.

**Explanation:** Clearly, each letter is given a numerical value that is twice the number indicating its place in the English alphabet.

So, RATE =  $R + A + T + E$

=  $(2 \times 18) + (2 \times 1) + (20 \times 2) + (5 \times 2) = 88$

## 4. Option (a) is correct.

**Explanation:** The number of letters in the code is two times greater than the number of letters in the specified word.

FLOWER  $\rightarrow 6 \times 2 + 2 = 14$

DISTASTE  $\rightarrow 8 \times 2 + 2 = 18$

So, BUREAUCRAT  $\rightarrow 10 \times 2 + 2 = 22$

## 5. Option (d) is correct.

**Explanation:** Each letter is replaced by a number that is three less than the letter's designation. The integer codes for the individual letters are physically

combined to create the word's code. As a result, the code for HEEL is now 5229.

## 6. Option (c) is correct.

**Explanation:** Letters are replaced by the place value of their reverse letters i.e.,  $Z = 1, Y = 2, B = 25, A = 26, M = 14$ .

Then, RUN =  $R + U + N = 9 + 6 + 13 = 28$

So, RAT =  $R + A + T = 9 + 26 + 7 = 42$

## 7. Option (b) is correct.

**Explanation:** The term given has interchangeable letter pairs.

S T R A I G H T  
 $\swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow \quad \searrow$   
 T S A R G I T H

Similarly

T H U R S D A Y  
 $\swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow \quad \searrow \quad \swarrow \quad \searrow$   
 H T R U D S Y A

## 8. Option (a) is correct.

**Explanation:** We can see, the word's initial letter is advanced one space to create the first letter of the code, but the other letters are unchanged.

## 9. Option (b) is correct.

**Explanation:** To get the corresponding letters of the code, the first, third, and fifth letters are each moved one step backward, while the second, fourth, and sixth letters are each moved one step ahead.

## 10. Option (d) is correct.

**Explanation:** Clearly, we have

| Letter | A | B | C | D | E | F  | G  | H  | I  | J  | K  | L  | M  | N  |
|--------|---|---|---|---|---|----|----|----|----|----|----|----|----|----|
| Value  | 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 |

So, INDIAN =  $I + N + D + I + A + N$

=  $17 + 27 + 7 + 17 + 1 + 27 = 96$

## 11. Option (c) is correct.

**Explanation:** According to their position in the English alphabet, the letters of the given word are coded. So, the code for "MAGNIFY" will be "1317149625."

## 12. Option (a) is correct.

**Explanation:** The middle (sixth) letter is moved two steps forward to obtain the matching letters of the code. The first, third, fifth, seventh, ninth, and eleventh letters in the word are all moved one step forward. The second, fourth, eighth, and tenth letters are all moved one step backward.

## 13. Option (a) is correct.

**Explanation:** The code letter for each letter in the

word is obtained by moving that letter one step backward in the word.

## 14. Option (c) is correct.

**Explanation:**

Taking,  $A = 1, B = 2, \dots, T = 20, \dots, Z = 26$

We have,

AT =  $A \times T = 1 \times 20 = 20$

CAT =  $C \times A \times T = 3 \times 1 \times 20 = 60$

So, MAT =  $M \times A \times T = 13 \times 1 \times 20 = 260$

## 15. Option (d) is correct.

**Explanation:** The provided word's letters are classified according to their position in the alphabet. The sum of the digits is written if the number has two digits:

$L \rightarrow 12 = 1 + 2 = 3$

$A \rightarrow 1$

$$R \rightarrow 18 = 1 + 8 = 9$$

$$V \rightarrow 22 = 2 + 2 = 4$$

$$A \rightarrow 1$$

$$E \rightarrow 5$$

Similarly,

$$I \rightarrow 9$$

$$N \rightarrow 14 = 1 + 4 = 5$$

$$S \rightarrow 19 = 1 + 9 = 10$$

$$E \rightarrow 5$$

$$C \rightarrow 3$$

$$T \rightarrow 20 = 2 + 0 = 2$$

So, INSECT will be coded as 9 - 5 - 10 - 5 - 3 - 2

**16. Option (c) is correct.**

*Explanation:*

We can see,

The pattern is forming,

|      |      |      |      |      |
|------|------|------|------|------|
| 4    | 3    | 7    | 2    | 0    |
| +1 ↓ | -1 ↓ | +1 ↓ | -1 ↓ | +1 ↓ |
| 5    | 2    | 8    | 1    | 1    |

Similarly,

|      |      |      |      |      |
|------|------|------|------|------|
| 7    | 5    | 1    | 6    | 3    |
| +1 ↓ | -1 ↓ | +1 ↓ | -1 ↓ | +1 ↓ |
| 8    | 4    | 2    | 5    | 4    |

**17. Option (b) is correct.**

*Explanation:* Moving the word's first letter to get the appropriate letters of the code, take one step back while moving the two middle letters each one step forward.

**18. Option (a) is correct.**

*Explanation:* To get the corresponding letter of the code, each letter in the word is two steps backward.

**19. Option (e) is correct.**

*Explanation:* Vowels A, E, I, O, and U are clearly coded as 1, 2, 3, 4 and 5, respectively. Each of the

word's consonants is shifted forward one position to provide the matching letter of the code. The ACID code is 1D3E.

**20. Option (b) is correct.**

*Explanation:* Every letter in the word is shifted one position backward, with the exception of the centre letter. To get the relevant letter of the code, move forward one step.

**21. Option (b) is correct.**

*Explanation:*

Each letter's numerical position in the English alphabet clearly serves as a code for that letter. Thus, A is coded as 1, B as 2, C as 3, D as 4, L as 12, M as 13, Y as 25, and Z as 26.

So, the code for CODE is 3 - 15 - 4 - 5.

**22. Option (a) is correct.**

*Explanation:* In the given code,

$$Z = 1, Y = 2, X = 3, \dots, C = 24, B = 25, A = 26$$

$$\text{So, GO} = 20 + 12 = 32 \text{ and } \text{SHE} = 8 + 19 + 22 = 49$$

$$\text{Similarly, SOME} = S + O + M + E = 8 + 12 + 14 + 22 = 56$$

**23. Option (b) is correct.**

*Explanation:*

$$\text{Taking } Z = 2, Y = 3, \dots, N = 14, B = 26, A = 27$$

We have,

$$\text{TIP} = (T + I + P) \times 6 = (8 + 19 + 12) \times 6$$

$$= 39 \times 6 = 234$$

$$\text{So, MAP} = (M + A + P) \times 6 = (15 + 27 + 12) \times 6$$

$$= 54 \times 6 = 324$$

**24. Option (b) is correct.**

*Explanation:* The word's letters have each been moved one position forward to get the relevant code letter.

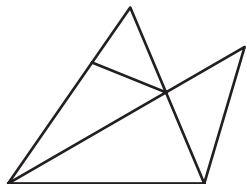
**25. Option (c) is correct.**

*Explanation:* The code letter is obtained by moving the first four letters of the word one step backward and the last five letters one step forward, respectively.

○○○

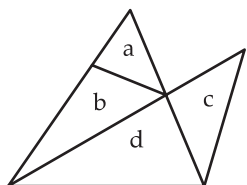
This section plays an important role in increasing the thinking level of students. In this section, geometrical figures are divided into some parts. A student need to identify the different parts by using the tagging and hence count the number of shapes so formed.

- **Example 1:** Find the number of triangles in the given figure.



- (a) 6      (b) 7      (c) 8      (d) 5

Sol. (b)

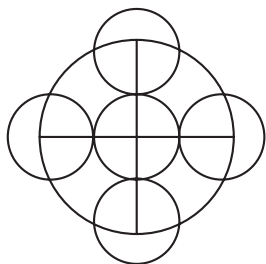


Triangles formed are:

a, b, c, d, ab, cd, abd

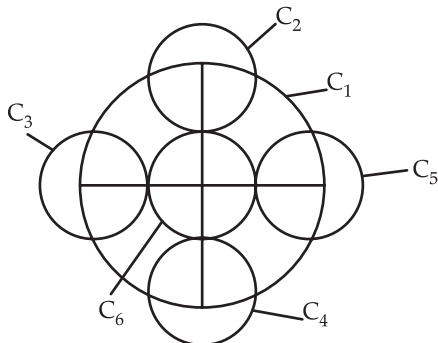
∴ Number of triangles = 7

- **Example 2:** Find the number of circles in the given figure.



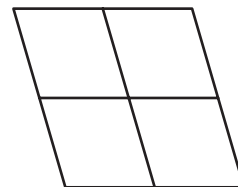
- (a) 6      (b) 5      (c) 4      (d) 7

Sol. (a)



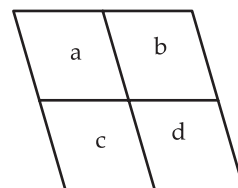
∴ Number of circles = 6

- **Example 3:** Find the number of parallelograms in the given figure.



- (a) 7      (b) 10      (c) 8      (d) 9

Sol. (d)



Parallelograms formed are :

a, b, c, d, ab, cd, ac, bd, abcd

∴ Number of parallelograms = 9

Series completion in verbal reasoning is a type of logical reasoning task that involves identifying the pattern or rule followed in a given series of words or phrases and determining the missing element that completes the series. It is commonly used in aptitude tests, entrance exams, and assessments to assess an individual's ability to recognise and understand patterns and sequences.

In series completion, you are typically given a sequence of words or phrases that follow a specific pattern. Your task is to identify the underlying rule or relationship between the elements and then applying that rule to determine the missing element or elements in the series.

### NUMBER SERIES

- Number series is a type of verbal reasoning task that involves identifying the pattern or rule followed in a given series of numbers and determining the missing number(s) that complete the series. It is a common component of aptitude tests, entrance exams, and assessments used to assess an individual's numerical reasoning and logical thinking abilities.

In number series, you are presented with a sequence of numbers, and your goal is to determine the underlying pattern or relationship between the numbers. By identifying the pattern, you can predict the missing number(s) and complete the series.

- **Example 1:** Which number would replace the question mark in the series?

3, 8, 13, ?, 23

- (a) 15      (b) 18  
(c) 16      (d) 21



**Sol. (b)** Clearly, the given sequence follows the pattern  
 $3 + 5 = 8, 8 + 5 = 13$   
 So, the missing number =  $13 + 5 = 18$

- Example 2:** Which is the number that comes next in the sequence?

3, 10, 29, 66, 127, ?

- (a) 215 (b) 196  
 (c) 218 (d) 225

**Sol. (c)** Clearly, the given series is,  
 $1^3 + 2, 2^3 + 2, 3^3 + 2, 4^3 + 2, 5^3 + 2$ .  
 So, next number =  $6^3 + 2 = 216 + 2 = 218$

### ALPHABET SERIES

- In verbal reasoning, alphabet series entail identifying the pattern or rule followed in a given series of letters or alphabets and determining the missing letter(s) to complete the series. It is a common component of aptitude tests, entrance exams, and assessments used to evaluate a person's logical thinking, pattern recognition, and reasoning abilities.

In alphabet series, you are shown a series of letters and your task is to figure out the underlying pattern or relationship between the letters. By recognising the pattern, you can predict the missing letter(s) and complete the series.

- Example 3:** Find the next two terms in the series.

A, C, E, J, ?, ?

- (a) L, P (b) M, O  
 (c) O, U (d) R, V

**Sol. (c)** We have the following sequence,

A  $\xrightarrow{+2}$  C  $\xrightarrow{+3}$  E  $\xrightarrow{+4}$  J  $\xrightarrow{+5}$  O  $\xrightarrow{+6}$  U

So, missing terms are O and U.

- Example 4:** In the following letter series, some of the letters are missing which are given in that order as one of the alternatives below it. Choose the correct alternative.

\_ xy \_ yyx \_ xxy

- (a) xxyy (b) yxyx  
 (c) yyxy (d) xxyy

**Sol. (c)** The pattern is xyy/xyx/xyx/xyx  
 The missing terms are y, y, x and y.

### MIXED SERIES

- A mixed series is a type of reasoning task in which multiple patterns or rules are combined within a single series. It entails recognising and analysing various patterns, relationships, or sequences within a given series of numbers, letters, or other elements, as well as determining the missing element(s) that complete the series.

Mixed series may contain a mix of number sequences, alphabetical sequences, or patterns involving symbols or other elements. The series may include alternating patterns, overlapping patterns, or a pattern change after a certain number of elements.

- Example 5:** Find the next term in the alpha-numeric series:

Z1A, W3D, T5G, Q7J, N9M, K11P, ?

- (a) H13S (b) L13P  
 (c) H15N (d) L13S

**Sol. (a)** Clearly, the pattern follow by the letters are given:

**I<sup>st</sup> Letter:**

Z  $\xrightarrow{-3}$  W  $\xrightarrow{-3}$  T  $\xrightarrow{-3}$  Q  $\xrightarrow{-3}$  N  $\xrightarrow{-3}$  K  $\xrightarrow{-3}$  (H)

**II<sup>nd</sup> Letter:**

A  $\xrightarrow{+3}$  D  $\xrightarrow{+3}$  G  $\xrightarrow{+3}$  J  $\xrightarrow{+3}$  M  $\xrightarrow{+3}$  P  $\xrightarrow{+3}$  (S)

Follow the pattern

1  $\xrightarrow{+2}$  3  $\xrightarrow{+2}$  5  $\xrightarrow{+2}$  7  $\xrightarrow{+2}$  9  $\xrightarrow{+2}$  11  $\xrightarrow{+2}$  (13)

Hence, H13S is the right answer.

- Example 6:** In the following question, a series of letters and numbers is given the terms of which follow certain definite pattern in groups. However, some terms in the series are missing, which are given in the same order as one of the alternatives below the series. Choose the correct alternative.

Z \_ A X \_ C \_ 17ET13 \_

- (a) 2521VG (b) 2421TM  
 (c) 2127UH (d) 2520UM

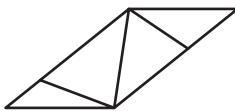
**Sol. (a)** Here, the pattern is:

Z - A = 26 - 1 = 25, X - C = 24 - 3 = 21

V - E = 22 - 5 = 17, T - G = 20 - 7 = 13

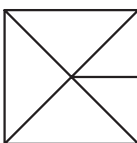
## EXERCISE

1.



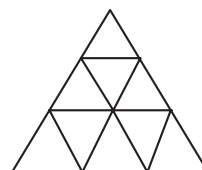
- (a) 6 (b) 7 (c) 5 (d) 8

2.



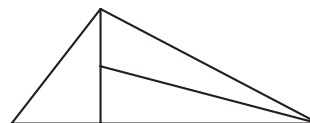
- (a) 9 (b) 8 (c) 10 (d) 7

3.



- (a) 11 (b) 13 (c) 14 (d) 12

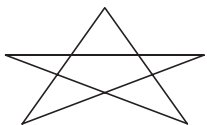
4.



- (a) 5 (b) 6 (c) 7 (d) 8

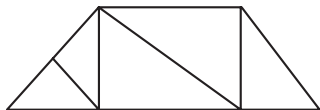


5.



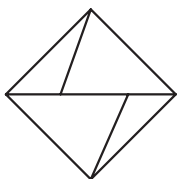
- (a) 5 (b) 10 (c) 7 (d) 8

6.



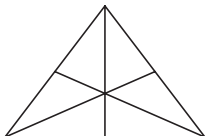
- (a) 6 (b) 8 (c) 7 (d) 10

7.



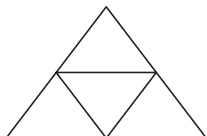
- (a) 7 (b) 8 (c) 6 (d) 5

8.



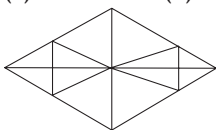
- (a) 16 (b) 13 (c) 9 (d) 7

9.



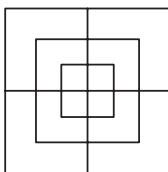
- (a) 4 (b) 5 (c) 6 (d) 7

10.



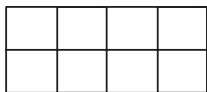
- (a) 16 (b) 22 (c) 28 (d) 32

11.



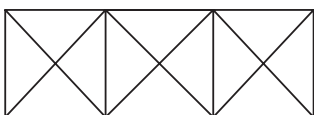
- (a) 8 (b) 12 (c) 15 (d) 18

12.



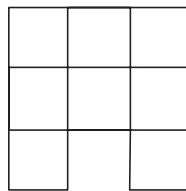
- (a) 11 (b) 10 (c) 12 (d) 15

13.



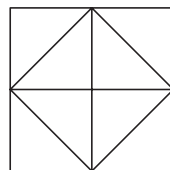
- (a) 4 (b) 6 (c) 5 (d) 8

14.



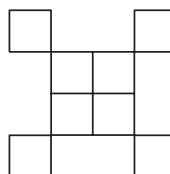
- (a) 10 (b) 4 (c) 8 (d) 12

15.



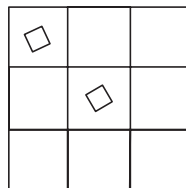
- (a) 7 (b) 5 (c) 8 (d) 6

16.



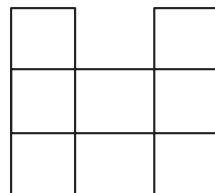
- (a) 9 (b) 8 (c) 10 (d) 12

17.



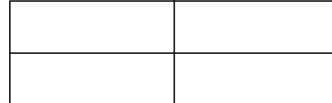
- (a) 12 (b) 14 (c) 18 (d) 16

18.



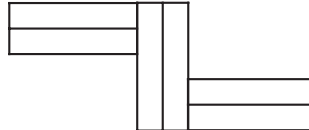
- (a) 8 (b) 7 (c) 9 (d) 6

19.



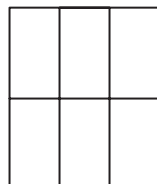
- (a) 6 (b) 7 (c) 9 (d) 8

20.



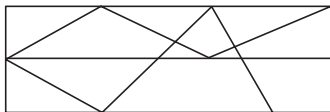
- (a) 9 (b) 8 (c) 10 (d) 12

21.



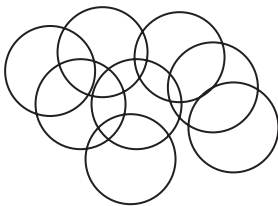
- (a) 17 (b) 16 (c) 15 (d) 18

22.



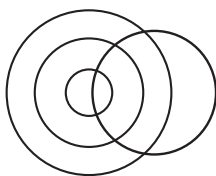
- (a) 4 (b) 3 (c) 2 (b) 5

23.



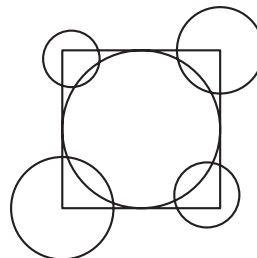
- (a) 6 (b) 8 (c) 9 (d) 7

24.



25.

- (a) 4 (b) 5 (c) 3 (d) 6



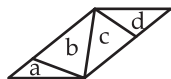
- (a) 4 (b) 2 (c) 3 (d) 5

## Answer Key

|     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1.  | (a) | 2.  | (c) | 3.  | (b) | 4.  | (a) | 5.  | (b) | 6.  | (c) | 7.  | (c) | 8.  | (a) | 9.  | (b) | 10. | (c) |
| 11. | (c) | 12. | (a) | 13. | (c) | 14. | (a) | 15. | (d) | 16. | (a) | 17. | (d) | 18. | (b) | 19. | (c) | 20. | (a) |
| 21. | (d) | 22. | (b) | 23. | (b) | 24. | (a) | 25. | (d) |     |     |     |     |     |     |     |     |     |     |

## Answers with Explanations

1. Option (a) is correct.

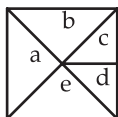


**Explanation:** Triangles formed are:

a, b, c, d, ab, cd

∴ Number of triangles = 6

2. Option (c) is correct.

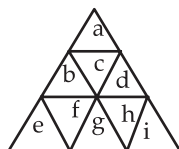


**Explanation:** Triangles formed are:

a, b, c, d, e, cd, ae, bcd, cde, ab

∴ Number of triangles = 10

3. Option (b) is correct.

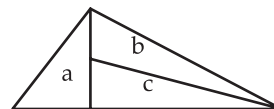


**Explanation:** Triangles formed are:

a, b, c, d, e, f, g, h, i, abcd, befg, dghi, abcdefghi

∴ Number of triangles = 13

4. Option (a) is correct.

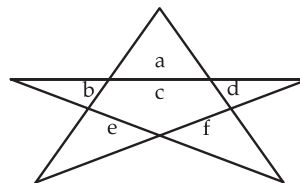


**Explanation:** Triangles formed are:

a, b, c, bc, abc

∴ Number of triangles = 5

5. Option (b) is correct.

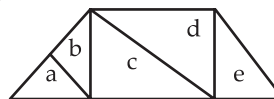


**Explanation:** Triangles formed are:

a, b, c, d, e, f, bcf, ecd, ace, acf, bcd

∴ Number of triangles = 10

6. Option (c) is correct.

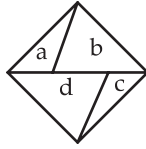


**Explanation:** Triangles formed are:

a, b, c, d, e, ab, abc

∴ Number of triangles = 7

7. Option (c) is correct.

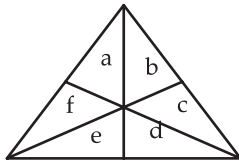


**Explanation:** Triangles formed are:

a, b, c, d, ab, cd

∴ Number of triangles = 6

8. Option (a) is correct.

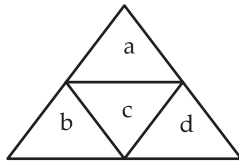


**Explanation:** Triangles formed are:

a, b, c, d, e, f, bc, de, af, abf, abc, cde, fed, afe, bcd, abcdef

∴ Number of triangles = 16

9. Option (b) is correct.

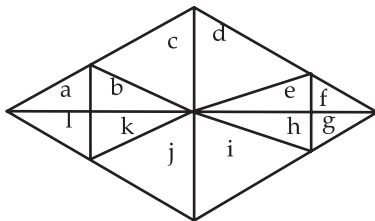


**Explanation:** Triangles formed are:

a, b, c, d, abcd

∴ Number of triangles = 5

10. Option (c) is correct.

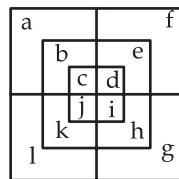


**Explanation:** Triangles formed are:

a, b, c, d, e, f, g, h, i, j, k, l, ab, ef, gh, lk, al, bk, eh, fg, abc, def, ghi, jkl, abcdef, lkjihg, abclkj and defghi

∴ Number of triangles = 28

11. Option (c) is correct.

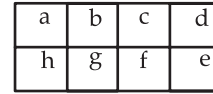


**Explanation:** Squares formed are:

c, d, i, j, bc, de, hi, jk, abc, def, ghi, jkl, cdji, bcdehijk, abcdefghijkl

∴ Number of squares = 15

12. Option (a) is correct.

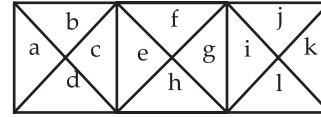


**Explanation:** Squares formed are:

a, b, c, d, e, f, g, h, abgh, bcgf, cdef

∴ Number of squares = 11

13. Option (c) is correct.

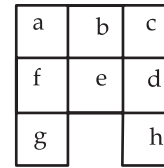


**Explanation:** Squares formed are:

abcd, efgh, ijkl, ce, gi

∴ Number of squares = 5

14. Option (a) is correct.

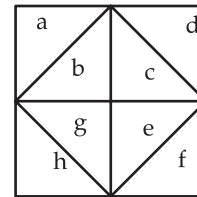


**Explanation:** Squares formed are:

a, b, c, d, e, f, g, h, abef, bcde

∴ Number of squares = 10

15. Option (d) is correct.

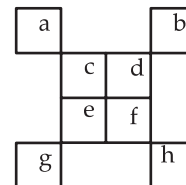


**Explanation:** Squares formed are:

ab, cd, ef, gh, bceg and abcdefgh

∴ Number of squares = 6

16. Option (a) is correct.

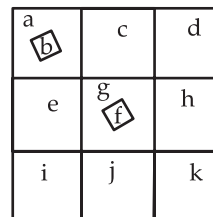


**Explanation:** Squares formed are:

a, b, c, d, e, f, g, h, cdfe

∴ Number of squares = 9

17. Option (d) is correct.



**Explanation:** Squares formed are:

b, f, ab, c, d, h, gf, e, i, j, k, abcegf, cdgfh, egfij, gfhjk, abcdefghijk

∴ Number of squares = 16

18. Option (b) is correct.

|   |   |   |
|---|---|---|
| a |   | e |
| b | d | f |
| c |   | g |

*Explanation:* Squares formed are:

a, b, c, d, e, f and g

∴ Number of squares = 7

19. Option (c) is correct.

|   |   |
|---|---|
| a | b |
| c | d |

*Explanation:* Rectangles formed are:

a, b, c, d, ab, cd, ac, bd and abcd

∴ Number of rectangles = 9

20. Option (a) is correct.

|   |   |   |   |
|---|---|---|---|
| a |   |   |   |
| b |   |   |   |
|   | c | d |   |
|   |   |   | e |
|   |   |   | f |

*Explanation:* Rectangles formed are:

a, b, c, d, e, f, ab, cd and ef

∴ Number of rectangles = 9

21. Option (d) is correct.

|   |   |   |
|---|---|---|
| a | b | c |
| f | e | d |

*Explanation:* Rectangles formed are:

a, b, c, d, e, f, ab, bc, fe, ed, af, be, cd, abc, def, abfe, bced, abcdef

∴ Number of rectangles = 18

22. Option (b) is correct.

*Explanation:*

∴ Number of rectangles = 3

23. Option (b) is correct.

*Explanation:*

∴ Number of circles = 8

24. Option (a) is correct.

*Explanation:*

∴ Number of circles = 4

25. Option (d) is correct.

*Explanation:*

∴ Number of circles = 5

# AGNIVEER NAVY (SSR)

## SAMPLE QUESTION PAPER - 1

### ENGLISH

#### Passage

The headmaster of our local primary school performs a magic show at this time of year, but the parents aren't invited. This child-centred approach is exactly what you would want from a school even though it's frustrating for a neuroscientist because magic is a really interesting way of studying the brain.

Optical illusions have given us very useful insights into how perception works. Tricking the visual system into thinking things are different shapes or colours than they actually tell us how the brain works in everyday situations as well. Our nervous system is really about filtering out rather than relaying information - if we were aware of all of our sensory inputs all the time, we would rapidly be overwhelmed. Attention is the way we direct the spotlight, ignoring the background and focusing on what matters.

Magicians rely on this to misdirect our attention. For example, there are basic circuits in the primary visual cortex that are tuned into where a pair of eyes are looking. Very handy when you're trying to pull a rabbit from a hat, or control a couple of hundred screaming kids - now that's real magic.

1. According to the author, which one of the following statements is true?
  - (a) The local primary school follows a child-centred approach.
  - (b) The author is probably a neuroscientist.
  - (c) It is difficult to trick our nervous system.
  - (d) Understanding magic is equivalent to understanding how brain works.
2. Why are optical illusions useful for a neuro-scientist?
  - (a) Optical illusions tell us how the visual system can be tricked into seeing something else which is not true.
  - (b) Optical illusions tell us how the visual system can be made fool-proof.
  - (c) Optical illusions explain how sight works.
  - (d) Optical illusions show how accurate vision can be.
3. Which of the following is a function of our nervous system?
  - (a) It overwhelms us with various sensory inputs.
  - (b) It relays all the information that is present around us.
  - (c) It filters out the background and focuses on the important information.
  - (d) It helps us focus on each and every detail around us.

4. How do magicians create magic?
  - (a) They surround the audience with different shapes or colours.
  - (b) They shift the audience's attention to certain details which helps the magicians hide their actions.
  - (c) They make sure that no detail is missed.
  - (d) They trick the audience into focusing on the background.
5. Which word in the passage means 'overpower'?
  - (a) Tricking
  - (b) Overwhelm
  - (c) Frustrating
  - (d) Misdirect

**Directions:** Each item in this section has a sentence with three underlined parts labelled as (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part and indicate your response on the Answer Sheet against the corresponding letter i.e., (a) or (b) or (c). If you find no error, your response should be indicated as (d).

6. A bread is a staple food prepared (a) from a dough of flour and water, (b) usually by baking. (c) No error (d).
7. The mascots bring in a lot of revenue (a) to finance the Olympic Games (b) through licensing and merchandising. (c) No error (d).

**Directions:** In this section, each question has a three parts labelled (a), (b), and (c). Read each sentence to find out whether there is any error in the underlined part and indicate your response in the Answer Sheet against the corresponding letter, i.e., (a), (b), (c), or (d). If you find "No error", your response should be indicated as (e).

8. (a) Despite of the simplicity  
(b) of production both Oldowan and Acheulian  
(c) tool-making process activate  
(d) different parts of the brain.  
(e) No error.
9. (a) The later tend to play  
(b) like an intro to his benighted  
(c) Celebrity Apprentice: stolid, explicit  
(d) and exhausting.  
(e) No error.

**Directions:** Each of the following sentences in this section has a blank space and four words or group of words are given after the sentence. Select the most appropriate word or group of words for the blank space and indicate your response on the Answer Sheet accordingly.

10. My grandparents \_\_\_\_ for a movie.
  - (a) took us out
  - (b) took off
  - (c) took after us
  - (d) took us back

11. Our cat \_\_\_\_\_ after we put posters up all over the neighbourhood.  
 (a) turned off (b) turned down  
 (c) turned up (d) turned on
12. The plot of the story thickens when the main character's \_\_\_\_\_ turns out to be a wanted criminal.  
 (a) lugubrious (b) doppelganger  
 (c) lovelorn (d) woeful

**Directions:** Each of the following sentences has a word or phrase underlined. Read the sentences carefully and find which part of speech the underlined word is. Indicate your response on the Answer Sheet accordingly.

13. He tripped clumsily on the rug and fell flat on his face.  
 (a) Conjunction (b) Interjection  
 (c) Adverb (d) Cause
14. Because she was feeling unwell, she decided to avoid the trip.  
 (a) Conjunction (b) Adjective  
 (c) Adverb (d) Interjection

**Directions:** In this section a word is spelt in four different ways. Identify the one which is correct. Choose the correct response (a), (b), (c) or (d) and indicate on the Answer Sheet accordingly.

15.  
 (a) Maintainence (b) Maintenance  
 (c) Maintenance (d) Maintainence
16.  
 (a) Questionnaire (b) Questionaire  
 (c) Questionairre (d) Questioniare

**Directions:** In this section sentences have been given in the direct speech and you are required to find the indirect form of the given sentences. Choose the correct response (a), (b), (c) or (d) and indicate on the Answer Sheet accordingly.

17. The owner said to the tenants, "We will keep the premises clean."  
 (a) The owner told the tenants that they would keep the premises clean.  
 (b) The owner ordered the tenants that they would the premises clean.  
 (c) The owner told the tenants that they will keep the premises clean.  
 (d) The owner wanted the tenants to kept the premises clean.
18. The fairy godmother said to the princess, "May your goodness win over evil."  
 (a) The fairy godmother blessed the princess that her goodness would win over evil.  
 (b) The fairy godmother asked the princess that her goodness would win over evil.  
 (c) The fairy godmother ordered the princess to make her goodness win over evil.  
 (d) The fairy godmother told the princess that her goodness will win over evil.

**Directions:** Each item in this section consists of a sentence with an underlined word followed by four words. Select the option that is opposite in meaning to the underlined word and mark your response on your Answer Sheet accordingly.

19. The nation was gaining ground at this time as the acceptable and legible idiom of collective political demands.  
 (a) incomprehensible (b) intelligible  
 (c) decipherable (d) readable
20. Among its distinctive features is a conflation between land, a national(ised) language, and a people.  
 (a) amalgamation (b) meld  
 (c) constituent (d) emulsion

**Directions:** Each item in this section consists of a sentence with an underlined word followed by four words. Select the option that is similar in meaning to the underlined word and mark your response on your Answer Sheet accordingly.

21. Recently, 23 of Watauga's best math students met virtually to test their mettle against their peers in the annual Math Counts competition.  
 (a) caliber (b) resolution  
 (c) placidity (d) adore
22. Since the callow baker was new to cake decorating, she did not know how to properly frost the multi-layer cake.  
 (a) puerile (b) childish  
 (c) immature (d) seasoned

**Directions:** Given below are some idioms/phrases followed by four alternative meanings to each. Choose the response (a), (b), (c) or (d), whichever is most appropriate expression and mark your response in your answer sheet accordingly.

23. Out of the woods  
 (a) Deliberately and without emotion  
 (b) Out of the forest  
 (c) Out of control  
 (d) Out of danger
24. "Could you tell me where the post office is?"  
 (a) She asked where the post office was.  
 (b) She asked me to tell her where the post office was.  
 (c) She asked me that where the post office was.  
 (d) She asked me to tell her where was the post office.
25. He said, 'I must leave early today.'  
 (a) He said that he must leave early today.  
 (b) He said that he had to leave early that day.  
 (c) He said that he must leave early that day.  
 (d) He said that he might leave early that day.

#### GENERAL AWARENESS

1. Jet streams are usually found in the \_\_\_\_\_.  
 (a) Ozonosphere (b) Mesosphere  
 (c) Troposphere (d) Ionosphere

2. The exceptionally high and low tides that occur at the time of the new moon or the full moon when the Sun, the Moon and the Earth are approximately aligned is called  
 (a) Spring (b) Fall  
 (c) Neap (d) Diurnal
3. Which is the general direction of cyclones formed in the Bay of Bengal?  
 (a) East to West (b) West to East  
 (c) West to South (d) North to South
4. Why the summer monsoon winds blow from South Western direction in the Northern hemisphere?  
 (a) The general direction of wind from the Indian Ocean is South Western.  
 (b) The presence of the doldrums around the equator.  
 (c) The low-pressure conditions in North-West India.  
 (d) Due to the effect of Coriolis force.
5. If the Earth's axis were perpendicular to the plane of its orbit, which one among the following would not have happened?  
 (a) The North pole will always lie in dark.  
 (b) Days and nights would be equal throughout the year.  
 (c) No change of seasons will take place.  
 (d) The Sun will be perpendicular to the equator.
6. Hot deserts like Sahara, Arabia etc, receive very negligible amount of rainfall. This is because they \_\_\_\_\_.  
 (a) Do not receive moisture-bearing wind from the oceans  
 (b) Are the most rocky and barren areas of the Earth  
 (c) Are located on the tropical high-pressure belt of the atmosphere  
 (d) are not on the path of the monsoons
7. The latitude is the angular distance of a point on the Earth's surface, north or South of the equator as measured from the \_\_\_\_\_.  
 (a) Centre of the Earth  
 (b) Tropic of Cancer or of Capricorn  
 (c) Equator  
 (d) Poles
8. Which one among the following features of the Constitution of India is indicative of the fact that real executive power is vested in the Council of Ministers headed by the Prime Minister?  
 (a) Federalism  
 (b) Representative legislature  
 (c) Universal adult franchise  
 (d) Parliamentary democracy
9. Match List-I with List-II and select the correct answer using the code given below the Lists:
- |                                |                                  |
|--------------------------------|----------------------------------|
| <b>List-I (Joint exercise)</b> | <b>List-II (Type)</b>            |
| (a) Indra                      | 1. Joint anti-terrorism exercise |

- (b) Cope India  
 (c) Ind-Indo Corpat  
 (d) Yudhabhyas
2. Joint air force exercise  
 3. Coordinated maritime patrolling  
 4. Joint army exercise

Code:

|     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 1 | 2 | 3 | 4 |
| (b) | 1 | 3 | 2 | 4 |
| (c) | 4 | 3 | 2 | 1 |
| (d) | 4 | 2 | 3 | 1 |

10. Match List-I with List-II and select the correct answer using the code given below the Lists:

**List-I (Ship)****List-II (Type)**

- |                  |                          |
|------------------|--------------------------|
| (a) INS Arihant  | 1. Aircraft Carrier      |
| (b) INS Vikrant  | 2. Landing platform Dock |
| (c) INS Shivalik | 3. Nuclear Submarine     |
| (d) INS Airavat  | 4. Stealth Frigate       |

Code:

|     | A | B | C | D |
|-----|---|---|---|---|
| (a) | 3 | 4 | 1 | 2 |
| (b) | 3 | 1 | 4 | 2 |
| (c) | 2 | 1 | 4 | 3 |
| (d) | 2 | 4 | 1 | 3 |

11. Which one among the following is not a reason for practising tank irrigation in the Peninsular India?  
 (a) The undulating relief and hard rocks.  
 (b) Little percolation of rain water due to impervious rock structure.  
 (c) Most of the rivers of Peninsular India are perennial.  
 (d) There are many streams which become torrential during rainy season.
12. Identify from the following states of India through which the Tropic of Cancer passes and arrange them from east to west:  
 1. Gujarat  
 2. West Bengal  
 3. Uttar Pradesh  
 4. Jharkhand  
 5. Madhya Pradesh  
 6. Bihar  
 7. Chhattisgarh

Select the correct answer using the code given below:

- |               |               |
|---------------|---------------|
| (a) 2-5-7-4-1 | (b) 2-4-7-5-1 |
| (c) 3-2-6-7-5 | (d) 3-7-4-6-2 |

13. Consider the following statements.

1. The Champaran Satyagraha marked Gandhiji's second appearance in Indian politics as a leader of the masses.  
 2. The Champaran Satyagraha was launched to address the problems faced by Indigo plantation workers.

Which of the statements given above is/are correct?

- |                  |                     |
|------------------|---------------------|
| (a) Only 1       | (b) Only 2          |
| (c) Both 1 and 2 | (d) Neither 1 nor 2 |

14. Consider the following statements

1. Charles Wood's Despatch of 1854 laid exclusive emphasis on the development of higher education in India and neglected primary and secondary education.



2. The Carlyle Circular issued by FIW Carlyle sought to check the Spread of revolutionary activities in educational institutions.  
Select the correct answer using the codes given below  
(a) Only 1 (b) Only 2  
(c) Both 1 and 2 (d) Neither 1 nor 2
15. Which one among the following is common to the Treaty of Yandaboo (1826), the Treaty of Salbai (1782) and the Treaty of Gandamak (1879)?  
(a) With these treaties, various Indian powers formed alliances to defeat the British  
(b) These treaties enabled the British to control the South Asian powers  
(c) These treaties expedited the spread of Indian culture abroad  
(d) These treaties gave an essential boost to enhanced trade in South Asia
16. Which one of the following Indian state does NOT share international border with two or more countries?  
(a) Arunachal Pradesh (b) Assam  
(c) Mizoram (d) Tripura
17. The duration of monsoon in India extends for an average period of:  
(a) 80 - 140 days (b) 100 - 120 days  
(c) 90 - 130 days (d) 100 - 140 days
18. Which one of the following is NOT a reason of decrease in biodiversity?  
(a) Large scale deforestation  
(b) Exploitation of forest produces  
(c) Maintaining sacred groves  
(d) Encroachment in forest areas
19. Which one of the following is NOT a cause of depletion in groundwater?  
(a) Afforestation  
(b) Loss of forests  
(c) Excessive pumping of groundwater  
(d) Construction of large scale concrete buildings
20. According to the Koppen climatic classification, the letter code Cfa denotes:  
(a) Tropical wet climate  
(b) Humid subtropical climate  
(c) Tundra climate  
(d) Tropical wet and dry climate
21. Consider the following places of India:  
(1) Itanagar (2) Imphal  
(3) Agartala (4) Aizawl  
Which one of the following is the correct chronological order of the above places in terms of sunrise time?  
(a) 3-2-1-4 (b) 2-1-4-3  
(c) 1-4-3-2 (d) 4-3-2-1
22. Consider the following Wildlife Sanctuaries of India:  
(1) Shikari Devi (2) Bhadra  
(3) Simlipal (4) Pachmarhi  
Which one of the following is the correct order of the above Wildlife Sanctuaries in terms of their location from south to north?
- (a) 1-2-3-4 (b) 2-4-3-1  
(c) 2-3-4-1 (d) 3-1-2-4
23. Spruce and Cedar are tree varieties of:  
(a) Equatorial forest  
(b) Temperate coniferous forest  
(c) Monsoon forest  
(d) Temperate deciduous forest
24. 'Sal' tree is a:  
(a) Tropical evergreen tree  
(b) Tropical semi-evergreen tree  
(c) Dry deciduous tree  
(d) Moist deciduous tree
25. In Thar region, the shifting sand dunes are locally known as:  
(a) Dhrian (b) Dauras  
(c) Dhoros (d) Dhaya

### SCIENCE

1. Solutions in test tubes containing  $H_2O$  and aqueous NaOH can be differentiated with the help of  
(a) Red litmus  
(b) Blue litmus  
(c)  $Na_2CO_3$   
(d) HCl (aqueous)
2. Which one among the following methods is not effective in removing arsenic from contaminated ground water?  
(a) Boiling water  
(b) Reverse osmosis  
(c) Ion exchange  
(d) Coagulation-adsorption
3. Sandalwood tree is considered a:  
(a) Total root parasite.  
(b) Total stem parasite.  
(c) Stem parasite.  
(d) Partial root parasite.
4. Which one among the following statements about stomach is not correct ?  
(a) Stomach acts as a temporary reservoir.  
(b) Stomach mixes food with gastric juice.  
(c) Stomach secretes lipase and amylase in gastric juice.  
(d) Rate of stomach emptying depends on the type of food.
5. Which one of the following organs breaks fat to produce cholesterol ?  
(a) Intestine (b) Liver  
(c) Lungs (d) Kidneys
6. An individual whose blood type is B may in an emergency donate blood to a person whose blood type is:  
(a) B or A (b) AB or A  
(c) A or O (d) AB or B
7. Which one among the following statements regarding cell is not correct?  
(a) Shape and size of cells are related to specific function.  
(b) Some cells have changing shapes.

- (c) Each cell has its own capacity to perform.  
(d) Same type of cells are present in all body tissues.
8. A jet plane flies through air with a velocity of 2 Mach. While the velocity of sound is 332 m/s in the air, speed of the plane is:  
(a) 166 m/s (b) 66.4 km/s  
(c) 332 m/s (d) 664 m/s
9. Which one among the following statements is correct?  
(a) Convex mirrors are used by doctors to examine oral cavity.  
(b) Concave mirrors are used as reflectors.  
(c) Convex mirrors are used as reflectors.  
(d) Convex mirrors should be used for shaving.
10. Which one among the following is the major cause of blurring and unsharp images of objects observed through very large telescope at the extreme limit of magnification?  
(a) Air turbulence of Earth's atmosphere.  
(b) Poor optical polish achievable on large mirrors.  
(c) Poor tracking capacities of telescope.  
(d) Varying density of air in the Earth's atmosphere.
11. When a moving bus suddenly applies brakes, the passengers sitting in it fall in the forward direction. This can be explained by:  
(a) The theory of relativity  
(b) Newton's first law  
(c) Newton's second law  
(d) Newton's third law
12. The material used for electric fuse is an alloy of tin and lead. This alloy should have:  
(a) High specific resistance and low melting point.  
(b) Low specific resistance and high melting point.  
(c) Low specific resistance and low melting point.  
(d) High specific resistance and high melting point.
13. Suppose you are standing 1 m in front of a plane mirror. What should be the minimum vertical size of the mirror so that you can see your full image in it?  
(a) 0.50 m (b) 2 m  
(c) Half of your height (d) Twice your height
14. The lines of force of a uniform magnetic field:  
(a) Must be convergent  
(b) Must be divergent  
(c) Must be parallel to each other  
(d) Intersect
15. A Jet engine works on the principle of conservation of:  
(a) Linear momentum (b) Angular momentum  
(c) Energy (d) Mass
16. The surface temperature of the Sun is nearly :  
(a) 2000 K (b) 4000 K  
(c) 6000 K (d) 8000 K
17. If the electrical resistance of a typical substance suddenly drops to zero, then the substance is called:  
(a) Superconductor (b) Semiconductor  
(c) Conductor (d) Insulator
18. "The stars seem to be higher on the sky than they actually are." This can be explained by :  
(a) Atmospheric refraction  
(b) Dispersion of light  
(c) Total internal reflection  
(d) Diffraction of light
19. Which one among the following is not a source of renewable energy?  
(a) Hydroelectricity (b) Solar energy  
(c) Fuel cell (d) Wind energy
20. A liquid rises to a certain length in a capillary tube. The tube is inclined to an angle of  $45^\circ$ . The length of the liquid column will:  
(a) Increase  
(b) Decrease  
(c) Remain unchanged  
(d) First decrease and then increase
21. Magnets attract magnetic substances such as iron, nickel, cobalt etc. They can also repel:  
(a) Paramagnetic substances  
(b) Ferromagnetic substances  
(c) Diamagnetic substances  
(d) Non magnetic substances
22. A car is moving towards a building after 10 seconds, its angle of elevation with the building becomes  $60^\circ$  from  $45^\circ$ . In how many seconds it will reach the foot of building from the initial position?  
(a) 21.66 sec (b) 22.65  
(c) 23.65 sec (d) 24.65 sec.
23. If  $b > a$ , then the equation  $(x - a)(x - b) = 1$  has root in which one of the following interval?  
(a)  $(-\infty, a)$  (b)  $(b, \infty)$   
(c)  $(-\infty, a), (b, \infty)$  (d)  $(a, b)$
24. What is the number of diagonals of a decagon?  
(a) 28 (b) 35  
(c) 38 (d) 42
25. If  $S_n = 5n + 27n^2$  for an A.P, then the value of common difference will be?  
(a) 6 (b) 12  
(c) 18 (d) 9

### MATHEMATICS

1. If set A and B has respectively 4 and 2 elements. What is the ratio of number of proper subset of A to the number of relation from set A to set B.  
(a) 15 : 256 (b) 8 : 256  
(c) 1 : 2 (d) None of these
2. If  $(x + 3)^3 + 27 = 0$  has three roots  $x_1, x_2$  and  $x_3$ , then the value of  $x_1^3 + x_2^3 + x_3^3$  will be.  
(a) -122 (b) -142  
(c) -162 (d) -182
3. Let  $t_n = \sin^n \theta + \cos^n \theta$   
then the value of  $\frac{t_{100} - t_{98}}{t_{98} - t_{96}}$  equal to?

- (a)  $\frac{t_{100}}{t_{96}}$  (b)  $\frac{t_{98}}{t_{96}}$
- (c)  $\frac{t_{96}}{t_{94}}$  (d)  $\frac{t_{100}}{t_{94}}$
4. If  $\alpha$  and  $\beta$  are the roots of the equation  $x^2 + 3x + 1 = 0$ , then the equation whose roots are  $\alpha^3 + \beta^3$  and  $\alpha^2 + \beta^2$  will be ?  
 (a)  $x^2 - 11x - 126 = 0$  (b)  $x^2 + 11x - 126 = 0$   
 (c)  $x^2 + 11x + 126 = 0$  (d)  $x^2 - 11x + 126 = 0$
5. If  $C(30, p + 2) = C(30, p + 6)$  and  $C(50, q + 5) = C(50, q + 7)$  the value of  $p^2 + q^2 = ?$   
 (a) 480 (b) 481  
 (c) 482 (d) 483
6. If  $S_n$  denotes the sum of  $n$  terms of A.P. where  $S_n = 2n^2 + 3n$ , then the value of  $\frac{t_9 + t_7}{t_5 - t_3}$ , where  $t_n$  denotes the  $n$ th term of the A.P.  
 (a)  $\frac{33}{4}$  (b)  $\frac{31}{4}$   
 (c)  $\frac{29}{4}$  (d)  $\frac{27}{4}$
7. The value of the limit  $\lim_{x \rightarrow 0} \frac{\tan^2 7x - \tan^2 3x}{\tan^2 5x - \tan^2 x}$   
 (a) 1 (b)  $\frac{7}{5}$   
 (c)  $\frac{21}{5}$  (d)  $\frac{5}{3}$
8. Solve the differential equation by suitable method.  
 $ydx - xdy = xy^2 dy + y^3 dx$   
 (a)  $\frac{y}{x} = xy + c$  (b)  $\frac{x}{y} = xy + c$   
 (c)  $x^2 = y^2 + c$  (d) None of these
9. If  $4x + 5y - 30 = 0$  and  $5x - 4y - 17 = 0$  are the two equations of diagonals of a square. Square has one vertex  $(-10, 2)$ . Find the Area of the square.  
 (a) 400 (b) 425  
 (c) 450 (d) 475
10. In a lottery of 20 tickets numbered from 11 to 30, three tickets are drawn simultaneously what is the probability that all the three number are prime number?  
 (a)  $\frac{1}{57}$  (b)  $\frac{2}{57}$   
 (c)  $\frac{3}{57}$  (d)  $\frac{20}{57}$
11. The function  $f(x) = |x| + x^2 + x^4$  will be?  
 (a) even  
 (b) odd  
 (c) both even and odd  
 (d) Neither even nor odd
12. Total number of terms in the expansion of  $(x^3 + 6x^2 + 12x + 8)^5 + (y^3 + 9y^2 + 27y + 27)^6$  will be  
 (a) 35 (b) 30  
 (c) 28 (d) 27
13. A man has 7 friends. In how many ways he can invite two or more than two friends for a party?  
 (a) 125 (b) 123  
 (c) 121 (d) 120
14. If one root of the equation  $x^2 + ax + b = 0$  is. Square of the other root, then which one is true?  
 (a)  $\frac{a}{b} = \frac{1+b}{a^2+3b}$  (b)  $\frac{a}{b} = \frac{1+b}{a^2-3b}$   
 (c)  $\frac{a}{b} = \frac{1-b}{a^2+3b}$  (d)  $\frac{a}{b} = \frac{1-b}{a^2-3b}$
15. A binary number is represented by  $(abacdc)_2$ , where  $a > b$  and  $c > d$ . What is its decimal equivalent to?  
 (a)  $(185)_{10}$  (b)  $(187)_{10}$   
 (c)  $(189)_{100}$  (d)  $(191)_{10}$
16. If  $A = 3 + \cos^4 \theta - \cos^2 \theta$ , then which one of the following is correct.  
 (a)  $\frac{11}{4} \leq A \leq 3$  (b)  $\frac{13}{4} \leq A \leq 3$   
 (c)  $\frac{11}{4} \leq A \leq 4$  (d)  $\frac{13}{4} \leq A \leq 4$
17. The value of  $\cot 3A - \operatorname{cosec} 3A$  will be equals to?  
 (a)  $\tan\left(\frac{3A}{2}\right)$  (b)  $-\tan\left(\frac{3A}{2}\right)$   
 (c)  $\cot\left(\frac{3A}{2}\right)$  (d)  $-\cot\left(\frac{3A}{2}\right)$
18. If  $a, b$  and  $c$  all are positive numbers, then  $(a + b + c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$  will be  
 (a)  $\geq 9$  (b)  $\geq 9\frac{1}{2}$   
 (c)  $\geq 10$  (d)  $10\frac{1}{2}$
19. If the ratio of G.M. to A.M. between two numbers  $a$  and  $b$  is  $2 : 3$ . Then the value of  $a : b = ?$   
 (a)  $\frac{7+3\sqrt{5}}{2}$  (b)  $\frac{7-3\sqrt{5}}{2}$   
 (c)  $\frac{7+3\sqrt{5}}{4}$  (d)  $\frac{7-3\sqrt{5}}{4}$
20. If the inverse of  $A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$  is equal to  $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ , then the value of  $pq + rs$  will be.  
 (a) 11 (b) 13  
 (c) 15 (d) 17

21. If  $p = \sin \theta + \tan \theta$ , then the value of  $p^3 + (p-1)^3 + (p-2)^2 + 4$  will be equals to?  
 (a) 1 (b) 3  
 (c) 5 (d) 7
22. Let  $A = \{1, 2, 3\}$ ,  $B = \{4, 5, 6, 7\}$  and let  $f = \{(1, 4), (2, 5), (3, 6)\}$  be a function from  $A$  to  $B$ . Based on the given information,  $f$  is best defined as :  
 (a) Surjective function (b) Injective function  
 (c) Bijective function (d) None of these
23. If the set  $A$  contains 5 elements and the set  $B$  contains 6 elements, then the number of one-one and onto mappings from  $A$  to  $B$  is  
 (a) 720 (b) 120  
 (c) 0 (d) None of these
24. Let  $A = \{1, 2, 3, \dots, n\}$  and  $B = \{a, b\}$ . Then the number of surjections from  $A$  into  $B$  is  
 (a)  ${}^n P_2$  (b)  $2^n - 2$   
 (c)  $2^n - 1$  (d) None of these
25. A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2 + x^2$  is  
 (a) not one-one  
 (b) one-one  
 (c) not onto  
 (d) neither one-one nor onto

## ANSWER KEY

## English

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (b)  | 3. (a)  | 4. (c)  | 5. (a)  |
| 6. (a)  | 7. (d)  | 8. (a)  | 9. (a)  | 10. (a) |
| 11. (c) | 12. (a) | 13. (c) | 14. (a) | 15. (b) |
| 16. (a) | 17. (c) | 18. (b) | 19. (b) | 20. (b) |
| 21. (a) | 22. (b) | 23. (b) | 24. (b) | 25. (b) |

## General Awareness

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (a)  | 3. (a)  | 4. (d)  | 5. (c)  |
| 6. (a)  | 7. (a)  | 8. (a)  | 9. (a)  | 10. (b) |
| 11. (c) | 12. (b) | 13. (b) | 14. (c) | 15. (b) |
| 16. (d) | 17. (b) | 18. (c) | 19. (a) | 20. (b) |
| 21. (b) | 22. (c) | 23. (b) | 24. (b) | 25. (a) |

## Science

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (a)  | 2. (a)  | 3. (d)  | 4. (d)  | 5. (b)  |
| 6. (d)  | 7. (d)  | 8. (d)  | 9. (c)  | 10. (a) |
| 11. (b) | 12. (a) | 13. (c) | 14. (c) | 15. (a) |
| 16. (c) | 17. (a) | 18. (a) | 19. (c) | 20. (c) |
| 21. (c) | 22. (c) | 23. (c) | 24. (b) | 25. (a) |

## Mathematics

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (a)  | 2. (c)  | 3. (c)  | 4. (b)  | 5. (c)  |
| 6. (a)  | 7. (d)  | 8. (b)  | 9. (c)  | 10. (a) |
| 11. (a) | 12. (a) | 13. (d) | 14. (a) | 15. (b) |
| 16. (a) | 17. (b) | 18. (a) | 19. (a) | 20. (d) |
| 21. (d) | 22. (b) | 23. (c) | 24. (b) | 25. (d) |

○○○

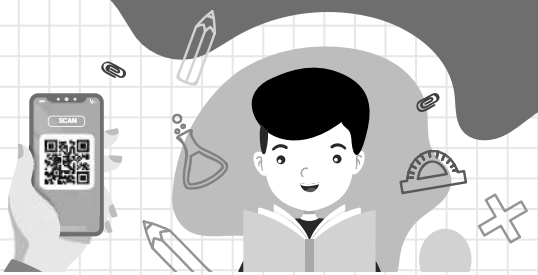
**Finished Solving the Paper ?**  
Time to evaluate yourself !

**SCAN THE CODE**



**For elaborate Solutions**

**OSWAAL COGNITIVE**  
**LEARNING TOOLS**



# AGNIVEER NAVY (SSR)

## SAMPLE QUESTION PAPER - 2

### ENGLISH

#### Passage

Global warming, largely caused by industrial development and consumer demands, has been causing havoc across the world. Temperatures are shooting up, floods have been ravaging South China and Northeast India, unseasonal rains and, ironically, delayed and poor monsoon rains are experienced. A major solution to mitigate such climate changes is to reduce the levels of greenhouse gases, particularly carbon dioxide, which cause this warming. In an effort to try and limit this warming, many countries across the world are gathering together and agreeing to make efforts to limit the rise in temperature to no higher than 1.5 degrees by the year 2050.

The major way to do so is to increase the number of plants, trees and forests across the world. They all absorb carbon dioxide from the air, and with the help of sunlight and water, produce food (staple for us) and oxygen (which we breathe). And the wood and timber they offer are used by us in buildings and furniture. They are thus justly named in Sanskrit as Kalpataru — the wish-giving tree.

Yet, we kill them: deforestation has been going on decade after decade across the world, affecting the weather as well as the lives of plants, animals, microbes and the livelihood of human tribes that live in forests. The total surface area of our Earth is 52 billion hectares (Ha), and 31% of this has been forest cover. But the huge Amazon forests of South America are being chopped off for commercial reasons. Peru and Bolivia in the western Amazon region are the worst affected by such deforestation; so are Mexico and its neighbours in Mesoamerica. Russia, with forests occupying 45% of its land area, is chopping off trees. Large scale deforestation has contributed to global warming over the years.

- Severe climate changes are a result of
  - Increase in poisonous gases
  - Global warming
  - Destruction of Kalpataru
  - Increased use of timber
- Forests are being destroyed for
  - legal reasons
  - political reasons
  - commercial reasons
  - scientific reasons
- What contributes to global warming?
  - Limiting the rise in temperature
  - Human tribes living in forests
  - Severe climate changes
  - Large scale deforestation

**Directions:** Each item in this section has a sentence with three underlined parts labelled as (a), (b) and (c). Read each sentence to find out whether there is any error in any underlined part and indicate your response on the

Answer Sheet against the corresponding letter i.e., (a) or (b) or (c). If you find no error, your response should be indicated as (d).

- Sometime around last (a) week, the baby begun (b) to walk. (c) No error (d)
- By the time the structure is complete, (a) the workers will have spent more than (b) a year working on them. (c) No error (d)
- A python named (a) Medusa is longer (b) then any other snake in the world. (c) No error (d)

**Directions:** In this section, each question has three parts labelled as (a), (b), and (c). Read each sentence to find out whether there is any error in any underlined part and indicate your response in the Answer Sheet against the corresponding letter, i.e., (a), (b), or (c). If you find "No error", your response should be indicated as (d).

- The devastating effects  
(a)  
of COVID-19 have brought greater public attention for  
(b)  
this centuries-long pattern of inequity in healthcare  
systems  
(c)  
No error  
(d)
- An economist and a professor,  
(a)  
Ila Patnaik, works at  
(b)  
National Institute of Public Finance and Policy.  
(c)  
No error.  
(d)

**Directions:** Given below are some idioms/phrases followed by four alternative meanings to each. Choose the response (a), (b), (c) or (d) which is the most appropriate expression and mark your response in the Answer Sheet accordingly.

- Miss the boat
  - Lose an opportunity by being slow to act
  - Not taking advantage of the situation
  - Not choosing the best alternative
  - Not being punctual
- Break a leg
  - To meet with an accident
  - To cause an accident
  - To wish good luck
  - To give an award-winning performance

11. Joker in the pack  
 (a) A person or thing that can change the situation in unexpected way.  
 (b) The person you're just talking about actually appears at that moment.  
 (c) Something that will never happen.  
 (d) To not judge someone or something based solely on appearance.

**Directions:** Each of the following items in this section consists of a sentence, parts of which have been jumbled. These parts have been labelled as P, Q, R and S. Given below each sentence are four sequences namely (a), (b), (c) and (d). You are required to re-arrange the jumbled parts of the sentence and mark your response accordingly.

12. small-scale industries and individuals (P) the COVID pandemic (Q) took the world by storm, creating (R) a domino effect on several medium-scale, (S)  
 (a) SQPR (b) SPRQ  
 (c) QRSP (d) PRSQ
13. risk for babies (P) and round foods (Q) have a significant choking (R) small, hard (S)  
 (a) PRSQ (b) QSPR  
 (c) RPSQ (d) SQRP

**Directions:** Each of the following sentences in this section has a blank space and four words or group of words are given after the sentence. Select the most appropriate word or group of words for the blank space and indicate your response on the Answer Sheet accordingly.

14. India \_\_\_\_ boosting infrastructure near the China border to facilitate faster movement of troops.  
 (a) will (b) have been  
 (c) has been (d) will be
15. If he had left the house on time, he \_\_\_\_ the train.  
 (a) wouldn't have missed  
 (b) won't have  
 (c) could have  
 (d) can have
16. Seema tried to \_\_\_\_ his family members about the recent shopping excursions of her husband, to find out what he bought for her birthday.  
 (a) probe (b) proscribe  
 (c) investigation (d) revere
17. The maestro dropped the instruments down the stairs and caused a terrible \_\_\_\_ that could be heard across the street.  
 (a) music (b) sonorous sound  
 (c) cacophony (d) euphony

**Directions:** Following items have one part of the sentence followed by four alternatives. Complete the sentences by choosing the correct alternative.

18. Though light showers will bring some respite,  
 (a) the air quality will again improve.  
 (b) the air quality will again deteriorate.  
 (c) the air quality will progress.  
 (d) the air quality will remain excellent.
19. It's been a long time  
 (a) since we meet.  
 (b) since we have met.  
 (c) for we met.  
 (d) for we have meet.

**Directions:** Each of the following sentences in this section has a blank space with four options. Select whichever preposition or determiner you consider the most appropriate for the blank space and indicate your response on the Answer Sheet accordingly.

20. You will have to run faster than that if you want to catch \_\_\_\_ with her.  
 (a) on (b) up  
 (c) of (d) in
21. The post office is \_\_\_\_ the street from the grocery store.  
 (a) on (b) in  
 (c) across (d) at

**Directions:** Each item in this section consists of a sentence with an underlined word followed by four words. Select the option that is similar in meaning to the underlined word and mark your response on your Answer Sheet accordingly.

22. The first priority is to persuade the management of the urgency of this matter.  
 (a) convince (b) dissuade  
 (c) encourage (d) deter

**Directions:** Each item in this section consists of a sentence with an underlined word followed by four words. Select the option that is opposite in meaning to the underlined word and mark your response on your Answer Sheet accordingly.

23. The army was forced to retreat after suffering heavy losses.  
 (a) advance (b) withdraw  
 (c) recoil (d) pull back
24. Even after earning millions of money, his parents do not have a proper home to stay.  
 (a) Even after earning  
 (b) millions of money,  
 (c) his parents do not have  
 (d) a proper home to stay.
25. The accused was asked for his last wish before he was hung to death.  
 (a) The accused was asked for  
 (b) his last wish  
 (c) before he was hung to death.  
 (d) No improvement required

### GENERAL AWARENESS

1. In the 10th Mandala of the Rigveda, which one of the following hymns reflects upon the marriage ceremonies?  
 (a) Surya Sukta (b) Purusha Sukta  
 (c) Dana Stutis (d) Urna Sutra
2. Under the PRASAD Tourism Scheme, which one of the following has not been identified as a religious site for development?  
 (a) Ajmer (Rajasthan)  
 (b) Haridwar (Uttarakhand)  
 (c) Somnath (Gujrat)  
 (d) Velankanni (Tamil Nadu)



3. Which of the following statements about the musical culture in 18<sup>th</sup> and 19<sup>th</sup> century South India is/are correct?

1. Musical developments were spearheaded by the Arcot court.
2. Tanjavur replaced Madras as the cultural capital of classical music in the second half of the nineteenth century.
3. Three great composers, Tyagaraja, Diksitar and Syama Sastri experimented with the kriti form to set the foundations for modern Carnatic music.

Select the correct answer using the code given below:

- (a) 1 and 3 only                      (b) 2 only  
(c) 3 only                              (d) 1, 2 and 3

4. Match List I with List II and select the correct answer using the code given below the Lists :

| List-I<br>(Molecule) |               | List-II<br>(Product of digestion) |                                      |
|----------------------|---------------|-----------------------------------|--------------------------------------|
| A.                   | Proteins      | (1)                               | Nitrogenous bases and pentose sugars |
| B.                   | Carbohydrates | (2)                               | Fatty acids and glycerol             |
| C.                   | Nucleic acids | (3)                               | Monosaccharides                      |
| D.                   | Lipids        | (4)                               | Amino acids                          |

Code:

- |          |          |          |          |
|----------|----------|----------|----------|
| <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> |
| (a) 2    | 3        | 1        | 4        |
| (b) 2    | 1        | 3        | 4        |
| (c) 4    | 1        | 3        | 2        |
| (d) 4    | 3        | 1        | 2        |

5. Precursor of which one of the following vitamins comes from  $\beta$ -carotene?

- (a) Vitamin A                      (b) Vitamin C  
(c) Vitamin D                      (d) Vitamin K

6. Which among the following about Mahatma Gandhi Non-Cooperation Movement are correct?

1. Refusal to attend Government Durbars and official functions.
2. Participation in elections.
3. Participation in rallies for the boycott of foreign goods.
4. Surrender of titles.

Select the correct answer using the codes given below.

- (a) 1, 2 and 3                      (b) 1, 3 and 4  
(c) 2, 3 and 4                      (d) 1 and 4

7. Gandhiji's response for breaking salt laws was in response to

- (a) Non-Cooperation Movement  
(b) Civil Disobedience Movement  
(c) Khilafat Movement  
(d) Quit India Movement

8. The Suez Canal, the Strait of Hormuz and the Strait of Gibraltar are important because they:

- (a) prevent attacks on bordering nations.  
(b) prohibit the movement of ships carrying nuclear weapons.  
(c) unite Russian access to warm water points.  
(d) control access to vital trade routes.

9. According to SIPRI's International Arms Transfer, 2021 Report, India imported maximum arms from which country?

- (a) Russia                              (b) France  
(c) United States                      (d) South Korea

10. Which among the following islands in the South China Sea are the source of conflict between several countries?

- (a) Marshall Islands                      (b) Solomon Islands  
(c) Kuril Islands                              (d) Spratly Islands

11. The government has decided to decommissioned which of the following Indian Naval ships in 2022?

1. Nishank                              2. Akshay  
3. Rana                                  4. Rajput

Code:

- (a) 1 and 2 only                      (b) 2 and 3 only  
(c) 1 and 4 only                      (d) 3 and 4 only

12. INS Kadamba which is an Indian Navy base is located in which of the following states?

- (a) Tamil Nadu                              (b) Odisha  
(c) Andhra Pradesh                      (d) Karnataka

13. Who among the following led India to their first overseas test victory in cricket?

- (a) C. K. Nayudu                              (b) M. A. K. Pataudi  
(c) Sunil Gavaskar                              (d) Bishen Singh Bedi

14. Who among the following was conferred the highest Bangladesh Award - Bangladesh Swadhinata Sammanona in the year 2011?

- (a) Indira Gandhi                              (b) Rajiv Gandhi  
(c) Jyoti Basu                                  (d) Satyajit Ray

15. Consider the following statements with regard to Nirbhay Missile

1. It is a supersonic cruise missile
2. Can carry nuclear warheads
3. It can be launched from multiple platforms

Which of the above statements is/are correct?

- (a) 1 and 2 only                      (b) 2 and 3 only  
(c) 1,2 and 3                              (d) 1 and 3 only

16. Consider the following statements with regard to INS Surat

1. Is the last ship of the Visakhapatnam-class stealth guided-missile destroyers.
2. It is expected to be commissioned by 2025.

Which of the above statements is/are correct?

- (a) 1 only                                  (b) 2 only  
(c) Both 1 and 2                              (d) Neither 1 nor 2

17. Which of the following statements is/are correct?

1. The Indian Army is organized into five regional commands.
2. The headquarters of Northern Command is located at Lucknow.

Code:

- (a) 1 only                                  (b) 2 only  
(c) Both 1 and 2                              (d) Neither 1 nor 2

18. Consider the following coal mines of India.

1. Bokaro                                  2. Adilabad  
3. Raniganj                                  4. Bishrampur



Select the correct sequence of the above from east to west

- (a) 1-3-4-2 (b) 3-1-4-2  
(c) 3-1-2-4 (d) 1-3-2-4
19. Which one among the following is the major reason for low agricultural productivity in dry regions of India?  
(a) Constraints of financial resources  
(b) Rural indebtedness  
(c) Dependence on erratic monsoon  
(d) Small size of landholdings
20. The Damuda series of Gondwana system has three stages, namely, Raniganj, Barren Rocks and Barakar. The middle stage is called Barren because  
(a) it consists of coal and not iron  
(b) it consists of iron and not coal  
(c) it consists of neither coal nor iron  
(d) it is a sedimentary layer consisting of shale and clay
21. Which one of the following statements is correct about effect of antibiotics on viruses?  
(a) Viruses are "non-living" entities but it can interact with antibiotics.  
(b) Taking antibiotics cures viral infections.  
(c) Viruses do not possess metabolic pathways on which antibiotics can function, whereas bacteria have such pathways.  
(d) Viruses are resistant to antibiotics.
22. Which one of the following statements regarding penicillin is correct?  
(a) Penicillin resistant bacteria can store this antibiotic in vacuole.  
(b) Penicillin resistant bacteria can degrade this antibiotic by an enzyme called  $\beta$ -lactamase.  
(c) Penicillin resistant bacteria can degrade this antibiotic by an enzyme called lactic acid dehydrogenase.  
(d) Penicillin is not absorbed by bacteria; so most bacteria are resistant.
23. Who among the following discovered the antibiotic producing fungus from *Penicillium* genus?  
(a) Louis Pasteur  
(b) Sir Alexander Fleming  
(c) Stanley Prusiner  
(d) Robert Hooke
24. At which one of the following places was a Shiva temple **not** constructed under the patronage of the Chola rulers?  
(a) Chidambaram  
(b) Thanjavur  
(c) Gangaikonda Cholapuram  
(d) Naneghat
25. Who among the following was the virashaiva Movement in Karnataka?  
(a) Basanava (b) Appar  
(c) Sambandar (d) Sundarar

## SCIENCE

1. Which one among the following statements is correct?  
(a) All arteries carry oxygenated blood.  
(b) All veins carry oxygenated blood.  
(c) Except the pulmonary artery, all other arteries carry oxygenated blood.  
(d) Except the pulmonary vein, all other veins carry oxygenated blood.
2. Which one among the following is correct regarding  $^{20}\text{Ne}$ ,  $^{23}\text{Na}^+$ ,  $^{19}\text{F}^-$ ,  $^{24}\text{Mg}^{2+}$ ?  
(a) They are isomers of each other.  
(b) They are isotopes of each other.  
(c) They are isoelectronic with each other.  
(d) All of the above.
3. Sometimes indigestion is caused by the secretion of too much hydrochloric acid in the stomach. To ease the pain caused, a tablet can be taken that reacts to reduce the amount of acid present. Which one among the following would be inappropriate for a manufacturer to include as a major reactant in the tablet?  
(a)  $\text{CaCO}_3$  (b)  $\text{MgCO}_3$   
(c)  $\text{NaOH}$  (d)  $\text{Mg}(\text{OH})_2$
4. A refracting telescope consists of :  
(a) one convex mirror and one convex lens.  
(b) two convex lenses of equal focal length.  
(c) two concave mirrors of different focal lengths.  
(d) two convex lenses of unequal focal lengths.
5. Two similarly charged bodies are kept 5 cm apart in air. If the second body is shifted away from the first by another 5 cm, their force of repulsion will be  
(a) doubled  
(b) halved  
(c) quadrupled  
(d) reduced to one-fourth
6. The main power supply in India is at 220 V, whereas that in the US is at 110 V. Which one among the following statement in this regard is correct?  
(a) 110 V is safer but more expensive to maintain.  
(b) 110 V is safer and cheaper to maintain.  
(c) 110 V leads to lower power loss.  
(d) 110 V works better at higher latitudes.
7. When a solid object is immersed in water, there is a loss in its weight. This loss is  
(a) equal to the weight of the water displaced.  
(b) less than the weight of the water displaced.  
(c) greater than the weight of the water displaced.  
(d) not related to the weight of the water displaced.
8. Body A of mass 2 kg and body B of mass 4 kg are of same material and are kept in sunshine for some interval of time. If the rise in temperature is equal for both the bodies, then which one among the following in this regard is correct?  
(a) Heat absorbed by B is double because its mass is double.  
(b) Heat absorbed by A is double because its mass is half.

- (c) Heat absorbed by both A and B is equal because the quantity of heat absorbed does not depend upon the mass.
- (d) Heat absorbed by B is four times than the heat absorbed by A because the quantity of heat absorbed is proportional to square of the mass.
9. For a steel boat floating on a lake, the weight of the water displaced by the boat is:
- Less than the weight of the boat.
  - More than the weight of the boat.
  - Equal to the weight of the part of the boat which is below the water level of the lake.
  - Equal to the weight of the boat.
10. Which one among the following groups is referred to as types of precipitation?
- Fog, Dew and Rain
  - Air, Water and Soil
  - Fog, Water and Rain
  - Dew, Soil and Rain
11. Which one among the following is not produced by sound wave in air?
- Polarization
  - Diffraction
  - Reflection
  - Refraction
12. A body attached to a spring balance weighs 10 kg on the Earth. The body attached to the same spring balance is taken to a planet where gravity is half of that of the Earth. The balance will read
- 20 kg
  - 10 kg
  - 5 kg
  - 2.5 kg
13. If the focal length of the biconvex lens is 25 cm, then the power of the lens will be:
- + 4 dioptre
  - 4 dioptre
  - + 0.04 dioptre
  - 0.04 dioptre
14. The thermal conductivity of copper is 4 times that of brass. Two rods of copper and brass having same length and cross-section are joined end to end. The free end of copper is at  $0^{\circ}\text{C}$  and the free end of brass is at  $100^{\circ}\text{C}$ . The temperature of the junction is
- $20^{\circ}\text{C}$
  - $40^{\circ}\text{C}$
  - $60^{\circ}\text{C}$
  - $10^{\circ}\text{C}$
15. The torque on a rectangular coil placed in a uniform magnetic field is large when the
- number of turn is large.
  - number of turns is less.
  - plane of the coil is perpendicular to the magnetic field.
  - area of the coil is small.
16. Two metallic wires A and B are of same material and have equal length. If the cross-sectional area of B is double that of A, then which one among the following is the electrical resistance of B?
- Twice that of A
  - Four times that of A
  - $\frac{1}{4}$  that of A
  - $\frac{1}{2}$  that of A
17. Two thin convex lenses of focal lengths 4 cm and 8 cm are separated by a distance of 4 cm in air. The combination will have the focal length of
- 4 cm
  - 8 cm
  - 12 cm
  - 32 cm
18. For which among the following house appliances, magnet is an essential part?
- Calling bell
  - Fan
  - Washing machine
  - All of the above
19. Consider the following statements :
- Hypermetropia is a defect of vision in which
- a person cannot see the distant objects clearly.
  - a person cannot see the near objects clearly.
  - the near point of the eye gets shifted away from the normal position.
  - the far point of the eye gets shifted towards the eye.
- Which of the statement given above are correct?
- (i) and (iii)
  - (ii) and (iv)
  - (i) and (iv)
  - (ii) and (iii)
20. To obtain the powerful parallel beams of light from a vehicle's headlight, one must use
- front surface silvered plane mirror
  - back surface silvered plane mirror
  - concave mirror
  - convex mirror
21. If the sum of first  $n$  term of a series is  $2n + 7$ , then what is the sum of first term, fifth term and  $10^{\text{th}}$  term ?
- 13
  - 15
  - 17
  - 21
22. If  $m = \sin \theta + \tan \theta$ ,  $n = \cos \theta + \sec \theta$ , then the value  $m^2 + n^2 + (m - n)^3$  will be ?
- 0
  - 2
  - 4
  - 8
23. Let  $U$  be the universal set and  $A \cup B \cup C = U$ . Then  $\{(A - B) \cup (B - C) \cup (C - A)\}'$  will be equals to ?
- $A \cap B \cap C$
  - $A \cap B \cup C$
  - $A \cup B \cap C$
  - $A \cup B \cup C$
24. If set A and B has 3 and 5 element respectively, then the total number of relation from set A to set B will be?
- B
  - 15
  - $2^8$
  - $2^{15}$
25. A fair coin is tossed  $n$  times. If the probability that head occurs 6 times is equal to the probability that head occurs 8 times, then  $n$  will be equal to?
- 7
  - 14
  - 16
  - 21

## MATHEMATICS

1. The value of  $\sqrt{3+4i} + \sqrt{3-4i}$  will be equals to ?

(a) 2 (b)  $2i$   
(c) 4 (d)  $4i$

2. If  $z_1, z_2$  and  $z_3$  be the three vertices of an equilateral triangle, then which one of the following is correct.

I.  $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$

II.  $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

(a) Only I (b) Only II  
(c) both I and II (d) neither I nor II.

3. The value of  $p$  for which the function  $f(x) = p \sin x +$

$\frac{1}{3} \sin 3x$  has an extremum at  $x = \frac{\pi}{3}$  will be ?

(a) 0 (b) 1  
(c) 2 (d) 3

4. The maximum and minimum value of the function  $f(x) = \sin^6 x + \cos^6 x$  will be ?

(a) 1 and  $\frac{1}{4}$  (b)  $\frac{1}{4}$  and  $\frac{1}{3}$

(c) 1 and  $\frac{3}{4}$  (d)  $\frac{1}{2}$  and  $\frac{3}{4}$

5. If  $\omega$  and  $\omega^2$  are the cube root of unity then the value of  $(3 - \omega^7)(3 - \omega^8)(3 - \omega^{16})(3 - \omega^{17})$  will be?

(a) 169 (b) 144  
(c) 121 (d) 100

6. The value of  $I = \int_{-3}^3 (ax^5 + bx^3 + cx + d) dx$  depends only on?

(a)  $a$  (b)  $b$   
(c)  $c$  (d)  $d$

7. The value of  $I = \int_0^1 \frac{\log(1+x)}{(1+x^2)} dx$  will be equals to ?

(a)  $\pi \log 2$  (b)  $\frac{\pi}{4} \log 2$

(c)  $\frac{\pi}{8} \log 2$  (d)  $\frac{\pi}{16} \log 2$

8. If  $|z + 2| = \sqrt{2} |z - 2|$ , then the locus of  $z$  will be?

(a) a line (b) a circle  
(c) an ellipse (d) a hyperbola

9.  $\int \frac{\tan x}{\sec x + \tan x} dx$  will be equal to ?

(a)  $\sec x + \tan x + x + c$   
(b)  $\sec x - \tan x + x + c$   
(c)  $\sec x - \tan x - x + c$   
(d)  $\sec x + \tan x - x + c$

10.  $\int \left( \frac{x^6 + 1}{x^2 + 1} \right) dx$  will be equals to ?

(a)  $\frac{x^5}{5} + \frac{x^3}{3} + x + c$  (b)  $\frac{x^5}{5} - \frac{x^3}{3} - x + c$

(c)  $\frac{x^5}{5} - \frac{x^3}{3} + x + c$  (d)  $\frac{x^5}{5} + \frac{x^3}{3} - x + c$

11. The function  $f(x) = x \left( \frac{a^x - 1}{a^x + 1} \right)$  will be?

(a) even function  
(b) odd function  
(c) both even and odd function  
(d) neither even nor odd function

12. The value of  ${}^{10}C_0^2 + {}^{10}C_1^2 + {}^{10}C_2^2 + \dots + {}^{10}C_{10}^2$  will be ?

(a)  $(10)^2$  (b)  $2(10)$

(c)  $\frac{20}{10}$  (d)  $\frac{20}{10}$

13. The sum of all number that can be formed using the digits 2, 3, 4 and 5 taken all at a time will be?

(a) 92324 (b) 93324  
(c) 92224 (d) 93224

14. If  $a, b, c$  are distinct positive number, then the expression  $3abc - (a + b + c)(b + c - a)(c + a - b)$  will be.

(a) positive (b) Negative  
(c) zero (d) always equals to 1

15. Conversion of  $(43.375)_{10}$  will be equal to?

(a)  $(101011.011)_2$  (b)  $(101010.011)_2$   
(c)  $(101011.101)_2$  (d)  $(101010.101)_2$

16. What is the minimum value of  $f(\theta) = 8\sin \theta - 12\sin^2 \theta$ ?

(a)  $\frac{16}{3}$  (b)  $\frac{3}{16}$

(c)  $\frac{4}{3}$  (d)  $\frac{3}{4}$

17. The value of  $\tan 5A + \sec 5A$  will be equals to?

(a)  $\frac{\cot\left(\frac{5A}{2}\right) - 1}{\cot\left(\frac{5A}{2}\right) + 1}$  (b)  $\frac{\cot\left(\frac{5A}{2}\right) + 1}{\cot\left(\frac{5A}{2}\right) - 1}$

(c)  $\cot\left(\frac{5A}{2}\right) + 1$  (d)  $\cot\left(\frac{5A}{2}\right) - 1$

18. If the roots of the equation  $(5p - 2q)x^2 + (2q - 9r)x + (9r - 5p) = 0$  are real and equal, then which one is correct ?

(a)  $r = \frac{9}{2}(5p + q)$  (b)  $r = \frac{2}{9}(5p - q)$   
(c)  $r = \frac{9}{2}(5p - q)$  (d)  $r = \frac{2}{9}(5p + q)$

19. If  $a, b, c$  are the  $p^{\text{th}}, q^{\text{th}}$  and  $r^{\text{th}}$  term of an H.P., then the

$$\text{value of } \begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} \text{ will be}$$

- (a)  $abc$  (b)  $pqr$   
 (c) 1 (d) 0
20. In the given figure, a flag cast some shadow on the ground. The angle of sun will be?

- (a)  $\frac{\pi}{12}$  (b)  $\frac{\pi}{8}$   
 (c)  $\frac{3\pi}{8}$  (d)  $\frac{5\pi}{12}$

21. 141, 120, 101, 84, 69, ?

- (a) 45 (b) 50  
 (c) 56 (d) 52

22. 11, 13, 17, 19, 23, 25, ?

- (a) 26 (b) 27 (c) 29 (d) 37

23. 2315, 3420, 4525, 5630, ?

- (a) 6150 (b) 6735  
 (c) 5950 (d) 6045

24. 2, 5, 9, ? 20, 27

- (a) 14 (b) 16 (c) 18 (d) 24

25. 0, 4, 8, 16, 24, 36, 48, ?

- (a) 64 (b) 72  
 (c) 50 (d) 68

## ANSWER KEY

## English

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (b)  | 2. (c)  | 3. (d)  | 4. (b)  | 5. (c)  |
| 6. (c)  | 7. (b)  | 8. (a)  | 9. (a)  | 10. (c) |
| 11. (a) | 12. (c) | 13. (d) | 14. (c) | 15. (a) |
| 16. (a) | 17. (c) | 18. (a) | 19. (b) | 20. (b) |
| 21. (c) | 22. (a) | 23. (a) | 24. (d) | 25. (c) |

## General Awareness

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (a)  | 2. (b)  | 3. (c)  | 4. (d)  | 5. (a)  |
| 6. (b)  | 7. (b)  | 8. (d)  | 9. (c)  | 10. (d) |
| 11. (a) | 12. (d) | 13. (b) | 14. (a) | 15. (b) |
| 16. (c) | 17. (d) | 18. (b) | 19. (c) | 20. (b) |
| 21. (c) | 22. (b) | 23. (b) | 24. (d) | 25. (a) |

## Science

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (c)  | 3. (d)  | 4. (d)  | 5. (d)  |
| 6. (a)  | 7. (a)  | 8. (c)  | 9. (c)  | 10. (a) |
| 11. (a) | 12. (c) | 13. (a) | 14. (a) | 15. (a) |
| 16. (d) | 17. (a) | 18. (c) | 19. (d) | 20. (c) |
| 21. (a) | 22. (c) | 23. (a) | 24. (d) | 25. (b) |

## Mathematics

- |         |         |         |         |         |
|---------|---------|---------|---------|---------|
| 1. (c)  | 2. (c)  | 3. (c)  | 4. (a)  | 5. (a)  |
| 6. (d)  | 7. (c)  | 8. (b)  | 9. (b)  | 10. (c) |
| 11. (a) | 12. (d) | 13. (b) | 14. (a) | 15. (a) |
| 16. (c) | 17. (b) | 18. (b) | 19. (d) | 20. (d) |
| 21. (c) | 22. (c) | 23. (b) | 24. (a) | 25. (a) |

○○○

Finished Solving the Paper ?  
Time to evaluate yourself !

**OSWAAL COGNITIVE  
LEARNING TOOLS**

SCAN THE CODE

For elaborate  
Solutions

# WRITING YOUR NOTES

**Just in case you have forgotten today, takedown your notes!**

**But why is it so important?**

Tools for the hands are tools for the brain writes Hetty Roessingh.

Handwritten notes are a powerful tool for encrypting embodied cognition and in turn supporting the brain's capacity for recuperation of information. If that sounds so scientific then in simple words:

Writing notes by hand help you in:

- ◆ Increasing your comprehension   ◆ Strengthening your memory   ◆ Igniting your creativity
- ◆ Engaging your mind   ◆ Increasing your attention span

Are these reasons enough to get you started?

1. ....
2. ....
3. ....
4. ....
5. ....
6. ....
7. ....
8. ....
9. ....
10. ....
11. ....
12. ....
13. ....
14. ....
15. ....
16. ....
17. ....
18. ....
19. ....
20. ....